

glucat

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# Chapter 1

## Namespace Index

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## Chapter 5

# Namespace Documentation

### 5.1 cga3 Namespace Reference

Definitions for 3D Conformal Geometric Algebra [DL].

#### Functions

- `template<typename Multivector_T >`  
`Multivector_T cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T >`  
`Multivector_T cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T >`  
`Multivector_T agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

#### 5.1.1 Detailed Description

Definitions for 3D Conformal Geometric Algebra [DL].

#### 5.1.2 Function Documentation

##### 5.1.2.1 agc3()

```
template<typename Multivector_T >
Multivector_T cga3::agc3 (
    const Multivector_T & X ) [inline]
```

Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Definition at line 126 of file [PyClical.h](#).

References [cga3std\(\)](#).

### 5.1.2.2 cga3()

```
template<typename Multivector_T >
Multivector_T cga3::cga3 (
    const Multivector_T & x ) [inline]
```

Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].

Definition at line 103 of file [PyClical.h](#).

### 5.1.2.3 cga3std()

```
template<typename Multivector_T >
Multivector_T cga3::cga3std (
    const Multivector_T & X ) [inline]
```

Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].

Definition at line 114 of file [PyClical.h](#).

Referenced by [agc3\(\)](#).

## 5.2 glucat Namespace Reference

### Namespaces

- namespace [gen](#)
- namespace [matrix](#)
- namespace [timing](#)

### Classes

- class [basis\\_table](#)  
*Table of basis elements used as a cache by basis\_element()*
- class [bool\\_to\\_type](#)  
*Bool to type.*
- class [clifford\\_algebra](#)  
*clifford\_algebra<> declares the operations of a Clifford algebra*
- class [compare\\_types](#)  
*Type comparison.*
- class [compare\\_types< T, T >](#)
- class [control\\_t](#)  
*Parameters to control tests.*
- struct [CTAssertion](#)  
*Compile time assertion.*
- struct [CTAssertion< true >](#)
- class [error](#)  
*Specific exception class.*

- class [framed\\_multi](#)  
*A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.*
- class [glucat\\_error](#)  
*Abstract exception class.*
- class [index\\_set](#)  
*Index set class based on std::bitset<> in Gnu standard C++ library.*
- class [index\\_set\\_hash](#)
- class [matrix\\_multi](#)  
*A matrix\_multi<Scalar\_T,LO,HI,Tune\_P> is a matrix approximation to a multivector.*
- class [numeric\\_traits](#)  
*Extra traits which extend numeric limits.*
- class [random\\_generator](#)  
*Random number generator with single instance per Scalar\_T.*
- class [sorted\\_range](#)  
*Sorted range for use with output.*
- class [sorted\\_range< Sorted\\_Map\\_T, Sorted\\_Map\\_T >](#)

## Typedefs

- using [index\\_t](#) = int  
*Size of index\_t should be enough to represent LO, HI.*
- using [set\\_value\\_t](#) = unsigned long  
*Size of set\_value\_t should be enough to contain index\_set<LO,HI>*
- typedef int(\* [intfn](#)) ()  
*For exception catching: pointer to function returning int.*
- typedef int(\* [intintfn](#)) (int)  
*For exception catching: pointer to function of int returning int.*
- using [tuning\\_slow](#) = tuning< [Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#), [Tuning\\_Default\\_Div\\_Max\\_Steps](#), [Tuning\\_Default\\_CR\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Default\\_DB\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Default\\_Log\\_Max\\_Outer\\_Steps](#), [Tuning\\_Default\\_Log\\_Max\\_Inner\\_Steps](#), [Tuning\\_Slow\\_Basis\\_Max\\_Count](#), [Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#), [Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#), [Tuning\\_Slow\\_Products\\_Size\\_Threshold](#), [Tuning\\_Default\\_Denom\\_Different\\_Bits](#), [Tuning\\_Default\\_Extra\\_Different\\_Bits](#), [Tuning\\_Default\\_Function\\_Precision](#) >
- using [tuning\\_naive](#) = tuning< [Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#), [Tuning\\_Default\\_Div\\_Max\\_Steps](#), [Tuning\\_Default\\_CR\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Default\\_DB\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Default\\_Log\\_Max\\_Outer\\_Steps](#), [Tuning\\_Default\\_Log\\_Max\\_Inner\\_Steps](#), [Tuning\\_Naive\\_Basis\\_Max\\_Count](#), [Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#), [Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#), [Tuning\\_Default\\_Products\\_Size\\_Threshold](#), [Tuning\\_Default\\_Denom\\_Different\\_Bits](#), [Tuning\\_Default\\_Extra\\_Different\\_Bits](#), [Tuning\\_Default\\_Function\\_Precision](#) >
- using [tuning\\_fast](#) = tuning< [Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#), [Tuning\\_Fast\\_Div\\_Max\\_Steps](#), [Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#), [Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#), [Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#), [Tuning\\_Fast\\_Basis\\_Max\\_Count](#), [Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#), [Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#), [Tuning\\_Fast\\_Products\\_Size\\_Threshold](#), [Tuning\\_Default\\_Denom\\_Different\\_Bits](#), [Tuning\\_Default\\_Extra\\_Different\\_Bits](#), [Tuning\\_Default\\_Function\\_Precision](#) >

## Functions

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of multivectors.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> bool  
*Test for inequality of multivector and scalar.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator!=](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of scalar and multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [error\\_squared\\_tol](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T  
*Quadratic norm error tolerance relative to a specific multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [error\\_squared](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold) -> Scalar\_T  
*Relative or absolute error using the quadratic norm.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold, const Scalar\_T tolerance) -> bool  
*Test for approximate equality of multivectors.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for approximate equality of multivectors.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Geometric sum of multivector and scalar.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator+](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Geometric sum of scalar and multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Geometric sum.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Geometric difference of multivector and scalar.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator-](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >  
*Geometric difference of scalar and multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
 auto [operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >





*Transformation via twisted adjoint action.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric multiplicative inverse.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Integer power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Multivector power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Outer product power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Real part: synonym for scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Imaginary part: deprecated (always 0)*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Pure part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`

*Vector part of multivector, as a vector\_t with respect to frame()*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}\{2\} \rightarrow \{2\}\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation,  $rev \circ invo == invo \circ rev$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value ==  $\sqrt{\text{norm}}$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`





*Inverse hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`static void check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_←`  
`_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that i is a valid complexifier for val.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator\* (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator^ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Outer product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator& (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator% (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto star (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator| (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator>> (std::istream &s, framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`

*Read multivector from input.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator<< (std::ostream &os, const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream`  
`&`

*Write multivector to output.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`auto operator<< (std::ostream &os, const std::pair< const index\_set< LO, HI >, Scalar_T > &term) ->`  
`std::ostream &`

*Write term to output.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T, LO, HI,`  
`Tune_P >`

*Exponential of multivector.*

- `template<typename Scalar_T, const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`

*Coordinate of product of terms.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`auto operator\* (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index\_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `\_GLUCAT\_CTAssert (std::numeric_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const index\_t BITS_PER_CHAR`  
*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*
- `\_GLUCAT\_CTAssert (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const index\_t DEFAULT_LO`  
*Default lowest index in an index set.*
- `template<typename LHS_T , typename RHS_T >`  
`auto pos\_mod (LHS_T lhs, RHS_T rhs) -> LHS_T`  
*Modulo function which works reliably for lhs < 0.*
- `template<const index\_t LO, const index\_t HI>`  
`auto operator^ (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Symmetric set difference: exclusive or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto operator& (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set intersection: and.*
- `template<const index\_t LO, const index\_t HI>`  
`auto operator| (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set union: or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto compare (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `\_GLUCAT\_CTAssert (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >), Default_index_set_too_big_for_value) template< const index\_t LO`  
*Size of set\_value\_t should be enough to contain bitset<DEFAULT\_HI-DEFAULT\_LO>*
- `const index\_t HI auto operator<< (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &`
- `template<const index\_t LO, const index\_t HI>`  
`auto operator>> (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &`  
*Read in index set.*
- `auto sign\_of\_square (index\_t j) -> int`  
*Square of generator {j}.*
- `template<const index\_t LO, const index\_t HI>`  
`auto min\_neg (const index\_set< LO, HI > &ist) -> index\_t`  
*Minimum negative index, or 0 if none.*
- `template<const index\_t LO, const index\_t HI>`  
`auto max\_pos (const index\_set< LO, HI > &ist) -> index\_t`



- Maximum positive index, or 0 if none.*

  - template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [operator](#)<< (std::ostream &os, const [index\\_set](#)< LO, HI > &ist) -> std::ostream &

*Write out index set.*
- static auto [inverse\\_reversed\\_gray](#) (unsigned long x) -> unsigned long
- Inverse reversed Gray code.*

  - static auto [inverse\\_gray](#) (unsigned long x) -> unsigned long

*Inverse Gray code.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)\* (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >
- Geometric product.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)^ (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Outer product.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)& (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >
- Inner product.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)% (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Left contraction.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [star](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T
- Hestenes scalar product.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)/ (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Geometric quotient.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)| (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >
- Transformation via twisted adjoint action.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)>> (std::istream &s, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &

*Read multivector from input.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator](#)<< (std::ostream &os, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &
- Write multivector to output.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [reframe](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs\_reframed, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs\_reframed) -> const [index\\_set](#)< LO, HI >

*Find a common frame for operands of a binary operator.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >
- Square root of multivector with specified complexifier.*

  - template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [matrix\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >



*Square root of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [matrix\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [exp](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

- auto [offset\\_level](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> [index\\_t](#)

*Determine the log2 dim corresponding to signature p, q.*

- template<typename Matrix\_Index\_T , const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [folded\\_dim](#) (const [index\\_set](#)< LO, HI > &sub) -> Matrix\_Index\_T

*Determine the matrix dimension of the fold of a subalgebra.*

- template<typename Multivector\_T , typename Matrix\_T , typename Basis\_Matrix\_T >  
static auto [fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T

*Inverse generalized Fast Fourier Transform.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P , const size\_t Size>  
static auto [pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static void [db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)

*Single step of product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Cyclic reduction square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation of log.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Incomplete square root cascade and Pade' approximation of log.*

- template<typename Scalar\_T >  
auto [log2](#) (const Scalar\_T &x) -> Scalar\_T

*Log base 2 of scalar.*

- template<typename Scalar\_T >  
auto [to\\_promote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::promoted::type

*Cast to promote.*

- template<typename Scalar\_T >  
auto [to\\_demote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::demoted::type

- *Cast to demote.*  
• int [try\\_catch](#) (intfn f)  
*Exception catching for functions returning int.*
- int [try\\_catch](#) (intintfn f, int arg)  
*Exception catching for functions of int returning int.*

## Variables

- const double [MS\\_PER\\_S](#) = 1000.0  
*Timing constant: deprecated here - moved to [test/timing.h](#).*
- const [index\\_t](#) [BITS\\_PER\\_SET\\_VALUE](#) = std::numeric\_limits<[set\\_value\\_t](#)>::digits  
*Number of bits in [set\\_value\\_t](#).*
- const [index\\_t](#) [DEFAULT\\_HI](#) = [index\\_t](#)([BITS\\_PER\\_SET\\_VALUE](#) / 2)  
*Default highest index in an index set.*
- static const long double [I\\_pi](#) = 3.1415926535897932384626433832795029L
- static const long double [I\\_ln2](#) = 0.6931471805599453094172321214581766L
- const unsigned int [Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 5.2.1 Typedef Documentation

### 5.2.1.1 [index\\_t](#)

using [glucat::index\\_t](#) = typedef int

Size of [index\\_t](#) should be enough to represent LO, HI.

Definition at line 77 of file [global.h](#).

### 5.2.1.2 intfn

```
typedef int(* glucat::intfn) ()
```

For exception catching: pointer to function returning int.

Definition at line 37 of file [try\\_catch.h](#).

### 5.2.1.3 intintfn

```
typedef int(* glucat::intintfn) (int)
```

For exception catching: pointer to function of int returning int.

Definition at line 40 of file [try\\_catch.h](#).

### 5.2.1.4 set\_value\_t

```
using glucat::set_value_t = typedef unsigned long
```

Size of set\_value\_t should be enough to contain index\_set<LO,HI>

Definition at line 79 of file [global.h](#).

### 5.2.1.5 tuning\_fast

```
using glucat::tuning_fast = typedef tuning < Tuning_Fast_Mult_Matrix_Threshold, Tuning_Fast_Div_Max_Steps,
Tuning_Fast_CR_Sqrt_Max_Steps, Tuning_Fast_DB_Sqrt_Max_Steps, Tuning_Fast_Log_Max_Outer_Steps,
Tuning_Fast_Log_Max_Inner_Steps, Tuning_Fast_Basis_Max_Count, Tuning_Fast_Fast_Size_Threshold,
Tuning_Fast_Inv_Fast_Dim_Threshold, Tuning_Fast_Products_Size_Threshold, Tuning_Default_↔
Denom_Different_Bits, Tuning_Default_Extra_Different_Bits, Tuning_Default_Function_Precision
>
```

Definition at line 97 of file [tuning.h](#).

### 5.2.1.6 tuning\_naive

```
using glucat::tuning_naive = typedef tuning < Tuning_Naive_Mult_Matrix_Threshold, Tuning↔
_Default_Div_Max_Steps, Tuning_Default_CR_Sqrt_Max_Steps, Tuning_Default_DB_Sqrt_Max_Steps,
Tuning_Default_Log_Max_Outer_Steps, Tuning_Default_Log_Max_Inner_Steps, Tuning_Naive_Basis_Max_Count,
Tuning_Naive_Fast_Size_Threshold, Tuning_Naive_Inv_Fast_Dim_Threshold, Tuning_Default_Products↔
_Size_Threshold, Tuning_Default_Denom_Different_Bits, Tuning_Default_Extra_Different_Bits,
Tuning_Default_Function_Precision >
```

Definition at line 69 of file [tuning.h](#).

### 5.2.1.7 tuning\_slow

```
using glucat::tuning_slow = typedef tuning < Tuning_Slow_Mult_Matrix_Threshold, Tuning_↵
Default_Div_Max_Steps, Tuning_Default_CR_Sqrt_Max_Steps, Tuning_Default_DB_Sqrt_Max_Steps,
Tuning_Default_Log_Max_Outer_Steps, Tuning_Default_Log_Max_Inner_Steps, Tuning_Slow_Basis_Max_Count,
Tuning_Slow_Fast_Size_Threshold, Tuning_Slow_Inv_Fast_Dim_Threshold, Tuning_Slow_Products_Size_Threshold,
Tuning_Default_Denom_Different_Bits, Tuning_Default_Extra_Different_Bits, Tuning_Default_↵
Function_Precision >
```

Definition at line 47 of file [tuning.h](#).

## 5.2.2 Function Documentation

### 5.2.2.1 \_GLUCAT\_CTAssert() [1/3]

```
glucat::_GLUCAT_CTAssert (
    _GLUCAT_BITS_PER_ULONG = BITS\_PER\_SET\_VALUE,
    BitsPerULongDoesNotMatchSetValueT ) const
```

Default lowest index in an index set.

### 5.2.2.2 \_GLUCAT\_CTAssert() [2/3]

```
glucat::_GLUCAT_CTAssert (
    sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT_LO > ) ,
    Default_index_set_too_big_for_value ) const
```

Size of `set_value_t` should be enough to contain `bitset<DEFAULT_HI-DEFAULT_LO>`

Write out index set

### 5.2.2.3 \_GLUCAT\_CTAssert() [3/3]

```
glucat::_GLUCAT_CTAssert (
    std::numeric_limits< unsigned char >::radix = 2,
    CannotDetermineBitsPerChar ) const
```

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

Number of bits per char is used to determine number of bits in `set_value_t`

### 5.2.2.4 abs()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Absolute value == sqrt(norm)

Definition at line 577 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::sqrt\(\)](#).

Referenced by [PyCical.clifford::abs\(\)](#), [clifford\\_to\\_str\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

### 5.2.2.5 acos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse cosine of multivector.

Definition at line 903 of file [clifford\\_algebra\\_imp.h](#).

References [acos\(\)](#), and [complexifier\(\)](#).

### 5.2.2.6 acos() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse cosine of multivector with specified complexifier.

Definition at line 883 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [acos\(\)](#).

### 5.2.2.7 `acosh()` [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic cosine of multivector.

Definition at line 844 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [complexifier\(\)](#).

### 5.2.2.8 `acosh()` [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic cosine of multivector with specified complexifier.

Definition at line 825 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [acos\(\)](#), and [acosh\(\)](#).

### 5.2.2.9 `approx_equal()` [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 169 of file [clifford\\_algebra\\_imp.h](#).

References [approx\\_equal\(\)](#), and [error\\_squared\\_tol\(\)](#).

**5.2.2.10 approx\_equal()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold,
    const Scalar_T tolerance ) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 154 of file [clifford\\_algebra\\_imp.h](#).

References [error\\_squared\(\)](#).

Referenced by [approx\\_equal\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.11 asin()** [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse sine of multivector.

Definition at line 1008 of file [clifford\\_algebra\\_imp.h](#).

References [asin\(\)](#), and [complexifier\(\)](#).

**5.2.2.12 asin()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse sine of multivector with specified complexifier.

Definition at line 988 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [asin\(\)](#).

**5.2.2.13 asinh()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic sine of multivector.

Definition at line 949 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [complexifier\(\)](#).

**5.2.2.14 asinh()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic sine of multivector with specified complexifier.

Definition at line 930 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [asin\(\)](#), and [asinh\(\)](#).

**5.2.2.15 atan()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse tangent of multivector.

Definition at line 1108 of file [clifford\\_algebra\\_imp.h](#).

References [atan\(\)](#), and [complexifier\(\)](#).



**5.2.2.16 atan()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse tangent of multivector with specified complexifier.

Definition at line 1088 of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [atan\(\)](#).

**5.2.2.17 atanh()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic tangent of multivector.

Definition at line 1052 of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [complexifier\(\)](#).

**5.2.2.18 atanh()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic tangent of multivector with specified complexifier.

Definition at line 1035 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), and [norm\(\)](#).

Referenced by [atan\(\)](#), and [atanh\(\)](#).

### 5.2.2.19 cascade\_log()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::cascade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Incomplete square root cascade and Pade' approximation of log.

Definition at line [1920](#) of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), [epsilon](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), [norm\(\)](#), and [pade\\_log\(\)](#).

Referenced by [matrix\\_log\(\)](#).

### 5.2.2.20 check\_complex()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static void glucat::check_complex (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) [inline], [static]
```

Check that i is a valid complexifier for val.

Definition at line [652](#) of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [cos\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.21 clifford\_exp()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::clifford_exp (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line [690](#) of file [clifford\\_algebra\\_imp.h](#).

References [log2\(\)](#).

Referenced by [exp\(\)](#).

### 5.2.2.22 compare()

```
template<const index_t LO, const index_t HI>
auto glucat::compare (
    const index_set< LO, HI > & a,
    const index_set< LO, HI > & b ) -> int [inline]
```

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

Lexicographic ordering of two sets: -1 if a<b, +1 if a>b, 0 if a==b.

Definition at line 574 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical::index\\_set\\_hidden\\_doctests\(\)](#).

### 5.2.2.23 complexifier()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::complexifier (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,
LO,HI,Tune_P>
```

Square root of -1 which commutes with all members of the frame of the given multivector.

Definition at line 592 of file [clifford\\_algebra\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [check\\_complex\(\)](#), [cos\(\)](#), [elliptic\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.24 conj()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::conj (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,
LO,HI,Tune_P> [inline]
```

Conjugation, rev o invo == invo o rev.

Definition at line 553 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.25 cos() [1/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Cosine of multivector.

Definition at line 874 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [cos\(\)](#).

**5.2.2.26 cos() [2/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Cosine of multivector with specified complexifier.

Definition at line 851 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [cos\(\)](#), and [tan\(\)](#).

**5.2.2.27 cosh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::cosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Hyperbolic cosine of multivector.

Definition at line 807 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

**5.2.2.28 cr\_sqrt()**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
static auto glucat::cr_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_Y_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1) ) ->
const matrix_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Cyclic reduction square root iteration.

Definition at line 1349 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.29 crd\_of\_mult() [1/2]**

```
template<typename Scalar_T , const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> Scalar_T
T [inline], [static]
```

Coordinate of product of terms.

Referenced by [operator%\(\)](#), [operator&\(\)](#), [operator\\*\(\)](#), and [operator^\(\)](#).

**5.2.2.30 crd\_of\_mult() [2/2]**

```
template<typename Scalar_T , const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> Scalar_T
T [inline], [static]
```

Coordinate of product of terms.

Definition at line 1709 of file [framed\\_multi\\_imp.h](#).

### 5.2.2.31 db\_sqrt()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::db_sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_tol = std::pow(std::numeric\_limits<Scalar_T>::epsilon(), 4) ) ->
const matrix\_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Product form of Denman-Beavers square root iteration.

Definition at line 1320 of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

### 5.2.2.32 db\_step()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static void glucat::db_step (
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & M,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & Y ) [inline], [static]
```

Single step of product form of Denman-Beavers square root iteration.

Definition at line 1308 of file [matrix\\_multi\\_imp.h](#).

References [inv\(\)](#).

Referenced by [cascade\\_log\(\)](#), and [db\\_sqrt\(\)](#).

### 5.2.2.33 elliptic()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::elliptic (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar↔
_T,LO,HI,Tune_P> [inline]
```

Square root of -1 which commutes with all members of the frame of the given multivector The name "elliptic" is now deprecated: use "complexifier" instead.

Definition at line 643 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

### 5.2.2.34 error\_squared()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::error_squared (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold ) -> Scalar_T [inline]
```

Relative or absolute error using the quadratic norm.

Definition at line 134 of file [clifford\\_algebra\\_imp.h](#).

References [norm\(\)](#).

Referenced by [approx\\_equal\(\)](#), and [PyClical::clifford\\_hidden\\_doctests\(\)](#).

### 5.2.2.35 error\_squared\_tol()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::error_squared_tol (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T
```

Quadratic norm error tolerance relative to a specific multivector.

Definition at line 112 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::pow\(\)](#).

Referenced by [approx\\_equal\(\)](#), and [PyClical::clifford\\_hidden\\_doctests\(\)](#).

### 5.2.2.36 even()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::even (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Even part.

Definition at line 513 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.37 exp()** [1/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::exp (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const framed\_multi<Scalar_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line [1750](#) of file [framed\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [exp\(\)](#), and [scalar\(\)](#).

Referenced by [cos\(\)](#), [cosh\(\)](#), [exp\(\)](#), [matrix\\_log\(\)](#), [matrix\\_sqrt\(\)](#), [pow\(\)](#), [sin\(\)](#), and [sinh\(\)](#).

**5.2.2.38 exp()** [2/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::exp (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line [2086](#) of file [matrix\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

**5.2.2.39 fast()**

```
template<typename Multivector_T , typename Matrix_T , typename Basis_Matrix_T >
static auto glucat::fast (
    const Matrix_T & X,
    index\_t level ) -> Multivector_T [static]
```

Inverse generalized Fast Fourier Transform.

Definition at line [1027](#) of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix::signed\\_perm\\_nork\(\)](#).



**5.2.2.40 folded\_dim()**

```
template<typename Matrix_Index_T , const index_t LO, const index_t HI>
static auto glucat::folded_dim (
    const index_set< LO, HI > & sub ) -> Matrix_Index_T    [inline], [static]
```

Determine the matrix dimension of the fold of a subalgebra.

Definition at line 101 of file [matrix\\_multi\\_imp.h](#).

References [offset\\_level\(\)](#).

**5.2.2.41 imag()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::imag (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T    [inline]
```

Imaginary part: deprecated (always 0)

Definition at line 497 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.42 inv()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::inv (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P>    [inline]
```

Geometric multiplicative inverse.

Definition at line 400 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [db\\_step\(\)](#), and [matrix\\_log\(\)](#).

**5.2.2.43 inverse\_gray()**

```
static auto glucat::inverse_gray (
    unsigned long x ) -> unsigned long    [inline], [static]
```

Inverse Gray code.

Definition at line 863 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).

#### 5.2.2.44 `inverse_reversed_gray()`

```
static auto glucat::inverse_reversed_gray (
    unsigned long x ) -> unsigned long    [inline], [static]
```

Inverse reversed Gray code.

Definition at line 846 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).

#### 5.2.2.45 `involute()`

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::involute (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,
LO,HI,Tune_P>    [inline]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

Definition at line 537 of file [clifford\\_algebra\\_imp.h](#).

#### 5.2.2.46 `log()` [1/4]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::log (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1800 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [log\(\)](#).

#### 5.2.2.47 `log()` [2/4]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 2045 of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_log\(\)](#).

**5.2.2.48 log()** [3/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Natural logarithm of multivector.

Definition at line 799 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [log\(\)](#).

**5.2.2.49 log()** [4/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Natural logarithm of multivector with specified complexifier.

Definition at line 791 of file [clifford\\_algebra\\_imp.h](#).

References [log\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [log\(\)](#), and [pow\(\)](#).

**5.2.2.50 log2()**

```
template<typename Scalar_T >
auto glucat::log2 (
    const Scalar_T & x ) -> Scalar_T [inline]
```

Log base 2 of scalar.

Definition at line 303 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

Referenced by [clifford\\_exp\(\)](#).

### 5.2.2.51 `matrix_log()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line [1967](#) of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [cascade\\_log\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [exp\(\)](#), [inv\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, matrix\\_log\(\), norm\(\), and glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [log\(\)](#), and [matrix\\_log\(\)](#).

### 5.2.2.52 `matrix_sqrt()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix_sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line [1571](#) of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [approx\\_equal\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, matrix\\_sqrt\(\), norm\(\), pade\\_approx\(\), pow\(\), and glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#), and [sqrt\(\)](#).

### 5.2.2.53 `max_abs()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::max_abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

Definition at line [585](#) of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.54 max\_pos()**

```
template<const index_t LO, const index_t HI>
auto glucat::max_pos (
    const index_set< LO, HI > & ist ) -> index_t [inline]
```

Maximum positive index, or 0 if none.

Definition at line 977 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical::index\\_set\\_hidden\\_doctests\(\)](#).

**5.2.2.55 min\_neg()**

```
template<const index_t LO, const index_t HI>
auto glucat::min_neg (
    const index_set< LO, HI > & ist ) -> index_t [inline]
```

Minimum negative index, or 0 if none.

Definition at line 970 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical::index\\_set\\_hidden\\_doctests\(\)](#).

**5.2.2.56 norm()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::norm (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar\_T norm == sum of norm of coordinates.

Definition at line 569 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [cascade\\_log\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [error\\_squared\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.57 odd()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::odd (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Odd part.

Definition at line 521 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#).

**5.2.2.58 offset\_level()**

```
auto glucat::offset_level (
    const index\_t p,
    const index\_t q ) -> index\_t    [inline]
```

Determine the log2 dim corresponding to signature p, q.

Definition at line 86 of file [matrix\\_multi\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), and [folded\\_dim\(\)](#).

**5.2.2.59 operator"!="() [1/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator!= (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool    [inline]
```

Test for inequality of multivectors.

Definition at line 86 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.60 operator"!="() [2/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator!= (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> bool    [inline]
```

Test for inequality of multivector and scalar.

Definition at line 94 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.61 operator"!="() [3/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator!= (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool    [inline]
```

Test for inequality of scalar and multivector.

Definition at line 102 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.62 operator%() [1/3]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator% (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Left contraction.

Definition at line 597 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.63 operator%() [2/3]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator% (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 581 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.64 operator%() [3/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator% (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 322 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.65 operator&() [1/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator& (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Inner product.

Definition at line 495 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.66 operator&() [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator& (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs ) -> const index_set<LO,HI> [inline]
```

Set intersection: and.

Definition at line 186 of file [index\\_set\\_imp.h](#).

**5.2.2.67 operator&() [3/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator& (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 562 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.68 operator&() [4/4]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator& (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 307 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.69 operator\*() [1/6]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Geometric product.

Definition at line 374 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#).



**5.2.2.70 operator\*() [2/6]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix_multi<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 502 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [reframe\(\)](#).

**5.2.2.71 operator\*() [3/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_←
T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 277 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.72 operator\*() [4/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Product of multivector and scalar.

Definition at line 251 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.73 operator\*() [5/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Product of scalar and multivector.

Definition at line 262 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.74 operator\*() [6/6]**

```
template<typename Scalar_T , const index_t LO, const index_t HI>
auto glucat::operator* (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> const std::pair<const index_set<LO,HI>, Scalar_T> [inline]
```

Product of terms.

Definition at line 1717 of file [framed\\_multi\\_imp.h](#).

References [crd\\_of\\_mult\(\)](#).

**5.2.2.75 operator+() [1/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum.

Definition at line 206 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.76 operator+() [2/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum of multivector and scalar.

Definition at line 181 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.77 operator+() [3/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum of scalar and multivector.

Definition at line 192 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.78 operator-() [1/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference.

Definition at line 240 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.79 operator-() [2/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference of multivector and scalar.

Definition at line 217 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.80 operator-() [3/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator- (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference of scalar and multivector.

Definition at line 228 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.81 operator/() [1/5]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator/ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed_multi<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 734 of file [framed\\_multi\\_imp.h](#).

**5.2.2.82 operator/() [2/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_←
_T,LO,HI,Tune_P>
```

Geometric quotient.

Definition at line 614 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix::isnan\(\)](#), and [reframe\(\)](#).

**5.2.2.83 operator/() [3/5]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_←
T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 374 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.84 operator/() [4/5]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Quotient of multivector and scalar.

Definition at line 348 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.85 operator/() [5/5]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Quotient of scalar and multivector.

Definition at line 359 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.86 operator<<() [1/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator<< (
    std::ostream & os,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::ostream&
```

Write multivector to output.

Definition at line [1148](#) of file [framed\\_multi\\_imp.h](#).

References [scalar\(\)](#).

**5.2.2.87 operator<<() [2/5]**

```
const index\_t HI auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist ) -> std::ostream &
```

**5.2.2.88 operator<<() [3/5]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const index\_set< LO, HI > & ist ) -> std::ostream&
```

Write out index set.

Definition at line [611](#) of file [index\\_set\\_imp.h](#).

**5.2.2.89 operator<<() [4/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator<< (
    std::ostream & os,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::ostream& \[inline\]
```

Write multivector to output.

Definition at line [955](#) of file [matrix\\_multi\\_imp.h](#).

**5.2.2.90 operator<<() [5/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const std::pair< const index\_set< LO, HI >, Scalar_T > & term ) -> std::ostream&
```

Write term to output.

Definition at line [1209](#) of file [framed\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**5.2.2.91 operator>>() [1/3]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator>> (
    std::istream & s,
    framed\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::istream&
```

Read multivector from input.

Definition at line [1248](#) of file [framed\\_multi\\_imp.h](#).

**5.2.2.92 operator>>() [2/3]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator>> (
    std::istream & s,
    index\_set< LO, HI > & ist ) -> std::istream&
```

Read in index set.

Definition at line [634](#) of file [index\\_set\\_imp.h](#).

**5.2.2.93 operator>>() [3/3]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator>> (
    std::istream & s,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::istream& [inline]
```

Read multivector from input.

Definition at line [966](#) of file [matrix\\_multi\\_imp.h](#).

**5.2.2.94 operator^() [1/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator^ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Outer product.

Definition at line 416 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.95 operator^() [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator^ (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs ) -> const index_set<LO,HI> [inline]
```

Symmetric set difference: exclusive or.

Definition at line 161 of file [index\\_set\\_imp.h](#).

**5.2.2.96 operator^() [3/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator^ (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 543 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.97 operator^() [4/4]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator^ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 292 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.98 operator" | () [1/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 760 of file [framed\\_multi\\_imp.h](#).

**5.2.2.99 operator" | () [2/4]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator| (
    const index\_set< LO, HI > & lhs,
    const index\_set< LO, HI > & rhs ) -> const index\_set<LO,HI> [inline]
```

Set union: or.

Definition at line 211 of file [index\\_set\\_imp.h](#).

**5.2.2.100 operator" | () [3/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 717 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.101 operator" | () [4/4]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 389 of file [clifford\\_algebra\\_imp.h](#).



### 5.2.2.102 outer\_pow()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::outer_pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Outer product power of multivector.

Definition at line 470 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.103 pade\_approx()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P , const size_t Size>
static auto glucat::pade_approx (
    const std::array< Scalar_T, Size > & numer,
    const std::array< Scalar_T, Size > & denom,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & X ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P> [inline], [static]
```

Pade' approximation.

Definition at line 1245 of file [matrix\\_multi\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#), and [pade\\_log\(\)](#).

### 5.2.2.104 pade\_log()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::pade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Pade' approximation of log.

Definition at line 1900 of file [matrix\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [pade\\_approx\(\)](#).

Referenced by [cascade\\_log\(\)](#).

### 5.2.2.105 pos\_mod()

```
template<typename LHS_T , typename RHS_T >
auto glucat::pos_mod (
    LHS_T lhs,
    RHS_T rhs ) -> LHS_T    [inline]
```

Modulo function which works reliably for lhs < 0.

Definition at line 117 of file [global.h](#).

Referenced by [complexifier\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_vector\(\)](#), [offset\\_level\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)](#).

### 5.2.2.106 pow() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P>    [inline]
```

Multivector power of multivector.

Definition at line 446 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#), and [log\(\)](#).

### 5.2.2.107 pow() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Integer power of multivector.

Definition at line 407 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.108 pure()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::pure (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Pure part.

Definition at line 505 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.109 quad()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::quad (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

Definition at line 561 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.110 real()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::real (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Real part: synonym for scalar part.

Definition at line 486 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.111 reframe()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::reframe (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs_reframed,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs_reframed ) -> const index\_set<LO,HI>
[inline]
```

Find a common frame for operands of a binary operator.

Definition at line 345 of file [matrix\\_multi\\_imp.h](#).

Referenced by [operator\\*\(\)](#), and [operator/\(\)](#).

### 5.2.2.112 reverse()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::reverse (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .

Definition at line 545 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.113 scalar()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::scalar (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar part.

Definition at line 478 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [exp\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#), and [operator<<\(\)](#).

### 5.2.2.114 sign\_of\_square()

```
auto glucat::sign_of_square (
    index\_t j ) -> int [inline]
```

Square of generator {j}.

Square of generator index j.

Definition at line 963 of file [index\\_set\\_imp.h](#).

### 5.2.2.115 sin() [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Sine of multivector.

Definition at line 979 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sin\(\)](#).

**5.2.2.116 sin()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Sine of multivector with specified complexifier.

Definition at line 956 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [sin\(\)](#), and [tan\(\)](#).

**5.2.2.117 sinh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Hyperbolic sine of multivector.

Definition at line 911 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

**5.2.2.118 sqrt()** [1/4]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const framed\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1727 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [sqrt\(\)](#).

**5.2.2.119 sqrt()** [2/4]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line [1667](#) of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.120 sqrt()** [3/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Square root of multivector.

Definition at line [683](#) of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sqrt\(\)](#).

**5.2.2.121 sqrt()** [4/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Square root of multivector with specified complexifier.

Definition at line [675](#) of file [clifford\\_algebra\\_imp.h](#).

References [sqrt\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), and [sqrt\(\)](#).

**5.2.2.122 star()** [1/3]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T
```

Hestenes scalar product.

Definition at line [684](#) of file [framed\\_multi\\_imp.h](#).

**5.2.2.123 star()** [2/3]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line [600](#) of file [matrix\\_multi\\_imp.h](#).

**5.2.2.124 star()** [3/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line [337](#) of file [clifford\\_algebra\\_imp.h](#).

References [star\(\)](#).

Referenced by [star\(\)](#).

**5.2.2.125 tan()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Tangent of multivector.

Definition at line [1079](#) of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [tan\(\)](#).

**5.2.2.126 tan()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Tangent of multivector with specified complexifier.

Definition at line [1060](#) of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [cos\(\)](#), and [sin\(\)](#).

Referenced by [tan\(\)](#).

**5.2.2.127 tanh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Hyperbolic tangent of multivector.

Definition at line [1016](#) of file [clifford\\_algebra\\_imp.h](#).

References [cosh\(\)](#), and [sinh\(\)](#).

**5.2.2.128 to\_demote()**

```
template<typename Scalar_T >
auto glucat::to_demote (
    const Scalar_T & val ) -> typename numeric\_traits<Scalar_T>::demoted::type [inline]
```

Cast to demote.

Definition at line [135](#) of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).



**5.2.2.129 to\_promote()**

```
template<typename Scalar_T >
auto glucat::to_promote (
    const Scalar_T & val ) -> typename numeric_traits<Scalar_T>::promoted::type
[inline]
```

Cast to promote.

Definition at line 125 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**5.2.2.130 try\_catch()** [1/2]

```
int glucat::try_catch (
    intfn f )
```

Exception catching for functions returning int.

Definition at line 49 of file [try\\_catch.h](#).

Referenced by [glucat::control\\_t::call\(\)](#).

**5.2.2.131 try\_catch()** [2/2]

```
int glucat::try_catch (
    intintfn f,
    int arg )
```

Exception catching for functions of int returning int.

Definition at line 64 of file [try\\_catch.h](#).

**5.2.2.132 vector\_part()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::vector_part (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const std::vector<Scalar↔
_T> [inline]
```

Vector part of multivector, as a [vector\\_t](#) with respect to [frame\(\)](#)

Definition at line 529 of file [clifford\\_algebra\\_imp.h](#).

## 5.2.3 Variable Documentation

### 5.2.3.1 BITS\_PER\_SET\_VALUE

```
const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits
```

Number of bits in set\_value\_t.

Definition at line 103 of file [global.h](#).

### 5.2.3.2 DEFAULT\_HI

```
const index_t glucat::DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)
```

Default highest index in an index set.

Definition at line 111 of file [global.h](#).

### 5.2.3.3 l\_ln2

```
const long double glucat::l_ln2 = 0.6931471805599453094172321214581766L [static]
```

Definition at line 44 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::ln\\_2\(\)](#).

### 5.2.3.4 l\_pi

```
const long double glucat::l_pi = 3.1415926535897932384626433832795029L [static]
```

Definition at line 43 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::pi\(\)](#).

### 5.2.3.5 MS\_PER\_S

```
const double glucat::MS_PER_S = 1000.0
```

Timing constant: deprecated here - moved to [test/timing.h](#).

Definition at line 83 of file [global.h](#).

#### 5.2.3.6 Tuning\_Fast\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
```

Definition at line 92 of file [tuning.h](#).

#### 5.2.3.7 Tuning\_Fast\_CR\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
```

Definition at line 88 of file [tuning.h](#).

#### 5.2.3.8 Tuning\_Fast\_DB\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
```

Definition at line 89 of file [tuning.h](#).

#### 5.2.3.9 Tuning\_Fast\_Div\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
```

Definition at line 87 of file [tuning.h](#).

#### 5.2.3.10 Tuning\_Fast\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
```

Definition at line 93 of file [tuning.h](#).

#### 5.2.3.11 Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
```

Definition at line 94 of file [tuning.h](#).

#### 5.2.3.12 Tuning\_Fast\_Log\_Max\_Inner\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
```

Definition at line 91 of file [tuning.h](#).

#### 5.2.3.13 Tuning\_Fast\_Log\_Max\_Outer\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
```

Definition at line 90 of file [tuning.h](#).

#### 5.2.3.14 Tuning\_Fast\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
```

Definition at line 86 of file [tuning.h](#).

#### 5.2.3.15 Tuning\_Fast\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0
```

Definition at line 95 of file [tuning.h](#).

#### 5.2.3.16 Tuning\_Int\_Digits

```
const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
```

Definition at line 36 of file [tuning.h](#).

#### 5.2.3.17 Tuning\_Max\_Threshold

```
const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning_Int_Digits
```

Definition at line 37 of file [tuning.h](#).

#### 5.2.3.18 Tuning\_Naive\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
```

Definition at line 65 of file [tuning.h](#).

#### 5.2.3.19 Tuning\_Naive\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 66 of file [tuning.h](#).

#### 5.2.3.20 Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 67 of file [tuning.h](#).

#### 5.2.3.21 Tuning\_Naive\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
```

Definition at line 64 of file [tuning.h](#).

#### 5.2.3.22 Tuning\_Slow\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
```

Definition at line 42 of file [tuning.h](#).

#### 5.2.3.23 Tuning\_Slow\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 43 of file [tuning.h](#).

#### 5.2.3.24 Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 44 of file [tuning.h](#).

#### 5.2.3.25 Tuning\_Slow\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
```

Definition at line 41 of file [tuning.h](#).

#### 5.2.3.26 Tuning\_Slow\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 45 of file [tuning.h](#).

## 5.3 glucat::gen Namespace Reference

### Classes

- class [generator\\_table](#)  
*Table of generators for specific signatures.*

### Typedefs

- using [signature\\_t](#) = std::pair< [index\\_t](#), [index\\_t](#) >  
*A signature is a pair of indices,  $p$ ,  $q$ , with  $p == \text{frame.max}()$ ,  $q == -\text{frame.min}()$*

### Variables

- static const std::array< [index\\_t](#), 8 > [offset\\_to\\_super](#) = {0,-1, 0,-1,-2, 3, 2, 1}  
*Offsets between the current signature and that of the real superalgebra.*

### 5.3.1 Typedef Documentation

### 5.3.1.1 signature\_t

```
using glucat::gen::signature_t = typedef std::pair<index_t, index_t>
```

A signature is a pair of indices, p, q, with  $p == \text{frame.max}()$ ,  $q == -\text{frame.min}()$

Definition at line 48 of file [generation.h](#).

## 5.3.2 Variable Documentation

### 5.3.2.1 offset\_to\_super

```
const std::array<index_t, 8> glucat::gen::offset_to_super = {0, -1, 0, -1, -2, 3, 2, 1} [static]
```

Offsets between the current signature and that of the real superalgebra.

Definition at line 86 of file [generation.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#) and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)](#).

## 5.4 glucat::matrix Namespace Reference

### Classes

- struct [eig\\_genus](#)  
*Structure containing classification of eigenvalues.*

### Typedefs

- using [eig\\_case\\_t](#) = enum { safe\_eigs, neg\_real\_eigs, both\_eigs}  
*Classification of eigenvalues of a matrix.*

## Functions

- `template<typename LHS_T, typename RHS_T >`  
`auto kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Kronecker tensor product of matrices - as per Matlab kron.*
- `template<typename LHS_T, typename RHS_T >`  
`auto mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Sparse Kronecker tensor product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T >`  
`auto nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T`  
*Left inverse of Kronecker product.*
- `template<typename LHS_T, typename RHS_T >`  
`auto signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- `template<typename Matrix_T >`  
`auto nnz (const Matrix_T &m) -> typename Matrix_T::size_type`  
*Number of non-zeros.*
- `template<typename Matrix_T >`  
`auto isinf (const Matrix_T &m) -> bool`  
*Infinite.*
- `template<typename Matrix_T >`  
`auto isnan (const Matrix_T &m) -> bool`  
*Not a Number.*
- `template<typename Matrix_T >`  
`auto unit (const typename Matrix_T::size_type n) -> const Matrix_T`  
*Unit matrix - as per Matlab eye.*
- `template<typename LHS_T, typename RHS_T >`  
`auto mono_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T >`  
`auto sparse_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of sparse matrices.*
- `template<typename LHS_T, typename RHS_T >`  
`auto prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of matrices.*
- `template<typename Scalar_T, typename LHS_T, typename RHS_T >`  
`auto inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T`  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$*
- `template<typename Matrix_T >`  
`auto norm_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Square of Frobenius norm.*
- `template<typename Matrix_T >`  
`auto trace (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Matrix trace.*
- `template<typename Matrix_T >`  
`auto eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >`  
*Eigenvalues of a matrix.*
- `template<typename Matrix_T >`  
`auto classify_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >`  
*Classify the eigenvalues of a matrix.*



- `template<typename LHS_T, typename RHS_T >`  
`void nork\_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const`  
`typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename Matrix_T >`  
`static auto to\_lapack (const Matrix_T &val) -> ublas::matrix< double, ublas::column_major >  
Convert matrix to LAPACK format.`

## 5.4.1 Typedef Documentation

### 5.4.1.1 eig\_case\_t

using `glucat::matrix::eig_case_t` = typedef enum { safe\_eigs, neg\_real\_eigs, both\_eigs}

Classification of eigenvalues of a matrix.

Definition at line 133 of file [matrix.h](#).

## 5.4.2 Function Documentation

### 5.4.2.1 classify\_eigenvalues()

```
template<typename Matrix_T >
auto glucat::matrix::classify_eigenvalues (
    const Matrix_T & val ) -> eig\_genus<Matrix_T>
```

Classify the eigenvalues of a matrix.

Definition at line 548 of file [matrix\\_imp.h](#).

References [eigenvalues\(\)](#), [epsilon](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_eig\\_case](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_eig\\_genus< Matrix\\_T >::m\\_safe\\_arg](#), and [glucat::numeric\\_traits< Scalar\\_T >::pi\(\)](#).

Referenced by [glucat::matrix\\_log\(\)](#), and [glucat::matrix\\_sqrt\(\)](#).

### 5.4.2.2 eigenvalues()

```
template<typename Matrix_T >
auto glucat::matrix::eigenvalues (
    const Matrix_T & val ) -> std::vector< std::complex<double> >
```

Eigenvalues of a matrix.

Definition at line 500 of file [matrix\\_imp.h](#).

References [to\\_lapack\(\)](#).

Referenced by [classify\\_eigenvalues\(\)](#).

#### 5.4.2.3 inner()

```
template<typename Scalar_T , typename LHS_T , typename RHS_T >
auto glucat::matrix::inner (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> Scalar_T
```

Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$

Inner product:  $\text{sum}(lhs(i,j)*rhs(i,j))/lhs.\text{nrows}()$

Definition at line 373 of file [matrix\\_imp.h](#).

#### 5.4.2.4 isinf()

```
template<typename Matrix_T >
auto glucat::matrix::isinf (
    const Matrix_T & m ) -> bool
```

Infinite.

Definition at line 275 of file [matrix\\_imp.h](#).

#### 5.4.2.5 isnan()

```
template<typename Matrix_T >
auto glucat::matrix::isnan (
    const Matrix_T & m ) -> bool
```

Not a Number.

Definition at line 292 of file [matrix\\_imp.h](#).

Referenced by [glucat::operator/\(\)](#).

#### 5.4.2.6 kron()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::kron (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Kronecker tensor product of matrices - as per Matlab kron.

Definition at line 83 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#).

#### 5.4.2.7 mono\_kron()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::mono_kron (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Sparse Kronecker tensor product of monomial matrices.

Definition at line 119 of file [matrix\\_imp.h](#).

Referenced by [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pm1\\_qm1\(\)](#).

#### 5.4.2.8 mono\_prod()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::mono_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type
```

Product of monomial matrices.

Definition at line 320 of file [matrix\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_f](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pp4\\_qm4\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_qp1\\_pm](#).

#### 5.4.2.9 nnz()

```
template<typename Matrix_T >
auto glucat::matrix::nnz (
    const Matrix_T & m ) -> typename Matrix_T::size_type
```

Number of non-zeros.

Definition at line 258 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 5.4.2.10 nork()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::nork (
    const LHS_T & lhs,
    const RHS_T & rhs,
    const bool mono = true ) -> const RHS_T
```

Left inverse of Kronecker product.

Definition at line 182 of file [matrix\\_imp.h](#).

References [norm\\_frob2\(\)](#).

#### 5.4.2.11 `nork_range()`

```
template<typename LHS_T , typename RHS_T >
void glucat::matrix::nork_range (
    RHS_T & result,
    const typename LHS_T::const_iterator2 lhs_it2,
    const RHS_T & rhs,
    const typename RHS_T::size_type res_s1,
    const typename RHS_T::size_type res_s2 )
```

Utility routine for nork: calculate result for a range of indices.

Definition at line 152 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

#### 5.4.2.12 `norm_frob2()`

```
template<typename Matrix_T >
auto glucat::matrix::norm_frob2 (
    const Matrix_T & val ) -> typename Matrix_T::value_type
```

Square of Frobenius norm.

Definition at line 395 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#).

Referenced by [nork\(\)](#).

#### 5.4.2.13 `prod()`

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type [inline]
```

Product of matrices.

Definition at line 361 of file [matrix\\_imp.h](#).

#### 5.4.2.14 signed\_perm\_nork()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::signed_perm_nork (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Left inverse of Kronecker product where lhs is a signed permutation matrix.

Definition at line 228 of file [matrix\\_imp.h](#).

Referenced by [glucat::fast\(\)](#).

#### 5.4.2.15 sparse\_prod()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::sparse_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type [inline]
```

Product of sparse matrices.

Definition at line 350 of file [matrix\\_imp.h](#).

#### 5.4.2.16 to\_lapack()

```
template<typename Matrix_T >
static auto glucat::matrix::to_lapack (
    const Matrix_T & val ) -> ublas::matrix<double, ublas::column_major> [static]
```

Convert matrix to LAPACK format.

Definition at line 440 of file [matrix\\_imp.h](#).

Referenced by [eigenvalues\(\)](#).

#### 5.4.2.17 trace()

```
template<typename Matrix_T >
auto glucat::matrix::trace (
    const Matrix_T & val ) -> typename Matrix_T::value_type
```

Matrix trace.

Definition at line 416 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#).

#### 5.4.2.18 unit()

```
template<typename Matrix_T >
auto glucat::matrix::unit (
    const typename Matrix_T::size_type n ) -> const Matrix_T [inline]
```

Unit matrix - as per Matlab eye.

Definition at line 310 of file [matrix\\_imp.h](#).

## 5.5 glucat::timing Namespace Reference

### Functions

- static double [elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

### Variables

- const double [MS\\_PER\\_SEC](#) = 1000.0  
*Timing constant: milliseconds per second.*
- const double [MS\\_PER\\_CLOCK](#) = [MS\\_PER\\_SEC](#) / double(CLOCKS\_PER\_SEC)  
*Timing constant: milliseconds per clock.*
- const int [EXTRA\\_TRIALS](#) = 2  
*Timing constant: trial expansion factor.*

### 5.5.1 Function Documentation

#### 5.5.1.1 elapsed()

```
static double glucat::timing::elapsed (
    clock_t cpu_time ) [inline], [static]
```

Elapsed time in milliseconds.

Definition at line 51 of file [timing.h](#).

References [MS\\_PER\\_CLOCK](#).

### 5.5.2 Variable Documentation

### 5.5.2.1 EXTRA\_TRIALS

```
const int glucat::timing::EXTRA_TRIALS = 2
```

Timing constant: trial expansion factor.

Definition at line 45 of file [timing.h](#).

### 5.5.2.2 MS\_PER\_CLOCK

```
const double glucat::timing::MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)
```

Timing constant: milliseconds per clock.

Definition at line 42 of file [timing.h](#).

Referenced by [elapsed\(\)](#).

### 5.5.2.3 MS\_PER\_SEC

```
const double glucat::timing::MS_PER_SEC = 1000.0
```

Timing constant: milliseconds per second.

Definition at line 39 of file [timing.h](#).

## 5.6 pade Namespace Reference

### Classes

- struct [pade\\_log\\_denom](#)

*Coefficients of denominator polynomials of Pade approximations produced by  $\text{Pade1}(\log(1+x), x, n, n)$*

- struct [pade\\_log\\_denom](#)< [dd\\_real](#) >
- struct [pade\\_log\\_denom](#)< [float](#) >
- struct [pade\\_log\\_denom](#)< [long double](#) >
- struct [pade\\_log\\_denom](#)< [qd\\_real](#) >
- struct [pade\\_log\\_numer](#)

*Coefficients of numerator polynomials of Pade approximations produced by  $\text{Pade1}(\log(1+x), x, n, n)$*

- struct [pade\\_log\\_numer](#)< [dd\\_real](#) >
- struct [pade\\_log\\_numer](#)< [float](#) >
- struct [pade\\_log\\_numer](#)< [long double](#) >
- struct [pade\\_log\\_numer](#)< [qd\\_real](#) >
- struct [pade\\_sqrt\\_denom](#)

*Coefficients of denominator polynomials of Pade approximations produced by  $\text{Pade1}(\sqrt{1+x}, x, n, n)$*

- struct [pade\\_sqrt\\_denom](#)< [dd\\_real](#) >
- struct [pade\\_sqrt\\_denom](#)< [float](#) >
- struct [pade\\_sqrt\\_denom](#)< [long double](#) >
- struct [pade\\_sqrt\\_denom](#)< [qd\\_real](#) >
- struct [pade\\_sqrt\\_numer](#)

*Coefficients of numerator polynomials of Pade approximations produced by  $\text{Pade1}(\sqrt{1+x}, x, n, n)$*

- struct [pade\\_sqrt\\_numer](#)< [dd\\_real](#) >
- struct [pade\\_sqrt\\_numer](#)< [float](#) >
- struct [pade\\_sqrt\\_numer](#)< [long double](#) >
- struct [pade\\_sqrt\\_numer](#)< [qd\\_real](#) >

## 5.7 PyClical Namespace Reference

### Classes

- class [clifford](#)
- class [index\\_set](#)

### Functions

- def [index\\_set\\_hidden\\_doctests](#) ()
- def [clifford\\_hidden\\_doctests](#) ()
- def [e](#) (obj)
- def [istpq](#) (p, q)
- def [\\_test](#) ()

### Variables

- [\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [lhs](#)
- [rhs](#)
- [threshold](#) = error\_squared\_tol([rhs](#)) if threshold is [None](#) else threshold
- [None](#)
- [tol](#) = error\_squared\_tol([rhs](#)) if tol is [None](#) else tol
- [obj](#)
- [i](#)
- [ixt](#)
- [fill](#)
- [scalar\\_epsilon](#) = [epsilon](#)
- float [pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [cl](#) = [clifford](#)
- [ist](#) = [index\\_set](#)
- def [ninf3](#) = [e](#)(4) + [e](#)(-1)
- def [nbar3](#) = [e](#)(4) - [e](#)(-1)

### 5.7.1 Function Documentation

#### 5.7.1.1 [\\_test\(\)](#)

```
def PyClical._test ( ) [private]
```

Definition at line [1962](#) of file [PyClical.pyx](#).

References [\\_test\(\)](#).

Referenced by [\\_test\(\)](#).



## 5.7.1.2 clifford\_hidden\_doctests()

```
def PyClical.clifford_hidden_doctests ( )
```

Tests for functions that Doctest cannot see.

For clifford.\_\_cinit\_\_: Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2, index_set ({1,2})))
2{1,2}
>>> print(clifford([2,3], index_set ({1,2})))
2{1}+3{2}
>>> print(clifford([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'list'>.
>>> print(clifford(None))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'NoneType'>.
>>> print(clifford(None, [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
>>> print(clifford([1,2], [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
>>> print(clifford(""))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string ''.
>>> print(clifford("{")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{'.
>>> print(clifford("{1")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1'.
>>> print(clifford("{+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{+'.
>>> print(clifford("{-")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{-'.
>>> print(clifford("{1}+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.

For clifford.__richcmp__: Compare objects of type clifford.

>>> clifford("{1}") == clifford("1{1}")
True
```

```
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 1253 of file [PyClicl.pyx](#).

References [glucat.error\\_squared\(\)](#), and [glucat.error\\_squared\\_tol\(\)](#).

### 5.7.1.3 e()

```
def PyClicl.e (
    obj )
```

Abbreviation for `clifford(index_set(obj))`.

```
>>> print(e(1))
{1}
>>> print(e(-1))
{-1}
>>> print(e(0))
1
```

Definition at line 1936 of file [PyClicl.pyx](#).

References [e\(\)](#).

Referenced by [e\(\)](#).

### 5.7.1.4 index\_set\_hidden\_doctests()

```
def PyClicl.index_set_hidden_doctests ( )
```

Tests for functions that Doctest cannot see.

For `index_set.__cinit__`: Construct `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
```

```

>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set(""))
{}
>>> print(index_set("{}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{}'.
>>> print(index_set("{1}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1}'.
>>> print(index_set("{1,2,100}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
>>> print(index_set({1,2,100}))
Traceback (most recent call last):
...
IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
>>> print(index_set([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize index_set object from <class 'list'>.

For index_set.__richcmp__: Compare two objects of class index_set.

>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
>>> None == index_set({1,2})
False
>>> None != index_set({1,2})
True
>>> None < index_set({1,2})
False
>>> None <= index_set({1,2})
False
>>> None > index_set({1,2})
False
>>> None >= index_set({1,2})
False
>>> index_set({1,2}) == None
False
>>> index_set({1,2}) != None
True
>>> index_set({1,2}) < None
False
>>> index_set({1,2}) <= None
False
>>> index_set({1,2}) > None
False
>>> index_set({1,2}) >= None
False

```

Definition at line 406 of file [PyClical.pyx](#).

References [glucat.compare\(\)](#), [glucat.max\\_pos\(\)](#), and [glucat.min\\_neg\(\)](#).

#### 5.7.1.5 istpq()

```
def PyClical.istpq (
    p,
    q )
```

Abbreviation for `index_set({-q,...p})`.

```
>>> print(istpq(2,3))
{-3,-2,-1,1,2}
```

Definition at line 1949 of file [PyClical.pyx](#).

References [istpq\(\)](#).

Referenced by [istpq\(\)](#).

### 5.7.2 Variable Documentation

#### 5.7.2.1 \_\_version\_\_

```
PyClical.__version__ = str(glucat_package_version,'utf-8') [private]
```

Definition at line 35 of file [PyClical.pyx](#).

#### 5.7.2.2 cl

```
PyClical.cl = clifford
```

Definition at line 1910 of file [PyClical.pyx](#).

#### 5.7.2.3 fill

```
PyClical.fill
```

Definition at line 1864 of file [PyClical.pyx](#).

#### 5.7.2.4 i

`PyClical.i`

Definition at line 1591 of file [PyClical.pyx](#).

#### 5.7.2.5 ist

`PyClical.ist = index_set`

Definition at line 1928 of file [PyClical.pyx](#).

#### 5.7.2.6 ixt

`PyClical.ixt`

Definition at line 1864 of file [PyClical.pyx](#).

#### 5.7.2.7 lhs

`PyClical.lhs`

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.8 nbar3

`def PyClical.nbar3 = e(4) - e(-1)`

Definition at line 1959 of file [PyClical.pyx](#).

#### 5.7.2.9 ninf3

`def PyClical.ninf3 = e(4) + e(-1)`

Definition at line 1958 of file [PyClical.pyx](#).

#### 5.7.2.10 None

`PyClical.None`

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.11 obj

`PyClical.obj`

Definition at line 1591 of file [PyClical.pyx](#).

#### 5.7.2.12 pi

```
float PyClical.pi = atan(clifford(1.0)) * 4.0
```

Definition at line 1907 of file [PyClical.pyx](#).

#### 5.7.2.13 rhs

`PyClical.rhs`

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.14 scalar\_epsilon

`PyClical.scalar_epsilon = epsilon`

Definition at line 1905 of file [PyClical.pyx](#).

#### 5.7.2.15 tau

```
float PyClical.tau = atan(clifford(1.0)) * 8.0
```

Definition at line 1908 of file [PyClical.pyx](#).

### 5.7.2.16 threshold

```
PyClical.threshold = error_squared_tol(rhs) if threshold is None else threshold
```

Definition at line 1359 of file [PyClical.pyx](#).

### 5.7.2.17 tol

```
PyClical.tol = error_squared_tol(rhs) if tol is None else tol
```

Definition at line 1359 of file [PyClical.pyx](#).

## 5.8 std Namespace Reference

### Classes

- struct [numeric\\_limits< glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >](#)  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*
- struct [numeric\\_limits< glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >](#)  
*Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.*





## Chapter 6

# Class Documentation

### 6.1 `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >` Class Template Reference

Table of basis elements used as a cache by `basis_element()`

```
#include <matrix_multi_imp.h>
```

Inheritance diagram for `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >`:

Collaboration diagram for `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >`:

#### Public Member Functions

- `basis_table` (const `basis_table` &)=delete
- auto `operator=` (const `basis_table` &) -> `basis_table` &=delete

#### Static Public Member Functions

- static auto `basis` () -> `basis_table` &  
*Single instance of basis table.*

#### Private Member Functions

- `basis_table` ()=default
- `~basis_table` ()=default

#### Friends

- class `friend_for_private_destructor`

### 6.1.1 Detailed Description

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >
```

Table of basis elements used as a cache by `basis_element()`

Definition at line [1162](#) of file [matrix\\_multi\\_imp.h](#).

### 6.1.2 Constructor & Destructor Documentation

#### 6.1.2.1 `basis_table()` [1/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table ( ) [private], [default]
```

#### 6.1.2.2 `~basis_table()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::~~basis_table ( ) [private], [default]
```

#### 6.1.2.3 `basis_table()` [2/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table (
    const basis\_table< Scalar_T, LO, HI, Matrix_T > & ) [delete]
```

### 6.1.3 Member Function Documentation

#### 6.1.3.1 `basis()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
static auto glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis ( ) -> basis\_table&
[inline], [static]
```

Single instance of basis table.

Definition at line [1168](#) of file [matrix\\_multi\\_imp.h](#).

### 6.1.3.2 operator=()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Matrix_T >
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::operator= (
    const basis_table< Scalar_T, LO, HI, Matrix_T > & ) -> basis_table &=delete
[delete]
```

## 6.1.4 Friends And Related Function Documentation

### 6.1.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Matrix_T >
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 1173 of file [matrix\\_multi\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.2 glucat::bool\_to\_type< truth\_value > Class Template Reference

Bool to type.

```
#include <global.h>
```

### Private Types

- enum { [value](#) = truth\_value }

### 6.2.1 Detailed Description

```
template<bool truth_value>
class glucat::bool_to_type< truth_value >
```

Bool to type.

Definition at line 69 of file [global.h](#).

### 6.2.2 Member Enumeration Documentation

#### 6.2.2.1 anonymous enum

```
template<bool truth_value>
anonymous enum [private]
```

## Enumerator

value	
-------	--

Definition at line 72 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.3 PyClical.clifford Class Reference

Inheritance diagram for PyClical.clifford:

Collaboration diagram for PyClical.clifford:

### Public Member Functions

- `def __cinit__(self, other=0, ixt=None)`
- `def __dealloc__(self)`
- `def __contains__(self, x)`
- `def __iter__(self)`
- `def reframe(self, ixt)`
- `def __richcmp__(lhs, rhs, int, op)`
- `def __getitem__(self, ixt)`
- `def __neg__(self)`
- `def __pos__(self)`
- `def __add__(lhs, rhs)`
- `def __iadd__(self, rhs)`
- `def __sub__(lhs, rhs)`
- `def __isub__(self, rhs)`
- `def __mul__(lhs, rhs)`
- `def __imul__(self, rhs)`
- `def __mod__(lhs, rhs)`
- `def __imod__(self, rhs)`
- `def __and__(lhs, rhs)`
- `def __iand__(self, rhs)`
- `def __xor__(lhs, rhs)`
- `def __ixor__(self, rhs)`
- `def __truediv__(lhs, rhs)`
- `def __idiv__(self, rhs)`
- `def inv(self)`
- `def __or__(lhs, rhs)`
- `def __ior__(self, rhs)`
- `def __pow__(self, m, dummy)`
- `def pow(self, m)`
- `def outer_pow(self, m)`
- `def __call__(self, grade)`
- `def scalar(self)`
- `def pure(self)`

- def [even](#) (self)
- def [odd](#) (self)
- def [vector\\_part](#) (self, frm=[None](#))
- def [involute](#) (self)
- def [reverse](#) (self)
- def [conj](#) (self)
- def [quad](#) (self)
- def [norm](#) (self)
- def [abs](#) (self)
- def [max\\_abs](#) (self)
- def [truncated](#) (self, limit)
- def [isinf](#) (self)
- def [isnan](#) (self)
- def [frame](#) (self)
- def [\\_\\_repr\\_\\_](#) (self)
- def [\\_\\_str\\_\\_](#) (self)

## Public Attributes

- [instance](#)

### 6.3.1 Detailed Description

Python class `clifford` wraps C++ class `Clifford`.

Definition at line [532](#) of file [PyClical.pyx](#).

### 6.3.2 Member Function Documentation

#### 6.3.2.1 `__add__()`

```
def PyClical.clifford.__add__ (
    lhs,
    rhs )
```

Geometric sum.

```
>>> print(clifford(1) + clifford("{2}"))
1+{2}
>>> print(clifford("{1}") + clifford("{2}"))
{1}+{2}
```

Definition at line [740](#) of file [PyClical.pyx](#).

### 6.3.2.2 `__and__()`

```
def PyClical.clifford.__and__ (
    lhs,
    rhs )
```

Inner product.

```
>>> print(clifford("{1}") & clifford("{2}"))
0
>>> print(clifford(2) & clifford("{2}"))
0
>>> print(clifford("{1}") & clifford("{1}"))
1
>>> print(clifford("{1}") & clifford("{1,2}"))
{2}
```

Definition at line 836 of file [PyClical.pyx](#).

### 6.3.2.3 `__call__()`

```
def PyClical.clifford.__call__ (
    self,
    grade )
```

Pure grade-vector part.

```
>>> print(clifford("{1}") (1))
{1}
>>> print(clifford("{1}") (0))
0
>>> print(clifford("1+{1}+{1,2}") (0))
1
>>> print(clifford("1+{1}+{1,2}") (1))
{1}
>>> print(clifford("1+{1}+{1,2}") (2))
{1,2}
>>> print(clifford("1+{1}+{1,2}") (3))
0
```

Definition at line 1020 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

#### 6.3.2.4 `__cinit__()`

```
def PyClical.clifford.__cinit__ (
    self,
    other = 0,
    ixt = None )
```

Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2,index_set({1,2})))
2{1,2}
>>> print(clifford([2,3],index_set({1,2})))
2{1}+3{2}
```

Definition at line 565 of file [PyClical.pyx](#).

#### 6.3.2.5 `__contains__()`

```
def PyClical.clifford.__contains__ (
    self,
    x )
```

Not applicable.

```
>>> x=clifford(index_set({-3,4,7})); -3 in x
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 627 of file [PyClical.pyx](#).

#### 6.3.2.6 `__dealloc__()`

```
def PyClical.clifford.__dealloc__ (
    self )
```

Clean up by deallocating the instance of C++ class Clifford.

Definition at line 621 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.3.2.7 `__getitem__()`

```
def PyClicl.clifford.__getitem__ (
    self,
    ixt )
```

Subscripting: map from index set to scalar coordinate.

```
>>> clifford("{1}") [index_set(1)]
1.0
>>> clifford("{1}") [index_set({1})]
1.0
>>> clifford("{1}") [index_set({1,2})]
0.0
>>> clifford("2{1,2}") [index_set({1,2})]
2.0
```

Definition at line 707 of file [PyClicl.pyx](#).

References [PyClicl.index\\_set.instance](#), and [PyClicl.clifford.instance](#).

### 6.3.2.8 `__iadd__()`

```
def PyClicl.clifford.__iadd__ (
    self,
    rhs )
```

Geometric sum.

```
>>> x = clifford(1); x += clifford("{2}"); print(x)
1+{2}
```

Definition at line 751 of file [PyClicl.pyx](#).

### 6.3.2.9 `__iand__()`

```
def PyClicl.clifford.__iand__ (
    self,
    rhs )
```

Inner product.

```
>>> x = clifford("{1}"); x &= clifford("{2}"); print(x)
0
>>> x = clifford(2); x &= clifford("{2}"); print(x)
0
>>> x = clifford("{1}"); x &= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
{2}
```

Definition at line 851 of file [PyClicl.pyx](#).



**6.3.2.10 `__idiv__()`**

```
def PyClical.clifford.__idiv__ (
    self,
    rhs )
```

Geometric quotient.

```
>>> x = clifford("{1}"); x /= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x /= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x /= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
-{2}
```

Definition at line 911 of file [PyClical.pyx](#).

**6.3.2.11 `__imod__()`**

```
def PyClical.clifford.__imod__ (
    self,
    rhs )
```

Contraction.

```
>>> x = clifford("{1}"); x %= clifford("{2}"); print(x)
0
>>> x = clifford(2); x %= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x %= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
{2}
```

Definition at line 821 of file [PyClical.pyx](#).

**6.3.2.12 `__imul__()`**

```
def PyClical.clifford.__imul__ (
    self,
    rhs )
```

Geometric product.

```
>>> x = clifford(2); x *= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x *= clifford("{2}"); print(x)
{1,2}
>>> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
{2}
```

Definition at line 793 of file [PyClical.pyx](#).

**6.3.2.13 `__ior__()`**

```
def PyClical.clifford.__ior__ (
    self,
    rhs )
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
-{1}
```

Definition at line 950 of file [PyClical.pyx](#).

**6.3.2.14 `__isub__()`**

```
def PyClical.clifford.__isub__ (
    self,
    rhs )
```

Geometric difference.

```
>>> x = clifford(1); x -= clifford("{2}"); print(x)
1-{2}
```

Definition at line 771 of file [PyClical.pyx](#).

**6.3.2.15 `__iter__()`**

```
def PyClical.clifford.__iter__ (
    self )
```

Not applicable.

```
>>> for a in clifford(index_set({-3,4,7})):print(a, end=",")
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 638 of file [PyClical.pyx](#).

**6.3.2.16 \_\_ixor\_\_()**

```
def PyClical.clifford.__ixor__ (
    self,
    rhs )
```

Outer product.

```
>>> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x ^= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
0
>>> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
0
```

Definition at line 881 of file [PyClical.pyx](#).

**6.3.2.17 \_\_mod\_\_()**

```
def PyClical.clifford.__mod__ (
    lhs,
    rhs )
```

Contraction.

```
>>> print(clifford("{1}") % clifford("{2}"))
0
>>> print(clifford(2) % clifford("{2}"))
2{2}
>>> print(clifford("{1}") % clifford("{1}"))
1
>>> print(clifford("{1}") % clifford("{1,2}"))
{2}
```

Definition at line 806 of file [PyClical.pyx](#).

**6.3.2.18 \_\_mul\_\_()**

```
def PyClical.clifford.__mul__ (
    lhs,
    rhs )
```

Geometric product.

```
>>> print(clifford("{1}") * clifford("{2}"))
{1,2}
>>> print(clifford(2) * clifford("{2}"))
2{2}
>>> print(clifford("{1}") * clifford("{1,2}"))
{2}
```

Definition at line 780 of file [PyClical.pyx](#).

### 6.3.2.19 `__neg__()`

```
def PyClical.clifford.__neg__ (
    self )
```

Unary `-`.

```
>>> print (-clifford("{1}"))
-{1}
```

Definition at line 722 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.3.2.20 `__or__()`

```
def PyClical.clifford.__or__ (
    lhs,
    rhs )
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
-{1}
```

Definition at line 939 of file [PyClical.pyx](#).

### 6.3.2.21 `__pos__()`

```
def PyClical.clifford.__pos__ (
    self )
```

Unary `+`.

```
>>> print (+clifford("{1}"))
{1}
```

Definition at line 731 of file [PyClical.pyx](#).

**6.3.2.22 `__pow__()`**

```
def PyClical.clifford.__pow__ (
    self,
    m,
    dummy )

Power: self to the m.

>>> x=clifford("{1}"); print(x ** 2)
1
>>> x=clifford("2"); print(x ** 2)
4
>>> x=clifford("2+{1}"); print(x ** 0)
1
>>> x=clifford("2+{1}"); print(x ** 1)
2+{1}
>>> x=clifford("2+{1}"); print(x ** 2)
5+4{1}
>>> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
1
```

Definition at line 961 of file [PyClical.pyx](#).

References [PyClical.clifford.pow\(\)](#).

**6.3.2.23 `__repr__()`**

```
def PyClical.clifford.__repr__ (
    self )

The "official" string representation of self.

>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
```

Definition at line 1235 of file [PyClical.pyx](#).

References [PyClical.clifford.\\_\\_repr\\_\\_\(\)](#), and [clifford\\_to\\_repr\(\)](#).

Referenced by [PyClical.clifford.\\_\\_repr\\_\\_\(\)](#).

**6.3.2.24 `__richcmp__()`**

```
def PyClical.clifford.__richcmp__ (
    lhs,
    rhs,
    int,
    op )
```

Compare objects of type clifford.

```
>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 672 of file [PyClical.pyx](#).

**6.3.2.25 `__str__()`**

```
def PyClical.clifford.__str__ (
    self )
```

The “informal” string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
'1+3{-1}+2{1,2}+4{-2,7}'
```

Definition at line 1244 of file [PyClical.pyx](#).

References [PyClical.clifford.\\_\\_str\\_\\_\(\)](#), and [clifford\\_to\\_str\(\)](#).

Referenced by [PyClical.clifford.\\_\\_str\\_\\_\(\)](#).

**6.3.2.26 `__sub__()`**

```
def PyClical.clifford.__sub__ (
    lhs,
    rhs )
```

Geometric difference.

```
>>> print(clifford(1) - clifford("{2}"))
1-{2}
>>> print(clifford("{1}") - clifford("{2}"))
{1}-{2}
```

Definition at line 760 of file [PyClical.pyx](#).

**6.3.2.27 `__truediv__()`**

```
def PyClical.clifford.__truediv__ (
    lhs,
    rhs )

Geometric quotient.

>>> print(clifford("{1}") / clifford("{2}"))
{1,2}
>>> print(clifford(2) / clifford("{2}"))
2{2}
>>> print(clifford("{1}") / clifford("{1}"))
1
>>> print(clifford("{1}") / clifford("{1,2}"))
-{2}
```

Definition at line 896 of file [PyClical.pyx](#).

**6.3.2.28 `__xor__()`**

```
def PyClical.clifford.__xor__ (
    lhs,
    rhs )

Outer product.

>>> print(clifford("{1}") ^ clifford("{2}"))
{1,2}
>>> print(clifford(2) ^ clifford("{2}"))
2{2}
>>> print(clifford("{1}") ^ clifford("{1}"))
0
>>> print(clifford("{1}") ^ clifford("{1,2}"))
0
```

Definition at line 866 of file [PyClical.pyx](#).

**6.3.2.29 `abs()`**

```
def PyClical.clifford.abs (
    self )

Absolute value: square root of norm.

>>> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
2.0
```

Definition at line 1175 of file [PyClical.pyx](#).

References [glucat.abs\(\)](#), and [PyClical.clifford.abs\(\)](#).

Referenced by [PyClical.clifford.abs\(\)](#).

### 6.3.2.30 conj()

```
def PyClical.clifford.conj (
    self )

Conjugation, reverse o involute == involute o reverse.

>>> print((clifford("{1}")).conj())
-{1}
>>> print((clifford("{2}") * clifford("{1}")).conj())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).conj())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").conj())
1-{1}-{1,2}
```

Definition at line 1138 of file [PyClical.pyx](#).

References [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.conj\(\)](#).

### 6.3.2.31 even()

```
def PyClical.clifford.even (
    self )

Even part of multivector, sum of even grade terms.

>>> print(clifford("1+{1}+{1,2}").even())
1+{1,2}
```

Definition at line 1061 of file [PyClical.pyx](#).

References [PyClical.clifford.even\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.even\(\)](#).

### 6.3.2.32 frame()

```
def PyClical.clifford.frame (
    self )

Subalgebra generated by all generators of terms of given multivector.

>>> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
{-2,-1,1,2,7}
>>> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
<class 'PyClical.index_set'>
```

Definition at line 1224 of file [PyClical.pyx](#).

References [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.frame\(\)](#), and [PyClical.clifford.reframe\(\)](#).



**6.3.2.33 inv()**

```
def PyClical.clifford.inv (
    self )

Geometric multiplicative inverse.

>>> x = clifford("{1}"); print(x.inv())
{1}
>>> x = clifford(2); print(x.inv())
0.5
>>> x = clifford("{1,2}"); print(x.inv())
-1,2}
```

Definition at line 926 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.inv\(\)](#).

Referenced by [PyClical.clifford.inv\(\)](#).

**6.3.2.34 involute()**

```
def PyClical.clifford.involute (
    self )

Main involution, each {i} is replaced by -{i} in each term,
eg. clifford("{1}") -> -clifford("{1}").

>>> print(clifford("{1}").involute())
-1
>>> print((clifford("{2}") * clifford("{1}")).involute())
-1,2}
>>> print((clifford("{1}") * clifford("{2}")).involute())
1,2}
>>> print(clifford("1+{1}+{1,2}").involute())
1-1+{1,2}
```

Definition at line 1107 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.involute\(\)](#).

Referenced by [PyClical.clifford.involute\(\)](#).

**6.3.2.35 isinf()**

```
def PyClical.clifford.isinf (
    self )

Check if a multivector contains any infinite values.

>>> clifford().isinf()
False
```

Definition at line 1206 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), [PyClical.clifford.isinf\(\)](#), and [PyClical.clifford.isnan\(\)](#).

Referenced by [PyClical.clifford.isinf\(\)](#).

### 6.3.2.36 isnan()

```
def PyClical.clifford.isnan (
    self )
```

Check if a multivector contains any IEEE NaN values.

```
>>> clifford().isnan()
False
```

Definition at line 1215 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.isnan\(\)](#).

Referenced by [PyClical.clifford.isinf\(\)](#), and [PyClical.clifford.isnan\(\)](#).

### 6.3.2.37 max\_abs()

```
def PyClical.clifford.max_abs (
    self )
```

Maximum of absolute values of components of multivector: multivector infinity norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
1.0
>>> clifford("3+2{1}+{1,2}").max_abs()
3.0
```

Definition at line 1184 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.max\\_abs\(\)](#).

Referenced by [PyClical.clifford.max\\_abs\(\)](#).

### 6.3.2.38 norm()

```
def PyClical.clifford.norm (
    self )
```

Norm == sum of squares of coordinates.

```
>>> clifford("1+{1}+{1,2}").norm()
3.0
>>> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
4.0
```

Definition at line 1164 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.norm\(\)](#).

Referenced by [PyClical.clifford.norm\(\)](#).

### 6.3.2.39 odd()

```
def PyClical.clifford.odd (
    self )
```

Odd part of multivector, sum of odd grade terms.

```
>>> print (clifford("1+{1}+{1,2}").odd())
{1}
```

Definition at line 1070 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.odd\(\)](#).

Referenced by [PyClical.clifford.odd\(\)](#).

### 6.3.2.40 outer\_pow()

```
def PyClical.clifford.outer_pow (
    self,
    m )
```

Outer product power.

```
>>> x=clifford("2+{1}"); print(x.outer_pow(0))
1
>>> x=clifford("2+{1}"); print(x.outer_pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.outer_pow(2))
4+4{1}
>>> print (clifford("1+{1}+{1,2}").outer_pow(3))
1+3{1}+3{1,2}
```

Definition at line 1004 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.outer\\_pow\(\)](#).

Referenced by [PyClical.clifford.outer\\_pow\(\)](#).

#### 6.3.2.41 pow()

```
def PyClical.clifford.pow (
    self,
    m )

Power: self to the m.

>>> x=clifford("{1}"); print(x.pow(2))
1
>>> x=clifford("2"); print(x.pow(2))
4
>>> x=clifford("2+{1}"); print(x.pow(0))
1
>>> x=clifford("2+{1}"); print(x.pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.pow(2))
5+4{1}
>>> print(clifford("1+{1}+{1,2}").pow(3))
1+3{1}+3{1,2}
>>> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
1
```

Definition at line 980 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.pow\(\)](#).

Referenced by [PyClical.clifford.\\_\\_pow\\_\\_\(\)](#), and [PyClical.clifford.pow\(\)](#).

#### 6.3.2.42 pure()

```
def PyClical.clifford.pure (
    self )

Pure part.

>>> print(clifford("1+{1}+{1,2}").pure())
{1}+{1,2}
>>> print(clifford("{1,2}").pure())
{1,2}
```

Definition at line 1050 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.pure\(\)](#).

Referenced by [PyClical.clifford.pure\(\)](#).

### 6.3.2.43 quad()

```
def PyCliclal.clifford.quad (
    self )

Quadratic form == (rev(x)*x)(0) .

>>> print(clifford("1+{1}+{1,2}") .quad())
3.0
>>> print(clifford("1+{-1}+{1,2}+{1,2,3}") .quad())
2.0
```

Definition at line 1153 of file PyClical.pyx.

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.quad\(\)](#).

Referenced by [PyClical.clifford.quad\(\)](#).

#### 6.3.2.44 reframe()

```
def PyCliclifford.reframe (
    self,
    ixt )

Put self into a larger frame, containing the union of self.frame() and index set ixt.
This can be used to make multiplication faster, by multiplying within a common frame.

>>> clifford("2+3{1}").reframe(index_set({1,2,3}))
clifford("2+3{1}")
>>> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);
True
```

Definition at line 649 of file PyClical.pyx.

**References** `glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI >>`,  
`glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuple< double, double> >>`,  
`glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >.frame()`, `PyClical.clifford.frame()`, and `PyClical.clifford.reframe()`.

Referenced by [PyClical.clifford.reframe\(\)](#).

### 6.3.2.45 reverse()

```
def PyClical.clifford.reverse (
    self )

Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").

>>> print(clifford("{1}").reverse())
{1}
>>> print((clifford("{2}") * clifford("{1}")).reverse())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).reverse())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").reverse())
1+{1}-{1,2}
```

Definition at line 1123 of file PyClical.pyx.

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.reverse\(\)](#).

Referenced by [PyClical.clifford.reverse\(\)](#).

**6.3.2.46 scalar()**

```
def PyClical.clifford.scalar (
    self )

Scalar part.

>>> clifford("1+{1}+{1,2}").scalar()
1.0
>>> clifford("{1,2}").scalar()
0.0
```

Definition at line 1039 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.scalar\(\)](#).

Referenced by [PyClical.clifford.scalar\(\)](#).

**6.3.2.47 truncated()**

```
def PyClical.clifford.truncated (
    self,
    limit )

Remove all terms of self with relative size smaller than limit.

>>> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
clifford("100000000")
>>> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
clifford("10000+{1}")
```

Definition at line 1195 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.truncated\(\)](#).

Referenced by [PyClical.clifford.truncated\(\)](#).

**6.3.2.48 vector\_part()**

```
def PyClical.clifford.vector_part (
    self,
    frm = None )

Vector part of multivector, as a Python list, with respect to frm.

>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
[2.0, 3.0]
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set((-1,1,2))))
[0.0, 2.0, 3.0]
```

Definition at line 1079 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.vector\\_part\(\)](#).

Referenced by [PyClical.clifford.vector\\_part\(\)](#).

### 6.3.3 Member Data Documentation

#### 6.3.3.1 instance

PyClical.clifford.instance

Definition at line 592 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.\\_\\_call\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_contains\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.clifford.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_getitem\\_\\_\(\)](#), [PyClical.clifford.\\_\\_getitem\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_invert\\_\\_\(\)](#), [PyClical.clifford.\\_\\_neg\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_setitem\\_\\_\(\)](#), [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [PyClical.index\\_set.max\(\)](#), [PyClical.clifford.max\\_abs\(\)](#), [PyClical.index\\_set.min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer\\_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [PyClical.index\\_set.sign\\_of\\_mult\(\)](#), [PyClical.index\\_set.sign\\_of\\_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)

## 6.4 glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T > Class Template Reference

clifford\_algebra<> declares the operations of a Clifford algebra

```
#include <clifford_algebra.h>
```

Inheritance diagram for glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >:

### Public Types

- using [scalar\\_t](#) = Scalar\_T
- using [index\\_set\\_t](#) = Index\_Set\_T
- using [multivector\\_t](#) = Multivector\_T
- using [pair\\_t](#) = std::pair< const [index\\_set\\_t](#), Scalar\_T >
- using [vector\\_t](#) = std::vector< Scalar\_T >

## Public Member Functions

- virtual `~clifford_algebra()` = default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0  
*Test for equality of multivectors.*
- virtual auto `operator==` (const `Scalar_T` &scr) const -> bool=0  
*Test for equality of multivector and scalar.*
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric sum.*
- virtual auto `operator+=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Geometric sum of multivector and scalar.*
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric difference.*
- virtual auto `operator-=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Geometric difference of multivector and scalar.*
- virtual auto `operator-` () const -> const `multivector_t`=0  
*Unary -.*
- virtual auto `operator*=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Product of multivector and scalar.*
- virtual auto `operator*=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric product.*
- virtual auto `operator%=>` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Contraction.*
- virtual auto `operator&=>` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Inner product.*
- virtual auto `operator^=>` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Outer product.*
- virtual auto `operator/=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Quotient of multivector and scalar.*
- virtual auto `operator/=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric quotient.*
- virtual auto `operator|=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Transformation via twisted adjoint action.*
- virtual auto `inv` () const -> const `multivector_t`=0  
*Geometric multiplicative inverse.*
- virtual auto `pow` (int m) const -> const `multivector_t`=0  
*\*this to the m*
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0  
*Outer product power.*
- virtual auto `frame` () const -> const `index_set_t`=0  
*Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade` () const -> `index_t`=0  
*Maximum of the grades of each term.*
- virtual auto `operator[]` (const `index_set_t` ist) const -> `Scalar_T`=0  
*Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t`=0  
*Pure grade-vector part.*
- virtual auto `scalar` () const -> `Scalar_T`=0  
*Scalar part.*
- virtual auto `pure` () const -> const `multivector_t`=0  
*Pure part.*



- virtual auto [even](#) () const -> const [multivector\\_t](#)=0  
*Even part of multivector, sum of even grade terms.*
- virtual auto [odd](#) () const -> const [multivector\\_t](#)=0  
*Odd part of multivector, sum of odd grade terms.*
- virtual auto [vector\\_part](#) () const -> const [vector\\_t](#)=0  
*Vector part of multivector, as a [vector\\_t](#) with respect to [frame\(\)](#)*
- virtual auto [vector\\_part](#) (const [index\\_set\\_t](#) frm, const bool prechecked) const -> const [vector\\_t](#)=0  
*Vector part of multivector, as a [vector\\_t](#) with respect to frm.*
- virtual auto [involute](#) () const -> const [multivector\\_t](#)=0  
*Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto [reverse](#) () const -> const [multivector\\_t](#)=0  
*Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto [conj](#) () const -> const [multivector\\_t](#)=0  
*Conjugation, reverse o involute == involute o reverse.*
- virtual auto [quad](#) () const -> [Scalar\\_T](#)=0  
*[Scalar\\_T](#) quadratic form == (rev(x)\*x)(0)*
- virtual auto [norm](#) () const -> [Scalar\\_T](#)=0  
*[Scalar\\_T](#) norm == sum of norm of coordinates.*
- virtual auto [max\\_abs](#) () const -> [Scalar\\_T](#)=0  
*Maximum of absolute values of components of multivector: multivector infinity norm.*
- virtual auto [truncated](#) (const [Scalar\\_T](#) &limit=[default\\_truncation](#)) const -> const [multivector\\_t](#)=0  
*Remove all terms with relative size smaller than limit.*
- virtual auto [isinf](#) () const -> bool=0  
*Check if a multivector contains any infinite values.*
- virtual auto [isnan](#) () const -> bool=0  
*Check if a multivector contains any IEEE NaN values.*
- virtual void [write](#) (const std::string &msg="") const =0  
*Write formatted multivector to output.*
- virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const =0  
*Write formatted multivector to file.*

## Static Public Member Functions

- static auto [classname](#) () -> const std::string

## Static Public Attributes

- static const [index\\_t v\\_lo](#) = [index\\_set\\_t::v\\_lo](#)
- static const [index\\_t v\\_hi](#) = [index\\_set\\_t::v\\_hi](#)
- static const [Scalar\\_T default\\_truncation](#) = std::numeric\_limits<[Scalar\\_T](#)>::epsilon()  
*Default for truncation.*

### 6.4.1 Detailed Description

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
class glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >
```

[clifford\\_algebra<>](#) declares the operations of a Clifford algebra

Definition at line 45 of file [clifford\\_algebra.h](#).

## 6.4.2 Member Typedef Documentation

### 6.4.2.1 index\_set\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::index_set_t = Index_↵
Set_T
```

Definition at line 49 of file [clifford\\_algebra.h](#).

### 6.4.2.2 multivector\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::multivector_t = Multivector_↵
_T
```

Definition at line 52 of file [clifford\\_algebra.h](#).

### 6.4.2.3 pair\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::pair_t = std::pair<const
index_set_t, Scalar_T>
```

Definition at line 53 of file [clifford\\_algebra.h](#).

### 6.4.2.4 scalar\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar_t = Scalar_T
```

Definition at line 48 of file [clifford\\_algebra.h](#).

### 6.4.2.5 vector\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_t = std::vector<Scalar_↵
_T>
```

Definition at line 54 of file [clifford\\_algebra.h](#).

### 6.4.3 Constructor & Destructor Documentation

#### 6.4.3.1 ~clifford\_algebra()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::~~clifford_algebra (
) [virtual], [default]
```

### 6.4.4 Member Function Documentation

#### 6.4.4.1 classname()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::classname [static]
```

Definition at line 66 of file [clifford\\_algebra\\_imp.h](#).

#### 6.4.4.2 conj()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::conj ( ) const
-> const multivector_t [pure virtual]
```

Conjugation, reverse o involute == involute o reverse.

#### 6.4.4.3 even()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::even ( ) const
-> const multivector_t [pure virtual]
```

Even part of multivector, sum of even grade terms.

#### 6.4.4.4 frame()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame ( ) const
-> const index\_set\_t [pure virtual]
```

Subalgebra generated by all generators of terms of given multivector.

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [PyClical.clifford::reframe\(\)](#).

#### 6.4.4.5 grade()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::grade ( ) const
-> index\_t [pure virtual]
```

Maximum of the grades of each term.

#### 6.4.4.6 inv()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::inv ( ) const
-> const multivector\_t [pure virtual]
```

Geometric multiplicative inverse.

#### 6.4.4.7 involute()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::involute ( )
const -> const multivector\_t [pure virtual]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

#### 6.4.4.8 isinf()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isinf ( ) const
-> bool [pure virtual]
```

Check if a multivector contains any infinite values.

#### 6.4.4.9 `isnan()`

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan ( ) const
-> bool [pure virtual]
```

Check if a multivector contains any IEEE NaN values.

Referenced by `glucat::cascade_log()`, `glucat::exp()`, `glucat::log()`, `glucat::matrix_log()`, `glucat::matrix_sqrt()`, `glucat::pade_log()`, and `glucat::sqrt()`.

#### 6.4.4.10 `max_abs()`

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::max_abs ( )
const -> Scalar_T [pure virtual]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

#### 6.4.4.11 `norm()`

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::norm ( ) const
-> Scalar_T [pure virtual]
```

`Scalar_T` norm == sum of norm of coordinates.

Referenced by `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi()`.

#### 6.4.4.12 `odd()`

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::odd ( ) const
-> const multivector_t [pure virtual]
```

Odd part of multivector, sum of odd grade terms.

#### 6.4.4.13 `operator%=( )`

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator%= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Contraction.

**6.4.4.14 operator&=()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator&= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Inner product.

**6.4.4.15 operator>()()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator() (
    index_t grade ) const -> const multivector_t [pure virtual]
```

Pure grade-vector part.

**6.4.4.16 operator\*=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric product.

**6.4.4.17 operator\*=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Product of multivector and scalar.

**6.4.4.18 operator+=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric sum.

**6.4.4.19 operator+=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Geometric sum of multivector and scalar.

**6.4.4.20 operator-( )**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator- ( )
const -> const multivector_t [pure virtual]
```

Unary -.

**6.4.4.21 operator-=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric difference.

**6.4.4.22 operator-=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Geometric difference of multivector and scalar.

**6.4.4.23 operator/=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric quotient.

**6.4.4.24 operator/=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Quotient of multivector and scalar.

**6.4.4.25 operator==( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
    const multivector_t & val ) const -> bool [pure virtual]
```

Test for equality of multivectors.

**6.4.4.26 operator==( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
    const Scalar_T & scr ) const -> bool [pure virtual]
```

Test for equality of multivector and scalar.

**6.4.4.27 operator[]()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator[] (
    const index_set_t ist ) const -> Scalar_T [pure virtual]
```

Subscripting: map from index set to scalar coordinate.

**6.4.4.28 operator^=( )**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator^= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Outer product.



**6.4.4.29 operator" |=(**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator|= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Transformation via twisted adjoint action.

**6.4.4.30 outer\_pow()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::outer_pow (
    int m ) const -> const multivector_t [pure virtual]
```

Outer product power.

**6.4.4.31 pow()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pow (
    int m ) const -> const multivector_t [pure virtual]
```

\*this to the m

**6.4.4.32 pure()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pure ( ) const
-> const multivector_t [pure virtual]
```

Pure part.

**6.4.4.33 quad()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::quad ( ) const
-> Scalar_T [pure virtual]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

**6.4.4.34 reverse()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::reverse ( )
const -> const multivector\_t [pure virtual]
```

Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .

**6.4.4.35 scalar()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar ( )
const -> Scalar_T [pure virtual]
```

Scalar part.

Referenced by [glucat::exp\(\)](#), [glucat::matrix\\_log\(\)](#), and [glucat::matrix\\_sqrt\(\)](#).

**6.4.4.36 truncated()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated (
    const Scalar_T & limit = default\_truncation ) const -> const multivector\_t [pure
virtual]
```

Remove all terms with relative size smaller than limit.

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

**6.4.4.37 vector\_part() [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part ( )
const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to [frame\(\)](#)

**6.4.4.38 vector\_part() [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
    const index\_set\_t frm,
    const bool prechecked ) const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to frm.

**6.4.4.39 write()** [1/2]

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    const std::string & msg = "" ) const [pure virtual]
```

Write formatted multivector to output.

**6.4.4.40 write()** [2/2]

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    std::ofstream & ofile,
    const std::string & msg = "" ) const [pure virtual]
```

Write formatted multivector to file.

**6.4.5 Member Data Documentation****6.4.5.1 default\_truncation**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation = std::numeric_limits<Scalar_T>::epsilon() [static]
```

Default for truncation.

Definition at line 59 of file [clifford\\_algebra.h](#).

**6.4.5.2 v\_hi**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_hi = index_set_t::v_hi [static]
```

Definition at line 51 of file [clifford\\_algebra.h](#).

### 6.4.5.3 v\_lo

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_lo = index↵
_set_t::v_lo [static]
```

Definition at line 50 of file [clifford\\_algebra.h](#).

The documentation for this class was generated from the following files:

- [glucat/clifford\\_algebra.h](#)
- [glucat/clifford\\_algebra\\_imp.h](#)

## 6.5 glucat::compare\_types< LHS\_T, RHS\_T > Class Template Reference

Type comparison.

```
#include <global.h>
```

### Public Types

- enum { [are\\_same](#) = false }

### 6.5.1 Detailed Description

```
template<typename LHS_T, typename RHS_T>
class glucat::compare_types< LHS_T, RHS_T >
```

Type comparison.

Definition at line 54 of file [global.h](#).

### 6.5.2 Member Enumeration Documentation

#### 6.5.2.1 anonymous enum

```
template<typename LHS_T , typename RHS_T >
anonymous enum
```

#### Enumerator

<a href="#">are_same</a>	
--------------------------	--

Definition at line 57 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.6 glucat::compare\_types< T, T > Class Template Reference

```
#include <global.h>
```

### Public Types

- enum { [are\\_same](#) = true }

### 6.6.1 Detailed Description

```
template<typename T>  
class glucat::compare_types< T, T >
```

Definition at line 60 of file [global.h](#).

### 6.6.2 Member Enumeration Documentation

#### 6.6.2.1 anonymous enum

```
template<typename T >  
anonymous enum
```

##### Enumerator

<a href="#">are_same</a>	
--------------------------	--

Definition at line 63 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.7 glucat::control\_t Class Reference

Parameters to control tests.

```
#include <control.h>
```

## Public Member Functions

- int [call](#) ([intfn](#) f) const  
*Call a function that returns int.*
- int [call](#) ([intintfn](#) f, int arg) const  
*Call a function of int that returns int.*

## Static Public Member Functions

- static const [control\\_t](#) & [control](#) (int argc, char \*\*argv)
- static bool [verbose](#) ()  
*Produce more detailed output from tests.*

## Private Member Functions

- bool [valid](#) () const
- bool [catch\\_exceptions](#) () const
- [control\\_t](#) (int argc, char \*\*argv)  
*Constructor from program arguments.*
- [control\\_t](#) ()=default
- [~control\\_t](#) ()=default
- [control\\_t](#) (const [control\\_t](#) &)=delete
- [control\\_t](#) & [operator=](#) (const [control\\_t](#) &)=delete

## Private Attributes

- bool [m\\_valid](#)  
*Test parameters are valid.*
- bool [m\\_catch\\_exceptions](#)  
*Catch exceptions.*

## Static Private Attributes

- static bool [m\\_verbose\\_output](#) = false  
*Produce more detailed output from tests.*

## Friends

- class [friend\\_for\\_private\\_destructor](#)

### 6.7.1 Detailed Description

Parameters to control tests.

Definition at line 39 of file [control.h](#).

## 6.7.2 Constructor & Destructor Documentation

### 6.7.2.1 control\_t() [1/3]

```
glucat::control_t::control_t (
    int argc,
    char ** argv ) [private]
```

Constructor from program arguments.

Test control constructor from program arguments.

Definition at line 88 of file [control.h](#).

References [GLUCAT\\_PACKAGE\\_NAME](#), [GLUCAT\\_VERSION](#), [m\\_catch\\_exceptions](#), [m\\_valid](#), [m\\_verbose\\_output](#), and [valid\(\)](#).

### 6.7.2.2 control\_t() [2/3]

```
glucat::control_t::control_t ( ) [private], [default]
```

### 6.7.2.3 ~control\_t()

```
glucat::control_t::~~control_t ( ) [private], [default]
```

### 6.7.2.4 control\_t() [3/3]

```
glucat::control_t::control_t (
    const control_t & ) [private], [delete]
```

## 6.7.3 Member Function Documentation

### 6.7.3.1 `call()` [1/2]

```
int glucat::control_t::call (
    intfn f ) const [inline]
```

Call a function that returns int.

Definition at line [136](#) of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

### 6.7.3.2 `call()` [2/2]

```
int glucat::control_t::call (
    intintfn f,
    int arg ) const [inline]
```

Call a function of int that returns int.

Definition at line [150](#) of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

### 6.7.3.3 `catch_exceptions()`

```
bool glucat::control_t::catch_exceptions ( ) const [inline], [private]
```

Definition at line [49](#) of file [control.h](#).

References [m\\_catch\\_exceptions](#).

Referenced by [call\(\)](#).

### 6.7.3.4 `control()`

```
static const control\_t & glucat::control_t::control (
    int argc,
    char ** argv ) [inline], [static]
```

Single instance Ref: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.

Definition at line [71](#) of file [control.h](#).



### 6.7.3.5 operator=()

```
control_t & glucat::control_t::operator= (
    const control_t & ) [private], [delete]
```

### 6.7.3.6 valid()

```
bool glucat::control_t::valid ( ) const [inline], [private]
```

Definition at line 44 of file [control.h](#).

References [m\\_valid](#).

Referenced by [call\(\)](#), and [control\\_t\(\)](#).

### 6.7.3.7 verbose()

```
static bool glucat::control_t::verbose ( ) [inline], [static]
```

Produce more detailed output from tests.

Definition at line 80 of file [control.h](#).

References [m\\_verbose\\_output](#).

## 6.7.4 Friends And Related Function Documentation

### 6.7.4.1 friend\_for\_private\_destructor

```
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 67 of file [control.h](#).

## 6.7.5 Member Data Documentation

### 6.7.5.1 m\_catch\_exceptions

```
bool glucat::control_t::m_catch_exceptions [private]
```

Catch exceptions.

Definition at line 48 of file [control.h](#).

Referenced by [catch\\_exceptions\(\)](#), and [control\\_t\(\)](#).

### 6.7.5.2 m\_valid

```
bool glucat::control_t::m_valid [private]
```

Test parameters are valid.

Definition at line 43 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [valid\(\)](#).

### 6.7.5.3 m\_verbose\_output

```
bool glucat::control_t::m_verbose_output = false [static], [private]
```

Produce more detailed output from tests.

Definition at line 53 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [verbose\(\)](#).

The documentation for this class was generated from the following file:

- [test/control.h](#)

## 6.8 glucat::CTAssertion< bool > Struct Template Reference

Compile time assertion.

### 6.8.1 Detailed Description

```
template<bool>
struct glucat::CTAssertion< bool >
```

Compile time assertion.

Definition at line 46 of file [global.h](#).

The documentation for this struct was generated from the following file:

- [glucat/global.h](#)

## 6.9 glucat::CTAssertion< true > Struct Reference

```
#include <global.h>
```

### 6.9.1 Detailed Description

Definition at line 47 of file [global.h](#).

The documentation for this struct was generated from the following file:

- [glucat/global.h](#)

## 6.10 glucat::numeric\_traits< Scalar\_T >::demoted Struct Reference

Demoted type for long double.

```
#include <promotion.h>
```

### Public Types

- using [type](#) = float
- using [type](#) = float

### 6.10.1 Detailed Description

```
template<typename Scalar_T>  
struct glucat::numeric_traits< Scalar_T >::demoted
```

Demoted type for long double.

Demoted type.

Definition at line 76 of file [promotion.h](#).

### 6.10.2 Member Typedef Documentation

#### 6.10.2.1 [type](#) [1/2]

```
template<typename Scalar_T >  
using glucat::numeric\_traits< Scalar_T >::demoted::type = float
```

Definition at line 78 of file [promotion.h](#).

### 6.10.2.2 type [2/2]

```
template<typename Scalar_T >
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 148 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

## 6.11 glucat::matrix::eig\_genus< Matrix\_T > Struct Template Reference

Structure containing classification of eigenvalues.

```
#include <matrix.h>
```

### Public Types

- using [Scalar\\_T](#) = typename Matrix\_T::value\_type

### Public Attributes

- bool [m\\_is\\_singular](#) = false  
*Is the matrix singular?*
- [eig\\_case\\_t](#) [m\\_eig\\_case](#) = safe\_eigs  
*What kind of eigenvalues does the matrix contain?*
- [Scalar\\_T](#) [m\\_safe\\_arg](#) = [Scalar\\_T](#)(0)  
*Argument such that  $\exp(\pi i m\_safe\_arg)$  lies between arguments of eigenvalues.*

### 6.11.1 Detailed Description

```
template<typename Matrix_T>
struct glucat::matrix::eig_genus< Matrix_T >
```

Structure containing classification of eigenvalues.

Definition at line 140 of file [matrix.h](#).

### 6.11.2 Member Typedef Documentation

### 6.11.2.1 Scalar\_T

```
template<typename Matrix_T >
using glucat::matrix::eig_genus< Matrix_T >::Scalar_T = typename Matrix_T::value_type
```

Definition at line 142 of file [matrix.h](#).

## 6.11.3 Member Data Documentation

### 6.11.3.1 m\_eig\_case

```
template<typename Matrix_T >
eig_case_t glucat::matrix::eig_genus< Matrix_T >::m_eig_case = safe_eigs
```

What kind of eigenvalues does the matrix contain?

Definition at line 146 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

### 6.11.3.2 m\_is\_singular

```
template<typename Matrix_T >
bool glucat::matrix::eig_genus< Matrix_T >::m_is_singular = false
```

Is the matrix singular?

Definition at line 144 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

### 6.11.3.3 m\_safe\_arg

```
template<typename Matrix_T >
Scalar_T glucat::matrix::eig_genus< Matrix_T >::m_safe_arg = Scalar_T(0)
```

Argument such that  $\exp(\pi \cdot m\_safe\_arg)$  lies between arguments of eigenvalues.

Definition at line 148 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix.h](#)

## 6.12 glucat::error< Class\_T > Class Template Reference

Specific exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::error< Class\_T >:

Collaboration diagram for glucat::error< Class\_T >:

### Public Member Functions

- [error](#) (const std::string &msg)  
*Specific exception class.*
- [error](#) (const std::string &context, const std::string &msg)
- auto [heading](#) () const noexcept -> const std::string override
- auto [classname](#) () const noexcept -> const std::string override
- void [print\\_error\\_msg](#) () const override

### Additional Inherited Members

#### 6.12.1 Detailed Description

```
template<class Class_T>
class glucat::error< Class_T >
```

Specific exception class.

Definition at line 56 of file [errors.h](#).

#### 6.12.2 Constructor & Destructor Documentation

##### 6.12.2.1 error() [1/2]

```
template<class Class_T >
glucat::error< Class_T >::error (
    const std::string & msg )
```

Specific exception class.

Definition at line 44 of file [errors\\_imp.h](#).

### 6.12.2.2 error() [2/2]

```
template<class Class_T >
glucat::error< Class_T >::error (
    const std::string & context,
    const std::string & msg )
```

Definition at line 50 of file [errors\\_imp.h](#).

## 6.12.3 Member Function Documentation

### 6.12.3.1 classname()

```
template<class Class_T >
auto glucat::error< Class_T >::classname [override], [virtual], [noexcept]
```

Implements [glucat::glucat\\_error](#).

Definition at line 63 of file [errors\\_imp.h](#).

### 6.12.3.2 heading()

```
template<class Class_T >
auto glucat::error< Class_T >::heading [override], [virtual], [noexcept]
```

Implements [glucat::glucat\\_error](#).

Definition at line 57 of file [errors\\_imp.h](#).

### 6.12.3.3 print\_error\_msg()

```
template<class Class_T >
void glucat::error< Class_T >::print_error_msg [override], [virtual]
```

Implements [glucat::glucat\\_error](#).

Definition at line 69 of file [errors\\_imp.h](#).

The documentation for this class was generated from the following files:

- [glucat/errors.h](#)
- [glucat/errors\\_imp.h](#)

## 6.13 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > Class Template Reference

A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.

```
#include <framed_multi.h>
```

Inheritance diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >:

Collaboration diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >:

### Classes

- class [hash\\_size\\_t](#)
- class [var\\_term](#)

*Variable term.*

### Public Types

- using [multivector\\_t](#) = [framed\\_multi](#)
- using [framed\\_multi\\_t](#) = [multivector\\_t](#)
- using [scalar\\_t](#) = [Scalar\\_T](#)
- using [tune\\_p](#) = [Tune\\_P](#)
- using [index\\_set\\_t](#) = [index\\_set](#)< [LO](#), [HI](#) >
- using [term\\_t](#) = std::pair< const [index\\_set\\_t](#), [Scalar\\_T](#) >
- using [vector\\_t](#) = std::vector< [Scalar\\_T](#) >
- using [error\\_t](#) = [error](#)< [multivector\\_t](#) >
- using [matrix\\_multi\\_t](#) = [matrix\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) >

### Public Member Functions

- [~framed\\_multi](#) () override=default  
*Destructor.*
- [framed\\_multi](#) ()  
*Default constructor.*
- template<typename Other\_Scalar\_T >  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
*Construct a multivector from a multivector with a different scalar type.*
- template<typename Other\_Scalar\_T >  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P > &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [framed\\_multi\\_t](#) &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const [Scalar\\_T](#) &crd=[Scalar\\_T](#)(1))  
*Construct a multivector from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const [Scalar\\_T](#) &crd, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const [Scalar\\_T](#) &scr, const [index\\_set\\_t](#) frm=[index\\_set\\_t](#)())  
*Construct a multivector from a scalar (within a frame, if given)*



- `framed_multi` (const int scr, const `index_set_t` frm=`index_set_t`())  
*Construct a multivector from an int (within a frame, if given)*
- `framed_multi` (const `vector_t` &vec, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given vector.*
- `framed_multi` (const std::string &str)  
*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `framed_multi` (const std::string &str, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `framed_multi` (const char \*str)  
*Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `framed_multi` (const char \*str, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- template<typename Other\_Scalar\_T >  
`framed_multi` (const `matrix_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
*Construct a multivector from a matrix\_multi\_t.*
- template<typename Other\_Scalar\_T >  
auto `fast_matrix_multi` (const `index_set_t` frm) const -> const `matrix_multi`< Other\_Scalar\_T, LO, HI, Tune\_P >  
*Use generalized FFT to construct a matrix\_multi\_t.*
- auto `fast_framed_multi` () const -> const `framed_multi_t`  
*Use inverse generalized FFT to construct a framed\_multi\_t.*
- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `nbr_terms` () const -> unsigned long  
*Number of terms.*
- auto `operator+=` (const `term_t` &term) -> `multivector_t` &  
*Add a term, if non-zero.*

## Static Public Member Functions

- static auto `classname` () -> const std::string  
*Class name used in messages.*
- static auto `random` (const `index_set_t` frm, Scalar\_T fill=Scalar\_T(1)) -> const `multivector_t`  
*Random multivector within a frame.*

## Private Types

- using `var_term_t` = `var_term`
- using `matrix_t` = typename `matrix_multi_t::matrix_t`
- using `sorted_map_t` = std::map< `index_set_t`, Scalar\_T, std::less< const `index_set_t` > >
- using `map_t` = std::unordered\_map< `index_set_t`, Scalar\_T, `index_set_hash`< LO, HI > >
- using `framed_pair_t` = std::pair< const `multivector_t`, const `multivector_t` >
- using `size_type` = typename map\_t::size\_type
- using `iterator` = typename map\_t::iterator
- using `const_iterator` = typename map\_t::const\_iterator

## Private Member Functions

- `framed_multi` (const `hash_size_t` &hash\_size)  
*Private constructor using hash\_size.*
- auto `fold` (const `index_set_t` frm) const -> `multivector_t`  
*Subalgebra isomorphism: fold each term within the given frame.*
- auto `unfold` (const `index_set_t` frm) const -> `multivector_t`  
*Subalgebra isomorphism: unfold each term within the given frame.*
- auto `centre_pm4_qp4` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .*
- auto `centre_pp4_qm4` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .*
- auto `centre_qp1_pm1` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .*
- auto `divide` (const `index_set_t` ist) const -> const `framed_pair_t`  
*Divide multivector into part divisible by `index_set` and remainder.*
- auto `fast` (const `index_t` level, const bool odd) const -> const `matrix_t`  
*Generalized FFT from `multivector_t` to `matrix_t`.*

## Friends

- `template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >`  
class `matrix_multi`
- `template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >`  
class `framed_multi`
- auto `operator*` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator^` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator&` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator%` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `star` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> `Scalar_T`
- auto `operator/` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator|` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator>>` (std::istream &s, `multivector_t` &val) -> std::istream &
- auto `operator<<` (std::ostream &os, const `multivector_t` &val) -> std::ostream &
- auto `operator<<` (std::ostream &os, const `term_t` &term) -> std::ostream &
- auto `exp` (const `multivector_t` &val) -> const `multivector_t`

## Additional Inherited Members

### 6.13.1 Detailed Description

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P =
tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >
```

A `framed_multi<Scalar_T,LO,HI,Tune_P>` is a framed approximation to a multivector.

Definition at line 126 of file `framed_multi.h`.

## 6.13.2 Member Typedef Documentation

### 6.13.2.1 const\_iterator

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::const_iterator = typename map_t<
::const_iterator [private]
```

Definition at line 167 of file [framed\\_multi.h](#).

### 6.13.2.2 error\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector\_t>
```

Definition at line 138 of file [framed\\_multi.h](#).

### 6.13.2.3 framed\_multi\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = multivector\_t
```

Definition at line 132 of file [framed\\_multi.h](#).

### 6.13.2.4 framed\_pair\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_pair_t = std::pair<const multivector\_t,
const multivector\_t> [private]
```

Definition at line 164 of file [framed\\_multi.h](#).

### 6.13.2.5 index\_set\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 135 of file [framed\\_multi.h](#).

### 6.13.2.6 iterator

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::iterator = typename map_t::iterator
[private]
```

Definition at line 166 of file [framed\\_multi.h](#).

### 6.13.2.7 map\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::map_t = std::unordered_map<index_set_t,
Scalar_T, index_set_hash<LO, HI> > [private]
```

Definition at line 150 of file [framed\\_multi.h](#).

### 6.13.2.8 matrix\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = matrix_multi<Scalar_T, LO, HI, Tune_P >
```

Definition at line 139 of file [framed\\_multi.h](#).

### 6.13.2.9 matrix\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = typename matrix_multi_t::matrix_t
[private]
```

Definition at line 148 of file [framed\\_multi.h](#).

#### 6.13.2.10 `multivector_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = framed\_multi
```

Definition at line 131 of file [framed\\_multi.h](#).

#### 6.13.2.11 `scalar_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 133 of file [framed\\_multi.h](#).

#### 6.13.2.12 `size_type`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::size_type = typename map_t::size_type
[private]
```

Definition at line 165 of file [framed\\_multi.h](#).

#### 6.13.2.13 `sorted_map_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::sorted_map_t = std::map< index\_set\_t,
Scalar_T, std::less<const index\_set\_t> > [private]
```

Definition at line 149 of file [framed\\_multi.h](#).

#### 6.13.2.14 `term_t`

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index\_set\_t,
Scalar_T>
```

Definition at line 136 of file [framed\\_multi.h](#).

### 6.13.2.15 tune\_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 134 of file [framed\\_multi.h](#).

### 6.13.2.16 var\_term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term_t = var_term [private]
```

Definition at line 147 of file [framed\\_multi.h](#).

### 6.13.2.17 vector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 137 of file [framed\\_multi.h](#).

## 6.13.3 Constructor & Destructor Documentation

### 6.13.3.1 ~framed\_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucal::framed_multi< Scalar_T, LO, HI, Tune_P >::~~framed_multi ( ) [override], [default]
```

Destructor.

### 6.13.3.2 framed\_multi() [1/15]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
glucal::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi
```

Default constructor.

Definition at line 59 of file [framed\\_multi\\_imp.h](#).

**6.13.3.3 framed\_multi()** [2/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const hash\_size\_t & hash_size ) [private]
```

Private constructor using *hash\_size*.

Definition at line 66 of file [framed\\_multi\\_imp.h](#).

**6.13.3.4 framed\_multi()** [3/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 74 of file [framed\\_multi\\_imp.h](#).

**6.13.3.5 framed\_multi()** [4/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 85 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [Index\\_Set\\_T](#), [Multivector\\_T](#) >::frame().

**6.13.3.6 framed\_multi()** [5/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 98 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [Index\\_Set\\_T](#), [Multivector\\_T](#) >::frame().

**6.13.3.7 framed\_multi()** [6/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1) )
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 111 of file [framed\\_multi\\_imp.h](#).

**6.13.3.8 framed\_multi()** [7/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 121 of file [framed\\_multi\\_imp.h](#).

**6.13.3.9 framed\_multi()** [8/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index_set_t() )
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 134 of file [framed\\_multi\\_imp.h](#).

**6.13.3.10 framed\_multi()** [9/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const int scr,
    const index\_set\_t frm = index_set_t() )
```

Construct a multivector from an int (within a frame, if given)

Definition at line 144 of file [framed\\_multi\\_imp.h](#).



**6.13.3.11 framed\_multi()** [10/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 154 of file [framed\\_multi\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::index\\_set< LO, HI >::max\(\)](#), and [glucat::index\\_set< LO, HI >::min\(\)](#).

**6.13.3.12 framed\_multi()** [11/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str )
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 176 of file [framed\\_multi\\_imp.h](#).

**6.13.3.13 framed\_multi()** [12/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 192 of file [framed\\_multi\\_imp.h](#).

**6.13.3.14 framed\_multi()** [13/15]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str ) [inline]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 209 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi](#).

### 6.13.3.15 framed\_multi() [14/15]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str,
    const index_set_t frm,
    const bool prechecked = false ) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 212 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi](#).

### 6.13.3.16 framed\_multi() [15/15]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a matrix\_multi\_t.

Definition at line 205 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_max\(\)](#), [glucat::matrix::nnz\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::norm\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::norm\(\)](#).

## 6.13.4 Member Function Documentation

### 6.13.4.1 centre\_pm4\_qp4()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4 (
    index_t & p,
    index_t & q ) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .

Definition at line 1469 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.13.4.2 centre\_pp4\_qm4()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pp4_qm4 (
    index_t & p,
    index_t & q ) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .

Definition at line 1511 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.13.4.3 centre\_qp1\_pm1()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1 (
    index_t & p,
    index_t & q ) -> multivector_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .

Definition at line 1553 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.13.4.4 classname()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::classname [static]
```

Class name used in messages.

Definition at line 50 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.5 divide()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::divide (
    const index_set_t ist ) const -> const framed_pair_t [private]
```

Divide multivector into part divisible by [index\\_set](#) and remainder.

Divide multivector into quotient with terms divisible by index set, and remainder.

Definition at line 1586 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.6 fast()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast (
    const index\_t level,
    const bool odd ) const -> const matrix\_t [private]
```

Generalized FFT from [multivector\\_t](#) to [matrix\\_t](#).

Definition at line 1602 of file [framed\\_multi\\_imp.h](#).

References [glucat::matrix::kron\(\)](#), [glucat::odd\(\)](#), and [glucat::scalar\(\)](#).

#### 6.13.4.7 fast\_framed\_multi()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi [inline]
```

Use inverse generalized FFT to construct a [framed\\_multi\\_t](#).

Definition at line 1700 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.8 fast\_matrix\_multi()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index\_set\_t frm ) const -> const matrix\_multi<Other_Scalar_T,LO,HI,Tune_P
>
```

Use generalized FFT to construct a [matrix\\_multi\\_t](#).

Definition at line 1668 of file [framed\\_multi\\_imp.h](#).

References [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

#### 6.13.4.9 fold()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fold (
    const index\_set\_t frm ) const -> multivector\_t [private]
```

Subalgebra isomorphism: fold each term within the given frame.

Definition at line 1434 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.10 nbr\_terms()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::nbr_terms
```

Number of terms.

Definition at line 1356 of file framed\_multi\_imp.h.

#### 6.13.4.11 operator+=(())

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & term ) -> multivector_t& [inline]
```

Add a term, if non-zero.

Insert a term into a multivector, add terms with same index set.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 295 of file framed\_multi\_imp.h.

#### 6.13.4.12 random()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index_set_t frm,
    Scalar_T fill = Scalar_T(1) ) -> const multivector_t [static]
```

Random multivector within a frame.

Definition at line 1058 of file framed\_multi\_imp.h.

Referenced by glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >::random().

#### 6.13.4.13 unfold()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold (
    const index_set_t frm ) const -> multivector_t [private]
```

Subalgebra isomorphism: unfold each term within the given frame.

Definition at line 1451 of file framed\_multi\_imp.h.

Referenced by glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >::fast\_framed\_multi().

## 6.13.5 Friends And Related Function Documentation

### 6.13.5.1 exp

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto exp (
    const multivector\_t & val ) -> const multivector\_t [friend]
```

### 6.13.5.2 framed\_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
friend class framed\_multi [friend]
```

Definition at line [143](#) of file [framed\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

### 6.13.5.3 matrix\_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
friend class matrix\_multi [friend]
```

Definition at line [141](#) of file [framed\\_multi.h](#).

### 6.13.5.4 operator%

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const multivector\_t & lhs,
    const multivector\_t & rhs ) -> const multivector\_t [friend]
```

#### 6.13.5.5 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

#### 6.13.5.6 operator\*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

#### 6.13.5.7 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

#### 6.13.5.8 operator<< [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val ) -> std::ostream & [friend]
```

#### 6.13.5.9 operator<< [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const term_t & term ) -> std::ostream & [friend]
```

#### 6.13.5.10 operator>>

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector\_t & val ) -> std::istream & [friend]
```

#### 6.13.5.11 operator^

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const multivector\_t & lhs,
    const multivector\_t & rhs ) -> const multivector\_t [friend]
```

#### 6.13.5.12 operator"|

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const multivector\_t & lhs,
    const multivector\_t & rhs ) -> const multivector\_t [friend]
```

#### 6.13.5.13 star

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const multivector\_t & lhs,
    const multivector\_t & rhs ) -> Scalar_T [friend]
```

The documentation for this class was generated from the following files:

- [glucat/framed\\_multi.h](#)
- [glucat/framed\\_multi\\_imp.h](#)

## 6.14 [glucat::gen::generator\\_table< Matrix\\_T >](#) Class Template Reference

Table of generators for specific signatures.

```
#include <generation.h>
```

Inheritance diagram for [glucat::gen::generator\\_table< Matrix\\_T >](#):

Collaboration diagram for [glucat::gen::generator\\_table< Matrix\\_T >](#):



## Public Member Functions

- auto [operator\(\)](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> const Matrix\_T \*  
*Pointer to generators for a specific signature.*
- [generator\\_table](#) (const [generator\\_table](#) &)=delete
- auto [operator=](#) (const [generator\\_table](#) &) -> [generator\\_table](#) &=delete

## Static Public Member Functions

- static auto [generator](#) () -> [generator\\_table](#)< Matrix\_T > &  
*Single instance of generator table.*

## Private Member Functions

- auto [gen\\_vector](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> const std::vector< Matrix\_T > &  
*Construct a vector of generators for a specific signature.*
- void [gen\\_from\\_pm1\\_qm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-1,q-1.*
- void [gen\\_from\\_pm4\\_qp4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-4,q+4.*
- void [gen\\_from\\_pp4\\_qm4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p+4,q-4.*
- void [gen\\_from\\_qp1\\_pm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for q+1,p-1.*
- [generator\\_table](#) ()=default
- [~generator\\_table](#) ()=default

## Friends

- class [friend\\_for\\_private\\_destructor](#)

### 6.14.1 Detailed Description

```
template<class Matrix_T>
class glucat::gen::generator_table< Matrix_T >
```

Table of generators for specific signatures.

Definition at line 52 of file [generation.h](#).

### 6.14.2 Constructor & Destructor Documentation

**6.14.2.1 generator\_table() [1/2]**

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::generator_table ( ) [private], [default]
```

**6.14.2.2 ~generator\_table()**

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::~~generator_table ( ) [private], [default]
```

**6.14.2.3 generator\_table() [2/2]**

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::generator_table (
    const generator_table< Matrix_T > & ) [delete]
```

**6.14.3 Member Function Documentation****6.14.3.1 gen\_from\_pm1\_qm1()**

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p-1,q-1.

Definition at line 127 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_kron\(\)](#).

**6.14.3.2 gen\_from\_pm4\_qp4()**

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pm4_qp4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p-4,q+4.

Definition at line 165 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

### 6.14.3.3 gen\_from\_pp4\_qm4()

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p+4,q-4.

Definition at line 198 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

### 6.14.3.4 gen\_from\_qp1\_pm1()

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for q+1,p-1.

Definition at line 231 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

### 6.14.3.5 gen\_vector()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::gen_vector (
    const index_t p,
    const index_t q ) -> const std::vector<Matrix_T>& [private]
```

Construct a vector of generators for a specific signature.

Definition at line 79 of file [generation\\_imp.h](#).

References [glucat::pos\\_mod\(\)](#).

### 6.14.3.6 generator()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::generator [static]
```

Single instance of generator table.

Definition at line 49 of file [generation\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#).

### 6.14.3.7 operator()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::operator() (
    const index_t p,
    const index_t q ) -> const Matrix_T* [inline]
```

Pointer to generators for a specific signature.

Definition at line 58 of file [generation\\_imp.h](#).

References [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

### 6.14.3.8 operator=()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::operator= (
    const generator_table< Matrix_T > & ) -> generator_table &=delete [delete]
```

## 6.14.4 Friends And Related Function Documentation

### 6.14.4.1 friend\_for\_private\_destructor

```
template<class Matrix_T >
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 75 of file [generation.h](#).

The documentation for this class was generated from the following files:

- [glucat/generation.h](#)
- [glucat/generation\\_imp.h](#)

## 6.15 glucat::glucat\_error Class Reference

Abstract exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::glucat\_error:

Collaboration diagram for glucat::glucat\_error:

## Public Member Functions

- [glucat\\_error](#) (const std::string &context, const std::string &msg)
- [~glucat\\_error](#) () noexcept override=default
- virtual auto [heading](#) () const noexcept -> const std::string=0
- virtual auto [classname](#) () const noexcept -> const std::string=0
- virtual void [print\\_error\\_msg](#) () const =0

## Public Attributes

- std::string [name](#)

### 6.15.1 Detailed Description

Abstract exception class.

Definition at line 41 of file [errors.h](#).

### 6.15.2 Constructor & Destructor Documentation

#### 6.15.2.1 glucat\_error()

```
glucat::glucat_error::glucat_error (
    const std::string & context,
    const std::string & msg ) [inline]
```

Definition at line 44 of file [errors.h](#).

#### 6.15.2.2 ~glucat\_error()

```
glucat::glucat_error::~~glucat_error ( ) [override], [default], [noexcept]
```

### 6.15.3 Member Function Documentation

#### 6.15.3.1 classname()

```
virtual auto glucat::glucat_error::classname ( ) const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#).

### 6.15.3.2 heading()

```
virtual auto glucat::glucat_error::heading ( ) const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#).

### 6.15.3.3 print\_error\_msg()

```
virtual void glucat::glucat_error::print_error_msg ( ) const [pure virtual]
```

Implemented in [glucat::error< Class\\_T >](#).

## 6.15.4 Member Data Documentation

### 6.15.4.1 name

```
std::string glucat::glucat_error::name
```

Definition at line 51 of file [errors.h](#).

The documentation for this class was generated from the following file:

- [glucat/errors.h](#)

## 6.16 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t Class Reference

### Public Member Functions

- [hash\\_size\\_t](#) (size\_t hash\_size)
- auto [operator\(\)](#) ( ) const -> size\_t

### Private Attributes

- size\_t [n](#)

### 6.16.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P =
tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t
```

Definition at line 152 of file [framed\\_multi.h](#).

## 6.16.2 Constructor & Destructor Documentation

### 6.16.2.1 hash\_size\_t()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::hash_size_t (
    size_t hash_size ) [inline]
```

Definition at line 155 of file [framed\\_multi.h](#).

## 6.16.3 Member Function Documentation

### 6.16.3.1 operator()()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::operator() ( ) const ->
size_t [inline]
```

Definition at line 158 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t::n](#).

## 6.16.4 Member Data Documentation

### 6.16.4.1 n

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
size_t glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::n [private]
```

Definition at line 161 of file [framed\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t::operator\(\)\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi.h](#)

## 6.17 glucat::index\_set< LO, HI > Class Template Reference

Index set class based on std::bitset<> in Gnu standard C++ library.

```
#include <index_set.h>
```

Inheritance diagram for glucat::index\_set< LO, HI >:

Collaboration diagram for glucat::index\_set< LO, HI >:

### Classes

- class [reference](#)  
*Index set member reference.*

### Public Types

- using [index\\_set\\_t](#) = [index\\_set](#)
- using [index\\_pair\\_t](#) = std::pair< [index\\_t](#), [index\\_t](#) >

### Public Member Functions

- [index\\_set](#) ()=default  
*Default constructor creates an empty set.*
- [index\\_set](#) (const [bitset\\_t](#) bst)  
*Constructor from bitset\_t.*
- [index\\_set](#) (const [index\\_t](#) idx)  
*Constructor from index.*
- [index\\_set](#) (const [set\\_value\\_t](#) folded\_val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Constructor from set value of an index set folded within the given frame.*
- [index\\_set](#) (const [index\\_pair\\_t](#) &range, const bool prechecked=false)  
*Constructor from range of indices from range.first to range.second.*
- [index\\_set](#) (const std::string &str)  
*Constructor from string.*
- auto [operator==](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Equality.*
- auto [operator!=](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Inequality.*
- auto [operator~](#) () const -> [index\\_set\\_t](#)  
*Set complement: not.*
- auto [operator^](#) = (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Symmetric set difference: exclusive or.*
- auto [operator&=](#) (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Set intersection: and.*
- auto [operator|=](#) (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Set union: or.*
- auto [operator\[\]](#) (const [index\\_t](#) idx) const -> bool  
*Subscripting: Test idx for membership: test value of bit idx.*
- auto [test](#) (const [index\\_t](#) idx) const -> bool



- Test idx for membership: test value of bit idx.*
- auto `set` () -> `index_set_t` &
- Include all indices except 0: set all bits except 0.*
- auto `set` (const `index_t` idx) -> `index_set_t` &
- Include idx: Set bit at idx if idx != 0.*
- auto `set` (const `index_t` idx, const int val) -> `index_set_t` &
- Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.*
- auto `reset` () -> `index_set_t` &
- Make set empty: Set all bits to 0.*
- auto `reset` (const `index_t` idx) -> `index_set_t` &
- Exclude idx: Set bit at idx to 0.*
- auto `flip` () -> `index_set_t` &
- Set complement, except 0: flip all bits, except 0.*
- auto `flip` (const `index_t` idx) -> `index_set_t` &
- Complement membership of idx if idx != 0: flip bit at idx if idx != 0.*
- auto `count` () const -> `index_t`
- Cardinality: Number of indices included in set.*
- auto `count_neg` () const -> `index_t`
- Number of negative indices included in set.*
- auto `count_pos` () const -> `index_t`
- Number of positive indices included in set.*
- auto `min` () const -> `index_t`
- Minimum member.*
- auto `max` () const -> `index_t`
- Maximum member.*
- auto `operator<` (const `index_set_t` rhs) const -> bool
- Less than operator used for comparisons, map, etc.*
- auto `is_contiguous` () const -> bool
- Determine if the index set is contiguous, ie. has no gaps.*
- auto `fold` () const -> const `index_set_t`
- Fold this index set within itself as a frame.*
- auto `fold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`
- Fold this index set within the given frame.*
- auto `unfold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`
- Unfold this index set within the given frame.*
- auto `value_of_fold` (const `index_set_t` frm) const -> `set_value_t`
- The set value of the fold of this index set within the given frame.*
- auto `sign_of_mult` (const `index_set_t` ist) const -> int
- Sign of geometric product of two Clifford basis elements.*
- auto `sign_of_square` () const -> int
- Sign of geometric square of a Clifford basis element.*
- auto `hash_fn` () const -> `size_t`
- Hash function.*
- auto `operator[]` (`index_t` idx) -> `reference`
- Subscripting: Element access.*

## Static Public Member Functions

- static auto `classname` () -> const std::string

## Static Public Attributes

- static const [index\\_t v\\_lo](#) = LO
- static const [index\\_t v\\_hi](#) = HI

## Private Types

- using [bitset\\_t](#) = std::bitset< HI - LO >
- using [error\\_t](#) = error< [index\\_set](#) >

## Private Member Functions

- [BOOST\\_STATIC\\_ASSERT](#) ((LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI-LO<=\_GLUCAT\_BITS\_PER\_ULONG))
- auto [lex\\_less\\_than](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Lexicographic ordering of two sets: \*this < rhs.*

## Friends

- class [reference](#)
- auto [operator^](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator&](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator|](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [compare](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> int

### 6.17.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >
```

Index set class based on std::bitset<> in Gnu standard C++ library.

Definition at line 73 of file [index\\_set.h](#).

### 6.17.2 Member Typedef Documentation

#### 6.17.2.1 [bitset\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index\_set< LO, HI >::bitset_t = std::bitset<HI - LO> [private]
```

Definition at line 81 of file [index\\_set.h](#).

### 6.17.2.2 error\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::error_t = error<index_set> [private]
```

Definition at line 82 of file [index\\_set.h](#).

### 6.17.2.3 index\_pair\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_pair_t = std::pair<index_t, index_t>
```

Definition at line 85 of file [index\\_set.h](#).

### 6.17.2.4 index\_set\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_set_t = index_set
```

Definition at line 84 of file [index\\_set.h](#).

## 6.17.3 Constructor & Destructor Documentation

### 6.17.3.1 index\_set() [1/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set ( ) [default]
```

Default constructor creates an empty set.

### 6.17.3.2 index\_set() [2/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const bitset_t bst )
```

Constructor from bitset\_t.

Definition at line 61 of file [index\\_set\\_imp.h](#).

**6.17.3.3 `index_set()` [3/6]**

```
template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::index\_set (
    const index\_t idx )
```

Constructor from index.

Constructor from index value.

Definition at line 55 of file [index\\_set\\_imp.h](#).

**6.17.3.4 `index_set()` [4/6]**

```
template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::index\_set (
    const set\_value\_t folded_val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Constructor from set value of an index set folded within the given frame.

Definition at line 68 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::index\\_set< LO, HI >::fold\(\)](#), [glucat::index\\_set< LO, HI >::min\(\)](#), and [glucat::index\\_set< LO, HI >::unfold\(\)](#).

**6.17.3.5 `index_set()` [5/6]**

```
template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::index\_set (
    const index\_pair\_t & range,
    const bool prechecked = false )
```

Constructor from range of indices from range.first to range.second.

Definition at line 82 of file [index\\_set\\_imp.h](#).

**6.17.3.6 `index_set()` [6/6]**

```
template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::index\_set (
    const std::string & str )
```

Constructor from string.

Definition at line 102 of file [index\\_set\\_imp.h](#).

## 6.17.4 Member Function Documentation

### 6.17.4.1 BOOST\_STATIC\_ASSERT()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::BOOST_STATIC_ASSERT (
    (LO<=0) && (0<=HI) && (LO< HI) && (-LO< _GLUCAT_BITS_PER_ULONG) && (HI< _GLUCAT_↵
    BITS_PER_ULONG) && (HI-LO<=_GLUCAT_BITS_PER_ULONG) ) [private]
```

### 6.17.4.2 classname()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::classname [inline], [static]
```

Definition at line 49 of file [index\\_set\\_imp.h](#).

### 6.17.4.3 count()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count [inline]
```

Cardinality: Number of indices included in set.

Definition at line 344 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::count\\_neg\(\)](#), [glucat::index\\_set< LO, HI >::count\\_pos\(\)](#), [glucat::framed\\_multi< Scalar\\_T, index\\_set< LO, HI >::index\\_set\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

### 6.17.4.4 count\_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_neg [inline]
```

Number of negative indices included in set.

Definition at line 364 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#).

#### 6.17.4.5 `count_pos()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_pos [inline]
```

Number of positive indices included in set.

Definition at line 376 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#).

#### 6.17.4.6 `flip()` [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip [inline]
```

Set complement, except 0: flip all bits, except 0.

Definition at line 319 of file [index\\_set\\_imp.h](#).

#### 6.17.4.7 `flip()` [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip (
    const index_t idx ) -> index_set_t& [inline]
```

Complement membership of idx if idx != 0: flip bit at idx if idx != 0.

Definition at line 330 of file [index\\_set\\_imp.h](#).

#### 6.17.4.8 `fold()` [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold [inline]
```

Fold this index set within itself as a frame.

Definition at line 747 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::index\\_set\(\)](#).

#### 6.17.4.9 fold() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold (
    const index_set_t frm,
    const bool prechecked = false ) const -> const index_set_t
```

Fold this index set within the given frame.

Definition at line 755 of file [index\\_set\\_imp.h](#).

#### 6.17.4.10 hash\_fn()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::hash_fn [inline]
```

Hash function.

Definition at line 950 of file [index\\_set\\_imp.h](#).

#### 6.17.4.11 is\_contiguous()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::is_contiguous [inline]
```

Determine if the index set is contiguous, ie. has no gaps.

Determine if the index set is contiguous, ie. has no gaps when 0 is included.

Definition at line 732 of file [index\\_set\\_imp.h](#).

#### 6.17.4.12 lex\_less\_than()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::lex_less_than (
    const index_set_t rhs ) const -> bool [inline], [private]
```

Lexicographic ordering of two sets: \*this < rhs.

Definition at line 588 of file [index\\_set\\_imp.h](#).

**6.17.4.13 max()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::max
```

Maximum member.

Maximum member, or 0 if none.

Definition at line 550 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

**6.17.4.14 min()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::min
```

Minimum member.

Minimum member, or 0 if none.

Definition at line 461 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::index\\_set< LO, HI >::index\\_set\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), and [glucat::index\\_set< LO, HI >::unfold\(\)](#).

**6.17.4.15 operator"!="()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator!= (
    const index_set_t rhs ) const -> bool [inline]
```

Inequality.

Definition at line 130 of file [index\\_set\\_imp.h](#).

**6.17.4.16 operator&=()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator&= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Set intersection: and.

Definition at line 174 of file [index\\_set\\_imp.h](#).



#### 6.17.4.17 operator<()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator< (
    const index_set_t rhs ) const -> bool [inline]
```

Less than operator used for comparisons, map, etc.

Definition at line 596 of file [index\\_set\\_imp.h](#).

#### 6.17.4.18 operator==( )

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator==(
    const index_set_t rhs ) const -> bool [inline]
```

Equality.

Definition at line 119 of file [index\\_set\\_imp.h](#).

#### 6.17.4.19 operator[]() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    const index_t idx ) const -> bool [inline]
```

Subscripting: Test idx for membership: test value of bit idx.

Definition at line 232 of file [index\\_set\\_imp.h](#).

#### 6.17.4.20 operator[]() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    index_t idx ) -> reference [inline]
```

Subscripting: Element access.

Definition at line 224 of file [index\\_set\\_imp.h](#).

#### 6.17.4.21 `operator^=()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator^= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Symmetric set difference: exclusive or.

Definition at line 149 of file [index\\_set\\_imp.h](#).

#### 6.17.4.22 `operator" |=()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator|= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Set union: or.

Definition at line 199 of file [index\\_set\\_imp.h](#).

#### 6.17.4.23 `operator~()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator~ [inline]
```

Set complement: not.

Definition at line 141 of file [index\\_set\\_imp.h](#).

#### 6.17.4.24 `reset()` [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset [inline]
```

Make set empty: Set all bits to 0.

Definition at line 294 of file [index\\_set\\_imp.h](#).

**6.17.4.25 reset()** [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset (
    const index_t idx ) -> index_set_t& [inline]
```

Exclude idx: Set bit at idx to 0.

Definition at line 305 of file [index\\_set\\_imp.h](#).

**6.17.4.26 set()** [1/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set [inline]
```

Include all indices except 0: set all bits except 0.

Definition at line 255 of file [index\\_set\\_imp.h](#).

**6.17.4.27 set()** [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx ) -> index_set_t& [inline]
```

Include idx: Set bit at idx if idx != 0.

Definition at line 266 of file [index\\_set\\_imp.h](#).

**6.17.4.28 set()** [3/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx,
    const int val ) -> index_set_t& [inline]
```

Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.

Definition at line 280 of file [index\\_set\\_imp.h](#).

#### 6.17.4.29 `sign_of_mult()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_mult (
    const index_set_t ist ) const -> int
```

Sign of geometric product of two Clifford basis elements.

Definition at line 880 of file [index\\_set\\_imp.h](#).

References [glucat::inverse\\_gray\(\)](#), and [glucat::inverse\\_reversed\\_gray\(\)](#).

#### 6.17.4.30 `sign_of_square()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_square [inline]
```

Sign of geometric square of a Clifford basis element.

Definition at line 930 of file [index\\_set\\_imp.h](#).

#### 6.17.4.31 `test()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::test (
    const index_t idx ) const -> bool [inline]
```

Test idx for membership: test value of bit idx.

Definition at line 240 of file [index\\_set\\_imp.h](#).

#### 6.17.4.32 `unfold()`

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::unfold (
    const index_set_t frm,
    const bool prechecked = false ) const -> const index_set_t
```

Unfold this index set within the given frame.

Definition at line 794 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::min\(\)](#).

Referenced by [glucat::index\\_set< LO, HI >::index\\_set\(\)](#).

### 6.17.4.33 value\_of\_fold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::value_of_fold (
    const index_set_t frm ) const -> set_value_t [inline]
```

The set value of the fold of this index set within the given frame.

Definition at line 829 of file [index\\_set\\_imp.h](#).

## 6.17.5 Friends And Related Function Documentation

### 6.17.5.1 compare

```
template<const index_t LO, const index_t HI>
auto compare (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> int [friend]
```

### 6.17.5.2 operator&

```
template<const index_t LO, const index_t HI>
auto operator& (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

### 6.17.5.3 operator^

```
template<const index_t LO, const index_t HI>
auto operator^ (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

### 6.17.5.4 operator"|"

```
template<const index_t LO, const index_t HI>
auto operator| (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

#### 6.17.5.5 reference

```
template<const index_t LO, const index_t HI>
friend class reference [friend]
```

Definition at line 174 of file [index\\_set.h](#).

### 6.17.6 Member Data Documentation

#### 6.17.6.1 v\_hi

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_hi = HI [static]
```

Definition at line 88 of file [index\\_set.h](#).

#### 6.17.6.2 v\_lo

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_lo = LO [static]
```

Definition at line 87 of file [index\\_set.h](#).

The documentation for this class was generated from the following files:

- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.18 PyClical.index\_set Class Reference

Inheritance diagram for PyClical.index\_set:

Collaboration diagram for PyClical.index\_set:

## Public Member Functions

- `def __cinit__(self, other=0)`
- `def __dealloc__(self)`
- `def __richcmp__(lhs, rhs, int, op)`
- `def __setitem__(self, idx, val)`
- `def __getitem__(self, idx)`
- `def __contains__(self, idx)`
- `def __iter__(self)`
- `def __invert__(self)`
- `def __xor__(lhs, rhs)`
- `def __ixor__(self, rhs)`
- `def __and__(lhs, rhs)`
- `def __iand__(self, rhs)`
- `def __or__(lhs, rhs)`
- `def __ior__(self, rhs)`
- `def count(self)`
- `def count_neg(self)`
- `def count_pos(self)`
- `def min(self)`
- `def max(self)`
- `def hash_fn(self)`
- `def sign_of_mult(self, rhs)`
- `def sign_of_square(self)`
- `def __repr__(self)`
- `def __str__(self)`

## Public Attributes

- `instance`

### 6.18.1 Detailed Description

Return the C++ `IndexSet` instance wrapped by `index_set(obj)`.

Python class `index_set` wraps C++ class `IndexSet`.

Definition at line 38 of file [PyClical.pyx](#).

### 6.18.2 Member Function Documentation

### 6.18.2.1 `__and__()`

```
def PyClical.index_set.__and__ (
    lhs,
    rhs )
```

Set intersection: and.

```
>>> print(index_set({1}) & index_set({2}))
{}
>>> print(index_set({1,2}) & index_set({2}))
{2}
```

Definition at line 271 of file [PyClical.pyx](#).

### 6.18.2.2 `__cinit__()`

```
def PyClical.index_set.__cinit__ (
    self,
    other = 0 )
```

Construct an object of type index\_set.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set("{1,2,1}"))
{1,2}
>>> print(index_set(""))
{}
```

Definition at line 74 of file [PyClical.pyx](#).

### 6.18.2.3 `__contains__()`

```
def PyClical.index_set.__contains__ (
    self,
    idx )
```



Check that an index\_set object contains the index idx: idx in self.

```
>>> 1 in index_set({1})
True
>>> 2 in index_set({1})
False
>>> -1 in index_set({2})
False
>>> 1 in index_set({2})
False
>>> 2 in index_set({2})
True
>>> 33 in index_set({2})
False
```

Definition at line 210 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

#### 6.18.2.4 \_\_dealloc\_\_()

```
def PyClical.index_set.__dealloc__ (
    self )
```

Clean up by deallocating the instance of C++ class IndexSet.

Definition at line 116 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

#### 6.18.2.5 \_\_getitem\_\_()

```
def PyClical.index_set.__getitem__ (
    self,
    idx )
```

Get the value of an index\_set object at an index.

```
>>> index_set({1})[1]
True
>>> index_set({1})[2]
False
>>> index_set({2})[-1]
False
>>> index_set({2})[1]
False
>>> index_set({2})[2]
True
>>> index_set({2})[33]
False
```

Definition at line 191 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.6 `__iand__()`

```
def PyClical.index_set.__iand__ (
    self,
    rhs )

Set intersection: and.

>>> x = index_set({1}); x &= index_set({2}); print(x)
{}
>>> x = index_set({1,2}); x &= index_set({2}); print(x)
{2}
```

Definition at line 282 of file [PyClical.pyx](#).

### 6.18.2.7 `__invert__()`

```
def PyClical.index_set.__invert__ (
    self )

Set complement: not.

>>> print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,
```

Definition at line 240 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.8 `__ior__()`

```
def PyClical.index_set.__ior__ (
    self,
    rhs )

Set union: or.

>>> x = index_set({1}); x |= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x |= index_set({2}); print(x)
{1,2}
```

Definition at line 304 of file [PyClical.pyx](#).

### 6.18.2.9 `__iter__()`

```
def PyClical.index_set.__iter__ (
    self )
```

Iterate over the indices of an index\_set.

```
>>> for i in index_set({-3,4,7}):print(i, end=",")
-3,4,7,
```

Definition at line 229 of file [PyClical.pyx](#).

References [glucat::index\\_set< LO, HI >.max\(\)](#), [glucat::index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >.max\(\)](#), [PyClical.index\\_set.max\(\)](#), [glucat::index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >.min\(\)](#), [glucat::index\\_set< LO, HI >.min\(\)](#), and [PyClical.index\\_set.min\(\)](#).

### 6.18.2.10 `__ixor__()`

```
def PyClical.index_set.__ixor__ (
    self,
    rhs )
```

Symmetric set difference: exclusive or.

```
>>> x = index_set({1}); x ^= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x ^= index_set({2}); print(x)
{1}
```

Definition at line 260 of file [PyClical.pyx](#).

### 6.18.2.11 `__or__()`

```
def PyClical.index_set.__or__ (
    lhs,
    rhs )
```

Set union: or.

```
>>> print(index_set({1}) | index_set({2}))
{1,2}
>>> print(index_set({1,2}) | index_set({2}))
{1,2}
```

Definition at line 293 of file [PyClical.pyx](#).

### 6.18.2.12 `__repr__()`

```
def PyClical.index_set.__repr__ (
    self )
```

The “official” string representation of self.

```
>>> index_set({1,2}).__repr__()
'index_set({1,2})'
>>> repr(index_set({1,2}))
'index_set({1,2})'
```

Definition at line 384 of file [PyClical.pyx](#).

References [PyClical.index\\_set.\\_\\_repr\\_\\_\(\)](#), and [index\\_set\\_to\\_repr\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_repr\\_\\_\(\)](#).

### 6.18.2.13 `__richcmp__()`

```
def PyClical.index_set.__richcmp__ (
    lhs,
    rhs,
    int,
    op )
```

Compare two objects of class `index_set`.

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
```

Definition at line 122 of file [PyClical.pyx](#).

#### 6.18.2.14 `__setitem__()`

```
def PyClical.index_set.__setitem__ (
    self,
    idx,
    val )
```

Set the value of an `index_set` object at index `idx` to value `val`.

```
>>> s=index_set({1}); s[2] = True; print(s)
{1,2}
>>> s=index_set({1,2}); s[1] = False; print(s)
{2}
```

Definition at line 179 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

#### 6.18.2.15 `__str__()`

```
def PyClical.index_set.__str__ (
    self )
```

The “informal” string representation of self.

```
>>> index_set({1,2}).__str__()
'{1,2}'
>>> str(index_set({1,2}))
'{1,2}'
```

Definition at line 395 of file [PyClical.pyx](#).

References [PyClical.index\\_set.\\_\\_str\\_\\_\(\)](#), and [index\\_set\\_to\\_str\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_str\\_\\_\(\)](#).

#### 6.18.2.16 `__xor__()`

```
def PyClical.index_set.__xor__ (
    lhs,
    rhs )
```

Symmetric set difference: exclusive or.

```
>>> print(index_set({1}) ^ index_set({2}))
{1,2}
>>> print(index_set({1,2}) ^ index_set({2}))
{1}
```

Definition at line 249 of file [PyClical.pyx](#).

### 6.18.2.17 count()

```
def PyClical.index_set.count (
    self )
```

Cardinality: Number of indices included in set.

```
>>> index_set({-1,1,2}).count()
3
```

Definition at line 315 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\(\)](#).

### 6.18.2.18 count\_neg()

```
def PyClical.index_set.count_neg (
    self )
```

Number of negative indices included in set.

```
>>> index_set({-1,1,2}).count_neg()
1
```

Definition at line 324 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\\_neg\(\)](#).

### 6.18.2.19 count\_pos()

```
def PyClical.index_set.count_pos (
    self )
```

Number of positive indices included in set.

```
>>> index_set({-1,1,2}).count_pos()
2
```

Definition at line 333 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\\_pos\(\)](#).

### 6.18.2.20 hash\_fn()

```
def PyClical.index_set.hash_fn (
    self )
```

Hash function.

Definition at line 360 of file [PyClical.pyx](#).

References [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.hash\\_fn\(\)](#).

### 6.18.2.21 max()

```
def PyClical.index_set.max (
    self )
```

Maximum member.

```
>>> index_set({-1,1,2}).max()
2
```

Definition at line 351 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.max\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_iter\\_\\_\(\)](#), and [PyClical.index\\_set.max\(\)](#).

### 6.18.2.22 min()

```
def PyClical.index_set.min (
    self )
```

Minimum member.

```
>>> index_set({-1,1,2}).min()
-1
```

Definition at line 342 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.min\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_iter\\_\\_\(\)](#), and [PyClical.index\\_set.min\(\)](#).

### 6.18.2.23 `sign_of_mult()`

```
def PyClical.index_set.sign_of_mult (
    self,
    rhs )

Sign of geometric product of two Clifford basis elements.

>>> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
1
```

Definition at line 366 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.sign\\_of\\_mult\(\)](#).

Referenced by [PyClical.index\\_set.sign\\_of\\_mult\(\)](#).

### 6.18.2.24 `sign_of_square()`

```
def PyClical.index_set.sign_of_square (
    self )

Sign of geometric square of a Clifford basis element.

>>> s = index_set({1,2}); s.sign_of_square()
-1
```

Definition at line 375 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.sign\\_of\\_square\(\)](#).

Referenced by [PyClical.index\\_set.sign\\_of\\_square\(\)](#).

## 6.18.3 Member Data Documentation

### 6.18.3.1 `instance`

`PyClical.index_set.instance`

Definition at line 95 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.\\_\\_call\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_contains\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.clifford.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_getitem\\_\\_\(\)](#), [PyClical.clifford.\\_\\_getitem\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_invert\\_\\_\(\)](#), [PyClical.clifford.\\_\\_neg\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_setitem\\_\\_\(\)](#), [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [PyClical.index\\_set.max\(\)](#), [PyClical.clifford.max\\_abs\(\)](#), [PyClical.index\\_set.min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer\\_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [PyClical.index\\_set.sign\\_of\\_mult\(\)](#), [PyClical.index\\_set.sign\\_of\\_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)



## 6.19 glucat::index\_set\_hash< LO, HI > Class Template Reference

```
#include <framed_multi.h>
```

### Public Types

- using [index\\_set\\_t](#) = [index\\_set](#)< LO, HI >

### Public Member Functions

- auto [operator\(\)](#) ([index\\_set\\_t](#) val) const -> [size\\_t](#)

#### 6.19.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>  
class glucat::index_set_hash< LO, HI >
```

Definition at line 117 of file [framed\\_multi.h](#).

#### 6.19.2 Member Typedef Documentation

##### 6.19.2.1 index\_set\_t

```
template<const index\_t LO, const index\_t HI>  
using glucat::index\_set\_hash< LO, HI >::index_set_t = index\_set<LO, HI>
```

Definition at line 120 of file [framed\\_multi.h](#).

#### 6.19.3 Member Function Documentation

##### 6.19.3.1 operator()

```
template<const index\_t LO, const index\_t HI>  
auto glucat::index\_set\_hash< LO, HI >::operator() (  
    index\_set\_t val ) const -> size\_t    [inline]
```

Definition at line 121 of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi.h](#)

## 6.20 `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >` Class Template Reference

A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.

```
#include <matrix_multi.h>
```

Inheritance diagram for `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`:

Collaboration diagram for `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`:

### Public Types

- using `multivector_t` = `matrix_multi`
- using `matrix_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set< LO, HI >`
- using `term_t` = `std::pair< const index_set_t, Scalar_T >`
- using `vector_t` = `std::vector< Scalar_T >`
- using `error_t` = `error< multivector_t >`
- using `framed_multi_t` = `framed_multi< Scalar_T, LO, HI, Tune_P >`

### Public Member Functions

- `~matrix_multi()` *override=default*  
*Destructor.*
- `matrix_multi()`  
*Default constructor.*
- `template<typename Other_Scalar_T > matrix_multi(const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > &val)`  
*Construct a multivector from a multivector with a different scalar type.*
- `template<typename Other_Scalar_T > matrix_multi(const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > &val, const index_set_t frm, const bool prechecked=false)`  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi(const multivector_t &val, const index_set_t frm, const bool prechecked=false)`  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi(const index_set_t ist, const Scalar_T &crd=Scalar_T(1))`  
*Construct a multivector from an index set and a scalar coordinate.*
- `matrix_multi(const index_set_t ist, const Scalar_T &crd, const index_set_t frm, const bool prechecked=false)`  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- `matrix_multi(const Scalar_T &scr, const index_set_t frm=index_set_t())`  
*Construct a multivector from a scalar (within a frame, if given)*
- `matrix_multi(const int scr, const index_set_t frm=index_set_t())`  
*Construct a multivector from an int (within a frame, if given)*
- `matrix_multi(const vector_t &vec, const index_set_t frm, const bool prechecked=false)`  
*Construct a multivector, within a given frame, from a given vector.*
- `matrix_multi(const std::string &str)`  
*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*

- **matrix\_multi** (const std::string &str, const **index\_set\_t** frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- **matrix\_multi** (const char \*str)  
*Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- **matrix\_multi** (const char \*str, const **index\_set\_t** frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- template<typename Other\_Scalar\_T >  
**matrix\_multi** (const **framed\_multi**< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
*Construct a multivector from a framed\_multi\_t.*
- template<typename Other\_Scalar\_T >  
**matrix\_multi** (const **framed\_multi**< Other\_Scalar\_T, LO, HI, Tune\_P > &val, const **index\_set\_t** frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a framed\_multi\_t.*
- auto **fast\_matrix\_multi** (const **index\_set\_t** frm) const -> const **matrix\_multi\_t**  
*Use generalized FFT to construct a matrix\_multi\_t.*
- template<typename Other\_Scalar\_T >  
auto **fast\_framed\_multi** () const -> const **framed\_multi**< Other\_Scalar\_T, LO, HI, Tune\_P >  
*Use inverse generalized FFT to construct a framed\_multi\_t.*
- **\_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS** auto **operator=** (const **multivector\_t** &rhs) -> **multivector\_t** &  
*Assignment operator.*
- auto **operator+=** (const **term\_t** &rhs) -> **multivector\_t** &  
*Add a term, if non-zero.*

## Static Public Member Functions

- static auto **classname** () -> const std::string  
*Class name used in messages.*
- static auto **random** (const **index\_set\_t** frm, Scalar\_T fill=Scalar\_T(1)) -> const **matrix\_multi\_t**  
*Random multivector within a frame.*

## Private Types

- using **orientation\_t** = ublas::row\_major
- using **basis\_matrix\_t** = ublas::compressed\_matrix< int, **orientation\_t** >
- using **matrix\_t** = ublas::matrix< Scalar\_T, **orientation\_t** >
- using **matrix\_index\_t** = typename matrix\_t::size\_type

## Private Member Functions

- template<typename Matrix\_T >  
**matrix\_multi** (const Matrix\_T &mtx, const **index\_set\_t** frm)  
*Construct a multivector within a given frame from a given matrix.*
- **matrix\_multi** (const **matrix\_t** &mtx, const **index\_set\_t** frm)  
*Construct a multivector within a given frame from a given matrix.*
- auto **basis\_element** (const **index\_set**< LO, HI > &ist) const -> const **basis\_matrix\_t**  
*Create a basis element matrix within the current frame.*

## Private Attributes

- [index\\_set\\_t m\\_frame](#)  
*Index set representing the frame for the subalgebra which contains the multivector.*
- [matrix\\_t m\\_matrix](#)  
*Matrix value representing the multivector within the folded frame.*

## Friends

- `template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >`  
`class framed\_multi`
- `template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >`  
`class matrix\_multi`
- `auto operator\* (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto operator^ (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto operator& (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto operator% (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto star (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> Scalar\_T`
- `auto operator/ (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto operator| (const matrix\_multi\_t &lhs, const matrix\_multi\_t &rhs) -> const matrix\_multi\_t`
- `auto operator>> (std::istream &s, multivector\_t &val) -> std::istream &`
- `auto operator<< (std::ostream &os, const multivector\_t &val) -> std::ostream &`
- `template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >`  
`auto reframe (const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs, const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs, matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs_reframed, matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs_reframed) -> const index\_set< Other_LO, Other_HI >`
- `template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >`  
`auto matrix\_sqrt (const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const index\_t level) -> const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >`
- `template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >`  
`auto matrix\_log (const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const index\_t level) -> const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >`

## Additional Inherited Members

### 6.20.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P =
tuning<>>
class glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >
```

A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.

Definition at line 137 of file [matrix\\_multi.h](#).

### 6.20.2 Member Typedef Documentation

### 6.20.2.1 `basis_matrix_t`

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_matrix_t = ublas::compressed_matrix<int, orientation_t> [private]
```

Definition at line 157 of file [matrix\\_multi.h](#).

### 6.20.2.2 `error_t`

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 148 of file [matrix\\_multi.h](#).

### 6.20.2.3 `framed_multi_t`

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = framed_multi<Scalar_T, LO, HI, Tune_P>
```

Definition at line 149 of file [matrix\\_multi.h](#).

### 6.20.2.4 `index_set_t`

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 145 of file [matrix\\_multi.h](#).

### 6.20.2.5 `matrix_index_t`

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_index_t = typename matrix_t::size_type [private]
```

Definition at line 159 of file [matrix\\_multi.h](#).

### 6.20.2.6 matrix\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = multivector_t
```

Definition at line 142 of file [matrix\\_multi.h](#).

### 6.20.2.7 matrix\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = ublas::matrix<Scalar_T,
orientation_t> [private]
```

Definition at line 158 of file [matrix\\_multi.h](#).

### 6.20.2.8 multivector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = matrix_multi
```

Definition at line 141 of file [matrix\\_multi.h](#).

### 6.20.2.9 orientation\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::orientation_t = ublas::row_major
[private]
```

Definition at line 156 of file [matrix\\_multi.h](#).

### 6.20.2.10 scalar\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 143 of file [matrix\\_multi.h](#).

### 6.20.2.11 term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 146 of file [matrix\\_multi.h](#).

### 6.20.2.12 tune\_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 144 of file [matrix\\_multi.h](#).

### 6.20.2.13 vector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 147 of file [matrix\\_multi.h](#).

## 6.20.3 Constructor & Destructor Documentation

### 6.20.3.1 ~matrix\_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::~~matrix_multi ( ) [override], [default]
```

Destructor.

### 6.20.3.2 matrix\_multi() [1/17]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi
```

Default constructor.

Definition at line 106 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.3 matrix\_multi()** [2/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 115 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.20.3.4 matrix\_multi()** [3/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 134 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.20.3.5 matrix\_multi()** [4/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const multivector\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 159 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).



**6.20.3.6 matrix\_multi()** [5/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1) )
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 171 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_](#)

**6.20.3.7 matrix\_multi()** [6/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 183 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.8 matrix\_multi()** [7/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t\(\) )
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 197 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.9 matrix\_multi()** [8/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t\(\) )
```

Construct a multivector from an int (within a frame, if given)

Definition at line 209 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.10 matrix\_multi()** [9/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 215 of file [matrix\\_multi\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), [glucat::index\\_set< LO, HI >::max\(\)](#), and [glucat::index\\_set< LO, HI >::min\(\)](#).

**6.20.3.11 matrix\_multi()** [10/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str )
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 240 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.12 matrix\_multi()** [11/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 246 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.13 matrix\_multi()** [12/17]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str ) [inline]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 196 of file [matrix\\_multi.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi](#).

**6.20.3.14 matrix\_multi()** [13/17]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str,
    const index_set_t frm,
    const bool prechecked = false ) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 199 of file [matrix\\_multi.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi](#).

**6.20.3.15 matrix\_multi()** [14/17]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a framed\_multi\_t.

Definition at line 253 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m](#)

**6.20.3.16 matrix\_multi()** [15/17]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index_set_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a framed\_multi\_t.

Definition at line 277 of file [matrix\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::truncated\(\)](#).

**6.20.3.17 matrix\_multi()** [16/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Matrix_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Matrix_T & mtx,
    const index\_set\_t frm ) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 303 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**6.20.3.18 matrix\_multi()** [17/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_t & mtx,
    const index\_set\_t frm ) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 322 of file [matrix\\_multi\\_imp.h](#).

**6.20.4 Member Function Documentation****6.20.4.1 basis\_element()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::basis_element (
    const index\_set< LO, HI > & ist ) const -> const basis\_matrix\_t [private]
```

Create a basis element matrix within the current frame.

Definition at line 1186 of file [matrix\\_multi\\_imp.h](#).

References [glucat::gen::generator\\_table< Matrix\\_T >::generator\(\)](#), [glucat::matrix::mono\\_prod\(\)](#), and [glucat::offset\\_level\(\)](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 6.20.4.2 classname()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::classname [static]
```

Class name used in messages.

Definition at line 78 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.3 fast\_framed\_multi()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi
```

Use inverse generalized FFT to construct a framed\_multi\_t.

Definition at line 1109 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_pm4\\_qp4\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_qp1\\_pm1\(\)](#), [glucat::gen::offset\\_to\\_super](#), [glucat::pos\\_mod\(\)](#), and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::unfold\(\)](#).

#### 6.20.4.4 fast\_matrix\_multi()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm ) const -> const matrix_multi_t [inline]
```

Use generalized FFT to construct a matrix\_multi\_t.

Definition at line 1096 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.5 operator+=()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & rhs ) -> multivector_t& [inline]
```

Add a term, if non-zero.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 416 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.6 operator=()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::operator= (
    const multivector\_t & rhs ) -> multivector\_t&
```

Assignment operator.

Definition at line 330 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.7 random()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index\_set\_t frm,
    Scalar_T fill = Scalar_T(1) ) -> const matrix\_multi\_t [static]
```

Random multivector within a frame.

Definition at line 926 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) >::random().

### 6.20.5 Friends And Related Function Documentation

#### 6.20.5.1 framed\_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
friend class framed\_multi [friend]
```

Definition at line 151 of file [matrix\\_multi.h](#).

#### 6.20.5.2 matrix\_log

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
auto matrix_log (
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index\_t level ) -> const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

### 6.20.5.3 matrix\_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
friend class matrix_multi [friend]
```

Definition at line 153 of file [matrix\\_multi.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

### 6.20.5.4 matrix\_sqrt

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
auto matrix_sqrt (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level ) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

### 6.20.5.5 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.6 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.7 operator\*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.8 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.9 operator<<

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val ) -> std::ostream & [friend]
```

### 6.20.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val ) -> std::istream & [friend]
```

### 6.20.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```



### 6.20.5.12 operator" |

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.13 reframe

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
auto reframe (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs_reframed,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs_reframed )
-> const index_set< Other_LO, Other_HI > [friend]
```

### 6.20.5.14 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> Scalar_T [friend]
```

## 6.20.6 Member Data Documentation

### 6.20.6.1 m\_frame

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
index_set_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_frame [private]
```

Index set representing the frame for the subalgebra which contains the multivector.

Definition at line 278 of file [matrix\\_multi.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

### 6.20.6.2 m\_matrix

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
matrix\_t glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::m_matrix [private]
```

Matrix value representing the multivector within the folded frame.

Definition at line 280 of file [matrix\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

The documentation for this class was generated from the following files:

- [glucat/framed\\_multi.h](#)
- [glucat/matrix\\_multi.h](#)
- [glucat/matrix\\_multi\\_imp.h](#)

## 6.21 std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > > Struct Template Reference

Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

```
#include <framed_multi.h>
```

Inheritance diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:

Collaboration diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:

### 6.21.1 Detailed Description

```
template<typename Scalar_T, const glucat::index\_t LO, const glucat::index\_t HI, typename Tune_P>
struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

Definition at line 345 of file [framed\\_multi.h](#).

The documentation for this struct was generated from the following file:

- [glucat/framed\\_multi.h](#)

## 6.22 std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > > Struct Template Reference

Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.

```
#include <matrix_multi.h>
```

Inheritance diagram for std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > >:

Collaboration diagram for std::numeric\_limits< glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > >:

### 6.22.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.

Definition at line 296 of file [matrix\\_multi.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi.h](#)

## 6.23 glucat::numeric\_traits< Scalar\_T > Class Template Reference

Extra traits which extend numeric limits.

```
#include <scalar.h>
```

### Classes

- struct [demoted](#)  
*Demoted type for long double.*
- struct [promoted](#)  
*Extra traits which extend numeric limits.*

### Public Member Functions

- auto [pi](#) () -> long double  
*Pi for long double.*
- auto [ln\\_2](#) () -> long double  
*log(2) for long double*
- auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> float  
*Extra traits which extend numeric limits.*
- auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> double  
*Cast to double.*
- auto [to\\_scalar\\_t](#) (const dd\_real &val) -> long double  
*Cast to long double.*
- auto [to\\_scalar\\_t](#) (const qd\_real &val) -> long double  
*Cast to long double.*
- auto [to\\_scalar\\_t](#) (const long double &val) -> dd\_real  
*Cast to dd\_real.*
- auto [to\\_scalar\\_t](#) (const qd\_real &val) -> dd\_real  
*Cast to dd\_real.*
- auto [to\\_scalar\\_t](#) (const long double &val) -> qd\_real  
*Cast to qd\_real.*
- auto [to\\_scalar\\_t](#) (const dd\_real &val) -> qd\_real  
*Cast to qd\_real.*

## Static Public Member Functions

- static auto [isInf](#) (const Scalar\_T &val) -> bool  
*Smart isinf.*
- static auto [isNaN](#) (const Scalar\_T &val) -> bool  
*Smart isnan.*
- static auto [isNaN\\_or\\_isInf](#) (const Scalar\_T &val) -> bool  
*Smart isnan or isinf.*
- static auto [NaN](#) () -> Scalar\_T  
*Smart NaN.*
- static auto [to\\_int](#) (const Scalar\_T &val) -> int  
*Cast to int.*
- static auto [to\\_double](#) (const Scalar\_T &val) -> double  
*Cast to double.*
- template<typename Other\_Scalar\_T >  
static auto [to\\_scalar\\_t](#) (const Other\_Scalar\_T &val) -> Scalar\_T  
*Cast to Scalar\_T.*
- static auto [fmod](#) (const Scalar\_T &lhs, const Scalar\_T &rhs) -> Scalar\_T  
*Modulo function for scalar.*
- static auto [conj](#) (const Scalar\_T &val) -> Scalar\_T  
*Complex conjugate of scalar.*
- static auto [real](#) (const Scalar\_T &val) -> Scalar\_T  
*Real part of scalar.*
- static auto [imag](#) (const Scalar\_T &val) -> Scalar\_T  
*Imaginary part of scalar.*
- static auto [abs](#) (const Scalar\_T &val) -> Scalar\_T  
*Absolute value of scalar.*
- static auto [pi](#) () -> Scalar\_T  
*Pi.*
- static auto [ln\\_2](#) () -> Scalar\_T  
*log(2)*
- static auto [pow](#) (const Scalar\_T &val, int n) -> Scalar\_T  
*Integer power.*
- static auto [sqrt](#) (const Scalar\_T &val) -> Scalar\_T  
*Square root of scalar.*
- static auto [exp](#) (const Scalar\_T &val) -> Scalar\_T  
*Exponential.*
- static auto [log](#) (const Scalar\_T &val) -> Scalar\_T  
*Logarithm of scalar.*
- static auto [log2](#) (const Scalar\_T &val) -> Scalar\_T  
*Log base 2.*
- static auto [cos](#) (const Scalar\_T &val) -> Scalar\_T  
*Cosine of scalar.*
- static auto [acos](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse cosine of scalar.*
- static auto [cosh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic cosine of scalar.*
- static auto [sin](#) (const Scalar\_T &val) -> Scalar\_T  
*Sine of scalar.*
- static auto [asin](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse sine of scalar.*

- static auto [sinh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic sine of scalar.*
- static auto [tan](#) (const Scalar\_T &val) -> Scalar\_T  
*Tangent of scalar.*
- static auto [atan](#) (const Scalar\_T &val) -> Scalar\_T  
*Inverse tangent of scalar.*
- static auto [tanh](#) (const Scalar\_T &val) -> Scalar\_T  
*Hyperbolic tangent of scalar.*

## Static Private Member Functions

- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isinf specialised for Scalar\_T without infinity.*
- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isinf specialised for Scalar\_T with infinity.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isnan specialised for Scalar\_T without quiet NaN.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isnan specialised for Scalar\_T with quiet NaN.*

### 6.23.1 Detailed Description

```
template<typename Scalar_T>
class glucat::numeric_traits< Scalar_T >
```

Extra traits which extend numeric limits.

Definition at line 47 of file [scalar.h](#).

### 6.23.2 Member Function Documentation

#### 6.23.2.1 [abs\(\)](#)

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::abs (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Absolute value of scalar.

Definition at line 182 of file [scalar.h](#).

### 6.23.2.2 `acos()`

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::acos (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse cosine of scalar.

Definition at line [245](#) of file [scalar.h](#).

### 6.23.2.3 `asin()`

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::asin (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse sine of scalar.

Definition at line [266](#) of file [scalar.h](#).

### 6.23.2.4 `atan()`

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::atan (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse tangent of scalar.

Definition at line [287](#) of file [scalar.h](#).

### 6.23.2.5 `conj()`

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::conj (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Complex conjugate of scalar.

Definition at line [161](#) of file [scalar.h](#).

### 6.23.2.6 cos()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::cos (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Cosine of scalar.

Definition at line 238 of file [scalar.h](#).

### 6.23.2.7 cosh()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::cosh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic cosine of scalar.

Definition at line 252 of file [scalar.h](#).

### 6.23.2.8 exp()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::exp (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Exponential.

Definition at line 217 of file [scalar.h](#).

### 6.23.2.9 fmod()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::fmod (
    const Scalar_T & lhs,
    const Scalar_T & rhs ) -> Scalar_T    [inline], [static]
```

Modulo function for scalar.

Definition at line 154 of file [scalar.h](#).

**6.23.2.10 imag()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::imag (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Imaginary part of scalar.

Definition at line 175 of file [scalar.h](#).

**6.23.2.11 isInf() [1/3]**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isinf.

Definition at line 83 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#).

**6.23.2.12 isInf() [2/3]**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart isinf specialised for Scalar\_T without infinity.

Definition at line 54 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::isNaN\\_or\\_isInf\(\)](#).

**6.23.2.13 isInf() [3/3]**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart isinf specialised for Scalar\_T with infinity.

Definition at line 61 of file [scalar.h](#).

References [\\_GLUCAT\\_ISINF](#).



**6.23.2.14** `isNaN()` [1/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isnan.

Definition at line 93 of file `scalar.h`.

References `glucat::numeric_traits< Scalar_T >::isNaN()`.

**6.23.2.15** `isNaN()` [2/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for `Scalar_T` without quiet NaN.

Definition at line 68 of file `scalar.h`.

Referenced by `glucat::numeric_traits< Scalar_T >::isNaN()`, and `glucat::numeric_traits< Scalar_T >::isNaN_or_isInf()`.

**6.23.2.16** `isNaN()` [3/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for `Scalar_T` with quiet NaN.

Definition at line 75 of file `scalar.h`.

References `_GLUCAT_ISNAN`.

**6.23.2.17** `isNaN_or_isInf()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN_or_isInf (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isnan or isinf.

Definition at line 103 of file `scalar.h`.

References `glucat::numeric_traits< Scalar_T >::isInf()`, and `glucat::numeric_traits< Scalar_T >::isNaN()`.

### 6.23.2.18 `ln_2()` [1/2]

```
auto glucat::numeric_traits< longdouble >::ln_2 ( ) -> long double    [inline]
```

`log(2)` for long double

Definition at line 59 of file [long\\_double.h](#).

References [glucat::l\\_ln2](#).

### 6.23.2.19 `ln_2()` [2/2]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::ln_2 ( ) -> Scalar_T    [inline], [static]
```

`log(2)`

Definition at line 196 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

### 6.23.2.20 `log()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::log (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Logarithm of scalar.

Definition at line 224 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

### 6.23.2.21 `log2()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::log2 (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Log base 2.

Definition at line 231 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::ln\\_2\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::log\(\)](#).

Referenced by [glucat::log2\(\)](#).

### 6.23.2.22 `NaN()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::NaN ( ) -> Scalar_T    [inline], [static]
```

Smart NaN.

Definition at line 115 of file `scalar.h`.

Referenced by `glucat::cr_sqrt()`, `glucat::matrix::norm_frob2()`, `glucat::operator*()`, and `glucat::matrix::trace()`.

### 6.23.2.23 `pi()` [1/2]

```
auto glucat::numeric_traits< longdouble >::pi ( ) -> long double    [inline]
```

Pi for long double.

Definition at line 51 of file `long_double.h`.

References `glucat::l_pi`.

### 6.23.2.24 `pi()` [2/2]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::pi ( ) -> Scalar_T    [inline], [static]
```

Pi.

Definition at line 189 of file `scalar.h`.

Referenced by `glucat::matrix::classify_eigenvalues()`.

### 6.23.2.25 `pow()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::pow (
    const Scalar_T & val,
    int n ) -> Scalar_T    [inline], [static]
```

Integer power.

Definition at line 203 of file `scalar.h`.

Referenced by `glucat::error_squared_tol()`.

#### 6.23.2.26 real()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::real (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Real part of scalar.

Definition at line [168](#) of file [scalar.h](#).

#### 6.23.2.27 sin()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::sin (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Sine of scalar.

Definition at line [259](#) of file [scalar.h](#).

#### 6.23.2.28 sinh()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::sinh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic sine of scalar.

Definition at line [273](#) of file [scalar.h](#).

#### 6.23.2.29 sqrt()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::sqrt (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Square root of scalar.

Definition at line [210](#) of file [scalar.h](#).

Referenced by [glucat::abs\(\)](#).

### 6.23.2.30 tan()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::tan (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Tangent of scalar.

Definition at line 280 of file [scalar.h](#).

### 6.23.2.31 tanh()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::tanh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic tangent of scalar.

Definition at line 294 of file [scalar.h](#).

### 6.23.2.32 to\_double()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_double (
    const Scalar_T & val ) -> double    [inline], [static]
```

Cast to double.

Definition at line 133 of file [scalar.h](#).

Referenced by [glucat::operator<<\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

### 6.23.2.33 to\_int()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_int (
    const Scalar_T & val ) -> int    [inline], [static]
```

Cast to int.

Definition at line 126 of file [scalar.h](#).

**6.23.2.34 to\_scalar\_t()** [1/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const dd_real & val ) -> long double    [inline]
```

Cast to long double.

Definition at line 71 of file [scalar\\_imp.h](#).

**6.23.2.35 to\_scalar\_t()** [2/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const dd_real & val ) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 116 of file [scalar\\_imp.h](#).

**6.23.2.36 to\_scalar\_t()** [3/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const long double & val ) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 89 of file [scalar\\_imp.h](#).

**6.23.2.37 to\_scalar\_t()** [4/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const long double & val ) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 107 of file [scalar\\_imp.h](#).

**6.23.2.38 to\_scalar\_t()** [5/9]

```
auto glucat::numeric_traits< double >::to_scalar_t (
    const Other_Scalar_T & val ) -> double    [inline]
```

Cast to double.

Definition at line 61 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**6.23.2.39 to\_scalar\_t()** [6/9]

```
auto glucat::numeric_traits< float >::to_scalar_t (
    const Other_Scalar_T & val ) -> float    [inline]
```

Extra traits which extend numeric limits.

Cast to float

Definition at line 52 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**6.23.2.40 to\_scalar\_t()** [7/9]

```
template<typename Scalar_T >
template<typename Other_Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_scalar_t (
    const Other_Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Cast to Scalar\_T.

Definition at line 141 of file [scalar.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix::nork\\_range\(\)](#), [glucat::to\\_demote\(\)](#), and [glucat::to\\_promote\(\)](#).

**6.23.2.41 to\_scalar\_t()** [8/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const qd_real & val ) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 98 of file [scalar\\_imp.h](#).

**6.23.2.42 to\_scalar\_t()** [9/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const qd_real & val ) -> long double    [inline]
```

Cast to long double.

Definition at line 80 of file [scalar\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/scalar.h](#)

## 6.24 pade::pade\_log\_denom< Scalar\_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< Scalar\_T, 14 >

### Static Public Attributes

- static const [array](#) [denom](#)

#### 6.24.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

Definition at line [1731](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.24.2 Member Typedef Documentation

##### 6.24.2.1 array

```
template<typename Scalar_T >
using pade::pade\_log\_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line [1733](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.24.3 Member Data Documentation



### 6.24.3.1 `denom`

```
template<typename Scalar_T >
const pade_log_denom< longdouble >::array pade::pade_log_denom< Scalar_T >::denom [static]
```

**Initial value:**

```
=
{
    1.0,                13.0/2.0,                468.0/25.0,                1573.0/50.0,
    1573.0/46.0,        11583.0/460.0,            10296.0/805.0,            2574.0/575.0,
    11583.0/10925.0,    143.0/874.0,                572.0/37145.0,            117.0/148580.0,
    13.0/742900.0,      1.0/10400600.0
}
```

Definition at line 1734 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.25 `pade::pade_log_denom< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

**Public Types**

- using [array](#) = `std::array< dd_real, 22 >`

**Static Public Attributes**

- static const [array](#) [denom](#)

### 6.25.1 Detailed Description

Definition at line 1820 of file [matrix\\_multi\\_imp.h](#).

### 6.25.2 Member Typedef Documentation

#### 6.25.2.1 `array`

```
using pade::pade_log_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1822 of file [matrix\\_multi\\_imp.h](#).

### 6.25.3 Member Data Documentation

#### 6.25.3.1 denom

```
const pade\_log\_denom< dd_real >::array pade::pade\_log\_denom< dd_real >::denom [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("2100")/dd_real("41"),
    dd_real("341145")/dd_real("1066"),
    dd_real("11069856")/dd_real("19721"),
    dd_real("6918660")/dd_real("19721"),
    dd_real("1410864")/dd_real("16687"),
    dd_real("734825")/dd_real("94054"),
    dd_real("348840")/dd_real("1363783"),
    dd_real("6783")/dd_real("2727566"),
    dd_real("266")/dd_real("53187537"),
    dd_real("7")/dd_real("8155422340"),
    dd_real("21")/dd_real("2"),
    dd_real("12635")/dd_real("82"),
    dd_real("1037799")/dd_real("2132"),
    dd_real("9883800")/dd_real("19721"),
    dd_real("293930")/dd_real("1517"),
    dd_real("88179")/dd_real("3034"),
    dd_real("305235")/dd_real("188108"),
    dd_real("40698")/dd_real("1363783"),
    dd_real("9975")/dd_real("70916716"),
    dd_real("7")/dd_real("70916716"),
    dd_real("1")/dd_real("538257874440")
}
```

Definition at line 1823 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.26 [pade::pade\\_log\\_denom](#)< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< float, 10 >

### Static Public Attributes

- static const [array](#) [denom](#)

#### 6.26.1 Detailed Description

Definition at line 1758 of file [matrix\\_multi\\_imp.h](#).

#### 6.26.2 Member Typedef Documentation

### 6.26.2.1 array

```
using pade::pade_log_denom< float >::array = std::array<float, 10>
```

Definition at line 1760 of file [matrix\\_multi\\_imp.h](#).

## 6.26.3 Member Data Documentation

### 6.26.3.1 denom

```
const pade_log_denom< float >::array pade::pade_log_denom< float >::denom [static]
```

#### Initial value:

```
=
{
    1.0,          9.0/2.0,      144.0/17.0,   147.0/17.0,
    441.0/85.0,   63.0/34.0,    84.0/221.0,   9.0/221.0,
    9.0/4862.0,   1.0/48620.0
}
```

Definition at line 1761 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.27 `pade::pade_log_denom< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< long double, 18 >

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.27.1 Detailed Description

Definition at line 1785 of file [matrix\\_multi\\_imp.h](#).

### 6.27.2 Member Typedef Documentation

### 6.27.2.1 array

```
using pade::pade_log_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1787 of file [matrix\\_multi\\_imp.h](#).

## 6.27.3 Member Data Documentation

### 6.27.3.1 denom

```
const array pade::pade_log_denom< long double >::denom [static]
```

Definition at line 1788 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.28 pade::pade\_log\_denom< qd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< qd\_real, 34 >

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.28.1 Detailed Description

Definition at line 1867 of file [matrix\\_multi\\_imp.h](#).

### 6.28.2 Member Typedef Documentation

### 6.28.2.1 array

```
using pade::pade_log_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1869 of file [matrix\\_multi\\_imp.h](#).

## 6.28.3 Member Data Documentation

### 6.28.3.1 denom

```
const pade_log_denom< qd_real >::array pade::pade_log_denom< qd_real >::denom [static]
```

**Initial value:**

```
=
{
    qd_real("1"),
    qd_real("33")/qd_real("2"),
    qd_real("8448")/qd_real("65"),
    qd_real("42284")/qd_real("65"),
    qd_real("211420")/qd_real("91"),
    qd_real("573562")/qd_real("91"),
    qd_real("32119472")/qd_real("2379"),
    qd_real("92917044")/qd_real("3965"),
    qd_real("603960786")/qd_real("17995"),
    qd_real("144626625")/qd_real("3599"),
    qd_real("2776831200")/qd_real("68381"),
    qd_real("16692542100")/qd_real("478667"),
    qd_real("12241197540")/qd_real("478667"),
    qd_real("1098569010")/qd_real("68381"),
    qd_real("31387686000")/qd_real("3624193"),
    qd_real("9939433900")/qd_real("2479711"),
    qd_real("67091178825")/qd_real("42155087"),
    qd_real("2683647153")/qd_real("4959422"),
    qd_real("19083713088")/qd_real("121505839"),
    qd_real("4708152900")/qd_real("121505839"),
    qd_real("941630580")/qd_real("116546417"),
    qd_real("88704330")/qd_real("62755763"),
    qd_real("12902448")/qd_real("62755763"),
    qd_real("1542684")/qd_real("62755763"),
    qd_real("6427850")/qd_real("2698497809"),
    qd_real("3471039")/qd_real("18889484663"),
    qd_real("8544096")/qd_real("774468871183"),
    qd_real("39556")/qd_real("79027435835"),
    qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
    qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
    qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
}
```

Definition at line 1870 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.29 pade::pade\_log\_numer< Scalar\_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

## Public Types

- using [array](#) = std::array< Scalar\_T, 14 >

## Static Public Attributes

- static const [array](#) [numer](#)

### 6.29.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

Definition at line [1714](#) of file [matrix\\_multi\\_imp.h](#).

### 6.29.2 Member Typedef Documentation

#### 6.29.2.1 array

```
template<typename Scalar_T >
using pade::pade\_log\_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line [1716](#) of file [matrix\\_multi\\_imp.h](#).

### 6.29.3 Member Data Documentation

#### 6.29.3.1 numer

```
template<typename Scalar_T >
const pade\_log\_numer< longdouble >::array pade::pade\_log\_numer< Scalar_T >::numer [static]
```

**Initial value:**

```
=
{
    0.0,          1.0,          6.0,          4741.0/300.0,
    1441.0/60.0,  107091.0/4600.0,  8638.0/575.0,  263111.0/40250.0,
    153081.0/80500.0,  395243.0/1101240.0,  28549.0/688275.0,  605453.0/228813200.0,
    785633.0/10296594000.0,  1145993.0/1873980108000.0
}
```

Definition at line [1717](#) of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.30 `pade::pade_log_numer< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array< dd_real, 22 >`

### Static Public Attributes

- static const `array number`

#### 6.30.1 Detailed Description

Definition at line 1800 of file `matrix_multi_imp.h`.

#### 6.30.2 Member Typedef Documentation

##### 6.30.2.1 `array`

```
using pade::pade_log_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1802 of file `matrix_multi_imp.h`.

#### 6.30.3 Member Data Documentation

##### 6.30.3.1 `number`

```
const pade_log_numer< dd_real >::array pade::pade_log_numer< dd_real >::number [static]
```

Initial value:

```
=
{
    dd_real("0"),
    dd_real("10"),
    dd_real("21603")/dd_real("164"),
    dd_real("978724")/dd_real("2665"),
    dd_real("12874933")/dd_real("39442"),
    dd_real("2406734")/dd_real("22755"),
    dd_real("30653165")/dd_real("2402928"),
    dd_real("25346331")/dd_real("47074027"),
    dd_real("105689791")/dd_real("15601677520"),
    dd_real("969715")/dd_real("53502994116"),
    dd_real("118999")/dd_real("26204577562592"),
    dd_real("1"),
    dd_real("22781")/dd_real("492"),
    dd_real("5492649")/dd_real("21320"),
    dd_real("4191605")/dd_real("10619"),
    dd_real("11473457")/dd_real("54612"),
    dd_real("166770367")/dd_real("4004880"),
    dd_real("647746389")/dd_real("215195552"),
    dd_real("278270613")/dd_real("3900419380"),
    dd_real("606046475")/dd_real("1379188292768"),
    dd_real("11098301")/dd_real("26204577562592"),
    dd_real("18858053")/dd_real("1392249205900512960")
}
```

Definition at line 1803 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.31 pade::pade\_log\_numer< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< float, 10 >

### Static Public Attributes

- static const [array numer](#)

#### 6.31.1 Detailed Description

Definition at line [1746](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.31.2 Member Typedef Documentation

##### 6.31.2.1 array

```
using pade::pade\_log\_numer< float >::array = std::array<float, 10>
```

Definition at line [1748](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.31.3 Member Data Documentation

##### 6.31.3.1 numer

```
const pade\_log\_numer< float >::array pade::pade\_log\_numer< float >::numer [static]
```

##### Initial value:

```
=
{
    0.0,          1.0,          4.0,          1337.0/204.0,
    385.0/68.0,    1879.0/680.0,    193.0/255.0,    197.0/1820.0,
    419.0/61880.0, 7129.0/61261200.0
}
```

Definition at line [1749](#) of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)



## 6.32 `pade::pade_log_numer< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array< long double, 18 >`

### Static Public Attributes

- static const `array numer`

#### 6.32.1 Detailed Description

Definition at line 1771 of file `matrix_multi_imp.h`.

#### 6.32.2 Member Typedef Documentation

##### 6.32.2.1 `array`

```
using pade::pade_log_numer< long double >::array = std::array<long double, 18>
```

Definition at line 1773 of file `matrix_multi_imp.h`.

#### 6.32.3 Member Data Documentation

##### 6.32.3.1 `numer`

```
const array pade::pade_log_numer< long double >::numer [static]
```

Definition at line 1774 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.33 `pade::pade_log_numer< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

## Public Types

- using [array](#) = std::array< qd\_real, 34 >

## Static Public Attributes

- static const [array](#) [numer](#)

### 6.33.1 Detailed Description

Definition at line 1841 of file [matrix\\_multi\\_imp.h](#).

### 6.33.2 Member Typedef Documentation

#### 6.33.2.1 array

```
using pade::pade\_log\_numer< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1843 of file [matrix\\_multi\\_imp.h](#).

### 6.33.3 Member Data Documentation

#### 6.33.3.1 numer

```
const pade\_log\_numer< qd_real >::array pade::pade\_log\_numer< qd_real >::numer [static]
```

Initial value:

```
=
{
    qd_real("0"),
    qd_real("16"),
    qd_real("95201")/qd_real("780"),
    qd_real("30721")/qd_real("52"),
    qd_real("7416257")/qd_real("3640"),
    qd_real("1039099")/qd_real("195"),
    qd_real("6097772319")/qd_real("555100"),
    qd_real("1564058073")/qd_real("85400"),
    qd_real("30404640205")/qd_real("1209264"),
    qd_real("725351278")/qd_real("25193"),
    qd_real("4092322670789")/qd_real("147429436"),
    qd_real("4559713849589")/qd_real("201040140"),
    qd_real("5049361751189")/qd_real("320023080"),
    qd_real("74979677195")/qd_real("8000577"),
    qd_real("16569850691873")/qd_real("3481514244"),
    qd_real("1065906022369")/qd_real("515779888"),
    qd_real("335956770855841")/qd_real("438412904800"),
    qd_real("1462444287585964")/qd_real("6041877844275"),
    qd_real("397242326339851")/qd_real("6122436215532"),
    qd_real("64211291334131")/qd_real("4373168725380"),
    qd_real("142322343550859")/qd_real("51080680851480"),
    qd_real("154355972958659")/qd_real("351179680853925"),
    qd_real("167483568676259")/qd_real("2937139148960100"),
    qd_real("1"),
```

```

    qd_real("4230788929433")/qd_real("704913395750424"),
    qd_real("197968763176019")/qd_real("392923948371995600"),
    qd_real("10537522306718")/qd_real("319250708052246425"),
    qd_real("236648286272519")/qd_real("144249197475035425500"),
    qd_real("260715545088119")/qd_real("4375558990076074573500"),
    qd_real("289596255666839")/qd_real("192874640282553367199880"),
    qd_real("8802625510547")/qd_real("361639950529787563499775"),
    qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
    qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
    qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
}

```

Definition at line 1844 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.34 pade::pade\_sqrt\_denom< Scalar\_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< Scalar\_T, 14 >

### Static Public Attributes

- static const [array](#) [denom](#)

#### 6.34.1 Detailed Description

```

template<typename Scalar_T>
struct pade::pade_sqrt_denom< Scalar_T >

```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

Definition at line 1401 of file [matrix\\_multi\\_imp.h](#).

#### 6.34.2 Member Typedef Documentation

##### 6.34.2.1 array

```

template<typename Scalar_T>
using pade::pade\_sqrt\_denom< Scalar_T >::array = std::array<Scalar_T, 14>

```

Definition at line 1403 of file [matrix\\_multi\\_imp.h](#).

### 6.34.3 Member Data Documentation

#### 6.34.3.1 denom

```
template<typename Scalar_T >
const pade_sqrt_denom< longdouble >::array pade::pade_sqrt_denom< Scalar_T >::denom [static]
```

##### Initial value:

```
=
{
    1.0,                25.0/4.0,        69.0/4.0,        1771.0/64.0,
    7315.0/256.0,       20349.0/1024.0,   4845.0/512.0,    12597.0/4096.0,
    21879.0/32768.0,    12155.0/131072.0,  1001.0/131072.0,  1365.0/4194304.0,
    91.0/16777216.0,   1.0/67108864.0
}
```

Definition at line 1404 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.35 pade::pade\_sqrt\_denom< dd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< dd\_real, 22 >

### Static Public Attributes

- static const [array](#) [denom](#)

#### 6.35.1 Detailed Description

Definition at line 1491 of file [matrix\\_multi\\_imp.h](#).

#### 6.35.2 Member Typedef Documentation

##### 6.35.2.1 array

```
using pade::pade_sqrt_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1493 of file [matrix\\_multi\\_imp.h](#).

### 6.35.3 Member Data Documentation

#### 6.35.3.1 `denom`

```
const pade_sqrt_denom< dd_real >::array pade::pade_sqrt_denom< dd_real >::denom [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("195")/dd_real("4"),
    dd_real("73815")/dd_real("256"),
    dd_real("121737")/dd_real("256"),
    dd_real("4539051")/dd_real("16384"),
    dd_real("4032015")/dd_real("65536"),
    dd_real("86493225")/dd_real("16777216"),
    dd_real("5014575")/dd_real("33554432"),
    dd_real("5311735")/dd_real("4294967296"),
    dd_real("33649")/dd_real("17179869184"),
    dd_real("231")/dd_real("1099511627776"),
    dd_real("41")/dd_real("4"),
    dd_real("9139")/dd_real("64"),
    dd_real("435897")/dd_real("1024"),
    dd_real("840565")/dd_real("2048"),
    dd_real("9641775")/dd_real("65536"),
    dd_real("84672315")/dd_real("4194304"),
    dd_real("67863915")/dd_real("67108864"),
    dd_real("4345965")/dd_real("268435456"),
    dd_real("1081575")/dd_real("17179869184"),
    dd_real("8855")/dd_real("274877906944"),
    dd_real("1")/dd_real("4398046511104")
}
```

Definition at line 1494 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.36 `pade::pade_sqrt_denom< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array< float, 10 >`

### Static Public Attributes

- static const `array` `denom`

#### 6.36.1 Detailed Description

Definition at line 1428 of file `matrix_multi_imp.h`.

#### 6.36.2 Member Typedef Documentation

### 6.36.2.1 array

```
using pade::pade_sqrt_denom< float >::array = std::array<float, 10>
```

Definition at line 1430 of file [matrix\\_multi\\_imp.h](#).

## 6.36.3 Member Data Documentation

### 6.36.3.1 denom

```
const pade_sqrt_denom< float >::array pade::pade_sqrt_denom< float >::denom [static]
```

#### Initial value:

```
=
{
    1.0,          17.0/4.0,      15.0/2.0,      455.0/64.0,
    1001.0/256.0,  1287.0/1024.0,  231.0/1024.0,  165.0/8192.0,
    45.0/65536,   1.0/262144.0
}
```

Definition at line 1431 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.37 pade::pade\_sqrt\_denom< long double > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< long double, 18 >

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.37.1 Detailed Description

Definition at line 1455 of file [matrix\\_multi\\_imp.h](#).

### 6.37.2 Member Typedef Documentation

### 6.37.2.1 array

```
using pade::pade_sqrt_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1457 of file [matrix\\_multi\\_imp.h](#).

## 6.37.3 Member Data Documentation

### 6.37.3.1 denom

```
const array pade::pade_sqrt_denom< long double >::denom [static]
```

Definition at line 1458 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.38 `pade::pade_sqrt_denom< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< qd\_real, 34 >

### Static Public Attributes

- static const [array](#) [denom](#)

### 6.38.1 Detailed Description

Definition at line 1538 of file [matrix\\_multi\\_imp.h](#).

### 6.38.2 Member Typedef Documentation

### 6.38.2.1 array

```
using pade::pade_sqrt_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1540 of file [matrix\\_multi\\_imp.h](#).

## 6.38.3 Member Data Documentation

### 6.38.3.1 denom

```
const pade_sqrt_denom< qd_real >::array pade::pade_sqrt_denom< qd_real >::denom [static]
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("126"),
    qd_real("557845")/qd_real("256"),
    qd_real("12515965")/qd_real("1024"),
    qd_real("1916797311")/qd_real("65536"),
    qd_real("4450881435")/qd_real("131072"),
    qd_real("171503444385")/qd_real("8388608"),
    qd_real("221120793075")/qd_real("33554432"),
    qd_real("4923689695575")/qd_real("4294967296"),
    qd_real("456864812569")/qd_real("4294967296"),
    qd_real("3486599885395")/qd_real("137438953472"),
    qd_real("2804116503573")/qd_real("549755813888"),
    qd_real("1886827875075")/qd_real("219902325552"),
    qd_real("263012370465")/qd_real("219902325552"),
    qd_real("240141729555")/qd_real("17592186044416"),
    qd_real("176848560525")/qd_real("140737488355328"),
    qd_real("51538723353")/qd_real("562949953421312"),
    qd_real("1450433115")/qd_real("281474976710656"),
    qd_real("977699359")/qd_real("4503599627370496"),
    qd_real("118183439")/qd_real("18014398509481984"),
    qd_real("9652005")/qd_real("72057594037927936"),
    qd_real("121737")/qd_real("72057594037927936"),
    qd_real("6545")/qd_real("576460752303423488"),
    qd_real("561")/qd_real("18446744073709551616"),
    qd_real("1")/qd_real("73786976294838206464")
}
```

Definition at line 1541 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.39 pade::pade\_sqrt\_numer< Scalar\_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< Scalar\_T, 14 >



## Static Public Attributes

- static const [array](#) `number`

### 6.39.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by `Pade1(sqrt(1+x),x,n,n)`

Definition at line [1384](#) of file [matrix\\_multi\\_imp.h](#).

### 6.39.2 Member Typedef Documentation

#### 6.39.2.1 `array`

```
template<typename Scalar_T >
using pade::pade\_sqrt\_numer< Scalar\_T >::array = std::array<Scalar_T, 14>
```

Definition at line [1386](#) of file [matrix\\_multi\\_imp.h](#).

### 6.39.3 Member Data Documentation

#### 6.39.3.1 `number`

```
template<typename Scalar_T >
const pade\_sqrt\_numer< longdouble >::array pade::pade\_sqrt\_numer< Scalar\_T >::number [static]
```

**Initial value:**

```
=
{
    1.0,          27.0/4.0,          81.0/4.0,          2277.0/64.0,
  10395.0/256.0,  32319.0/1024.0,   8721.0/512.0,   26163.0/4096.0,
  53703.0/32768.0, 36465.0/131072.0, 3861.0/131072.0, 7371.0/4194304.0,
    819.0/16777216.0, 27.0/67108864.0
}
```

Definition at line [1387](#) of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.40 pade::pade\_sqrt\_numer< dd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using [array](#) = std::array< dd\_real, 22 >

### Static Public Attributes

- static const [array](#) [number](#)

#### 6.40.1 Detailed Description

Definition at line [1471](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.40.2 Member Typedef Documentation

##### 6.40.2.1 array

```
using pade::pade\_sqrt\_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line [1473](#) of file [matrix\\_multi\\_imp.h](#).

#### 6.40.3 Member Data Documentation

##### 6.40.3.1 number

```
const pade\_sqrt\_numer< dd_real >::array pade::pade\_sqrt\_numer< dd_real >::number [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("215")/dd_real("4"),
    dd_real("90687")/dd_real("256"),
    dd_real("168861")/dd_real("256"),
    dd_real("7228859")/dd_real("16384"),
    dd_real("7538115")/dd_real("65536"),
    dd_real("195747825")/dd_real("16777216"),
    dd_real("14375115")/dd_real("33554432"),
    dd_real("20764055")/dd_real("4294967296"),
    dd_real("206701")/dd_real("17179869184"),
    dd_real("3311")/dd_real("1099511627776"),
    dd_real("43")/dd_real("4"),
    dd_real("10621")/dd_real("64"),
    dd_real("567987")/dd_real("1024"),
    dd_real("1246355")/dd_real("2048"),
    dd_real("16583853")/dd_real("65536"),
    dd_real("173376645")/dd_real("4194304"),
    dd_real("171655785")/dd_real("67108864"),
    dd_real("14375115")/dd_real("268435456"),
    dd_real("5167525")/dd_real("17179869184"),
    dd_real("76153")/dd_real("274877906944"),
    dd_real("43")/dd_real("4398046511104")
}
```

Definition at line [1474](#) of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.41 `pade::pade_sqrt_numer< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array< float, 10 >`

### Static Public Attributes

- static const `array numer`

#### 6.41.1 Detailed Description

Definition at line 1416 of file `matrix_multi_imp.h`.

#### 6.41.2 Member Typedef Documentation

##### 6.41.2.1 `array`

```
using pade::pade_sqrt_numer< float >::array = std::array<float, 10>
```

Definition at line 1418 of file `matrix_multi_imp.h`.

#### 6.41.3 Member Data Documentation

##### 6.41.3.1 `numer`

```
const pade_sqrt_numer< float >::array pade::pade_sqrt_numer< float >::numer [static]
```

##### Initial value:

```
=  
{  
    1.0,          19.0/4.0,      19.0/2.0,      665.0/64.0,  
    1729.0/256.0,  2717.0/1024.0,  627.0/1024.0,  627.0/8192.0,  
    285.0/65536.0, 19.0/262144.0  
}
```

Definition at line 1419 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.42 `pade::pade_sqrt_numer< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- using `array` = `std::array< long double, 18 >`

### Static Public Attributes

- static const `array numer`

#### 6.42.1 Detailed Description

Definition at line 1441 of file `matrix_multi_imp.h`.

#### 6.42.2 Member Typedef Documentation

##### 6.42.2.1 `array`

```
using pade::pade_sqrt_numer< long double >::array = std::array<long double, 18>
```

Definition at line 1443 of file `matrix_multi_imp.h`.

#### 6.42.3 Member Data Documentation

##### 6.42.3.1 `numer`

```
const array pade::pade_sqrt_numer< long double >::numer [static]
```

Definition at line 1444 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.43 `pade::pade_sqrt_numer< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

## Public Types

- using [array](#) = std::array< qd\_real, 34 >

## Static Public Attributes

- static const [array number](#)

### 6.43.1 Detailed Description

Definition at line 1512 of file [matrix\\_multi\\_imp.h](#).

### 6.43.2 Member Typedef Documentation

#### 6.43.2.1 array

using [pade::pade\\_sqrt\\_numer< qd\\_real >::array](#) = std::array<qd\_real, 34>

Definition at line 1514 of file [matrix\\_multi\\_imp.h](#).

### 6.43.3 Member Data Documentation

#### 6.43.3.1 numer

const [pade\\_sqrt\\_numer< qd\\_real >::array pade::pade\\_sqrt\\_numer< qd\\_real >::numer](#) [static]

**Initial value:**

```
=
{
    qd_real("1"),
    qd_real("134"),
    qd_real("633485")/qd_real("256"),
    qd_real("15246721")/qd_real("1024"),
    qd_real("2518145487")/qd_real("65536"),
    qd_real("6344873535")/qd_real("131072"),
    qd_real("267226297065")/qd_real("8388608"),
    qd_real("379874182975")/qd_real("33554432"),
    qd_real("9425348845815")/qd_real("4294967296"),
    qd_real("987417498133")/qd_real("4294967296"),
    qd_real("8055248011085")/qd_real("137438953472"),
    qd_real("6958363175533")/qd_real("549755813888"),
    qd_real("5056698705201")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("17592186044416"),
    qd_real("623623871325")/qd_real("140737488355328"),
    qd_real("203123203803")/qd_real("562949953421312"),
    qd_real("6478601247")/qd_real("281474976710656"),
    qd_real("5038912081")/qd_real("4503599627370496"),
    qd_real("719844583")/qd_real("18014398509481984"),
    qd_real("71853815")/qd_real("72057594037927936"),
    qd_real("1165197")/qd_real("72057594037927936"),
    qd_real("87703")/qd_real("576460752303423488"),
    qd_real("12529")/qd_real("18446744073709551616"),
    qd_real("67")/qd_real("73786976294838206464"),
    qd_real("67")/qd_real("4"),
    qd_real("43617")/qd_real("64"),
    qd_real("6992857")/qd_real("1024"),
    qd_real("215632197")/qd_real("8192"),
    qd_real("12301285425")/qd_real("262144"),
    qd_real("89075432355")/qd_real("2097152"),
    qd_real("687479618945")/qd_real("33554432"),
    qd_real("1443521895305")/qd_real("268435456"),
    qd_real("13195488384141")/qd_real("17179869184"),
}
```

Definition at line 1515 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.44 `glucat::numeric_traits< Scalar_T >::promoted` Struct Reference

Extra traits which extend numeric limits.

```
#include <promotion.h>
```

### Public Types

- using `type` = double
- using `type` = long double
- using `type` = double

#### 6.44.1 Detailed Description

```
template<typename Scalar_T>  
struct glucat::numeric_traits< Scalar_T >::promoted
```

Extra traits which extend numeric limits.

Promoted type.

Promoted type for long double.

Promoted type for double

Definition at line 70 of file [promotion.h](#).

#### 6.44.2 Member Typedef Documentation

##### 6.44.2.1 `type` [1/3]

```
template<typename Scalar_T >  
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 72 of file [promotion.h](#).

##### 6.44.2.2 `type` [2/3]

```
template<typename Scalar_T >  
using glucat::numeric_traits< Scalar_T >::promoted::type = long double
```

Definition at line 86 of file [promotion.h](#).

### 6.44.2.3 type [3/3]

```
template<typename Scalar_T >
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 145 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

## 6.45 glucat::random\_generator< Scalar\_T > Class Template Reference

Random number generator with single instance per `Scalar_T`.

```
#include <random.h>
```

### Public Member Functions

- [random\\_generator](#) (const [random\\_generator](#) &)=delete
- auto [operator=](#) (const [random\\_generator](#) &) -> [random\\_generator](#) &=delete
- auto [uniform](#) () -> `Scalar_T`
- auto [normal](#) () -> `Scalar_T`

### Static Public Member Functions

- static auto [generator](#) () -> [random\\_generator](#) &  
*Single instance of Random number generator.*

### Private Member Functions

- [random\\_generator](#) ()
- [~random\\_generator](#) ()=default

### Private Attributes

- `std::mt19937` [uint\\_gen](#)
- `std::uniform_real_distribution< double >` [uniform\\_dist](#)
- `std::normal_distribution< double >` [normal\\_dist](#)

### Static Private Attributes

- static const unsigned long [seed](#) = 19590921UL

## Friends

- class [friend\\_for\\_private\\_destructor](#)

### 6.45.1 Detailed Description

```
template<typename Scalar_T>
class glucat::random_generator< Scalar_T >
```

Random number generator with single instance per Scalar\_T.

Definition at line 42 of file [random.h](#).

### 6.45.2 Constructor & Destructor Documentation

#### 6.45.2.1 random\_generator() [1/2]

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::random_generator (
    const random_generator< Scalar_T > & ) [delete]
```

#### 6.45.2.2 random\_generator() [2/2]

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::random_generator ( ) [inline], [private]
```

Definition at line 61 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::seed](#).

#### 6.45.2.3 ~random\_generator()

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::~~random_generator ( ) [private], [default]
```

### 6.45.3 Member Function Documentation



### 6.45.3.1 generator()

```
template<typename Scalar_T >
static auto glucat::random_generator< Scalar_T >::generator ( ) -> random_generator&    [inline],
[static]
```

Single instance of Random number generator.

Definition at line 51 of file [random.h](#).

### 6.45.3.2 normal()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::normal ( ) -> Scalar_T    [inline]
```

Definition at line 70 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::normal\\_dist](#).

### 6.45.3.3 operator=()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::operator= (
    const random_generator< Scalar_T > & ) -> random_generator &=delete    [delete]
```

### 6.45.3.4 uniform()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::uniform ( ) -> Scalar_T    [inline]
```

Definition at line 68 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::uniform\\_dist](#).

## 6.45.4 Friends And Related Function Documentation

### 6.45.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T >
friend class friend_for_private_destructor    [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 48 of file [random.h](#).

## 6.45.5 Member Data Documentation

### 6.45.5.1 normal\_dist

```
template<typename Scalar_T >
std::normal_distribution<double> glucat::random_generator< Scalar_T >::normal_dist [private]
```

Definition at line 59 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::normal\(\)](#).

### 6.45.5.2 seed

```
template<typename Scalar_T >
const unsigned long glucat::random_generator< Scalar_T >::seed = 19590921UL [static], [private]
```

Definition at line 55 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::random\\_generator\(\)](#).

### 6.45.5.3 uint\_gen

```
template<typename Scalar_T >
std::mt19937 glucat::random_generator< Scalar_T >::uint_gen [private]
```

Definition at line 57 of file [random.h](#).

### 6.45.5.4 uniform\_dist

```
template<typename Scalar_T >
std::uniform_real_distribution<double> glucat::random_generator< Scalar_T >::uniform_dist
[private]
```

Definition at line 58 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::uniform\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/random.h](#)

## 6.46 glucat::index\_set< LO, HI >::reference Class Reference

Index set member reference.

```
#include <index_set.h>
```

Collaboration diagram for glucat::index\_set< LO, HI >::reference:

### Public Member Functions

- [reference](#) ()=delete  
*Default constructor is deleted.*
- [reference](#) ([index\\_set\\_t](#) &ist, [index\\_t](#) idx)  
*index\_set reference*
- [~reference](#) ()=default
- auto [operator==](#) (const [reference](#) &c\_j) const -> bool  
*for b[i] == c[j];*
- auto [operator=](#) (const bool x) -> [reference](#) &  
*for b[i] = x;*
- auto [operator=](#) (const [reference](#) &c\_j) -> [reference](#) &  
*for b[i] = c[j];*
- auto [operator~](#) () const -> bool  
*Flips a bit.*
- [operator bool](#) () const  
*for x = b[i];*
- auto [flip](#) () -> [reference](#) &  
*for b[i].flip();*

### Private Attributes

- [index\\_set\\_t](#) \* m\_pst
- [index\\_t](#) m\_idx

### Friends

- class [index\\_set](#)

#### 6.46.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set< LO, HI >::reference
```

Index set member reference.

Definition at line 177 of file [index\\_set.h](#).

## 6.46.2 Constructor & Destructor Documentation

### 6.46.2.1 `reference()` [1/2]

```
template<const index\_t LO, const index\_t HI>  
glucat::index\_set< LO, HI >::reference::reference ( ) [delete]
```

Default constructor is deleted.

### 6.46.2.2 `reference()` [2/2]

```
template<const index\_t LO, const index\_t HI>  
glucat::index\_set< LO, HI >::reference::reference (   
    index\_set\_t & ist,  
    index\_t idx ) [inline]
```

[index\\_set](#) reference

Definition at line 985 of file [index\\_set\\_imp.h](#).

### 6.46.2.3 `~reference()`

```
template<const index\_t LO, const index\_t HI>  
glucat::index\_set< LO, HI >::reference::~reference ( ) [default]
```

## 6.46.3 Member Function Documentation

### 6.46.3.1 `flip()`

```
template<const index\_t LO, const index\_t HI>  
auto glucat::index\_set< LO, HI >::reference::flip [inline]
```

for `b[i].flip()`;

Definition at line 1049 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set](#)< LO, HI >::reference::flip().

Referenced by [glucat::index\\_set](#)< LO, HI >::reference::flip().

### 6.46.3.2 operator bool()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::operator bool  [inline]
```

for x = b[i];

Definition at line 1041 of file [index\\_set\\_imp.h](#).

### 6.46.3.3 operator=() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const bool x ) -> reference&  [inline]
```

for b[i] = x;

Definition at line 1003 of file [index\\_set\\_imp.h](#).

### 6.46.3.4 operator=() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const reference & c_j ) -> reference&  [inline]
```

for b[i] = c[j];

Definition at line 1017 of file [index\\_set\\_imp.h](#).

### 6.46.3.5 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator== (
    const reference & c_j ) const -> bool  [inline]
```

for b[i] == c[j];

Definition at line 995 of file [index\\_set\\_imp.h](#).

### 6.46.3.6 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator~ [inline]
```

Flips a bit.

flips the bit

Definition at line 1034 of file [index\\_set\\_imp.h](#).

## 6.46.4 Friends And Related Function Documentation

### 6.46.4.1 index\_set

```
template<const index_t LO, const index_t HI>
friend class index_set [friend]
```

Definition at line 178 of file [index\\_set.h](#).

## 6.46.5 Member Data Documentation

### 6.46.5.1 m\_idx

```
template<const index_t LO, const index_t HI>
index_t glucat::index_set< LO, HI >::reference::m_idx [private]
```

Definition at line 200 of file [index\\_set.h](#).

### 6.46.5.2 m\_pst

```
template<const index_t LO, const index_t HI>
index_set_t* glucat::index_set< LO, HI >::reference::m_pst [private]
```

Definition at line 199 of file [index\\_set.h](#).

The documentation for this class was generated from the following files:

- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.47 glucat::sorted\_range< Map\_T, Sorted\_Map\_T > Class Template Reference

Sorted range for use with output.

```
#include <framed_multi_imp.h>
```

### Public Types

- using [map\\_t](#) = Map\_T
- using [sorted\\_map\\_t](#) = Sorted\_Map\_T
- using [sorted\\_iterator](#) = typename Sorted\_Map\_T::const\_iterator

### Public Member Functions

- [sorted\\_range](#) (Sorted\_Map\_T &sorted\_val, const Map\_T &val)

### Public Attributes

- [sorted\\_iterator](#) [sorted\\_begin](#)
- [sorted\\_iterator](#) [sorted\\_end](#)

#### 6.47.1 Detailed Description

```
template<typename Map_T, typename Sorted_Map_T>  
class glucat::sorted_range< Map_T, Sorted_Map_T >
```

Sorted range for use with output.

Definition at line 1112 of file [framed\\_multi\\_imp.h](#).

#### 6.47.2 Member Typedef Documentation

##### 6.47.2.1 map\_t

```
template<typename Map_T , typename Sorted_Map_T >  
using glucat::sorted\_range< Map_T, Sorted_Map_T >::map_t = Map_T
```

Definition at line 1115 of file [framed\\_multi\\_imp.h](#).

### 6.47.2.2 sorted\_iterator

```
template<typename Map_T , typename Sorted_Map_T >
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T↵
::const_iterator
```

Definition at line 1117 of file [framed\\_multi\\_imp.h](#).

### 6.47.2.3 sorted\_map\_t

```
template<typename Map_T , typename Sorted_Map_T >
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1116 of file [framed\\_multi\\_imp.h](#).

## 6.47.3 Constructor & Destructor Documentation

### 6.47.3.1 sorted\_range()

```
template<typename Map_T , typename Sorted_Map_T >
glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Map_T & val ) [inline]
```

Definition at line 1119 of file [framed\\_multi\\_imp.h](#).

References [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::sorted\\_begin](#), and [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::s](#)

## 6.47.4 Member Data Documentation

### 6.47.4.1 sorted\_begin

```
template<typename Map_T , typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1126 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::sorted\\_range\(\)](#).



#### 6.47.4.2 `sorted_end`

```
template<typename Map_T , typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1127 of file `framed_multi_imp.h`.

Referenced by `glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range()`.

The documentation for this class was generated from the following file:

- `glucat/framed_multi_imp.h`

## 6.48 `glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >` Class Template Reference

```
#include <framed_multi_imp.h>
```

### Public Types

- using `map_t` = `Sorted_Map_T`
- using `sorted_map_t` = `Sorted_Map_T`
- using `sorted_iterator` = `typename Sorted_Map_T::const_iterator`

### Public Member Functions

- `sorted_range` (`Sorted_Map_T &sorted_val, const Sorted_Map_T &val`)

### Public Attributes

- `sorted_iterator sorted_begin`
- `sorted_iterator sorted_end`

#### 6.48.1 Detailed Description

```
template<typename Sorted_Map_T>
class glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >
```

Definition at line 1131 of file `framed_multi_imp.h`.

#### 6.48.2 Member Typedef Documentation

### 6.48.2.1 map\_t

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::map_t = Sorted_Map_T
```

Definition at line 1134 of file [framed\\_multi\\_imp.h](#).

### 6.48.2.2 sorted\_iterator

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T::const_iterator
```

Definition at line 1136 of file [framed\\_multi\\_imp.h](#).

### 6.48.2.3 sorted\_map\_t

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1135 of file [framed\\_multi\\_imp.h](#).

## 6.48.3 Constructor & Destructor Documentation

### 6.48.3.1 sorted\_range()

```
template<typename Sorted_Map_T >
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Sorted_Map_T & val ) [inline]
```

Definition at line 1138 of file [framed\\_multi\\_imp.h](#).

## 6.48.4 Member Data Documentation

#### 6.48.4.1 sorted\_begin

```
template<typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1142 of file [framed\\_multi\\_imp.h](#).

#### 6.48.4.2 sorted\_end

```
template<typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1143 of file [framed\\_multi\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi\\_imp.h](#)

## 6.49 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term Class Reference

Variable term.

Inheritance diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:

Collaboration diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:

### Public Types

- using [var\\_pair\\_t](#) = std::pair< [index\\_set](#)< LO, HI >, Scalar\_T >

### Public Member Functions

- [~var\\_term](#) ()=default  
*Destructor.*
- [var\\_term](#) ()  
*Default constructor.*
- [var\\_term](#) (const [index\\_set\\_t](#) ist, const Scalar\_T &crd=Scalar\_T(1))  
*Construct a variable term from an index set and a scalar coordinate.*
- auto [operator\\*=](#) (const [term\\_t](#) &rhs) -> [var\\_term\\_t](#) &  
*Product of variable term and term.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*

### 6.49.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P =
tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term
```

Variable term.

Definition at line 279 of file [framed\\_multi.h](#).

### 6.49.2 Member Typedef Documentation

#### 6.49.2.1 [var\\_pair\\_t](#)

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_pair_t = std::pair<index\_set<LO,
HI>, Scalar_T>
```

Definition at line 283 of file [framed\\_multi.h](#).

### 6.49.3 Constructor & Destructor Documentation

#### 6.49.3.1 [~var\\_term\(\)](#)

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::~~var_term ( ) [default]
```

Destructor.

#### 6.49.3.2 [var\\_term\(\)](#) [1/2]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term ( ) [inline]
```

Default constructor.

Definition at line 291 of file [framed\\_multi.h](#).

### 6.49.3.3 var\_term() [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term (
    const index_set_t ist,
    const Scalar_T & crd = Scalar_T(1) ) [inline]
```

Construct a variable term from an index set and a scalar coordinate.

Definition at line 295 of file [framed\\_multi.h](#).

## 6.49.4 Member Function Documentation

### 6.49.4.1 classname()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
static auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::classname ( ) ->
const std::string [inline], [static]
```

Class name used in messages.

Definition at line 286 of file [framed\\_multi.h](#).

### 6.49.4.2 operator\*=( )

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::operator*= (
    const term_t & rhs ) -> var_term_t& [inline]
```

Product of variable term and term.

Definition at line 299 of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi.h](#)



# Chapter 7

## File Documentation

### 7.1 glucat/clifford\_algebra.h File Reference

```
#include "glucat/global.h"
#include <limits>
#include <string>
#include <utility>
#include <vector>
```

Include dependency graph for clifford\_algebra.h: This graph shows which files directly or indirectly include this file:

#### Classes

- class [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >](#)  
*clifford\_algebra<> declares the operations of a Clifford algebra*

#### Namespaces

- namespace [glucat](#)

#### Macros

- [#define \\_GLUCAT\\_CLIFFORD\\_ALGEBRA\\_OPERATIONS](#)

#### Functions

- [template<template< typename, const index\\_t, const index\\_t, typename > class Multivector, template< typename, const index\\_t, const index\\_t, typename > class RHS, typename Scalar\\_T , const index\\_t LO, const index\\_t HI, typename Tune\\_P >](#)  
[auto glucat::operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of multivectors.*
- [template<template< typename, const index\\_t, const index\\_t, typename > class Multivector, typename Scalar\\_T , const index\\_t LO, const index\\_t HI, typename Tune\\_P >](#)  
[auto glucat::operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> bool  
*Test for inequality of multivector and scalar.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of scalar and multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Quadratic norm error tolerance relative to a specific multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`  
*Relative or absolute error using the quadratic norm.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of multivector and scalar.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of scalar and multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric difference of multivector and scalar.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric difference of scalar and multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric difference.*



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- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric multiplicative inverse.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Integer power of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Multivector power of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Outer product power of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Scalar part.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Real part: synonym for scalar part.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Imaginary part: deprecated (always 0)*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Pure part.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Even part.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Odd part.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`  
*Vector part of multivector, as a vector\_t with respect to frame()*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}\{2\} \rightarrow \{2\}\{1\}$ .*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value ==  $\sqrt{\text{norm}}$*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::max_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::clifford_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



*Inverse sine of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse sine of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic sine of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic sine of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic sine of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic tangent of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector.*

## 7.1.1 Macro Definition Documentation

### 7.1.1.1 \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

```
#define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
```

Definition at line 145 of file [clifford\\_algebra.h](#).

## 7.2 clifford\_algebra.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     clifford_algebra.h : Declare the operations of a Clifford algebra
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <limits>
00037 #include <string>
00038 #include <utility>
00039 #include <vector>
00040
00041 namespace glucat
00042 {
00043     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00044     class clifford_algebra
00045     {
00046     public:
00047         using scalar_t = Scalar_T;
00048         using index_set_t = Index_Set_T;
00049         static const index_t v_lo = index_set_t::v_lo;
00050         static const index_t v_hi = index_set_t::v_hi;
00051         using multivector_t = Multivector_T;
00052         using pair_t = std::pair<const index_set_t, Scalar_T>;
00053         using vector_t = std::vector<Scalar_T>;
00054
00055         static auto classname() -> const std::string;
00056
00057         static const Scalar_T default_truncation;
00058
00059         virtual ~clifford_algebra() = default;
00060
00061         // clifford_algebra operations
```

```

00065     virtual auto operator==(const multivector_t& val) const -> bool = 0;
00067     virtual auto operator==(const Scalar_T& scr) const -> bool = 0;
00069     virtual auto operator+=(const multivector_t& rhs) -> multivector_t& = 0;
00071     virtual auto operator+=(const Scalar_T& scr) -> multivector_t& = 0;
00073     virtual auto operator-=(const multivector_t& rhs) -> multivector_t& = 0;
00075     virtual auto operator-=(const Scalar_T& scr) -> multivector_t& = 0;
00077     virtual auto operator-() const -> const multivector_t = 0;
00079     virtual auto operator*=(const Scalar_T& scr) -> multivector_t& = 0;
00081     virtual auto operator*=(const multivector_t& rhs) -> multivector_t& = 0;
00083     virtual auto operator%=(const multivector_t& rhs) -> multivector_t& = 0;
00085     virtual auto operator&=(const multivector_t& rhs) -> multivector_t& = 0;
00087     virtual auto operator^=(const multivector_t& rhs) -> multivector_t& = 0;
00089     virtual auto operator/=(const Scalar_T& scr) -> multivector_t& = 0;
00091     virtual auto operator/=(const multivector_t& rhs) -> multivector_t& = 0;
00093     virtual auto operator|=(const multivector_t& rhs) -> multivector_t& = 0;
00095     virtual auto inv() const -> const multivector_t = 0;
00097     virtual auto pow(int m) const -> const multivector_t = 0;
00099     virtual auto outer_pow(int m) const -> const multivector_t = 0;
00101     virtual auto frame() const -> const index_set_t = 0;
00103     virtual auto grade() const -> index_t = 0;
00105     virtual auto operator[] (const index_set_t ist) const -> Scalar_T = 0;
00107     virtual auto operator() (index_t grade) const -> const multivector_t = 0;
00109     virtual auto scalar() const -> Scalar_T = 0;
00111     virtual auto pure() const -> const multivector_t = 0;
00113     virtual auto even() const -> const multivector_t = 0;
00115     virtual auto odd() const -> const multivector_t = 0;
00117     virtual auto vector_part() const -> const vector_t = 0;
00119     virtual auto vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t =
0;
00121     virtual auto involute() const -> const multivector_t = 0;
00123     virtual auto reverse() const -> const multivector_t = 0;
00125     virtual auto conj() const -> const multivector_t = 0;
00127     virtual auto quad() const -> Scalar_T = 0;
00129     virtual auto norm() const -> Scalar_T = 0;
00131     virtual auto max_abs() const -> Scalar_T = 0;
00133     virtual auto truncated(const Scalar_T& limit = default_truncation) const -> const multivector_t
= 0;
00135     virtual auto isinf() const -> bool = 0;
00137     virtual auto isnan() const -> bool = 0;
00139     virtual void write(const std::string& msg="") const = 0;
00141     virtual void write(std::ofstream& ofile, const std::string& msg="") const = 0;
00142 };
00143
00144 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00145 #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS \
00146     auto operator==(const multivector_t& val) const -> bool override;\
00147     auto operator==(const Scalar_T& scr) const -> bool override;\
00148     auto operator+=(const multivector_t& rhs) -> multivector_t& override;\
00149     auto operator+=(const Scalar_T& scr) -> multivector_t& override;\
00150     auto operator-=(const multivector_t& rhs) -> multivector_t& override;\
00151     auto operator-=(const Scalar_T& scr) -> multivector_t& override;\
00152     auto operator-() const -> const multivector_t override;\
00153     auto operator*=(const Scalar_T& scr) -> multivector_t& override;\
00154     auto operator*=(const multivector_t& rhs) -> multivector_t& override;\
00155     auto operator%=(const multivector_t& rhs) -> multivector_t& override;\
00156     auto operator&=(const multivector_t& rhs) -> multivector_t& override;\
00157     auto operator^=(const multivector_t& rhs) -> multivector_t& override;\
00158     auto operator/=(const Scalar_T& scr) -> multivector_t& override;\
00159     auto operator/=(const multivector_t& rhs) -> multivector_t& override;\
00160     auto operator|=(const multivector_t& rhs) -> multivector_t& override;\
00161     auto inv() const -> const multivector_t override;\
00162     auto pow(int m) const -> const multivector_t override;\
00163     auto outer_pow(int m) const -> const multivector_t override;\
00164     auto frame() const -> const index_set_t override;\
00165     auto grade() const -> index_t override;\
00166     auto operator[] (const index_set_t ist) const -> Scalar_T override;\
00167     auto operator() (index_t grade) const -> const multivector_t override;\
00168     auto scalar() const -> Scalar_T override;\
00169     auto pure() const -> const multivector_t override;\
00170     auto even() const -> const multivector_t override;\
00171     auto odd() const -> const multivector_t override;\
00172     auto vector_part() const -> const vector_t override;\
00173     auto vector_part(const index_set_t frm, const bool prechecked = false) const \
00174     -> const vector_t override;\
00175     auto involute() const -> const multivector_t override;\
00176     auto reverse() const -> const multivector_t override;\
00177     auto conj() const -> const multivector_t override;\
00178     auto quad() const -> Scalar_T override;\
00179     auto norm() const -> Scalar_T override;\
00180     auto max_abs() const -> Scalar_T override;\
00181     auto truncated(const Scalar_T& limit = multivector_t::default_truncation) const \
00182     -> const multivector_t override;\
00183     auto isinf() const -> bool override;\
00184     auto isnan() const -> bool override;\
00185     void write(const std::string& msg="") const override;\
00186     void write(std::ofstream& ofile, const std::string& msg="") const override;\
00187 #endif // _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS

```



```

00188
00190     template
00191     <
00192         template<typename, const index_t, const index_t, typename> class Multivector,
00193         template<typename, const index_t, const index_t, typename> class RHS,
00194         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00195     >
00196     auto
00197     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
bool;
00198
00200     template
00201     <
00202         template<typename, const index_t, const index_t, typename> class Multivector,
00203         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00204     >
00205     auto
00206     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool;
00207
00209     template
00210     <
00211         template<typename, const index_t, const index_t, typename> class Multivector,
00212         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00213     >
00214     auto
00215     operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00216
00218     template
00219     <
00220         template<typename, const index_t, const index_t, typename> class Multivector,
00221         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00222     >
00223     auto
00224     error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00225
00227     template
00228     <
00229         template<typename, const index_t, const index_t, typename> class Multivector,
00230         const index_t, const index_t, typename> class RHS,
00231         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232     >
00233     auto
00234     error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00235                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00236                 const Scalar_T threshold) -> Scalar_T;
00237
00239     template
00240     <
00241         template<typename, const index_t, const index_t, typename> class Multivector,
00242         template<typename, const index_t, const index_t, typename> class RHS,
00243         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00244     >
00245     auto
00246     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00247                const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00248                const Scalar_T threshold,
00249                const Scalar_T tolerance) -> bool;
00250
00252     template
00253     <
00254         template<typename, const index_t, const index_t, typename> class Multivector,
00255         template<typename, const index_t, const index_t, typename> class RHS,
00256         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00257     >
00258     auto
00259     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00260                const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00261
00263     template
00264     <
00265         template<typename, const index_t, const index_t, typename> class Multivector,
00266         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00267     >
00268     auto
00269     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00270
00272     template
00273     <
00274         template<typename, const index_t, const index_t, typename> class Multivector,
00275         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00276     >
00277     auto
00278     operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00279
00281     template

```



```

00282 <
00283     template<typename, const index_t, const index_t, typename> class Multivector,
00284     template<typename, const index_t, const index_t, typename> class RHS,
00285     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00286 >
00287 auto
00288 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00289
00291 template
00292 <
00293     template<typename, const index_t, const index_t, typename> class Multivector,
00294     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00295 >
00296 auto
00297 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00298
00300 template
00301 <
00302     template<typename, const index_t, const index_t, typename> class Multivector,
00303     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304 >
00305 auto
00306 operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00307
00309 template
00310 <
00311     template<typename, const index_t, const index_t, typename> class Multivector,
00312     template<typename, const index_t, const index_t, typename> class RHS,
00313     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00314 >
00315 auto
00316 operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00317
00319 template
00320 <
00321     template<typename, const index_t, const index_t, typename> class Multivector,
00322     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00323 >
00324 auto
00325 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00326
00328 template
00329 <
00330     template<typename, const index_t, const index_t, typename> class Multivector,
00331     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00332 >
00333 auto
00334 operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00335
00337 template
00338 <
00339     template<typename, const index_t, const index_t, typename> class Multivector,
00340     template<typename, const index_t, const index_t, typename> class RHS,
00341     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00342 >
00343 auto
00344 operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00345
00347 template
00348 <
00349     template<typename, const index_t, const index_t, typename> class Multivector,
00350     template<typename, const index_t, const index_t, typename> class RHS,
00351     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00352 >
00353 auto
00354 operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00355
00357 template
00358 <
00359     template<typename, const index_t, const index_t, typename> class Multivector,
00360     template<typename, const index_t, const index_t, typename> class RHS,
00361     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00362 >
00363 auto
00364 operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00365
00367 template
00368 <

```

```

00369     template<typename, const index_t, const index_t, typename> class Multivector,
00370     template<typename, const index_t, const index_t, typename> class RHS,
00371     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00372 >
00373     auto
00374     operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00375
00376     template
00377     <
00378         template<typename, const index_t, const index_t, typename> class Multivector,
00379         template<typename, const index_t, const index_t, typename> class RHS,
00380         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00381     >
00382     auto
00383     star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T;

00384
00385     template
00386     <
00387         template<typename, const index_t, const index_t, typename> class Multivector,
00388         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00389     >
00390     auto
00391     operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00392
00393     template
00394     <
00395         template<typename, const index_t, const index_t, typename> class Multivector,
00396         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00397     >
00398     auto
00399     operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00400
00401     template
00402     <
00403         template<typename, const index_t, const index_t, typename> class Multivector,
00404         template<typename, const index_t, const index_t, typename> class RHS,
00405         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00406     >
00407     auto
00408     operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00409
00410     template
00411     <
00412         template<typename, const index_t, const index_t, typename> class Multivector,
00413         template<typename, const index_t, const index_t, typename> class RHS,
00414         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00415     >
00416     auto
00417     operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;

00418
00419     template
00420     <
00421         template<typename, const index_t, const index_t, typename> class Multivector,
00422         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00423     >
00424     auto
00425     inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;

00426
00427     template
00428     <
00429         template<typename, const index_t, const index_t, typename> class Multivector,
00430         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00431     >
00432     auto
00433     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00434
00435     template
00436     <
00437         template<typename, const index_t, const index_t, typename> class Multivector,
00438         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00439     >
00440     auto
00441     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;

00442
00443     template< template<typename, const index_t, const index_t, typename> class Multivector,
00444             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00445     auto
00446     outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const

```

```

Multivector<Scalar_T,LO,HI,Tune_P>;
00457
00459     template
00460     <
00461         template<typename, const index_t, const index_t, typename> class Multivector,
00462         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00463     >
00464     auto
00465     scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00466
00468     template
00469     <
00470         template<typename, const index_t, const index_t, typename> class Multivector,
00471         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00472     >
00473     auto
00474     real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00475
00477     template
00478     <
00479         template<typename, const index_t, const index_t, typename> class Multivector,
00480         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00481     >
00482     auto
00483     imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00484
00486     template
00487     <
00488         template<typename, const index_t, const index_t, typename> class Multivector,
00489         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00490     >
00491     auto
00492     pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00493
00495     template
00496     <
00497         template<typename, const index_t, const index_t, typename> class Multivector,
00498         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00499     >
00500     auto
00501     even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00502
00504     template
00505     <
00506         template<typename, const index_t, const index_t, typename> class Multivector,
00507         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00508     >
00509     auto
00510     odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00511
00513     template
00514     <
00515         template<typename, const index_t, const index_t, typename> class Multivector,
00516         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00517     >
00518     auto
00519     vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>;
00520
00522     template
00523     <
00524         template<typename, const index_t, const index_t, typename> class Multivector,
00525         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00526     >
00527     auto
00528     involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00529
00531     template
00532     <
00533         template<typename, const index_t, const index_t, typename> class Multivector,
00534         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00535     >
00536     auto
00537     reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00538
00540     template
00541     <
00542         template<typename, const index_t, const index_t, typename> class Multivector,
00543         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00544     >
00545     auto
00546     conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00547
00549     template
00550     <
00551         template<typename, const index_t, const index_t, typename> class Multivector,
00552         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00553     >

```

```

00554     auto
00555     quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00556
00557     template
00558     <
00559         template<typename, const index_t, const index_t, typename> class Multivector,
00560         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00561     >
00562     auto
00563     norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00564
00565     template
00566     <
00567         template<typename, const index_t, const index_t, typename> class Multivector,
00568         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00569     >
00570     auto
00571     abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00572
00573     template
00574     <
00575         template<typename, const index_t, const index_t, typename> class Multivector,
00576         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00577     >
00578     auto
00579     max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00580
00581     template
00582     <
00583         template<typename, const index_t, const index_t, typename> class Multivector,
00584         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00585     >
00586     auto
00587     complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00588     Multivector<Scalar_T,LO,HI,Tune_P>;
00589
00590     template
00591     <
00592         template<typename, const index_t, const index_t, typename> class Multivector,
00593         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00594     >
00595     auto
00596     elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00597
00598     template
00599     <
00600         template<typename, const index_t, const index_t, typename> class Multivector,
00601         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00602     >
00603     auto
00604     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00605           const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00606           const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00607
00608     template
00609     <
00610         template<typename, const index_t, const index_t, typename> class Multivector,
00611         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00612     >
00613     auto
00614     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00615
00616     // Transcendental functions
00617
00618     template
00619     <
00620         template<typename, const index_t, const index_t, typename> class Multivector,
00621         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00622     >
00623     auto
00624     clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00625     Multivector<Scalar_T,LO,HI,Tune_P>;
00626
00627     template
00628     <
00629         template<typename, const index_t, const index_t, typename> class Multivector,
00630         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00631     >
00632     auto
00633     log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00634          const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00635          const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00636
00637     template
00638     <
00639         template<typename, const index_t, const index_t, typename> class Multivector,
00640         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00641     >
00642     auto
00643     log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00644          const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00645          const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00646
00647     template
00648     <
00649         template<typename, const index_t, const index_t, typename> class Multivector,
00650         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00651     >

```

```

00650     auto
00651     log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00652
00653     template
00654     <
00655         template<typename, const index_t, const index_t, typename> class Multivector,
00656         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00657     >
00658     auto
00659     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00660         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00661         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00662
00663     template
00664     <
00665         template<typename, const index_t, const index_t, typename> class Multivector,
00666         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00667     >
00668     auto
00669     cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00670
00671     template
00672     <
00673         template<typename, const index_t, const index_t, typename> class Multivector,
00674         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00675     >
00676     auto
00677     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00678         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00679         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00680
00681     template
00682     <
00683         template<typename, const index_t, const index_t, typename> class Multivector,
00684         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00685     >
00686     auto
00687     acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00688
00689     template
00690     <
00691         template<typename, const index_t, const index_t, typename> class Multivector,
00692         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00693     >
00694     auto
00695     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00696
00697     template
00698     <
00699         template<typename, const index_t, const index_t, typename> class Multivector,
00700         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00701     >
00702     auto
00703     cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00704
00705     template
00706     <
00707         template<typename, const index_t, const index_t, typename> class Multivector,
00708         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00709     >
00710     auto
00711     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00712         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00713         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00714
00715     template
00716     <
00717         template<typename, const index_t, const index_t, typename> class Multivector,
00718         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00719     >
00720     auto
00721     acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00722
00723     template
00724     <
00725         template<typename, const index_t, const index_t, typename> class Multivector,
00726         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00727     >
00728     auto
00729     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00730         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00731         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00732
00733     template
00734     <
00735         template<typename, const index_t, const index_t, typename> class Multivector,
00736         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00737     >
00738     auto
00739     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00740
00741     template
00742     <
00743         template<typename, const index_t, const index_t, typename> class Multivector,
00744         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P

```

```

00747 >
00748 auto
00749 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00750      const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00751      const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00752
00753 template
00754 <
00755     template<typename, const index_t, const index_t, typename> class Multivector,
00756     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00757 >
00758 auto
00759 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00760
00761 template
00762 <
00763     template<typename, const index_t, const index_t, typename> class Multivector,
00764     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00765 >
00766 auto
00767 sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00768
00769 template
00770 <
00771     template<typename, const index_t, const index_t, typename> class Multivector,
00772     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00773 >
00774 auto
00775 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00776       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00777       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00778
00779 template
00780 <
00781     template<typename, const index_t, const index_t, typename> class Multivector,
00782     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00783 >
00784 auto
00785 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00786
00787 template
00788 <
00789     template<typename, const index_t, const index_t, typename> class Multivector,
00790     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00791 >
00792 auto
00793 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00794     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00795     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00796
00797 template
00798 <
00799     template<typename, const index_t, const index_t, typename> class Multivector,
00800     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00801 >
00802 auto
00803 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00804
00805 template
00806 <
00807     template<typename, const index_t, const index_t, typename> class Multivector,
00808     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00809 >
00810 auto
00811 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00812      const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00813      const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00814
00815 template
00816 <
00817     template<typename, const index_t, const index_t, typename> class Multivector,
00818     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00819 >
00820 auto
00821 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00822
00823 template
00824 <
00825     template<typename, const index_t, const index_t, typename> class Multivector,
00826     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00827 >
00828 auto
00829 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00830
00831 template
00832 <
00833     template<typename, const index_t, const index_t, typename> class Multivector,
00834     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00835 >
00836 auto
00837 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00838
00839 template
00840 <
00841     template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00844     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00845 >
00846 auto
00847 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00848        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00849        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00850
00852 template
00853 <
00854     template<typename, const index_t, const index_t, typename> class Multivector,
00855     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00856 >
00857 auto
00858 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00859 }
00860 #endif // _GLUCAT_CLIFFORD_ALGEBRA_H

```

## 7.3 glucat/clifford\_algebra\_imp.h File Reference

```

#include "glucat/clifford_algebra.h"
#include "glucat/scalar.h"
#include <array>

```

Include dependency graph for clifford\_algebra\_imp.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace [glucat](#)

### Functions

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of multivectors.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool`  
*Test for inequality of multivector and scalar.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of scalar and multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Quadratic norm error tolerance relative to a specific multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`  
*Relative or absolute error using the quadratic norm.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P > auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`





*Outer product.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of multivector and scalar.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of scalar and multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric multiplicative inverse.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Integer power of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Multivector power of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::outer_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Outer product power of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar part.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Real part: synonym for scalar part.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Imaginary part: deprecated (always 0)*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Pure part.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T, HI, Tune_P >`

*Vector part of multivector, as a vector\_t with respect to frame()*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg. {1}\*{2} -> {2}\*{1}.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form == (rev(x)\*x)(0)*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value == sqrt(norm)*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::max_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static void glucat::check_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that i is a valid complexifier for val.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::clifford_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector.*

- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic cosine of multivector.*



- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse sine of multivector with specified complexifier.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse sine of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Hyperbolic tangent of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse hyperbolic tangent of multivector with specified complexifier.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse hyperbolic tangent of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Tangent of multivector with specified complexifier.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Tangent of multivector.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse tangent of multivector with specified complexifier.*
- `template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Inverse tangent of multivector.*

## 7.4 clifford\_algebra\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     clifford_algebra_imp.h : Implement common Clifford algebra functions
00006     *****/
```

```

00007      begin                : Sun 2001-12-09
00008      copyright             : (C) 2001-2021 by Paul C. Leopardi
00009      *****
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034      // References for algorithms:
00035      // [AS]:
00036      // Milton Abramowicz and Irene A. Stegun, "Handbook of mathematical functions",
00037      // Dover 1972, first published 1965.
00038      // [CHKL]:
00039      // Sheung Hun Cheng, Nicholas J. Higham, Charles S. Kenney and Alan J. Laub,
00040      // "Approximating the Logarithm of a Matrix to Specified Accuracy", 1999.
00041      // ftp://ftp.ma.man.ac.uk/pub/narep/narep353.ps.gz
00042      // [GL]:
00043      // Gene H. Golub and Charles F. van Loan,
00044      // "Matrix Computations", 3rd ed., Johns Hopkins UP, 1996.
00045      // [GW]:
00046      // C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis",
00047      // 6th Edition, Addison-Wesley, 1999.
00048      // [H]:
00049      // Nicholas J. Higham
00050      // "The Scaling and Squaring Method for the Matrix Exponential Revisited",
00051      // SIAM Journal on Matrix Analysis and Applications,
00052      // Vol. 26, Issue 4 (2005), pp. 1179-1193.
00053      // [Z]:
00054      // Doron Zeilberger, "PADE" (Maple code), 2002.
00055      // http://www.math.rutgers.edu/~zeilberg/tokhniot/PADE
00056
00057      #include "glucat/clifford_algebra.h"
00058      #include "glucat/scalar.h"
00059
00060      #include <array>
00061
00062      namespace glucat
00063      {
00064          template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00065          auto
00066          clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00067          classname() -> const std::string
00068          { return "clifford_algebra"; }
00069
00070          template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00071          const
00072          Scalar_T
00073          clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00074          default_truncation = std::numeric_limits<Scalar_T>::epsilon();
00075
00076          template
00077          <
00078              template<typename, const index_t, const index_t, typename> class Multivector,
00079              template<typename, const index_t, const index_t, typename> class RHS,
00080              typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00081          >
00082          inline
00083          auto
00084          operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00085          bool
00086          { return !(lhs == rhs); }
00087
00088          template< template<typename, const index_t, const index_t, typename> class Multivector,
00089                  typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00090          inline
00091          auto
00092          operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool
00093          { return !(lhs == scr); }
00094
00095

```

```

00096
00098 template< template<typename, const index_t, const index_t, typename> class Multivector,
00099           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00100 inline
00101 auto
00102 operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00103 { return !(rhs == scr); }
00104
00106 template
00107 <
00108     template<typename, const index_t, const index_t, typename> class Multivector,
00109     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00110 >
00111 auto
00112 error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00113 {
00114     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00115     static const auto scalar_eps = std::numeric_limits<Scalar_T>::epsilon();
00116     static const auto nbr_different_bits =
00117         std::numeric_limits<Scalar_T>::digits / Tune_P::denom_different_bits +
00118         Tune_P::extra_different_bits;
00119     static const auto abs_tol = scalar_eps *
00120         numeric_traits<Scalar_T>::pow(Scalar_T(2), nbr_different_bits);
00121     using framed_multi_t = typename multivector_t::framed_multi_t;
00122     const auto nbr_terms = double(framed_multi_t(val).truncated(scalar_eps).nbr_terms());
00123     return abs_tol * abs_tol * std::max(Scalar_T(nbr_terms), Scalar_T(1));
00124 }
00126 template
00127 <
00128     template<typename, const index_t, const index_t, typename> class Multivector,
00129     template<typename, const index_t, const index_t, typename> class RHS,
00130     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00131 >
00132 inline
00133 auto
00134 error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00135              const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00136              const Scalar_T threshold) -> Scalar_T
00137 {
00138     const auto relative = norm(rhs) > threshold;
00139     const auto abs_norm_diff = norm(rhs-lhs);
00140     return (relative)
00141         ? abs_norm_diff/norm(rhs)
00142         : abs_norm_diff;
00143 }
00144
00146 template
00147 <
00148     template<typename, const index_t, const index_t, typename> class Multivector,
00149     template<typename, const index_t, const index_t, typename> class RHS,
00150     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00151 >
00152 inline
00153 auto
00154 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00155             const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00156             const Scalar_T threshold,
00157             const Scalar_T tolerance) -> bool
00158 { return error_squared(lhs, rhs, threshold) < tolerance; }
00159
00161 template
00162 <
00163     template<typename, const index_t, const index_t, typename> class Multivector,
00164     template<typename, const index_t, const index_t, typename> class RHS,
00165     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00166 >
00167 inline
00168 auto
00169 approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00170             const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00171 {
00172     const Scalar_T rhs_tol = error_squared_tol(rhs);
00173     return approx_equal(lhs, rhs, rhs_tol, rhs_tol);
00174 }
00175
00177 template< template<typename, const index_t, const index_t, typename> class Multivector,
00178           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00179 inline
00180 auto
00181 operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00182 {
00183     auto result = lhs;
00184     return result += scr;
00185 }
00186

```



```

00188     template< template<typename, const index_t, const index_t, typename> class Multivector,
00189               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00190     inline
00191     auto
00192     operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00193     Multivector<Scalar_T,LO,HI,Tune_P>
00194     {
00195         return rhs + scr;
00196     }
00197
00198     template
00199     <
00200         template<typename, const index_t, const index_t, typename> class Multivector,
00201         template<typename, const index_t, const index_t, typename> class RHS,
00202         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00203     >
00204     inline
00205     auto
00206     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00207     const Multivector<Scalar_T,LO,HI,Tune_P>
00208     {
00209         auto result = lhs;
00210         return result += rhs;
00211     }
00212
00213     template< template<typename, const index_t, const index_t, typename> class Multivector,
00214               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215     inline
00216     auto
00217     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
00218     Multivector<Scalar_T,LO,HI,Tune_P>
00219     {
00220         auto result = lhs;
00221         return result -= scr;
00222     }
00223
00224     template< template<typename, const index_t, const index_t, typename> class Multivector,
00225               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00226     inline
00227     auto
00228     operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00229     Multivector<Scalar_T,LO,HI,Tune_P>
00230     { return -rhs + scr; }
00231
00232     template
00233     <
00234         template<typename, const index_t, const index_t, typename> class Multivector,
00235         template<typename, const index_t, const index_t, typename> class RHS,
00236         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00237     >
00238     inline
00239     auto
00240     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00241     const Multivector<Scalar_T,LO,HI,Tune_P>
00242     {
00243         auto result = lhs;
00244         return result -= rhs;
00245     }
00246
00247     template< template<typename, const index_t, const index_t, typename> class Multivector,
00248               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00249     inline
00250     auto
00251     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
00252     Multivector<Scalar_T,LO,HI,Tune_P>
00253     {
00254         auto result = lhs;
00255         return result *= scr;
00256     }
00257
00258     template< template<typename, const index_t, const index_t, typename> class Multivector,
00259               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00260     inline
00261     auto
00262     operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00263     Multivector<Scalar_T,LO,HI,Tune_P>
00264     {
00265         // Note: this assumes that scalar commutes with multivector.
00266         // This excludes Clifford algebras over non-commuting rings.
00267         return rhs * scr;
00268     }
00269
00270     template
00271     <
00272         template<typename, const index_t, const index_t, typename> class Multivector,
00273         template<typename, const index_t, const index_t, typename> class RHS,
00274         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00275     >

```



```

00275     inline
00276     auto
00277     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00278     {
00279         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00280         return lhs * multivector_t(rhs);
00281     }
00282
00283     template
00284     <
00285         template<typename, const index_t, const index_t, typename> class Multivector,
00286         template<typename, const index_t, const index_t, typename> class RHS,
00287         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00288     >
00289     inline
00290     auto
00291     operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00292     {
00293         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00294         return lhs ^ multivector_t(rhs);
00295     }
00296
00297     template
00298     <
00299         template<typename, const index_t, const index_t, typename> class Multivector,
00300         template<typename, const index_t, const index_t, typename> class RHS,
00301         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00302     >
00303     inline
00304     auto
00305     operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00306     {
00307         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00308         return lhs & multivector_t(rhs);
00309     }
00310
00311     template
00312     <
00313         template<typename, const index_t, const index_t, typename> class Multivector,
00314         template<typename, const index_t, const index_t, typename> class RHS,
00315         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00316     >
00317     inline
00318     auto
00319     operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00320     {
00321         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00322         return lhs % multivector_t(rhs);
00323     }
00324
00325     template
00326     <
00327         template<typename, const index_t, const index_t, typename> class Multivector,
00328         template<typename, const index_t, const index_t, typename> class RHS,
00329         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00330     >
00331     inline
00332     auto
00333     star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T
00334     {
00335         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00336         return star(lhs, multivector_t(rhs));
00337     }
00338
00339     template< template<typename, const index_t, const index_t, typename> class Multivector,
00340               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00341     inline
00342     auto
00343     operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00344     {
00345         auto result = lhs;
00346         return result /= scr;
00347     }
00348
00349     template< template<typename, const index_t, const index_t, typename> class Multivector,
00350               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00351     inline
00352     auto
00353     operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00354     {

```

```

00361     Multivector<Scalar_T,LO,HI,Tune_P> result = scr;
00362     return result /= rhs;
00363 }
00364
00365 template
00366 <
00367     template<typename, const index_t, const index_t, typename> class Multivector,
00368     template<typename, const index_t, const index_t, typename> class RHS,
00369     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00370 >
00371 inline
00372 auto
00373 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00374 const Multivector<Scalar_T,LO,HI,Tune_P>
00375 {
00376     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00377     return lhs / multivector_t(rhs);
00378 }
00379
00380 template
00381 <
00382     template<typename, const index_t, const index_t, typename> class Multivector,
00383     template<typename, const index_t, const index_t, typename> class RHS,
00384     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00385 >
00386 inline
00387 auto
00388 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00389 const Multivector<Scalar_T,LO,HI,Tune_P>
00390 {
00391     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00392     return lhs | multivector_t(rhs);
00393 }
00394
00395 template< template<typename, const index_t, const index_t, typename> class Multivector,
00396           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00397 inline
00398 auto
00399 inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00400 { return val.inv(); }
00401
00402 template< template<typename, const index_t, const index_t, typename> class Multivector,
00403           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00404 auto
00405 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
00406 Multivector<Scalar_T,LO,HI,Tune_P>
00407 {
00408     {
00409         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00410         if (lhs == Scalar_T(0))
00411         {
00412             using traits_t = numeric_traits<Scalar_T>;
00413             return
00414                 (rhs < 0)
00415                 ? traits_t::NaN()
00416                 : (rhs == 0)
00417                 ? Scalar_T(1)
00418                 : Scalar_T(0);
00419         }
00420         auto result = multivector_t(Scalar_T(1));
00421         auto power =
00422             (rhs < 0)
00423             ? lhs.inv()
00424             : lhs;
00425         for (auto
00426             k = std::abs(rhs);
00427             k != 0;
00428             k /= 2)
00429         {
00430             if (k % 2)
00431                 result *= power;
00432             power *= power;
00433         }
00434         return result;
00435     }
00436
00437 template
00438 <
00439     template<typename, const index_t, const index_t, typename> class Multivector,
00440     template<typename, const index_t, const index_t, typename> class RHS,
00441     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00442 >
00443 inline
00444 auto
00445 pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00446 Multivector<Scalar_T,LO,HI,Tune_P>
00447 {
00448     using traits_t = numeric_traits<Scalar_T>;

```

```

00449
00450     if (lhs == Scalar_T(0))
00451     {
00452         const Scalar_T m = rhs.scalar();
00453         if (rhs == m)
00454             return
00455                 (m < 0)
00456                 ? traits_t::NaN()
00457                 : (m == 0)
00458                 ? Scalar_T(1)
00459                 : Scalar_T(0);
00460         else
00461             return Scalar_T(0);
00462     }
00463     return exp(log(lhs) * rhs);
00464 }
00465
00466 template< template<typename, const index_t, const index_t, typename> class Multivector,
00467           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00468 auto
00469 outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
00470 Multivector<Scalar_T,LO,HI,Tune_P>
00471 { return lhs.outer_pow(rhs); }
00472
00473 template< template<typename, const index_t, const index_t, typename> class Multivector,
00474           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00475 inline
00476 auto
00477 scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00478 { return val.scalar(); }
00479
00480 template< template<typename, const index_t, const index_t, typename> class Multivector,
00481           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00482 inline
00483 auto
00484 real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00485 { return val.scalar(); }
00486
00487 template
00488 <
00489     template<typename, const index_t, const index_t, typename> class Multivector,
00490     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00491 >
00492 inline
00493 auto
00494 imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00495 { return Scalar_T(0); }
00496
00497 template< template<typename, const index_t, const index_t, typename> class Multivector,
00498           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00499 inline
00500 auto
00501 pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00502 { return val - val.scalar(); }
00503
00504 template< template<typename, const index_t, const index_t, typename> class Multivector,
00505           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00506 inline
00507 auto
00508 even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00509 { return val.even(); }
00510
00511 template< template<typename, const index_t, const index_t, typename> class Multivector,
00512           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00513 inline
00514 auto
00515 odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00516 { return val.odd(); }
00517
00518 template< template<typename, const index_t, const index_t, typename> class Multivector,
00519           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00520 inline
00521 auto
00522 vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00523 { return val.vector_part(); }
00524
00525 template< template<typename, const index_t, const index_t, typename> class Multivector,
00526           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00527 inline
00528 auto
00529 involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00530 { return val.involute(); }
00531
00532 template< template<typename, const index_t, const index_t, typename> class Multivector,
00533           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00534 inline
00535 auto
00536
```

```

00545 reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00546 { return val.reverse(); }
00547
00549 template< template<typename, const index_t, const index_t, typename> class Multivector,
00550           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00551 inline
00552 auto
00553 conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00554 { return val.conj(); }
00555
00557 template< template<typename, const index_t, const index_t, typename> class Multivector,
00558           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00559 inline
00560 auto
00561 quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00562 { return val.quad(); }
00563
00565 template< template<typename, const index_t, const index_t, typename> class Multivector,
00566           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00567 inline
00568 auto
00569 norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00570 { return val.norm(); }
00571
00573 template< template<typename, const index_t, const index_t, typename> class Multivector,
00574           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00575 inline
00576 auto
00577 abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00578 { return numeric_traits<Scalar_T>::sqrt(val.norm()); }
00579
00581 template< template<typename, const index_t, const index_t, typename> class Multivector,
00582           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00583 inline
00584 auto
00585 max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00586 { return val.max_abs(); }
00587
00589 template< template<typename, const index_t, const index_t, typename> class Multivector,
00590           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00591 auto
00592 complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00593 {
00594     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00595     using traits_t = numeric_traits<Scalar_T>;
00596
00597     auto frm = val.frame();
00598     using array_t = std::array<index_t, 4>;
00599     auto incp = array_t{0, 2, 1, 0};
00600     auto incq = array_t{1, 0, 0, 0};
00601     auto bott = pos_mod((frm.count_pos() - frm.count_neg()), 4);
00602     for (auto
00603         k = index_t(0);
00604         k != incp[bott];
00605         k++)
00606     for (auto
00607         idx = index_t(1);
00608         idx != HI+1;
00609         ++idx)
00610         if (!frm[idx])
00611         {
00612             frm.set(idx);
00613             break;
00614         }
00615     for (auto
00616         k = index_t(0);
00617         k != incq[bott];
00618         k++)
00619     for (auto
00620         idx = index_t(-1);
00621         idx != LO-1;
00622         --idx)
00623         if (!frm[idx])
00624         {
00625             frm.set(idx);
00626             break;
00627         }
00628     auto new_bott = pos_mod(frm.count_pos() - frm.count_neg(), 4);
00629
00630     if ((incp[new_bott] == 0) && (incq[new_bott] == 0))
00631         return multivector_t(frm, Scalar_T(1));
00632     else
00633         // Return IEEE NaN or -Inf
00634         return traits_t::NaN();
00635 }
00636

```

```

00639     template< template<typename, const index_t, const index_t, typename> class Multivector,
00640               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00641     inline
00642     auto
00643     elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00644     { return complexifier(val); }
00645
00646     template< template<typename, const index_t, const index_t, typename> class Multivector,
00647               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00648     inline
00649     static
00650     void
00651     check_complex(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00652                  const Multivector<Scalar_T,LO,HI,Tune_P>& i, const bool prechecked = false)
00653     {
00654     {
00655         if (!prechecked)
00656         {
00657             using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00658             using index_set_t = typename multivector_t::index_set_t;
00659             using error_t = typename multivector_t::error_t;
00660
00661             const auto i_frame = i.frame();
00662             // We need i to be a complexifier whose frame is large enough to represent val
00663             if (complexifier(i) != i ||
00664                 (val.frame() | i_frame) != i_frame ||
00665                 complexifier(val).frame().count() > i_frame.count())
00666                 throw error_t("check_complex(val, i): i is not a valid complexifier for val");
00667         }
00668     }
00669
00670     template< template<typename, const index_t, const index_t, typename> class Multivector,
00671               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00672     inline
00673     auto
00674     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00675          bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00676     { return sqrt(val, i, prechecked); }
00677
00678     template< template<typename, const index_t, const index_t, typename> class Multivector,
00679               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00680     inline
00681     auto
00682     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00683     { return sqrt(val, complexifier(val), true); }
00684
00685     template< template<typename, const index_t, const index_t, typename> class Multivector,
00686               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00687     auto
00688     clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00689     Multivector<Scalar_T,LO,HI,Tune_P>
00690     {
00691     {
00692         // Scaling and squaring Pade' approximation of matrix exponential
00693         // Reference: [GL], Section 11.3, p572-576
00694         // Reference: [H]
00695
00696         using traits_t = numeric_traits<Scalar_T>;
00697
00698         const auto scalar_val = val.scalar();
00699         const auto scalar_exp = traits_t::exp(scalar_val);
00700         if (traits_t::isNaN_or_isInf(scalar_exp))
00701             return traits_t::NaN();
00702         if (val == scalar_val)
00703             return scalar_exp;
00704
00705         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00706         auto A = val - scalar_val;
00707         const auto pure_scale2 = A.norm();
00708
00709         if (traits_t::isNaN_or_isInf(pure_scale2))
00710             return traits_t::NaN();
00711         if (pure_scale2 == Scalar_T(0))
00712             return scalar_exp;
00713
00714         const auto ilog2_scale =
00715             std::max(0, traits_t::to_int(ceil((log2(pure_scale2) +
00716             Scalar_T(A.frame().count())/Scalar_T(2))) - 3);
00717         const auto i_scale = traits_t::pow(Scalar_T(2), ilog2_scale);
00718         if (traits_t::isNaN_or_isInf(i_scale))
00719             return traits_t::NaN();
00720
00721         A /= i_scale;
00722         multivector_t pure_exp;
00723         {
00724             using limits_t = std::numeric_limits<Scalar_T>;
00725             const auto nbr_even_powers = 2*(limits_t::digits / 32) + 4;
00726             using nbr_t = decltype(nbr_even_powers);

```

```

00727     // Create an array of coefficients
00728     const auto max_power = 2*nr_even_powers + 1;
00729     static std::array<Scalar_T, max_power+1> c;
00730     if (c[0] != Scalar_T(1))
00731     {
00732         c[0] = Scalar_T(1);
00733         for (auto
00734             k = decltype(max_power)(0);
00735             k != max_power;
00736             ++k)
00737             c[k+1] = c[k]*(max_power-k) / ((2*max_power-k)*(k+1));
00738     }
00739
00740     // Create an array of even powers
00741     std::array<multivector_t, nr_even_powers> AA;
00742     AA[0] = A * A;
00743     AA[1] = AA[0] * AA[0];
00744     for (auto
00745         k = nr_t(2);
00746         k != nr_even_powers;
00747         ++k)
00748         AA[k] = AA[k-2] * AA[1];
00749
00750     // Use compensated summation to calculate U and AV
00751     auto residual = multivector_t();
00752     auto U = multivector_t(c[0]);
00753     for (auto
00754         k = nr_t(0);
00755         k != nr_even_powers;
00756         ++k)
00757     {
00758         const auto& term = AA[k]*c[2*k + 2] - residual;
00759         const auto& sum = U + term;
00760         residual = (sum - U) - term;
00761         U = sum;
00762     }
00763     residual = multivector_t();
00764     auto AV = multivector_t(c[1]);
00765     for (auto
00766         k = nr_t(0);
00767         k != nr_even_powers;
00768         ++k)
00769     {
00770         const auto& term = AA[k]*c[2*k + 3] - residual;
00771         const auto& sum = AV + term;
00772         residual = (sum - AV) - term;
00773         AV = sum;
00774     }
00775     AV *= A;
00776     pure_exp = (U+AV) / (U-AV);
00777 }
00778 for (auto
00779     k = decltype(ilog2_scale)(0);
00780     k != ilog2_scale;
00781     ++k)
00782     pure_exp *= pure_exp;
00783 return pure_exp * scalar_exp;
00784 }
00785
00786 template< template<typename, const index_t, const index_t, typename> class Multivector,
00787           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788 inline
00789 auto
00790 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00791 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00792 { return log(val, i, prechecked); }
00793
00794 template< template<typename, const index_t, const index_t, typename> class Multivector,
00795           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 inline
00797 auto
00798 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00799 { return log(val, complexifier(val), true); }
00800
00801
00802 template< template<typename, const index_t, const index_t, typename> class Multivector,
00803           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00804 inline
00805 auto
00806 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00807 {
00808     using traits_t = numeric_traits<Scalar_T>;
00809     if (val.isnan())
00810         return traits_t::NaN();
00811
00812     const auto& s = val.scalar();
00813     if (val == s)
00814         return traits_t::cosh(s);

```

```

00816     return (exp(val)+exp(-val)) / Scalar_T(2);
00817 }
00818
00820 // Reference: [AS], Section 4.6, p86-89
00821 template< template<typename, const index_t, const index_t, typename> class Multivector,
00822           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00823 inline
00824 auto
00825 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
{
00826 {
00827     using traits_t = numeric_traits<Scalar_T>;
00828     check_complex(val, i, prechecked);
00829     if (val.isnan())
00830         return traits_t::NaN();
00831
00832     const auto radical = sqrt(val*val - Scalar_T(1), i, true);
00833     return (norm(val + radical) >= norm(val))
00834         ? log(val + radical, i, true)
00835         : -log(val - radical, i, true);
00836 }
00837
00839 // Reference: [AS], Section 4.6, p86-89
00840 template< template<typename, const index_t, const index_t, typename> class Multivector,
00841           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00842 inline
00843 auto
00844 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00845 { return acosh(val, complexifier(val), true); }
00846
00848 template< template<typename, const index_t, const index_t, typename> class Multivector,
00849           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00850 auto
00851 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
{
00852 {
00853     using traits_t = numeric_traits<Scalar_T>;
00854     if (val.isnan())
00855         return traits_t::NaN();
00856
00857     const auto& s = val.scalar();
00858     if (val == s)
00859         return traits_t::cos(s);
00860
00861     check_complex(val, i, prechecked);
00862
00863     static const auto& twopi = Scalar_T(2) * traits_t::pi();
00864     const auto& z = i *
00865         (val - s + traits_t::fmod(s, twopi));
00866     return (exp(z)+exp(-z)) / Scalar_T(2);
00867 }
00868
00870 template< template<typename, const index_t, const index_t, typename> class Multivector,
00871           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00872 inline
00873 auto
00874 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00875 { return cos(val, complexifier(val), true); }
00876
00878 // Reference: [AS], Section 4.4, p79-83
00879 template< template<typename, const index_t, const index_t, typename> class Multivector,
00880           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00881 inline
00882 auto
00883 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
{
00884 {
00885     using traits_t = numeric_traits<Scalar_T>;
00886     if (val.isnan())
00887         return traits_t::NaN();
00888
00889     const auto& s = val.scalar();
00890     if (val == s && traits_t::abs(s) <= Scalar_T(1))
00891         return traits_t::acos(s);
00892
00893     check_complex(val, i, prechecked);
00894     return i * acosh(val, i, true);
00895 }
00896
00898 // Reference: [AS], Section 4.4, p79-83
00899 template< template<typename, const index_t, const index_t, typename> class Multivector,
00900           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00901 inline
00902 auto
00903 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00904 { return acos(val, complexifier(val), true); }
00905

```

```

00907     template< template<typename, const index_t, const index_t, typename> class Multivector,
00908               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00909     inline
00910     auto
00911     sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00912     {
00913         using traits_t = numeric_traits<Scalar_T>;
00914         if (val.isnan())
00915             return traits_t::NaN();
00916
00917         const auto& s = val.scalar();
00918         if (val == s)
00919             return traits_t::sinh(s);
00920
00921         return (exp(val)-exp(-val)) / Scalar_T(2);
00922     }
00923
00924     // Reference: [AS], Section 4.6, p86-89
00925     template< template<typename, const index_t, const index_t, typename> class Multivector,
00926               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00927     inline
00928     auto
00929     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00930           bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00931     {
00932         using traits_t = numeric_traits<Scalar_T>;
00933         check_complex(val, i, prechecked);
00934         if (val.isnan())
00935             return traits_t::NaN();
00936
00937         const auto radical = sqrt(val*val + Scalar_T(1), i, true);
00938         return (norm(val + radical) >= norm(val))
00939             ? log(val + radical, i, true)
00940             : -log(-val + radical, i, true);
00941     }
00942
00943     // Reference: [AS], Section 4.6, p86-89
00944     template< template<typename, const index_t, const index_t, typename> class Multivector,
00945               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00946     inline
00947     auto
00948     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00949     { return asinh(val, complexifier(val), true); }
00950
00951     template< template<typename, const index_t, const index_t, typename> class Multivector,
00952               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00953     auto
00954     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00955         prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00956     {
00957         using traits_t = numeric_traits<Scalar_T>;
00958         if (val.isnan())
00959             return traits_t::NaN();
00960
00961         const auto& s = val.scalar();
00962         if (val == s)
00963             return traits_t::sin(s);
00964
00965         check_complex(val, i, prechecked);
00966
00967         static const auto& twopi = Scalar_T(2) * traits_t::pi();
00968         const auto& z = i *
00969             (val - s + traits_t::fmod(s, twopi));
00970         return i * (exp(-z)-exp(z)) / Scalar_T(2);
00971     }
00972
00973     template< template<typename, const index_t, const index_t, typename> class Multivector,
00974               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00975     inline
00976     auto
00977     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00978     { return sin(val, complexifier(val), true); }
00979
00980     // Reference: [AS], Section 4.4, p79-83
00981     template< template<typename, const index_t, const index_t, typename> class Multivector,
00982               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00983     inline
00984     auto
00985     asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00986         bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00987     {
00988         using traits_t = numeric_traits<Scalar_T>;
00989         if (val.isnan())
00990             return traits_t::NaN();
00991
00992         const auto& s = val.scalar();
00993         if (val == s && traits_t::abs(s) <= Scalar_T(1))
00994             return traits_t::asin(s);
00995     }

```



```

00996         return traits_t::asin(s);
00997
00998     check_complex(val, i, prechecked);
00999     return -i * asinh(i * val, i, true);
01000 }
01001
01002 // Reference: [AS], Section 4.4, p79-83
01003 template< template<typename, const index_t, const index_t, typename> class Multivector,
01004           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01005 inline
01006 auto
01007 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01008 { return asin(val, complexifier(val), true); }
01009
01010 template< template<typename, const index_t, const index_t, typename> class Multivector,
01011           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01012 inline
01013 auto
01014 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01015 {
01016     using traits_t = numeric_traits<Scalar_T>;
01017     if (val.isnan())
01018         return traits_t::NaN();
01019
01020     const auto& s = val.scalar();
01021     if (val == s)
01022         return traits_t::tanh(s);
01023
01024     return sinh(val) / cosh(val);
01025 }
01026
01027 // Reference: [AS], Section 4.6, p86-89
01028 template< template<typename, const index_t, const index_t, typename> class Multivector,
01029           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01030 inline
01031 auto
01032 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01033        bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01034 {
01035     using traits_t = numeric_traits<Scalar_T>;
01036     check_complex(val, i, prechecked);
01037     return val.isnan()
01038         ? traits_t::NaN()
01039         : (norm(val + Scalar_T(1)) > norm(val - Scalar_T(1)))
01040           ? (log(val + Scalar_T(1), i, true) - log(-val + Scalar_T(1), i, true)) / Scalar_T(2)
01041           : log((val + Scalar_T(1)) / (-val + Scalar_T(1)), i, true) / Scalar_T(2);
01042 }
01043
01044 // Reference: [AS], Section 4.6, p86-89
01045 template< template<typename, const index_t, const index_t, typename> class Multivector,
01046           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01047 inline
01048 auto
01049 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01050 { return atanh(val, complexifier(val), true); }
01051
01052 template< template<typename, const index_t, const index_t, typename> class Multivector,
01053           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01054 inline
01055 auto
01056 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
01057      prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01058 {
01059     using traits_t = numeric_traits<Scalar_T>;
01060     if (val.isnan())
01061         return traits_t::NaN();
01062
01063     const auto& s = val.scalar();
01064     if (val == s)
01065         return traits_t::tan(s);
01066
01067     check_complex(val, i, prechecked);
01068     return sin(val, i, true) / cos(val, i, true);
01069 }
01070
01071 template< template<typename, const index_t, const index_t, typename> class Multivector,
01072           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01073 inline
01074 auto
01075 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01076 { return tan(val, complexifier(val), true); }
01077
01078 // Reference: [AS], Section 4.4, p79-83
01079 template< template<typename, const index_t, const index_t, typename> class Multivector,
01080           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01081 inline
01082 auto

```

```

01088     atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01089     {
01090         using traits_t = numeric_traits<Scalar_T>;
01091         if (val.isnan())
01092             return traits_t::NaN();
01093
01094         const auto& s = val.scalar();
01095         if (val == s)
01096             return traits_t::atan(s);
01097
01098         check_complex(val, i, prechecked);
01099         return -i * atanh(i * val, i, true);
01100     }
01101
01102 // Reference: [AS], Section 4.4, p79-83
01103 template< template<typename, const index_t, const index_t, typename> class Multivector,
01104           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01105 inline
01106 auto
01107 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01108 { return atan(val, complexifier(val), true); }
01109
01110 }
01111 #endif // _GLUCAT_CLIFFORD_ALGEBRA_IMP_H

```

## 7.5 glucat/errors.h File Reference

```

#include <string>
#include <exception>
#include <stdexcept>

```

Include dependency graph for errors.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::glucat\\_error](#)  
*Abstract exception class.*
- class [glucat::error< Class\\_T >](#)  
*Specific exception class.*

### Namespaces

- namespace [glucat](#)

## 7.6 errors.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_H
00002 #define _GLUCAT_ERRORS_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors.h : Declare error classes and functions
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright             : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,

```

```

00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include <string>
00035 #include <exception>
00036 #include <stdexcept>
00037
00038 namespace glucat
00039 {
00041     class glucat_error : public std::logic_error
00042     {
00043     public:
00044         glucat_error(const std::string& context, const std::string& msg)
00045             : logic_error(msg), name(context)
00046         { }
00047         ~glucat_error() noexcept override = default;
00048         virtual auto heading() const noexcept -> const std::string =0;
00049         virtual auto classname() const noexcept -> const std::string =0;
00050         virtual void print_error_msg() const =0;
00051         std::string name;
00052     };
00053
00055     template< class Class_T >
00056     class error : public glucat_error
00057     {
00058     public:
00059         error(const std::string& msg);
00060         error(const std::string& context, const std::string& msg);
00061         auto heading() const noexcept -> const std::string override;
00062         auto classname() const noexcept -> const std::string override;
00063         void print_error_msg() const override;
00064     };
00065 }
00066 #endif // _GLUCAT_ERRORS_H

```

## 7.7 glucat/errors\_imp.h File Reference

```

#include "glucat/errors.h"
#include <string>
#include <iostream>
#include <ostream>

```

Include dependency graph for errors\_imp.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace `glucat`

## 7.8 errors\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_IMP_H
00002 #define _GLUCAT_ERRORS_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors_imp.h : Define error functions

```

```

00006      -----
00007      begin                : Sun 2001-12-20
00008      copyright            : (C) 2001-2007 by Paul C. Leopardi
00009      *****
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034      #include "glucat/errors.h"
00035
00036      #include <string>
00037      #include <iostream>
00038      #include <ostream>
00039
00040      namespace glucat
00041      {
00042      template< class Class_T >
00043      error<Class_T>::
00044      error(const std::string& msg)
00045      : glucat_error(Class_T::classname(), msg)
00046      { }
00047
00048      template< class Class_T >
00049      error<Class_T>::
00050      error(const std::string& context, const std::string& msg)
00051      : glucat_error(context, msg)
00052      { }
00053
00054      template< class Class_T >
00055      auto
00056      error<Class_T>::
00057      heading() const noexcept -> const std::string
00058      { return "Error in glucat: "; }
00059
00060      template< class Class_T >
00061      auto
00062      error<Class_T>::
00063      classname() const noexcept -> const std::string
00064      { return name; }
00065
00066      template< class Class_T >
00067      void
00068      error<Class_T>::
00069      print_error_msg() const
00070      { std::cerr << heading() << classname() << std::endl << what() << std::endl; }
00071      }
00072      #endif // _GLUCAT_ERRORS_IMP_H

```

## 7.9 glucat/framed\_multi.h File Reference

```

#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include <string>
#include <utility>
#include <map>

```

```
#include <unordered_map>
#include <vector>
```

Include dependency graph for framed\_multi.h: This graph shows which files directly or indirectly include this file:

## Classes

- class [glucat::index\\_set\\_hash< LO, HI >](#)
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >](#)  
*A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.*
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t](#)
- class [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::var\\_term](#)  
*Variable term.*
- struct [std::numeric\\_limits< glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >](#)  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*

## Namespaces

- namespace [glucat](#)
- namespace [std](#)

## Functions

- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator\\*](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator^](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator&](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator%](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::star](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator/](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator|](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream`  
`&`  
*Read multivector from input.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::`  
`ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term)`  
`-> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T,`  
`LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`static auto glucat::crd\_of\_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::`  
`::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`auto glucat::operator\* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*

## 7.10 framed\_multi.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_FRAMED_MULTI_H
00002 #define _GLUCAT_FRAMED_MULTI_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   framed_multi.h : Declare a class for the framed representation of a multivector
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,

```

```

00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039
00040 #if defined(_GLUCAT_USE_BOOST_POOL_ALLOC)
00041 // Use the Boost pool allocator
00042 #include <boost/pool/poolfwd.hpp>
00043 #endif
00044
00045 #include <string>
00046 #include <utility>
00047 #include <map>
00048 #include <unordered_map>
00049 #include <vector>
00050
00051 namespace glucat
00052 {
00053     // Forward declarations for friends
00054
00055     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00056     class framed_multi; // forward
00057
00058     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059     class matrix_multi; // forward
00060
00061     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00062     auto
00063     operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00064 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00065
00066     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00067     auto
00068     operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00069 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00070
00071     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00072     auto
00073     operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00074 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00075
00076     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00077     auto
00078     operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00079 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00080
00081     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00082     auto
00083     star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00084 -> Scalar_T;
00085
00086     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00087     auto
00088     operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00089 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00090
00091     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092     auto
00093     operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00094 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00095
00096     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097     auto
00098     operator> (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00100     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00101     auto
00102     operator< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00103
00104     template< typename Scalar_T, const index_t LO, const index_t HI >
00105     auto
00106     operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
00107 std::ostream&;
00108
00109     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00110     auto
00111     exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00112
00113     template< const index_t LO, const index_t HI>

```

```

00117     class index_set_hash
00118     {
00119     public:
00120         using index_set_t = index_set<LO, HI>;
00121         inline auto operator()(index_set_t val) const -> size_t { return val.hash_fn(); }
00122     };
00123
00125     template< typename Scalar_T = double,    const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<> >
00126     class framed_multi :
00127     public clifford_algebra< Scalar_T, index_set<LO,HI>, framed_multi<Scalar_T,LO,HI,Tune_P> >,
00128     private std::unordered_map< index_set<LO,HI>, Scalar_T, index_set_hash<LO,HI> >
00129     {
00130     public:
00131         using multivector_t = framed_multi;
00132         using framed_multi_t = multivector_t;
00133         using scalar_t = Scalar_T;
00134         using tune_p = Tune_P;
00135         using index_set_t = index_set<LO, HI>;
00136         using term_t = std::pair<const index_set_t, Scalar_T>;
00137         using vector_t = std::vector<Scalar_T>;
00138         using error_t = error<multivector_t>;
00139         using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >;
00140         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00141             friend class matrix_multi;
00142         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00143             friend class framed_multi;
00144
00145     private:
00146         class var_term; // forward
00147         using var_term_t = class var_term;
00148         using matrix_t = typename matrix_multi_t::matrix_t;
00149         using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >;
00150         using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>;
00151
00152         class hash_size_t
00153         {
00154         public:
00155             hash_size_t(size_t hash_size)
00156             : n(hash_size)
00157             { };
00158             auto operator()() const -> size_t
00159             { return n; }
00160         private:
00161             size_t n;
00162         };
00163
00164         using framed_pair_t = std::pair<const multivector_t, const multivector_t>;
00165         using size_type = typename map_t::size_type;
00166         using iterator = typename map_t::iterator;
00167         using const_iterator = typename map_t::const_iterator;
00168
00169     public:
00171         static auto classname() -> const std::string;
00173         ~framed_multi() override = default;
00175         framed_multi();
00176
00177     private:
00179         framed_multi(const hash_size_t& hash_size);
00180     public:
00182         template< typename Other_Scalar_T >
00183         framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00185         template< typename Other_Scalar_T >
00186         framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00187             const index_set_t frm, const bool prechecked = false);
00189         framed_multi(const framed_multi_t& val,
00190             const index_set_t frm, const bool prechecked = false);
00192         framed_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00194         framed_multi(const index_set_t ist, const Scalar_T& crd,
00195             const index_set_t frm, const bool prechecked = false);
00197         framed_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00199         framed_multi(const int scr, const index_set_t frm = index_set_t());
00201         framed_multi(const vector_t& vec,
00202             const index_set_t frm, const bool prechecked = false);
00204         framed_multi(const std::string& str);
00206         framed_multi(const std::string& str,
00207             const index_set_t frm, const bool prechecked = false);
00209         framed_multi(const char* str)
00210         { *this = framed_multi(std::string(str)); };
00212         framed_multi(const char* str,
00213             const index_set_t frm, const bool prechecked = false)
00214         { *this = framed_multi(std::string(str), frm, prechecked); };
00216         template< typename Other_Scalar_T >
00217         framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P >& val);
00219         template< typename Other_Scalar_T >

```



```

00220     auto fast_matrix_multi(const index_set_t frm) const -> const
matrix_multi<Other_Scalar_T,LO,HI,Tune_P >;
00222     auto fast_framed_multi() const -> const framed_multi_t;
00223
00224     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00225
00227     auto nbr_terms() const -> unsigned long;
00228
00230     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const multivector_t;
00231
00232     // Friend declarations
00233
00234     friend auto
00235     operator* <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00236     friend auto
00237     operator^ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00238     friend auto
00239     operator& <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00240     friend auto
00241     operator% <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00242     friend auto
00243     star <>(const multivector_t& lhs, const multivector_t& rhs) -> Scalar_T;
00244     friend auto
00245     operator/ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00246     friend auto
00247     operator| <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00248
00249     friend auto
00250     operator>> <>(std::istream& s, multivector_t& val) -> std::istream&;
00251     friend auto
00252     operator<< <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00253     friend auto
00254     operator<< <>(std::ostream& os, const term_t& term) -> std::ostream&;
00255
00256     friend auto
00257     exp <>(const multivector_t& val) -> const multivector_t;
00258
00260     auto operator+= (const term_t& term) -> multivector_t&;
00261
00262 private:
00264     auto fold(const index_set_t frm) const -> multivector_t;
00266     auto unfold(const index_set_t frm) const -> multivector_t;
00268     auto centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&;
00270     auto centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&;
00272     auto centre_qp1_pm1(index_t& p, index_t& q) -> multivector_t&;
00274     auto divide(const index_set_t ist) const -> const framed_pair_t;
00276     auto fast(const index_t level, const bool odd) const -> const matrix_t;
00277
00279     class var_term :
00280     public std::pair<index_set<LO,HI>, Scalar_T>
00281     {
00282     public:
00283         using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>;
00284
00286         static auto classname() -> const std::string
00287         { return "var_term"; };
00289         ~var_term() = default;
00291         var_term()
00292         : var_pair_t(index_set_t(), Scalar_T(1))
00293         { };
00295         var_term(const index_set_t ist, const Scalar_T& crd = Scalar_T(1))
00296         : var_pair_t(ist, crd)
00297         { };
00299         auto operator*= (const term_t& rhs) -> var_term_t&
00300         {
00301             this->second *= rhs.second * this->first.sign_of_mult(rhs.first);
00302             this->first ^= rhs.first;
00303             return *this;
00304         }
00305     };
00306 };
00307
00308 // Non-members
00309
00311 template< typename Scalar_T, const index_t LO, const index_t HI >
00312 inline
00313 static
00314 auto
00315 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00316             const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T;
00317
00319 template< typename Scalar_T, const index_t LO, const index_t HI >
00320 auto
00321 operator*
00322 (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00323  const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const index_set<LO,HI>,
Scalar_T>;

```

```

00324
00326     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327     auto
00328     sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00329
00331     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00332     auto
00333     exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00334
00336     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00337     auto
00338     log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00339 }
00340
00341 namespace std
00342 {
00344     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00345     struct numeric_limits< glucat::framed_multi<Scalar_T,LO,HI,Tune_P> > :
00346     public numeric_limits<Scalar_T>
00347     { };
00348 }
00349 #endif // _GLUCAT_FRAMED_MULTI_H

```

## 7.11 glucat/framed\_multi\_imp.h File Reference

```

#include "glucat/framed_multi.h"
#include "glucat/scalar.h"
#include "glucat/random.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <sstream>
#include <fstream>

```

Include dependency graph for framed\_multi\_imp.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >](#)  
*Sorted range for use with output.*
- class [glucat::sorted\\_range< Sorted\\_Map\\_T, Sorted\\_Map\\_T >](#)

### Namespaces

- namespace [glucat](#)

### Macros

- [#define \\_GLUCAT\\_HASH\\_N\(x\) \(x\)](#)
- [#define \\_GLUCAT\\_HASH\\_SIZE\\_T\(x\) \(typename multivector\\_t::hash\\_size\\_t\)\(x\)](#)

## Functions

- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator\\*](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator^](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator&](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator%](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::star](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator/](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator|](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator<<](#) (std::ostream &os, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI>  
auto [glucat::operator<<](#) (std::ostream &os, const std::pair< const index\_set< LO, HI >, Scalar\_T > &term) -> std::ostream &  
*Write term to output.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator>>](#) (std::istream &s, framed\_multi< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI>  
static auto [glucat::crd\\_of\\_mult](#) (const std::pair< const index\_set< LO, HI >, Scalar\_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar\_T > &rhs) -> Scalar\_T  
*Coordinate of product of terms.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI>  
auto [glucat::operator\\*](#) (const std::pair< const index\_set< LO, HI >, Scalar\_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar\_T > &rhs) -> const std::pair< const index\_set< LO, HI >, Scalar\_T >  
*Product of terms.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::sqrt](#) (const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &val, const framed\_multi< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const framed\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*

## 7.11.1 Macro Definition Documentation

### 7.11.1.1 `_GLUCAT_HASH_N`

```
#define _GLUCAT_HASH_N(  
    x ) (x)
```

Definition at line 54 of file [framed\\_multi\\_imp.h](#).

### 7.11.1.2 `_GLUCAT_HASH_SIZE_T`

```
#define _GLUCAT_HASH_SIZE_T(  
    x ) (typename multivector_t::hash_size_t)(x)
```

Definition at line 55 of file [framed\\_multi\\_imp.h](#).

## 7.12 `framed_multi_imp.h`

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_FRAMED_MULTI_IMP_H
00002 #define _GLUCAT_FRAMED_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi_imp.h : Implement the coordinate map representation of a
00006     Clifford algebra element
00007     -----
00008     begin                : Sun 2001-12-09
00009     copyright            : (C) 2001-2021 by Paul C. Leopardi
00010     ****
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     ****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
```

```

00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****
00032 See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include "glucat/framed_multi.h"
00036
00037 #include "glucat/scalar.h"
00038 #include "glucat/random.h"
00039 #include "glucat/generation.h"
00040 #include "glucat/matrix.h"
00041
00042 #include <sstream>
00043 #include <fstream>
00044
00045 namespace glucat
00046 {
00047     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00048     auto
00049     framed_multi<Scalar_T,LO,HI,Tune_P>::
00050     classname() -> const std::string
00051     { return "framed_multi"; }
00052
00053 #define _GLUCAT_HASH_N(x) (x)
00054 #define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)
00055
00056     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00057     framed_multi<Scalar_T,LO,HI,Tune_P>::
00058     framed_multi()
00059     : map_t(_GLUCAT_HASH_N(0))
00060     { }
00061
00062     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063     framed_multi<Scalar_T,LO,HI,Tune_P>::
00064     framed_multi(const hash_size_t& hash_size)
00065     : map_t(_GLUCAT_HASH_N(hash_size()))
00066     { }
00067
00068     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00069     template< typename Other_Scalar_T >
00070     framed_multi<Scalar_T,LO,HI,Tune_P>::
00071     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00072     : map_t(_GLUCAT_HASH_N(val.size()))
00073     {
00074         for (auto& val_term : val)
00075             this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00076     }
00077
00078     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00079     template< typename Other_Scalar_T >
00080     framed_multi<Scalar_T,LO,HI,Tune_P>::
00081     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00082                 const index_set_t frm, const bool prechecked)
00083     : map_t(_GLUCAT_HASH_N(val.size()))
00084     {
00085         if (!prechecked && (val.frame() | frm) != frm)
00086             throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00087         for (auto& val_term : val)
00088             this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00089     }
00090
00091     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092     framed_multi<Scalar_T,LO,HI,Tune_P>::
00093     framed_multi(const multivector_t& val,
00094                 const index_set_t frm, const bool prechecked)
00095     : map_t(_GLUCAT_HASH_N(val.size()))
00096     {
00097         if (!prechecked && (val.frame() | frm) != frm)
00098             throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00099         for (auto& val_term : val)
00100             this->insert(val_term);
00101     }
00102
00103     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00104     framed_multi<Scalar_T,LO,HI,Tune_P>::
00105     framed_multi(const index_set_t ist, const Scalar_T& crd)
00106     : map_t(_GLUCAT_HASH_N(1))
00107     {
00108         if (crd != Scalar_T(0))
00109             this->insert(term_t(ist, crd));
00110     }
00111
00112     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113     framed_multi<Scalar_T,LO,HI,Tune_P>::
00114     framed_multi(const index_set_t ist, const Scalar_T& crd,

```

```

00123         const index_set_t frm, const bool prechecked)
00124 : map_t(_GLUCAT_HASH_N(1))
00125 {
00126     if (!prechecked && (ist | frm) != frm)
00127         throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00128     if (crd != Scalar_T(0))
00129         this->insert(term_t(ist, crd));
00130 }
00131
00132 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00133 framed_multi<Scalar_T,LO,HI,Tune_P>::
00134 framed_multi(const Scalar_T& scr, const index_set_t frm)
00135 : map_t(_GLUCAT_HASH_N(1))
00136 {
00137     if (scr != Scalar_T(0))
00138         this->insert(term_t(index_set_t(), scr));
00139 }
00140
00141 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00142 framed_multi<Scalar_T,LO,HI,Tune_P>::
00143 framed_multi(const int scr, const index_set_t frm)
00144 : map_t(_GLUCAT_HASH_N(1))
00145 {
00146     if (scr != Scalar_T(0))
00147         this->insert(term_t(index_set_t(), Scalar_T(scr)));
00148 }
00149
00150 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00151 framed_multi<Scalar_T,LO,HI,Tune_P>::
00152 framed_multi(const vector_t& vec,
00153             const index_set_t frm, const bool prechecked)
00154 : map_t(_GLUCAT_HASH_N(vec.size()))
00155 {
00156     if (!prechecked && index_t(vec.size()) != frm.count())
00157         throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00158     auto idx = frm.min();
00159     const auto frm_end = frm.max()+1;
00160     for (auto& crd : vec)
00161     {
00162         *this += term_t(index_set_t(idx), crd);
00163         for (
00164             ++idx;
00165             idx != frm_end && !frm[idx];
00166             ++idx)
00167             ;
00168     }
00169 }
00170
00171 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00172 framed_multi<Scalar_T,LO,HI,Tune_P>::
00173 framed_multi(const std::string& str)
00174 : map_t(_GLUCAT_HASH_N(0))
00175 {
00176     std::istringstream ss(str);
00177     ss >> *this;
00178     if (!ss)
00179         throw error_t("multivector_t(str): could not parse string");
00180     // Peek to see if the end of the string has been reached.
00181     ss.peek();
00182     if (!ss.eof())
00183         throw error_t("multivector_t(str): could not parse entire string");
00184 }
00185
00186 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00187 framed_multi<Scalar_T,LO,HI,Tune_P>::
00188 framed_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00189 : map_t(_GLUCAT_HASH_N(0))
00190 {
00191     if (prechecked)
00192         *this = multivector_t(str);
00193     else
00194         *this = multivector_t(multivector_t(str), frm, false);
00195 }
00196
00197 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00198 template< typename Other_Scalar_T >
00199 framed_multi<Scalar_T,LO,HI,Tune_P>::
00200 framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00201 : map_t(_GLUCAT_HASH_N(1))
00202 {
00203     if (val == Other_Scalar_T(0))
00204         return;
00205     const auto dim = val.m_matrix.size1();
00206     using traits_t = numeric_traits<Scalar_T>;
00207     if (dim == 1)
00208     {

```

```

00216         this->insert(term_t(index_set_t(), traits_t::to_scalar_t(val.m_matrix(0, 0))));
00217         return;
00218     }
00219     if (dim >= Tune_P::inv_fast_dim_threshold)
00220     try
00221     {
00222         *this = (val.template fast_framed_multi<Scalar_T>()).truncated();
00223         return;
00224     }
00225     catch (const glucat_error& e)
00226     { }
00227
00228     const auto val_norm = traits_t::to_scalar_t(val.norm());
00229     if (traits_t::isNaN_or_isInf(val_norm))
00230     {
00231         *this = traits_t::NaN();
00232         return;
00233     }
00234     const auto frm = val.frame();
00235     const auto algebra_dim = set_value_t(1) << frm.count();
00236     auto result = multivector_t(
00237         _GLUCAT_HASH_SIZE_T(std::min<size_t>(algebra_dim, matrix::nnz(val.m_matrix))));
00238     for (auto
00239         stv = set_value_t(0);
00240         stv != algebra_dim;
00241         stv++)
00242     {
00243         const auto ist = index_set_t(stv, frm, true);
00244         const auto crd =
00245             traits_t::to_scalar_t(matrix::inner<Other_Scalar_T>(val.basis_element(ist), val.m_matrix));
00246         if (crd != Scalar_T(0))
00247             result.insert(term_t(ist, crd));
00248     }
00249     *this = result.truncated();
00250 }
00251
00252 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00253 auto
00254 framed_multi<Scalar_T,LO,HI,Tune_P>::
00255 operator== (const multivector_t& rhs) const -> bool
00256 {
00257     if (this->size() != rhs.size())
00258         return false;
00259     const auto rhs_end = rhs.end();
00260     for (auto& this_term : *this)
00261     {
00262         const const_iterator& rhs_it = rhs.find(this_term.first);
00263         if (rhs_it == rhs_end || rhs_it->second != this_term.second)
00264             return false;
00265     }
00266     return true;
00267 }
00268
00269 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00270 inline
00271 auto
00272 framed_multi<Scalar_T,LO,HI,Tune_P>::
00273 operator== (const Scalar_T& scr) const -> bool
00274 {
00275     switch (this->size())
00276     {
00277     case 0:
00278         return scr == Scalar_T(0);
00279     case 1:
00280     {
00281         const auto& this_it = this->begin();
00282         return this_it->first == index_set_t() && this_it->second == scr;
00283     }
00284     default:
00285         return false;
00286     }
00287 }
00288
00289 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00290 inline
00291 auto
00292 framed_multi<Scalar_T,LO,HI,Tune_P>::
00293 operator+= (const Scalar_T& scr) -> multivector_t&
00294 {
00295     *this += term_t(index_set_t(), scr);
00296     return *this;
00297 }
00298
00299 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00300 inline
00301 auto
00302 framed_multi<Scalar_T,LO,HI,Tune_P>::

```

```

00307 operator+= (const multivector_t& rhs) -> multivector_t&
00308 { // simply add terms
00309     for (auto& rhs_term : rhs)
00310         *this += rhs_term;
00311     return *this;
00312 }
00313
00315 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00316 inline
00317 auto
00318 framed_multi<Scalar_T,LO,HI,Tune_P>::
00319 operator-= (const Scalar_T& scr) -> multivector_t&
00320 {
00321     *this += term_t(index_set_t(), -scr);
00322     return *this;
00323 }
00324
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 inline
00328 auto
00329 framed_multi<Scalar_T,LO,HI,Tune_P>::
00330 operator-= (const multivector_t& rhs) -> multivector_t&
00331 {
00332     for (auto& rhs_term : rhs)
00333         *this += term_t(rhs_term.first, -(rhs_term.second));
00334     return *this;
00335 }
00336
00338 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00339 inline
00340 auto
00341 framed_multi<Scalar_T,LO,HI,Tune_P>::
00342 operator- () const -> const multivector_t
00343 { // multiply coordinates of all terms by -1
00344     auto result = *this;
00345     for (auto& result_term : result)
00346         result_term.second *= Scalar_T(-1);
00347     return result;
00348 }
00349
00351 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00352 auto
00353 framed_multi<Scalar_T,LO,HI,Tune_P>::
00354 operator*= (const Scalar_T& scr) -> multivector_t&
00355 { // multiply coordinates of all terms by scalar
00356     using traits_t = numeric_traits<Scalar_T>;
00357
00358     if (traits_t::isNaN_or_isInf(scr))
00359         return *this = traits_t::NaN();
00360     if (scr == Scalar_T(0))
00361         if (this->isnan())
00362             *this = traits_t::NaN();
00363     else
00364         this->clear();
00365     else
00366         for (auto& this_term : *this)
00367             this_term.second *= scr;
00368     return *this;
00369 }
00370
00372 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00373 auto
00374 operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00375 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00376 {
00377     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00378     using traits_t = numeric_traits<Scalar_T>;
00379
00380     if (lhs.isnan() || rhs.isnan())
00381         return traits_t::NaN();
00382
00383     const double lhs_size = lhs.size();
00384     const double rhs_size = rhs.size();
00385     const auto our_frame = lhs.frame() | rhs.frame();
00386     const auto frm_count = our_frame.count();
00387     const auto algebra_dim = set_value_t(1) << frm_count;
00388     const auto direct_mult = lhs_size * rhs_size <= double(algebra_dim);
00389     if (direct_mult)
00390     { // If we have a sparse multiply, store the result directly
00391         auto result = multivector_t(
00392             _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00393         for (auto& lhs_term : lhs)
00394             for (auto& rhs_term : rhs)
00395                 result += lhs_term * rhs_term;
00396         return result;
00397     }
00398     else

```



```

00398     { // Past a certain threshold, the matrix algorithm is fastest
00399         using matrix_multi_t = typename multivector_t::matrix_multi_t;
00400         return matrix_multi_t(lhs, our_frame, true) *
00401             matrix_multi_t(rhs, our_frame, true);
00402     }
00403 }
00404
00406 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00407 inline
00408 auto
00409 framed_multi<Scalar_T,LO,HI,Tune_P>::
00410 operator*=(const multivector_t& rhs) -> multivector_t&
00411 { return *this = *this * rhs; }
00412
00414 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00415 auto
00416 operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00417 { // Arvind Raja's original reference:
00418     // "old clical, outerproduct(p,q:pterm):pterm in file compmod.pas"
00419
00420     if (lhs.empty() || rhs.empty())
00421         return Scalar_T(0);
00422
00423     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00424     using index_set_t = typename multivector_t::index_set_t;
00425     using term_t = typename multivector_t::term_t;
00426
00427     const auto empty_set = index_set_t();
00428
00429     const double lhs_size = lhs.size();
00430     const double rhs_size = rhs.size();
00431     const auto lhs_frame = lhs.frame();
00432     const auto rhs_frame = rhs.frame();
00433     const auto our_frame = lhs_frame | rhs_frame;
00434     const auto algebra_dim = set_value_t(1) << our_frame.count();
00435     auto result = multivector_t(
00436         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00437     const auto lhs_end = lhs.end();
00438     const auto rhs_end = rhs.end();
00439
00440     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00441     {
00442         for (auto
00443             result_stv = set_value_t(0);
00444             result_stv != algebra_dim;
00445             ++result_stv)
00446         {
00447             const auto result_ist = index_set_t(result_stv, our_frame, true);
00448             const auto lhs_result_frame = lhs_frame & result_ist;
00449             const auto lhs_result_dim = set_value_t(1) << lhs_result_frame.count();
00450             auto result_crd = Scalar_T(0);
00451             for (auto
00452                 lhs_stv = set_value_t(0);
00453                 lhs_stv != lhs_result_dim;
00454                 ++lhs_stv)
00455             {
00456                 const auto lhs_ist = index_set_t(lhs_stv, lhs_result_frame, true);
00457                 const auto rhs_ist = result_ist ^ lhs_ist;
00458                 if ((rhs_ist | rhs_frame) == rhs_frame)
00459                 {
00460                     const auto lhs_it = lhs.find(lhs_ist);
00461                     if (lhs_it != lhs_end)
00462                     {
00463                         const auto rhs_it = rhs.find(rhs_ist);
00464                         if (rhs_it != rhs_end)
00465                             result_crd += crd_of_mult(*lhs_it, *rhs_it);
00466                     }
00467                 }
00468             }
00469             if (result_crd != Scalar_T(0))
00470                 result.insert(term_t(result_ist, result_crd));
00471         }
00472         return result;
00473     }
00474     else
00475     {
00476         for (auto& lhs_term : lhs)
00477             for (auto& rhs_term : rhs)
00478                 if ((lhs_term.first & rhs_term.first) == empty_set)
00479                     result += lhs_term * rhs_term;
00480         return result;
00481     }
00482 }
00483
00485 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00486 inline

```

```

00487     auto
00488     framed_multi<Scalar_T,LO,HI,Tune_P>::
00489     operator^= (const multivector_t& rhs) -> multivector_t&
00490     { return *this = *this ^ rhs; }
00491
00492     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00493     auto
00494     operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00495     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00496     { // Arvind Raja's original reference:
00497       // "old clical, innerproduct(p,q:pterm):pterm in file compmod.pas"
00498
00499       if (lhs.empty() || rhs.empty())
00500         return Scalar_T(0);
00501
00502       using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00503       using index_set_t = typename multivector_t::index_set_t;
00504       using term_t = typename multivector_t::term_t;
00505
00506       const auto lhs_end = lhs.end();
00507       const auto rhs_end = rhs.end();
00508       const double lhs_size = lhs.size();
00509       const double rhs_size = rhs.size();
00510
00511       const auto lhs_frame = lhs.frame();
00512       const auto rhs_frame = rhs.frame();
00513
00514       const auto our_frame = lhs_frame | rhs_frame;
00515       const auto algebra_dim = set_value_t(1) << our_frame.count();
00516       auto result = multivector_t(
00517         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00518       if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00519       {
00520         for (auto
00521             result_stv = set_value_t(0);
00522             result_stv != algebra_dim;
00523             ++result_stv)
00524         {
00525           const auto result_ist = index_set_t(result_stv, our_frame, true);
00526           const auto comp_frame = our_frame & ~result_ist;
00527           const auto comp_dim = set_value_t(1) << comp_frame.count();
00528           auto result_crd = Scalar_T(0);
00529           for (auto
00530               comp_stv = set_value_t(1);
00531               comp_stv != comp_dim;
00532               ++comp_stv)
00533           {
00534             const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00535             const auto our_ist = result_ist ^ comp_ist;
00536             if ((our_ist | lhs_frame) == lhs_frame)
00537             {
00538               const auto lhs_it = lhs.find(our_ist);
00539               if (lhs_it != lhs_end)
00540               {
00541                 const auto rhs_it = rhs.find(comp_ist);
00542                 if (rhs_it != rhs_end)
00543                   result_crd += crd_of_mult(*lhs_it, *rhs_it);
00544               }
00545             }
00546             if (result_stv != 0)
00547             {
00548               if ((our_ist | rhs_frame) == rhs_frame)
00549               {
00550                 const auto rhs_it = rhs.find(our_ist);
00551                 if (rhs_it != rhs_end)
00552                 {
00553                   const auto lhs_it = lhs.find(comp_ist);
00554                   if (lhs_it != lhs_end)
00555                     result_crd += crd_of_mult(*lhs_it, *rhs_it);
00556                 }
00557             }
00558             }
00559           }
00560           if (result_crd != Scalar_T(0))
00561             result.insert(term_t(result_ist, result_crd));
00562         }
00563       }
00564       else
00565       {
00566         const auto empty_set = index_set_t();
00567         for (auto& lhs_term : lhs)
00568         {
00569           const auto lhs_ist = lhs_term.first;
00570           if (lhs_ist != empty_set)
00571             for (auto& rhs_term : rhs)
00572             {
00573               const auto rhs_ist = rhs_term.first;

```

```

00574         if (rhs_ist != empty_set)
00575         {
00576             const auto our_ist = lhs_ist | rhs_ist;
00577             if ((lhs_ist == our_ist) || (rhs_ist == our_ist))
00578                 result += lhs_term * rhs_term;
00579         }
00580     }
00581 }
00582 }
00583 return result;
00584 }
00585
00586 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00587 inline
00588 auto
00589 framed_multi<Scalar_T,LO,HI,Tune_P>::
00590 operator*= (const multivector_t& rhs) -> multivector_t&
00591 { return *this = *this & rhs; }
00592
00593 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00594 auto
00595 operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00596 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00597 {
00598     // Reference: Leo Dorst, "Honing geometric algebra for its use in the computer sciences",
00599     // in Geometric Computing with Clifford Algebras, ed. G. Sommer,
00600     // Springer 2001, Chapter 6, pp. 127-152.
00601     // http://staff.science.uva.nl/~leo/clifford/index.html
00602
00603     if (lhs.empty() || rhs.empty())
00604         return Scalar_T(0);
00605
00606     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00607     using index_set_t = typename multivector_t::index_set_t;
00608     using term_t = typename multivector_t::term_t;
00609     using map_t = typename multivector_t::map_t;
00610
00611     const auto lhs_end = lhs.end();
00612     const auto rhs_end = rhs.end();
00613     const double lhs_size = lhs.size();
00614     const double rhs_size = rhs.size();
00615     const auto lhs_frame = lhs.frame();
00616     const auto rhs_frame = rhs.frame();
00617
00618     const auto our_frame = lhs_frame | rhs_frame;
00619     const auto algebra_dim = set_value_t(1) << our_frame.count();
00620     auto result = multivector_t(
00621         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00622
00623     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00624     {
00625         for (auto
00626             result_stv = set_value_t(0);
00627             result_stv != algebra_dim;
00628             ++result_stv)
00629         {
00630             const auto result_ist = index_set_t(result_stv, our_frame, true);
00631             const auto comp_frame = lhs_frame & ~result_ist;
00632             const auto comp_dim = set_value_t(1) << comp_frame.count();
00633             auto result_crd = Scalar_T(0);
00634             for (auto
00635                 comp_stv = set_value_t(0);
00636                 comp_stv != comp_dim;
00637                 ++comp_stv)
00638             {
00639                 const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00640                 const auto rhs_ist = result_ist ^ comp_ist;
00641                 if ((rhs_ist | rhs_frame) == rhs_frame)
00642                 {
00643                     const auto rhs_it = rhs.find(rhs_ist);
00644                     if (rhs_it != rhs_end)
00645                     {
00646                         const auto lhs_it = lhs.find(comp_ist);
00647                         if (lhs_it != lhs_end)
00648                             result_crd += crd_of_mult(*lhs_it, *rhs_it);
00649                     }
00650                 }
00651             }
00652         }
00653         if (result_crd != Scalar_T(0))
00654             result.insert(term_t(result_ist, result_crd));
00655     }
00656     else
00657     {
00658         for (auto& rhs_term : rhs)
00659         {
00660             const auto rhs_ist = rhs_term.first;

```

```

00662         for (auto& lhs_term : lhs)
00663         {
00664             const index_set_t lhs_ist = lhs_term.first;
00665             if ((lhs_ist | rhs_ist) == rhs_ist)
00666                 result += lhs_term * rhs_term;
00667         }
00668     }
00669 }
00670 return result;
00671 }
00672
00673 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00674 inline
00675 auto
00676 framed_multi<Scalar_T,LO,HI,Tune_P>::
00677 operator%=(const multivector_t& rhs) -> multivector_t&
00678 { return *this = *this % rhs; }
00679
00680 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00681 auto
00682 star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00683 -> Scalar_T
00684 {
00685     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00686
00687     auto result = Scalar_T(0);
00688     const auto small_star_large = lhs.size() < rhs.size();
00689     const auto* smallp =
00690         small_star_large
00691         ? &lhs
00692         : &rhs;
00693     const auto* largep =
00694         small_star_large
00695         ? &rhs
00696         : &lhs;
00697
00698     for (auto& small_term : *smallp)
00699     {
00700         const auto small_ist = small_term.first;
00701         const auto large_crd = (*largep)[small_ist];
00702         if (large_crd != Scalar_T(0))
00703             result += small_ist.sign_of_square() * small_term.second * large_crd;
00704     }
00705     return result;
00706 }
00707
00708 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00709 auto
00710 framed_multi<Scalar_T,LO,HI,Tune_P>::
00711 operator/=(const Scalar_T& scr) -> multivector_t&
00712 { // Divide coordinates of all terms by scr
00713     using traits_t = numeric_traits<Scalar_T>;
00714
00715     if (traits_t::isNaN(scr))
00716         return *this = traits_t::NaN();
00717     if (traits_t::isInf(scr))
00718         if (this->isnan())
00719             *this = traits_t::NaN();
00720         else
00721             this->clear();
00722     else
00723         for (auto& this_term : *this)
00724             this_term.second /= scr;
00725     return *this;
00726 }
00727
00728 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00729 inline
00730 auto
00731 operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00732 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00733 {
00734     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00735     using traits_t = numeric_traits<Scalar_T>;
00736     using index_set_t = typename multivector_t::index_set_t;
00737     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00738
00739     if (rhs == Scalar_T(0))
00740         return traits_t::NaN();
00741
00742     const auto our_frame = lhs.frame() | rhs.frame();
00743     return matrix_multi_t(lhs, our_frame, true) / matrix_multi_t(rhs, our_frame, true);
00744 }
00745
00746 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00747 inline
00748 auto
00749 
```

```

00752     framed_multi<Scalar_T,LO,HI,Tune_P>::
00753     operator/= (const multivector_t& rhs) -> multivector_t&
00754     { return *this = *this / rhs; }
00755
00757     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00758     inline
00759     auto
00760     operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00761     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00762     {
00763         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00764         using matrix_multi_t = typename multivector_t::matrix_multi_t;
00765         return matrix_multi_t(rhs) * matrix_multi_t(lhs) / matrix_multi_t(rhs.involute());
00766     }
00767
00769     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00770     inline
00771     auto
00772     framed_multi<Scalar_T,LO,HI,Tune_P>::
00773     operator|= (const multivector_t& rhs) -> multivector_t&
00774     { return *this = *this | rhs; }
00775
00777     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00778     inline
00779     auto
00780     framed_multi<Scalar_T,LO,HI,Tune_P>::
00781     inv() const -> const multivector_t
00782     {
00783         auto result = matrix_multi_t(Scalar_T(1), this->frame());
00784         return result /= matrix_multi_t(*this);
00785     }
00786
00788     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00789     auto
00790     framed_multi<Scalar_T,LO,HI,Tune_P>::
00791     pow(int m) const -> const multivector_t
00792     { return glucat::pow(*this, m); }
00793
00795     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796     auto
00797     framed_multi<Scalar_T,LO,HI,Tune_P>::
00798     outer_pow(int m) const -> const multivector_t
00799     {
00800         if (m < 0)
00801             throw error_t("outer_pow(int): negative exponent");
00802         auto result = multivector_t(Scalar_T(1));
00803         auto a = *this;
00804         for (;
00805             m != 0;
00806             m >>= 1, a = a ^ a)
00807             if (m & 1)
00808                 result ^= a;
00809         return result;
00810     }
00811
00813     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00814     inline
00815     auto
00816     framed_multi<Scalar_T,LO,HI,Tune_P>::
00817     frame() const -> const index_set_t
00818     {
00819         auto result = index_set_t();
00820         for (auto& this_term : *this)
00821             result |= this_term.first;
00822         return result;
00823     }
00824
00826     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00827     inline
00828     auto
00829     framed_multi<Scalar_T,LO,HI,Tune_P>::
00830     grade() const -> index_t
00831     {
00832         auto result = index_t(0);
00833         for (auto& this_term : *this)
00834             result = std::max(result, this_term.first.count());
00835         return result;
00836     }
00837
00839     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840     inline
00841     auto
00842     framed_multi<Scalar_T,LO,HI,Tune_P>::
00843     operator[] (const index_set_t ist) const -> Scalar_T
00844     {
00845         const auto& this_it = this->find(ist);

```

```

00846     if (this_it == this->end())
00847         return Scalar_T(0);
00848     else
00849         return this_it->second;
00850 }
00851
00853 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00854 auto
00855 framed_multi<Scalar_T,LO,HI,Tune_P>::
00856 operator() (index_t grade) const -> const multivector_t
00857 {
00858     if ((grade < 0) || (grade > HI-LO))
00859         return Scalar_T(0);
00860     else
00861     {
00862         auto result = multivector_t();
00863         for (auto& this_term : *this)
00864             if (this_term.first.count() == grade)
00865                 result += this_term;
00866         return result;
00867     }
00868 }
00869
00871 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00872 inline
00873 auto
00874 framed_multi<Scalar_T,LO,HI,Tune_P>::
00875 scalar() const -> Scalar_T
00876 { return (*this)[index_set_t()]; }
00877
00879 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00880 inline
00881 auto
00882 framed_multi<Scalar_T,LO,HI,Tune_P>::
00883 pure() const -> const multivector_t
00884 { return *this - this->scalar(); }
00885
00887 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00888 auto
00889 framed_multi<Scalar_T,LO,HI,Tune_P>::
00890 even() const -> const multivector_t
00891 { // even part of x, sum of the pure(count) with even count
00892     auto result = multivector_t();
00893     for (auto& this_term : *this)
00894         if ((this_term.first.count() % 2) == 0)
00895             result.insert(this_term);
00896     return result;
00897 }
00898
00900 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00901 auto
00902 framed_multi<Scalar_T,LO,HI,Tune_P>::
00903 odd() const -> const multivector_t
00904 { // even part of x, sum of the pure(count) with even count
00905     auto result = multivector_t();
00906     for (auto& this_term : *this)
00907         if ((this_term.first.count() % 2) == 1)
00908             result.insert(this_term);
00909     return result;
00910 }
00911
00913 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00914 auto
00915 framed_multi<Scalar_T,LO,HI,Tune_P>::
00916 vector_part() const -> const vector_t
00917 { return this->vector_part(this->frame(), true); }
00918
00920 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00921 auto
00922 framed_multi<Scalar_T,LO,HI,Tune_P>::
00923 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00924 {
00925     if (!prechecked && (this->frame() | frm) != frm)
00926         throw error_t("vector_part(frm): value is outside of requested frame");
00927     auto result = vector_t();
00928     result.reserve(frm.count());
00929     const auto frm_end = frm.max()+1;
00930     for (auto
00931         idx = frm.min();
00932         idx != frm_end;
00933         ++idx)
00934         // Frame may contain indices which do not correspond to a grade 1 term but
00935         // frame cannot omit any index corresponding to a grade 1 term
00936         if (frm[idx])
00937             result.push_back((*this)[index_set_t(idx)]);
00938     return result;
00939 }

```

```

00940
00942 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00943 auto
00944 framed_multi<Scalar_T,LO,HI,Tune_P>::
00945 involute() const -> const multivector_t
00946 {
00947     auto result = *this;
00948     for (auto& result_term : result)
00949     { // for a k-vector u, involute(u) == (-1)^k * u
00950         if ((result_term.first.count() % 2) == 1)
00951             result_term.second *= Scalar_T(-1);
00952     }
00953     return result;
00954 }
00955
00957 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00958 auto
00959 framed_multi<Scalar_T,LO,HI,Tune_P>::
00960 reverse() const -> const multivector_t
00961 {
00962     auto result = *this;
00963     for (auto& result_term : result)
00964     { // For a k-vector u, reverse(u) = { -u, k == 2,3 (mod 4)
00965         // { u, k == 0,1 (mod 4)
00966         switch (result_term.first.count() % 4)
00967         {
00968             case 2:
00969             case 3:
00970                 result_term.second *= Scalar_T(-1);
00971                 break;
00972             default:
00973                 break;
00974         }
00975     }
00976     return result;
00977 }
00978
00979 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00980 auto
00981 framed_multi<Scalar_T,LO,HI,Tune_P>::
00982 conj() const -> const multivector_t
00983 {
00984     auto result = *this;
00985     for (auto& result_term : result)
00986     { // For a k-vector u, conj(u) = { -u, k == 1,2 (mod 4)
00987         // { u, k == 0,3 (mod 4)
00988         switch (result_term.first.count() % 4)
00989         {
00990             case 1:
00991             case 2:
00992                 result_term.second *= Scalar_T(-1);
00993                 break;
00994             default:
00995                 break;
00996         }
00997     }
00998     return result;
00999 }
01000
01001 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01002 auto
01003 framed_multi<Scalar_T,LO,HI,Tune_P>::
01004 quad() const -> Scalar_T
01005 {
01006     // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
01007     // ref: old clical: quadfunction(p:pterm):pterm in file compmod.pas
01008     auto result = Scalar_T(0);
01009     for (auto& this_term : *this)
01010     {
01011         const auto sign =
01012             (this_term.first.count_neg() % 2)
01013             ? -Scalar_T(1)
01014             : Scalar_T(1);
01015         result += sign * (this_term.second) * (this_term.second);
01016     }
01017     return result;
01018 }
01019
01021 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01022 auto
01023 framed_multi<Scalar_T,LO,HI,Tune_P>::
01024 norm() const -> Scalar_T
01025 {
01026     using traits_t = numeric_traits<Scalar_T>;
01027
01028     auto result = Scalar_T(0);
01029     for (auto& this_term : *this)
01030     {
01031         const auto abs_crd = traits_t::abs(this_term.second);

```

```

01032         result += abs_crd * abs_crd;
01033     }
01034     return result;
01035 }
01036
01037 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01038 auto
01039 framed_multi<Scalar_T,LO,HI,Tune_P>::
01040 max_abs() const -> Scalar_T
01041 {
01042     {
01043         using traits_t = numeric_traits<Scalar_T>;
01044
01045         auto result = Scalar_T(0);
01046         for (auto& this_term : *this)
01047         {
01048             const auto abs_crd = traits_t::abs(this_term.second);
01049             if (abs_crd > result)
01050                 result = abs_crd;
01051         }
01052         return result;
01053     }
01054
01055 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01056 auto
01057 framed_multi<Scalar_T,LO,HI,Tune_P>::
01058 random(const index_set_t frm, Scalar_T fill) -> const multivector_t
01059 {
01060     {
01061         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01062         using index_set_t = typename multivector_t::index_set_t;
01063         using term_t = typename multivector_t::term_t;
01064
01065         using random_generator_t = random_generator<Scalar_T>;
01066         auto& generator = random_generator_t::generator();
01067
01068         fill =
01069             (fill < Scalar_T(0))
01070             ? Scalar_T(0)
01071             : (fill > Scalar_T(1))
01072             ? Scalar_T(1)
01073             : fill;
01074         const auto algebra_dim = set_value_t(1) << frm.count();
01075         using traits_t = numeric_traits<Scalar_T>;
01076         const auto mean_abs = traits_t::sqrt(Scalar_T(double(algebra_dim)));
01077         auto result = multivector_t();
01078         for (auto
01079             stv = set_value_t(0);
01080             stv != algebra_dim;
01081             ++stv)
01082             if (generator.uniform() < fill)
01083             {
01084                 const auto& result_crd = generator.normal() / mean_abs;
01085                 result.insert(term_t(index_set_t(stv, frm, true), result_crd));
01086             }
01087         return result;
01088     }
01089
01090 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01091 inline
01092 void
01093 framed_multi<Scalar_T,LO,HI,Tune_P>::
01094 write(const std::string& msg) const
01095 { std::cout << msg << std::endl << " " << (*this) << std::endl; }
01096
01097 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01098 inline
01099 void
01100 framed_multi<Scalar_T,LO,HI,Tune_P>::
01101 write(std::ofstream& ofile, const std::string& msg) const
01102 {
01103     if (!ofile)
01104         throw error_t("write(ofile,msg): cannot write to output file");
01105     ofile << msg << std::endl << " " << (*this) << std::endl;
01106 }
01107
01108 template< typename Map_T,typename Sorted_Map_T >
01109 class sorted_range
01110 {
01111 public:
01112     using map_t = Map_T;
01113     using sorted_map_t = Sorted_Map_T;
01114     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01115
01116     sorted_range (Sorted_Map_T &sorted_val, const Map_T& val)
01117     {
01118         for (auto& val_term : val)
01119             sorted_val.insert(val_term);
01120         sorted_begin = sorted_val.begin();
01121     }
01122 }

```



```

01124         sorted_end = sorted_val.end();
01125     }
01126     sorted_iterator sorted_begin;
01127     sorted_iterator sorted_end;
01128 };
01129
01130 template< typename Sorted_Map_T >
01131 class sorted_range< Sorted_Map_T, Sorted_Map_T >
01132 {
01133 public:
01134     using map_t = Sorted_Map_T;
01135     using sorted_map_t = Sorted_Map_T;
01136     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01137
01138     sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T& val)
01139     : sorted_begin( val.begin() ),
01140       sorted_end( val.end() )
01141     { }
01142     sorted_iterator sorted_begin;
01143     sorted_iterator sorted_end;
01144 };
01145
01146 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01147 auto
01148 operator<< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
01149 {
01150     {
01151         using limits_t = std::numeric_limits<Scalar_T>;
01152         if (val.empty())
01153             os << 0;
01154         else if (val.isnan())
01155             os << limits_t::quiet_NaN();
01156         else if (val.isinf())
01157         {
01158             const Scalar_T& inf = limits_t::infinity();
01159             os << (scalar(val) < 0.0 ? -inf : inf);
01160         }
01161         else
01162         {
01163             using traits_t = numeric_traits<Scalar_T>;
01164             using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01165             Scalar_T truncation;
01166             switch (os.flags() & std::ios::floatfield)
01167             {
01168                 case std::ios_base::scientific:
01169                     truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()) + 1);
01170                     break;
01171                 case std::ios_base::fixed:
01172                     truncation = Scalar_T(1) / (traits_t::pow(Scalar_T(10), int(os.precision())) *
01173 val.max_abs());
01174                     break;
01175                 case std::ios_base::fixed | std::ios_base::scientific:
01176                     truncation = multivector_t::default_truncation;
01177                     break;
01178                 default:
01179                     truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()));
01180                     break;
01181             }
01182             auto truncated_val = val.truncated(truncation);
01183             if (truncated_val.empty())
01184                 os << 0;
01185             else
01186             {
01187                 using map_t = typename multivector_t::map_t;
01188                 using sorted_map_t = typename multivector_t::sorted_map_t;
01189                 using sorted_iterator = typename sorted_map_t::const_iterator;
01190                 auto sorted_val = sorted_map_t();
01191                 const auto sorted_val_range = sorted_range< map_t, sorted_map_t >(sorted_val, truncated_val);
01192                 auto sorted_it = sorted_val_range.sorted_begin;
01193                 os << *sorted_it;
01194                 for (++sorted_it;
01195                    sorted_it != sorted_val_range.sorted_end;
01196                    ++sorted_it)
01197                 {
01198                     const Scalar_T& scr = sorted_it->second;
01199                     if (scr >= 0.0)
01200                         os << '+';
01201                     os << *sorted_it;
01202                 }
01203             }
01204             return os;
01205         }
01206     }
01207
01208     template< typename Scalar_T, const index_t LO, const index_t HI >
01209     auto
01210     operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
01211     std::ostream&

```

```

01211 {
01212     const auto second_as_double = numeric_traits<Scalar_T>::to_double(term.second);
01213     const auto use_double =
01214         (os.precision() <= std::numeric_limits<double>::digits10) ||
01215         (term.second == Scalar_T(second_as_double));
01216     if (term.first.count() == 0)
01217         if (use_double)
01218             os << second_as_double;
01219         else
01220             os << term.second;
01221     else if (term.second == Scalar_T(-1))
01222     {
01223         os << '-';
01224         os << term.first;
01225     }
01226     else if (term.second != Scalar_T(1))
01227     {
01228         if (use_double)
01229         {
01230             auto tol = std::pow(10.0, -os.precision());
01231             if ( std::fabs(second_as_double + 1.0) < tol )
01232                 os << '-';
01233             else if ( std::fabs(second_as_double - 1.0) >= tol )
01234                 os << second_as_double;
01235         }
01236         else
01237             os << term.second;
01238         os << term.first;
01239     }
01240     else
01241         os << term.first;
01242     return os;
01243 }
01244
01246 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01247 auto
01248 operator<> (std::istream& s, framed_multi<Scalar_T, LO, HI, Tune_P> & val) -> std::istream&
01249 { // Input looks like 1.0-2.0{1,2}+3.2{3,4}.
01250     using multivector_t = framed_multi<Scalar_T, LO, HI, Tune_P>;
01251     // Parsing variables.
01252     auto local_val = multivector_t();
01253     auto c = 0;
01254     // Parsing control variables.
01255     auto negative = false;
01256     auto expect_term = true;
01257     // The multivector may begin with '+' or '-'. Check for this.
01258     c = s.peek();
01259     if (s.good() && (c == int('+') || c == int('-')))
01260     { // A '-' here negates the following term.
01261         negative = (c == int('-'));
01262         // Consume the '+' or '-'.
01263         s.get();
01264     }
01265     while (s.good())
01266     { // Parse a term.
01267         // A term consists of an optional scalar, followed by an optional index set.
01268         // At least one of the two must be present.
01269         // Default coordinate is Scalar_T(1).
01270         auto coordinate = Scalar_T(1);
01271         // Default index set is empty.
01272         auto ist = index_set<LO, HI>();
01273         // First, check for an opening brace.
01274         c = s.peek();
01275         if (s.good())
01276         { // If the character is not an opening brace,
01277             // a coordinate value is expected here.
01278             if (c != int('{'))
01279             { // Try to read a coordinate value.
01280                 double coordinate_as_double;
01281                 s >> coordinate_as_double;
01282                 // Reading the coordinate may have resulted in an end of file condition.
01283                 // This is not a failure.
01284                 if (s)
01285                     coordinate = Scalar_T(coordinate_as_double);
01286             }
01287         }
01288         else
01289         { // End of file here ends parsing while a term may still be expected.
01290             break;
01291         }
01292         // Coordinate is now Scalar_T(1) or a Scalar_T value.
01293         // Parse an optional index set.
01294         if (s.good())
01295         {
01296             c = s.peek();
01297             if (s.good() && c == int('{'))
01298                 // Try to read index set.

```

```

01299         s » ist;
01300     }
01301 }
01302 // Reading the term may have resulted in an end of file condition.
01303 // This is not a failure.
01304 if (s)
01305 {
01306     // Immediately after parsing a term, another term is not expected.
01307     expect_term = false;
01308     if (coordinate != Scalar_T(0))
01309     {
01310         // Add the term to the local multivector.
01311         coordinate =
01312             negative
01313             ? -coordinate
01314             : coordinate;
01315         using term_t = typename multivector_t::term_t;
01316         local_val += term_t(ist, coordinate);
01317     }
01318 }
01319 // Check if anything follows the current term.
01320 if (s.good())
01321 {
01322     c = s.peek();
01323     if (s.good())
01324     { // Only '+' and '-' are valid here.
01325         if (c == int('+') || c == int('-'))
01326         { // A '-' here negates the following term.
01327             negative = (c == int('-'));
01328             // Consume the '+' or '-'.
01329             s.get();
01330             // Immediately after '+' or '-',
01331             // expect another term.
01332             expect_term = true;
01333         }
01334         else
01335         { // Any other character here is a not failure,
01336             // but still ends the parsing of the multivector.
01337             break;
01338         }
01339     }
01340 }
01341 // If a term is still expected, this is a failure.
01342 if (expect_term)
01343     s.clear(std::istream::failbit);
01344 // End of file is not a failure.
01345 if (s)
01346 { // The multivector has been successfully parsed.
01347     val = local_val;
01348 }
01349 return s;
01350 }
01351 }
01352
01353 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01354 auto
01355 framed_multi<Scalar_T,LO,HI,Tune_P>::
01356 nbr_terms () const -> unsigned long
01357 { return this->size(); }
01358
01359 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01360 inline
01361 auto
01362 framed_multi<Scalar_T,LO,HI,Tune_P>::
01363 operator+= (const term_t& term) -> multivector_t&
01364 { // Do not insert terms with 0 coordinate
01365     if (term.second != Scalar_T(0))
01366     {
01367         const auto& this_it = this->find(term.first);
01368         if (this_it == this->end())
01369             this->insert(term);
01370         else if (this_it->second + term.second == Scalar_T(0))
01371             // Erase term if resulting coordinate is 0
01372             this->erase(this_it);
01373         else
01374             this_it->second += term.second;
01375     }
01376     return *this;
01377 }
01378
01379 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01380 auto
01381 framed_multi<Scalar_T,LO,HI,Tune_P>::
01382 isinf() const -> bool
01383 {
01384     using traits_t = numeric_traits<Scalar_T>;
01385 }

```

```

01389     if (std::numeric_limits<Scalar_T>::has_infinity)
01390     for (auto& this_term : *this)
01391     if (traits_t::isInf(this_term.second))
01392     return true;
01393     return false;
01394 }
01395
01396 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01397 auto
01398 framed_multi<Scalar_T,LO,HI,Tune_P>::
01399 isnan() const -> bool
01400 {
01401     using traits_t = numeric_traits<Scalar_T>;
01402
01403     if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
01404     for (auto& this_term : *this)
01405     if (traits_t::isNaN(this_term.second))
01406     return true;
01407     return false;
01408 }
01409
01410 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01411 auto
01412 framed_multi<Scalar_T,LO,HI,Tune_P>::
01413 truncated(const Scalar_T& limit) const -> const multivector_t
01414 {
01415     using traits_t = numeric_traits<Scalar_T>;
01416
01417     if (this->isnan() || this->isinf())
01418     return *this;
01419     const auto truncation = traits_t::abs(limit);
01420     const auto top = max_abs();
01421     auto result = multivector_t();
01422     if (top != Scalar_T(0))
01423     for (auto& this_term : *this)
01424     if (traits_t::abs(this_term.second) > top * truncation)
01425     result.insert(this_term);
01426     return result;
01427 }
01428
01429 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01430 auto
01431 framed_multi<Scalar_T,LO,HI,Tune_P>::
01432 fold(const index_set_t frm) const -> multivector_t
01433 {
01434     if (frm.is_contiguous())
01435     return *this;
01436     else
01437     {
01438         auto result = multivector_t();
01439         for (auto& this_term : *this)
01440         result.insert(term_t(this_term.first.fold(frm), this_term.second));
01441     }
01442 }
01443
01444 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01445 auto
01446 framed_multi<Scalar_T,LO,HI,Tune_P>::
01447 unfold(const index_set_t frm) const -> multivector_t
01448 {
01449     if (frm.is_contiguous())
01450     return *this;
01451     else
01452     {
01453         auto result = multivector_t();
01454         for (auto& this_term : *this)
01455         result.insert(term_t(this_term.first.unfold(frm), this_term.second));
01456     }
01457 }
01458
01459 // Reference: [L] 16.4 Periodicity of 8, p216
01460 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01461 auto
01462 framed_multi<Scalar_T,LO,HI,Tune_P>::
01463 centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&
01464 {
01465     // We add 4 to q by subtracting 4 from p
01466     if (q+4 > -LO)
01467     throw error_t("centre_pm4_qp4(p,q): LO is too high to represent this value");
01468     if (this->frame().max() > p-4)
01469     {
01470         using index_pair_t = typename index_set_t::index_pair_t;
01471         const auto pm3210 = index_set_t(index_pair_t(p-3,p), true);
01472         const auto qm4321 = index_set_t(index_pair_t(-q-4,-q-1), true);
01473         const auto& tqm4321 = term_t(qm4321, Scalar_T(1));
01474     }

```

```

01481     auto result = multivector_t();
01482     for (auto& this_term : *this)
01483     {
01484         const auto ist = this_term.first;
01485         if (ist.max() > p-4)
01486         {
01487             auto var_term = var_term_t();
01488             for (auto
01489                 n = index_t(0);
01490                 n != index_t(4);
01491                 ++n)
01492                 if (ist[n+p-3])
01493                     var_term *= term_t(index_set_t(n-q-4), Scalar_T(1)) * tqm4321;
01494             // Mask out {p-3}..{p}
01495             result.insert(term_t(ist & ~pm3210, this_term.second) *
01496                             term_t(var_term.first, var_term.second));
01497         }
01498         else
01499             result.insert(this_term);
01500     }
01501     *this = result;
01502 }
01503 p -=4; q += 4;
01504 return *this;
01505 }
01506
01507 // Reference: [L] 16.4 Periodicity of 8, p216
01508 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01509 auto
01510 framed_multi<Scalar_T,LO,HI,Tune_P>::
01511 centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&
01512 {
01513     // We add 4 to p by subtracting 4 from q
01514     if (p+4 > HI)
01515         throw error_t("centre_pp4_qm4(p,q): HI is too low to represent this value");
01516     if (this->frame().min() < -q+4)
01517     {
01518         using index_pair_t = typename index_set_t::index_pair_t;
01519         const auto qp0123 = index_set_t(index_pair_t(-q,-q+3), true);
01520         const auto pp1234 = index_set_t(index_pair_t(p+1,p+4), true);
01521         const auto& tpp1234 = term_t(pp1234, Scalar_T(1));
01522         auto result = multivector_t();
01523         for (auto& this_term : *this)
01524         {
01525             index_set_t ist = this_term.first;
01526             if (ist.min() < -q+4)
01527             {
01528                 auto var_term = var_term_t();
01529                 for (auto
01530                     n = index_t(0);
01531                     n != index_t(4);
01532                     ++n)
01533                     if (ist[n-q])
01534                         var_term *= term_t(index_set_t(n+p+1), Scalar_T(1)) * tpp1234;
01535                 // Mask out {-q}..{-q+3}
01536                 result.insert(term_t(var_term.first, var_term.second) *
01537                               term_t(ist & ~qp0123, this_term.second));
01538             }
01539             else
01540                 result.insert(this_term);
01541         }
01542         *this = result;
01543     }
01544     p +=4; q -= 4;
01545     return *this;
01546 }
01547 }
01548
01549 // Reference: [P] Proposition 15.20, p 131
01550 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01551 auto
01552 framed_multi<Scalar_T,LO,HI,Tune_P>::
01553 centre_qp1_pml(index_t& p, index_t& q) -> multivector_t&
01554 {
01555     if (q+1 > HI)
01556         throw error_t("centre_qp1_pml(p,q): HI is too low to represent this value");
01557     if (p-1 > -LO)
01558         throw error_t("centre_qp1_pml(p,q): LO is too high to represent this value");
01559     const auto qp1 = index_set_t(q+1);
01560     const auto& tqp1 = term_t(qp1, Scalar_T(1));
01561     auto result = multivector_t();
01562     for (auto& this_term : *this)
01563     {
01564         const auto ist = this_term.first;
01565         auto var_term = var_term_t(index_set_t(), this_term.second);
01566         for (auto
01567             n = -q;
01568             n != p;

```

```

01570         ++n)
01571         if (n != 0 && ist[n])
01572             var_term *= term_t(index_set_t(-n) | qpl, Scalar_T(1));
01573         if (p != 0 && ist[p])
01574             var_term *= tqpl;
01575         result.insert(term_t(var_term.first, var_term.second));
01576     }
01577     index_t orig_p = p;
01578     p = q+1;
01579     q = orig_p-1;
01580     return *this = result;
01581 }
01582
01583 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01584 auto
01585 framed_multi<Scalar_T,LO,HI,Tune_P>::
01586 divide(const index_set_t ist) const -> const framed_pair_t
01587 {
01588     auto quo = multivector_t();
01589     auto rem = multivector_t();
01590     for (auto& this_term : *this)
01591         if ((this_term.first | ist) == this_term.first)
01592             quo.insert(term_t(this_term.first ^ ist, this_term.second));
01593         else
01594             rem.insert(this_term);
01595     return framed_pair_t(quo, rem);
01596 }
01597
01598 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01599 auto
01600 framed_multi<Scalar_T,LO,HI,Tune_P>::
01601 fast(const index_t level, const bool odd) const -> const matrix_t
01602 {
01603     // Assume val is already folded and centred
01604     if (this->empty())
01605     {
01606         using matrix_index_t = typename matrix_multi_t::matrix_index_t;
01607         const auto dim = matrix_index_t(1) << level;
01608         auto result = matrix_t(dim, dim);
01609         result.clear();
01610         return result;
01611     }
01612     if (level == 0)
01613         return matrix::unit<matrix_t>(1) * this->scalar();
01614
01615     using basis_matrix_t = typename matrix_multi_t::basis_matrix_t;
01616     using basis_scalar_t = typename basis_matrix_t::value_type;
01617
01618     const auto& I = matrix::unit<basis_matrix_t>(2);
01619     auto J = basis_matrix_t(2,2,2);
01620     J.clear();
01621     J(0,1) = basis_scalar_t(-1);
01622     J(1,0) = basis_scalar_t( 1);
01623     auto K = J;
01624     K(0,1) = basis_scalar_t( 1);
01625     auto JK = I;
01626     JK(0,0) = basis_scalar_t(-1);
01627
01628     const auto ist_mn = index_set_t(-level);
01629     const auto ist_pn = index_set_t(level);
01630     if (level == 1)
01631     {
01632         if (odd)
01633             return matrix_t(J) * (*this)[ist_mn] + matrix_t(K) * (*this)[ist_pn];
01634         else
01635             return matrix_t(I) * this->scalar() + matrix_t(JK) * (*this)[ist_mn ^ ist_pn];
01636     }
01637     else
01638     {
01639         const auto& pair_mn = this->divide(ist_mn);
01640         const auto& quo_mn = pair_mn.first;
01641         const auto& rem_mn = pair_mn.second;
01642         const auto& pair_quo_mnpn = quo_mn.divide(ist_pn);
01643         const auto& val_mnpn = pair_quo_mnpn.first;
01644         const auto& val_mn = pair_quo_mnpn.second;
01645         const auto& pair_rem_mnpn = rem_mn.divide(ist_pn);
01646         const auto& val_pn = pair_rem_mnpn.first;
01647         const auto& val_l = pair_rem_mnpn.second;
01648         using matrix::kron;
01649         if (odd)
01650             return - kron(JK, val_l.fast (level-1, 1))
01651                 + kron(I, val_mnpn.fast (level-1, 1))
01652                 + kron(J, val_mn.fast (level-1, 0))
01653                 + kron(K, val_pn.fast (level-1, 0));
01654         else
01655             return kron(I, val_l.fast (level-1, 0))
01656                 + kron(JK, val_mnpn.fast (level-1, 0))

```

```

01659         + kron(K, val_mn.fast (level-1, 1))
01660         - kron(J, val_pn.fast (level-1, 1));
01661     }
01662 }
01663
01665 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01666 template< typename Other_Scalar_T >
01667 auto
01668 framed_multi<Scalar_T,LO,HI,Tune_P>::
01669 fast_matrix_multi(const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>
01670 {
01671     // Fold val
01672     auto val = this->fold(frm);
01673     auto p = frm.count_pos();
01674     auto q = frm.count_neg();
01675     const auto bott_offset = gen::offset_to_super[pos_mod(p - q, 8)];
01676     p += std::max(bott_offset, index_t(0));
01677     q -= std::min(bott_offset, index_t(0));
01678     if (p > HI)
01679         throw error_t("fast_matrix_multi(frm): HI is too low to represent this value");
01680     if (q > -LO)
01681         throw error_t("fast_matrix_multi(frm): LO is too high to represent this value");
01682     // Centre val
01683     while (p - q > 4)
01684         val.centre_pm4_qp4(p, q);
01685     while (p - q < -3)
01686         val.centre_pp4_qm4(p, q);
01687     if (p - q > 1)
01688         val.centre_qp1_pm1(p, q);
01689     const index_t level = (p + q)/2;
01690
01691     // Do the fast transform
01692     const auto& ev_val = val.even();
01693     const auto& od_val = val.odd();
01694     return matrix_multi<Other_Scalar_T,LO,HI,Tune_P>(ev_val.fast(level, 0) + od_val.fast(level, 1),
01695 frm);
01696 }
01697
01697 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01698 inline
01699 auto
01700 framed_multi<Scalar_T,LO,HI,Tune_P>::
01701 fast_framed_multi() const -> const multivector_t
01702 { return *this; }
01703
01705 template< typename Scalar_T, const index_t LO, const index_t HI >
01706 inline
01707 static
01708 auto
01709 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01710 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T
01711 { return lhs.first.sign_of_mult(rhs.first) * lhs.second * rhs.second; }
01712
01714 template< typename Scalar_T, const index_t LO, const index_t HI >
01715 inline
01716 auto
01717 operator* (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01718 const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const
index_set<LO,HI>, Scalar_T>
01719 {
01720     using term_t = std::pair<const index_set<LO,HI>, Scalar_T>;
01721     return term_t(lhs.first ^ rhs.first, crd_of_mult(lhs, rhs));
01722 }
01723
01725 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01726 auto
01727 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01728 {
01729     using traits_t = numeric_traits<Scalar_T>;
01730     if (val.isnan())
01731         return traits_t::NaN();
01732
01733     check_complex(val, i, prechecked);
01734
01735     const auto realval = val.scalar();
01736     if (val == realval)
01737     {
01738         if (realval < Scalar_T(0))
01739             return i * traits_t::sqrt(-realval);
01740         else
01741             return traits_t::sqrt(realval);
01742     }
01743     using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01744     return sqrt(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01745 }
01746

```

```

01748 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01749 auto
01750 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01751 {
01752     using traits_t = numeric_traits<Scalar_T>;
01753     if (val.isnan())
01754         return traits_t::NaN();
01755
01756     const auto s = scalar(val);
01757     if (val == s)
01758         return traits_t::exp(s);
01759
01760     const double size = val.size();
01761     const auto frm_count = val.frame().count();
01762     const auto algebra_dim = set_value_t(1) « frm_count;
01763
01764     if( (size * size <= double(algebra_dim)) || (frm_count < Tune_P::mult_matrix_threshold))
01765     {
01766         switch (Tune_P::function_precision)
01767         {
01768             case precision_demoted:
01769             {
01770                 using demoted_scalar_t = typename traits_t::demoted::type;
01771                 using demoted_multivector_t = framed_multi<demoted_scalar_t,LO,HI,Tune_P>;
01772
01773                 const auto& demoted_val = demoted_multivector_t(val);
01774                 return clifford_exp(demoted_val);
01775             }
01776             break;
01777             case precision_promoted:
01778             {
01779                 using promoted_scalar_t = typename traits_t::promoted::type;
01780                 using promoted_multivector_t = framed_multi<promoted_scalar_t,LO,HI,Tune_P>;
01781
01782                 const auto& promoted_val = promoted_multivector_t(val);
01783                 return clifford_exp(promoted_val);
01784             }
01785             break;
01786             default:
01787                 return clifford_exp(val);
01788         }
01789     }
01790     else
01791     {
01792         using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01793         return exp(matrix_multi_t(val));
01794     }
01795 }
01796
01798 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01799 auto
01800 log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01801 {
01802     using traits_t = numeric_traits<Scalar_T>;
01803     if (val == Scalar_T(0) || val.isnan())
01804         return traits_t::NaN();
01805
01806     check_complex(val, i, prechecked);
01807
01808     const auto realval = val.scalar();
01809     if (val == realval)
01810     {
01811         if (realval < Scalar_T(0))
01812             return i * traits_t::pi() + traits_t::log(-realval);
01813         else
01814             return traits_t::log(realval);
01815     }
01816     using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01817     return log(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01818 }
01819 }
01820 #endif // _GLUCAT_FRAMED_MULTI_IMP_H

```

## 7.13 glucat/generation.h File Reference

```

#include "glucat/global.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <utility>
#include <array>

```



```
#include <map>
#include <vector>
```

Include dependency graph for generation.h: This graph shows which files directly or indirectly include this file:

## Classes

- class `glucat::gen::generator_table< Matrix_T >`  
*Table of generators for specific signatures.*

## Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## Typedefs

- using `glucat::gen::signature_t = std::pair< index_t, index_t >`  
*A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min()*

## Variables

- static const `std::array< index_t, 8 > glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}`  
*Offsets between the current signature and that of the real superalgebra.*

## 7.14 generation.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GENERATION_H
00002 #define _GLUCAT_GENERATION_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation.h : Declare functions for generation of the matrix representation
00006     -----
00007     begin                : Wed Jan 23 2002
00008     copyright            : (C) 2002-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
```

```

00035
00036 #include <boost/numeric/ublas/fwd.hpp>
00037
00038 #include <utility>
00039 #include <array>
00040 #include <map>
00041 #include <vector>
00042
00043 namespace glucat { namespace gen
00044 {
00045     namespace ublas = boost::numeric::ublas;
00046
00047     using signature_t = std::pair<index_t, index_t>;
00048
00049     template< class Matrix_T >
00050     class generator_table :
00051     private std::map< signature_t, std::vector<Matrix_T> >
00052     {
00053     public:
00054         auto operator() (const index_t p, const index_t q) -> const Matrix_T*;
00055         static auto generator() -> generator_table<Matrix_T>&;
00056     private:
00057         auto gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&;
00058         void gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig);
00059         void gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig);
00060         void gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig);
00061         void gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig);
00062
00063         friend class friend_for_private_destructor;
00064         // Enforce singleton
00065         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00066         generator_table() = default;
00067         ~generator_table() = default;
00068     public:
00069         generator_table(const generator_table&) = delete;
00070         auto operator= (const generator_table&) -> generator_table& = delete;
00071     };
00072
00073     static const std::array<index_t, 8> offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1};
00074 } }
00075 #endif // _GLUCAT_GENERATION_H

```

## 7.15 glucat/generation\_imp.h File Reference

```

#include "glucat/global.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"

```

Include dependency graph for generation\_imp.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## 7.16 generation\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_IMP_H
00002 #define _GLUCAT_GENERATION_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation_imp.h : Implement functions for generation of the matrix representation
00006
00007     begin                : Wed Jan 23 2002
00008     copyright            : (C) 2002-2012 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify

```

```

00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/generation.h"
00036 #include "glucat/matrix.h"
00037
00038 namespace glucat { namespace gen
00039 {
00040     // References for algorithms:
00041     // [M]: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.
00042     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00043     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00044
00046     // Reference: [M] Item 47
00047     template< class Matrix_T >
00048     auto
00049     generator_table<Matrix_T>::
00050     generator() -> generator_table<Matrix_T>&
00051     { static generator_table<Matrix_T> g; return g; }
00052
00054     // Reference: [P] Table 15.27, p 133
00055     template< class Matrix_T >
00056     inline
00057     auto
00058     generator_table<Matrix_T>::
00059     operator() (const index_t p, const index_t q) -> const Matrix_T*
00060     {
00061         const auto bott = pos_mod(p-q, 8);
00062         switch(bott)
00063         {
00064             case 0:
00065             case 2:
00066                 // Construct generators
00067                 return &(gen_vector(p, q)[q]);
00068             default:
00069                 // Select generators from the vector for a larger frame
00070                 const auto super_p = p + std::max(offset_to_super[bott], index_t(0));
00071                 const auto super_q = q - std::min(offset_to_super[bott], index_t(0));
00072                 return &(gen_vector(super_p, super_q)[super_q]);
00073         }
00074     }
00075
00077     template< class Matrix_T >
00078     auto
00079     generator_table<Matrix_T>::
00080     gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&
00081     {
00082         using result_t = std::vector<Matrix_T>;
00083         const auto card = p + q;
00084         const auto bias = p - q;
00085         const auto bott = pos_mod(bias, 8);
00086         const auto sig = signature_t(p, q);
00087         if (this->find(sig) == this->end())
00088             switch(bott)
00089             {
00090                 case 0:
00091                     if (bias < 0)
00092                         // Construct generators for p,q given generators for p+4,q-4
00093                         gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00094                     else if (bias > 0)
00095                         // Construct generators for p,q given generators for p-4,q+4
00096                         gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00097                     else if (card == 0)
00098                         { // Base case. Save a generator vector containing one matrix, size 1.
00099                             auto result = result_t(1, matrix::unit<Matrix_T>(1));
00100                             this->insert(make_pair(sig, result));
00101                         }

```

```

00102         else
00103             // Construct generators for p,q given generators for p-1,q-1
00104             gen_from_pml_qml(gen_vector(p-1, q-1), sig);
00105         break;
00106     case 2:
00107         if (bias < 2)
00108             // Construct generators for p,q given generators for p+4,q-4
00109             gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00110         else if (bias > 2)
00111             // Construct generators for p,q given generators for p-4,q+4
00112             gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00113         else
00114             // Construct generators for p,q given generators for q+1,p-1
00115             gen_from_qp1_pml(gen_vector(q+1, p-1), sig);
00116         break;
00117     default:
00118         break;
00119     }
00120     return (*this)[sig];
00121 }
00122
00124 // Reference: [P] Proposition 15.17, p 131
00125 template< class Matrix_T >
00126 void
00127 generator_table<Matrix_T>::
00128 gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig)
00129 {
00130     const auto new_size = old.size() + 2;
00131     using size_t = decltype(new_size);
00132     using result_t = std::vector<Matrix_T>;
00133     auto result = result_t(new_size);
00134
00135     const auto old_dim = old[0].size1();
00136     const auto& eye = matrix::unit<Matrix_T>(old_dim);
00137
00138     auto neg = Matrix_T(2,2,2);
00139     neg(0,1) = -1;
00140     neg(1,0) = 1;
00141
00142     auto pos = neg;
00143     pos(0,1) = 1;
00144
00145     auto dup = Matrix_T(2,2,2);
00146     dup(0,0) = 1;
00147     dup(1,1) = -1;
00148
00149     result[0] = matrix::mono_kron(neg, eye);
00150     for (auto
00151         k = size_t(1);
00152         k != new_size-1;
00153         ++k)
00154         result[k] = matrix::mono_kron(dup, old[k-1]);
00155     result[new_size-1] = matrix::mono_kron(pos, eye);
00156
00157     // Save the resulting generator array.
00158     this->insert(make_pair(sig, result));
00159 }
00160
00162 // Reference: [L] 16.4 Periodicity of 8, p216
00163 template< class Matrix_T >
00164 void
00165 generator_table<Matrix_T>::
00166 gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig)
00167 {
00168     const auto old_size = old.size();
00169     using size_t = decltype(old_size);
00170     using result_t = std::vector<Matrix_T>;
00171     auto result = result_t(old_size);
00172
00173     auto h = old[0];
00174     for (auto
00175         k = size_t(1);
00176         k != size_t(4);
00177         ++k)
00178         h = matrix::mono_prod(old[k], h);
00179
00180     for (auto
00181         k = size_t(0);
00182         k != old_size-4;
00183         ++k)
00184         result[k] = old[k+4];
00185     for (auto
00186         k = old_size-4;
00187         k != old_size;
00188         ++k)
00189         result[k] = matrix::mono_prod(old[k+4-old_size], h);
00190     // Save the resulting generator array.

```

```

00191     this->insert(make_pair(sig, result));
00192 }
00193
00195 // Reference: [L] 16.4 Periodicity of 8, p216
00196 template< class Matrix_T >
00197 void
00198 generator_table<Matrix_T>::
00199 gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig)
00200 {
00201     const auto old_size = old.size();
00202     using size_t = decltype(old_size);
00203     using result_t = std::vector<Matrix_T>;
00204     auto result = result_t(old_size);
00205
00206     auto h = old[old_size-1];
00207     for (auto
00208         k = size_t(1);
00209         k != size_t(4);
00210         ++k)
00211         h = matrix::mono_prod(old[old_size-1-k], h);
00212
00213     for (auto
00214         k = size_t(0);
00215         k != size_t(4);
00216         ++k)
00217         result[k] = matrix::mono_prod(old[k+old_size-4], h);
00218     for (auto
00219         k = size_t(4);
00220         k != old_size;
00221         ++k)
00222         result[k] = old[k-4];
00223     // Save the resulting generator array.
00224     this->insert(make_pair(sig, result));
00225 }
00226
00228 // Reference: [P] Proposition 15.20, p 131
00229 template< class Matrix_T >
00230 void
00231 generator_table<Matrix_T>::
00232 gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig)
00233 {
00234     const auto old_size = old.size();
00235     using size_t = decltype(old_size);
00236     using result_t = std::vector<Matrix_T>;
00237     auto result = result_t(old_size);
00238
00239     const auto& h = old[old_size-1];
00240     for (auto
00241         k = size_t(0);
00242         k != old_size-1;
00243         ++k)
00244         result[k] = matrix::mono_prod(old[old_size-2-k], h);
00245     result[old_size-1] = h;
00246
00247     // Save the resulting generator array.
00248     this->insert(make_pair(sig, result));
00249 }
00250 } }
00252 #endif // _GLUCAT_GENERATION_IMP_H

```

## 7.17 glucat/global.h File Reference

```

#include "glucat/portability.h"
#include <limits>
#include <climits>

```

Include dependency graph for global.h: This graph shows which files directly or indirectly include this file:

### Classes

- struct [glucat::CTAssertion< true >](#)
- class [glucat::compare\\_types< LHS\\_T, RHS\\_T >](#)  
Type comparison.
- class [glucat::compare\\_types< T, T >](#)
- class [glucat::bool\\_to\\_type< truth\\_value >](#)  
Bool to type.

## Namespaces

- namespace [glucat](#)

## Macros

- `#define _GLUCAT_CTAssert(expr, msg) namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }`

## Typedefs

- using [glucat::index\\_t](#) = int  
*Size of index\_t should be enough to represent LO, HI.*
- using [glucat::set\\_value\\_t](#) = unsigned long  
*Size of set\_value\_t should be enough to contain index\_set<LO,HI>*

## Functions

- [glucat::GLUCAT\\_CTAssert](#) (std::numeric\_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const index\_t BITS\_PER\_CHAR  
*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*
- [glucat::GLUCAT\\_CTAssert](#) (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const index\_t DEFAULT\_LO  
*Default lowest index in an index set.*
- template<typename LHS\_T, typename RHS\_T >  
auto [glucat::pos\\_mod](#) (LHS\_T lhs, RHS\_T rhs) -> LHS\_T  
*Modulo function which works reliably for lhs < 0.*

## Variables

- const double [glucat::MS\\_PER\\_S](#) = 1000.0  
*Timing constant: deprecated here - moved to [test/timing.h](#).*
- const index\_t [glucat::BITS\\_PER\\_SET\\_VALUE](#) = std::numeric\_limits<set\_value\_t>::digits  
*Number of bits in set\_value\_t.*
- const index\_t [glucat::DEFAULT\\_HI](#) = index\_t(BITS\_PER\_SET\_VALUE / 2)  
*Default highest index in an index set.*

## 7.17.1 Macro Definition Documentation

### 7.17.1.1 \_GLUCAT\_CTAssert

```
#define _GLUCAT_CTAssert(  
    expr,  
    msg ) namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
```

Definition at line 48 of file [global.h](#).

## 7.18 global.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLOBAL_H
00002 #define _GLUCAT_GLOBAL_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     global.h : Global declarations
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/portability.h"
00035
00036 #include <limits>
00037 #include <climits>
00038
00039 namespace glucat
00040 {
00041     // References:
00042     // [AA]: A. Alexandrescu, "Modern C++ Design", Addison-Wesley, 2001.
00043
00044     // Reference: [AA], p. 25
00045     template<bool> struct CTAAssertion;
00046     template<> struct CTAAssertion<true> { };
00047     #define _GLUCAT_CTAssert(expr, msg) \
00048         namespace { struct msg { glucat::CTAAssertion<(expr)> ERROR_##msg; }; }
00049
00050     // Reference: [AA], pp. 34--37
00051     template < typename LHS_T, typename RHS_T >
00052     class compare_types
00053     {
00054     public:
00055         enum { are_same = false };
00056     };
00057     template < typename T >
00058     class compare_types<T, T>
00059     {
00060     public:
00061         enum { are_same = true };
00062     };
00063
00064     // Reference: [AA], 2.4, p. 29
00065     template< bool truth_value >
00066     class bool_to_type
00067     {
00068     private:
00069         enum { value = truth_value };
00070     };
00071
00072     // Global types which determine sizes
00073     using index_t = int;
00074     using set_value_t = unsigned long;
00075
00076     // Global constants
00077     const double MS_PER_S = 1000.0;
00078
00079     // Constants which determine sizes
00080
00081     // Bits per unsigned long
00082     #if (ULONG_MAX == (4294967295UL))

```

```

00089  #define _GLUCAT_BITS_PER_ULONG 32
00090  #elif (ULONG_MAX == (18446744073709551615UL))
00091  #define _GLUCAT_BITS_PER_ULONG 64
00092  #elif defined(__WORDSIZE)
00093  #define _GLUCAT_BITS_PER_ULONG __WORDSIZE
00094  #endif
00095
00097  _GLUCAT_CTAssert(std::numeric_limits<unsigned char>::radix == 2, CannotDetermineBitsPerChar)
00098
00099
00100  const index_t BITS_PER_CHAR = std::numeric_limits<unsigned char>::digits;
00101
00103  const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits;
00104
00105  _GLUCAT_CTAssert(_GLUCAT_BITS_PER_ULONG == BITS_PER_SET_VALUE, BitsPerULongDoesNotMatchSetValueT)
00106
00107  // Constants which are determined by size
00109  const index_t DEFAULT_LO = -index_t(BITS_PER_SET_VALUE / 2);
00111  const index_t DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2);
00112
00114  template< typename LHS_T, typename RHS_T >
00115  inline
00116  auto
00117  pos_mod(LHS_T lhs, RHS_T rhs) -> LHS_T
00118  { return lhs > 0? lhs % rhs : (-lhs) % rhs == 0 ? 0 : rhs - (-lhs) % rhs; }
00119
00120 }
00121 #endif // _GLUCAT_GLOBAL_H

```

## 7.19 glucat/glucat.h File Reference

```

#include "glucat/portability.h"
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/scalar.h"
#include "glucat/long_double.h"
#include "glucat/qd.h"
#include "glucat/promotion.h"
#include "glucat/random.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include "glucat/matrix_multi.h"

```

Include dependency graph for glucat.h: This graph shows which files directly or indirectly include this file:

## 7.20 glucat.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLUCAT_H
00002 #define _GLUCAT_GLUCAT_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  glucat.h : Organize GluCat header files for applications
00006  -----
00007  begin                : Sun 2001-12-09
00008  copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  *****/
00010
00011  This library is free software: you can redistribute it and/or modify
00012  it under the terms of the GNU Lesser General Public License as published
00013  by the Free Software Foundation, either version 3 of the License, or
00014  (at your option) any later version.
00015
00016  This library is distributed in the hope that it will be useful,
00017  but WITHOUT ANY WARRANTY; without even the implied warranty of

```



```

00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author.  See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     Arvind Raja's original header comments and references follow.
00032     *****
00033     // clifford algebra package, Arvind.Raja@hut.fi
00034     // ref: Press et.al. "Numerical Recipes in C", 2nd ed., C.U.P., 1992.
00035     // ref: LEDA, v 3.0, Stefan N\aher, Max-Planck-Institut f\ur Informatik
00036     // ref: Stroustrup B., "The C++ Programming Language", 2nd ed.,
00037     // Addison-Wesley, 1991.
00038     // ref: R. Sedgewick, "Algorithms in C++", Addison-Wesley, 1992.
00039     // ref: S. Meyers, "Effective C++ ", Addison-Wesley, 1992.
00040     *****/
00041
00042 #include "glucat/portability.h"
00043
00044 #include "glucat/global.h"
00045
00046 #include "glucat/errors.h"
00047
00048 #include "glucat/index_set.h"
00049
00050 #include "glucat/scalar.h"
00051
00052 #include "glucat/long_double.h"
00053
00054 #include "glucat/qd.h"
00055
00056 #include "glucat/promotion.h"
00057
00058 #include "glucat/random.h"
00059
00060 #include "glucat/clifford_algebra.h"
00061
00062 #include "glucat/tuning.h"
00063
00064 #include "glucat/framed_multi.h"
00065
00066 #include "glucat/generation.h"
00067
00068 #include "glucat/matrix.h"
00069
00070 #include "glucat/matrix_multi.h"
00071
00072 #endif // _GLUCAT_GLUCAT_H

```

## 7.21 glucat/glucat\_config.h File Reference

This graph shows which files directly or indirectly include this file:

### Macros

- #define GLUCAT\_HAVE\_CXX11 1
- #define GLUCAT\_HAVE\_INTTYPES\_H 1
- #define GLUCAT\_HAVE\_STDINT\_H 1
- #define GLUCAT\_HAVE\_STDIO\_H 1
- #define GLUCAT\_HAVE\_STDLIB\_H 1
- #define GLUCAT\_HAVE\_STRINGS\_H 1
- #define GLUCAT\_HAVE\_STRING\_H 1
- #define GLUCAT\_HAVE\_SYS\_STAT\_H 1
- #define GLUCAT\_HAVE\_SYS\_TYPES\_H 1
- #define GLUCAT\_HAVE\_UNISTD\_H 1

- `#define GLUCAT_PACKAGE "glucat"`
- `#define GLUCAT_PACKAGE_BUGREPORT ""`
- `#define GLUCAT_PACKAGE_NAME "glucat"`
- `#define GLUCAT_PACKAGE_STRING "glucat 0.12.0"`
- `#define GLUCAT_PACKAGE_TARNAME "glucat"`
- `#define GLUCAT_PACKAGE_URL ""`
- `#define GLUCAT_PACKAGE_VERSION "0.12.0"`
- `#define GLUCAT_STDC_HEADERS 1`
- `#define GLUCAT_VERSION "0.12.0"`

## 7.21.1 Macro Definition Documentation

### 7.21.1.1 GLUCAT\_HAVE\_CXX11

```
#define GLUCAT_HAVE_CXX11 1
```

Definition at line 20 of file [glucat\\_config.h](#).

### 7.21.1.2 GLUCAT\_HAVE\_INTTYPES\_H

```
#define GLUCAT_HAVE_INTTYPES_H 1
```

Definition at line 28 of file [glucat\\_config.h](#).

### 7.21.1.3 GLUCAT\_HAVE\_STDINT\_H

```
#define GLUCAT_HAVE_STDINT_H 1
```

Definition at line 39 of file [glucat\\_config.h](#).

### 7.21.1.4 GLUCAT\_HAVE\_STDIO\_H

```
#define GLUCAT_HAVE_STDIO_H 1
```

Definition at line 44 of file [glucat\\_config.h](#).

#### 7.21.1.5 GLUCAT\_HAVE\_STDLIB\_H

```
#define GLUCAT_HAVE_STDLIB_H 1
```

Definition at line 49 of file [glucat\\_config.h](#).

#### 7.21.1.6 GLUCAT\_HAVE\_STRING\_H

```
#define GLUCAT_HAVE_STRING_H 1
```

Definition at line 59 of file [glucat\\_config.h](#).

#### 7.21.1.7 GLUCAT\_HAVE\_STRINGS\_H

```
#define GLUCAT_HAVE_STRINGS_H 1
```

Definition at line 54 of file [glucat\\_config.h](#).

#### 7.21.1.8 GLUCAT\_HAVE\_SYS\_STAT\_H

```
#define GLUCAT_HAVE_SYS_STAT_H 1
```

Definition at line 64 of file [glucat\\_config.h](#).

#### 7.21.1.9 GLUCAT\_HAVE\_SYS\_TYPES\_H

```
#define GLUCAT_HAVE_SYS_TYPES_H 1
```

Definition at line 69 of file [glucat\\_config.h](#).

#### 7.21.1.10 GLUCAT\_HAVE\_UNISTD\_H

```
#define GLUCAT_HAVE_UNISTD_H 1
```

Definition at line 74 of file [glucat\\_config.h](#).

#### 7.21.1.11 GLUCAT\_PACKAGE

```
#define GLUCAT_PACKAGE "glucat"
```

Definition at line 79 of file [glucat\\_config.h](#).

#### 7.21.1.12 GLUCAT\_PACKAGE\_BUGREPORT

```
#define GLUCAT_PACKAGE_BUGREPORT ""
```

Definition at line 84 of file [glucat\\_config.h](#).

#### 7.21.1.13 GLUCAT\_PACKAGE\_NAME

```
#define GLUCAT_PACKAGE_NAME "glucat"
```

Definition at line 89 of file [glucat\\_config.h](#).

#### 7.21.1.14 GLUCAT\_PACKAGE\_STRING

```
#define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
```

Definition at line 94 of file [glucat\\_config.h](#).

#### 7.21.1.15 GLUCAT\_PACKAGE\_TARNAME

```
#define GLUCAT_PACKAGE_TARNAME "glucat"
```

Definition at line 99 of file [glucat\\_config.h](#).

#### 7.21.1.16 GLUCAT\_PACKAGE\_URL

```
#define GLUCAT_PACKAGE_URL ""
```

Definition at line 104 of file [glucat\\_config.h](#).

**7.21.1.17 GLUCAT\_PACKAGE\_VERSION**

```
#define GLUCAT_PACKAGE_VERSION "0.12.0"
```

Definition at line 109 of file [glucat\\_config.h](#).

**7.21.1.18 GLUCAT\_STDC\_HEADERS**

```
#define GLUCAT_STDC_HEADERS 1
```

Definition at line 116 of file [glucat\\_config.h](#).

**7.21.1.19 GLUCAT\_VERSION**

```
#define GLUCAT_VERSION "0.12.0"
```

Definition at line 121 of file [glucat\\_config.h](#).

**7.22 glucat\_config.h**

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLUCAT_CONFIG_H
00002 #define _GLUCAT_GLUCAT_CONFIG_H 1
00003
00004 /* glucat/glucat_config.h. Generated automatically at end of configure. */
00005 /* config.h. Generated from config.h.in by configure. */
00006 /* config.h.in. Generated from configure.ac by autoheader. */
00007
00008 /* Define to dummy 'main' function (if any) required to link to the Fortran
00009    libraries. */
00010 /* #undef F77_DUMMY_MAIN */
00011
00012 /* Define if F77 and FC dummy 'main' functions are identical. */
00013 /* #undef FC_DUMMY_MAIN_EQ_F77 */
00014
00015 /* Define if you have a BLAS library. */
00016 /* #undef HAVE_BLAS */
00017
00018 /* define if the compiler supports basic C++11 syntax */
00019 #ifndef GLUCAT_HAVE_CXX11
00020 #define GLUCAT_HAVE_CXX11 1
00021 #endif
00022
00023 /* define if the compiler supports basic C++14 syntax */
00024 /* #undef HAVE_CXX14 */
00025
00026 /* Define to 1 if you have the <inttypes.h> header file. */
00027 #ifndef GLUCAT_HAVE_INTTYPES_H
00028 #define GLUCAT_HAVE_INTTYPES_H 1
00029 #endif
00030
00031 /* Define if you have LAPACK library. */
00032 /* #undef HAVE_LAPACK */
00033
00034 /* Define to 1 if you have the 'lmf' library (-lmf). */
00035 /* #undef HAVE_LIBIMF */
00036
00037 /* Define to 1 if you have the <stdint.h> header file. */
00038 #ifndef GLUCAT_HAVE_STDINT_H
00039 #define GLUCAT_HAVE_STDINT_H 1
00040 #endif
```

```
00041
00042 /* Define to 1 if you have the <stdio.h> header file. */
00043 #ifndef GLUCAT_HAVE_STDIO_H
00044 #define GLUCAT_HAVE_STDIO_H 1
00045 #endif
00046
00047 /* Define to 1 if you have the <stdlib.h> header file. */
00048 #ifndef GLUCAT_HAVE_STDLIB_H
00049 #define GLUCAT_HAVE_STDLIB_H 1
00050 #endif
00051
00052 /* Define to 1 if you have the <strings.h> header file. */
00053 #ifndef GLUCAT_HAVE_STRINGS_H
00054 #define GLUCAT_HAVE_STRINGS_H 1
00055 #endif
00056
00057 /* Define to 1 if you have the <string.h> header file. */
00058 #ifndef GLUCAT_HAVE_STRING_H
00059 #define GLUCAT_HAVE_STRING_H 1
00060 #endif
00061
00062 /* Define to 1 if you have the <sys/stat.h> header file. */
00063 #ifndef GLUCAT_HAVE_SYS_STAT_H
00064 #define GLUCAT_HAVE_SYS_STAT_H 1
00065 #endif
00066
00067 /* Define to 1 if you have the <sys/types.h> header file. */
00068 #ifndef GLUCAT_HAVE_SYS_TYPES_H
00069 #define GLUCAT_HAVE_SYS_TYPES_H 1
00070 #endif
00071
00072 /* Define to 1 if you have the <unistd.h> header file. */
00073 #ifndef GLUCAT_HAVE_UNISTD_H
00074 #define GLUCAT_HAVE_UNISTD_H 1
00075 #endif
00076
00077 /* Name of package */
00078 #ifndef GLUCAT_PACKAGE
00079 #define GLUCAT_PACKAGE "glucat"
00080 #endif
00081
00082 /* Define to the address where bug reports for this package should be sent. */
00083 #ifndef GLUCAT_PACKAGE_BUGREPORT
00084 #define GLUCAT_PACKAGE_BUGREPORT ""
00085 #endif
00086
00087 /* Define to the full name of this package. */
00088 #ifndef GLUCAT_PACKAGE_NAME
00089 #define GLUCAT_PACKAGE_NAME "glucat"
00090 #endif
00091
00092 /* Define to the full name and version of this package. */
00093 #ifndef GLUCAT_PACKAGE_STRING
00094 #define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
00095 #endif
00096
00097 /* Define to the one symbol short name of this package. */
00098 #ifndef GLUCAT_PACKAGE_TARNAME
00099 #define GLUCAT_PACKAGE_TARNAME "glucat"
00100 #endif
00101
00102 /* Define to the home page for this package. */
00103 #ifndef GLUCAT_PACKAGE_URL
00104 #define GLUCAT_PACKAGE_URL ""
00105 #endif
00106
00107 /* Define to the version of this package. */
00108 #ifndef GLUCAT_PACKAGE_VERSION
00109 #define GLUCAT_PACKAGE_VERSION "0.12.0"
00110 #endif
00111
00112 /* Define to 1 if all of the C90 standard headers exist (not just the ones
00113    required in a freestanding environment). This macro is provided for
00114    backward compatibility; new code need not use it. */
00115 #ifndef GLUCAT_STDC_HEADERS
00116 #define GLUCAT_STDC_HEADERS 1
00117 #endif
00118
00119 /* Version number of package */
00120 #ifndef GLUCAT_VERSION
00121 #define GLUCAT_VERSION "0.12.0"
00122 #endif
00123
00124 /* once: _GLUCAT_GLUCAT_CONFIG_H */
00125 #endif
```

## 7.23 glucat/glucat\_imp.h File Reference

```
#include "glucat/errors_imp.h"
#include "glucat/index_set_imp.h"
#include "glucat/scalar_imp.h"
#include "glucat/clifford_algebra_imp.h"
#include "glucat/random.h"
#include "glucat/framed_multi_imp.h"
#include "glucat/matrix_imp.h"
#include "glucat/generation_imp.h"
#include "glucat/matrix_multi_imp.h"
```

Include dependency graph for glucat\_imp.h: This graph shows which files directly or indirectly include this file:

## 7.24 glucat\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_GLUCAT_IMP_H
00002 #define _GLUCAT_GLUCAT_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     glucat_imp.h : Organize GluCat template definitions which cannot be compiled separately
00006     -----
00007     begin                : Sun 2001-12-25
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     For Arvind Raja's original header comments, see glucat.h
00032     *****/
00033
00034 // Template definitions which cannot be compiled separately
00035
00036 #include "glucat/errors_imp.h"
00037
00038 #include "glucat/index_set_imp.h"
00039
00040 #include "glucat/scalar_imp.h"
00041
00042 #include "glucat/clifford_algebra_imp.h"
00043
00044 #include "glucat/random.h"
00045
00046 #include "glucat/framed_multi_imp.h"
00047
00048 #include "glucat/matrix_imp.h"
00049
00050 #include "glucat/generation_imp.h"
00051
00052 #include "glucat/matrix_multi_imp.h"
00053
00054 #endif // _GLUCAT_GLUCAT_IMP_H
```

## 7.25 glucat/index\_set.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include <boost/static_assert.hpp>
#include <bitset>
#include <utility>
```

Include dependency graph for index\_set.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::index\\_set< LO, HI >](#)  
*Index set class based on std::bitset<> in Gnu standard C++ library.*
- class [glucat::index\\_set< LO, HI >::reference](#)  
*Index set member reference.*

### Namespaces

- namespace [glucat](#)

### Functions

- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator^](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Symmetric set difference: exclusive or.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator&](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Set intersection: and.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator|](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Set union: or.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::compare](#) (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- [glucat::GLUCAT\\_CTAssert](#) (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >), Default\_index\_set\_too\_big\_for\_value) template< const index\_t LO  
*Size of set\_value\_t should be enough to contain bitset<DEFAULT\_HI-DEFAULT\_LO>*
- const index\_t HI auto [glucat::operator<<](#) (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator>>](#) (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &  
*Read in index set.*
- auto [glucat::sign\\_of\\_square](#) (index\_t j) -> int  
*Square of generator [j].*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::min\\_neg](#) (const index\_set< LO, HI > &ist) -> index\_t  
*Minimum negative index, or 0 if none.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::max\\_pos](#) (const index\_set< LO, HI > &ist) -> index\_t  
*Maximum positive index, or 0 if none.*



## 7.26 index\_set.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_INDEX_SET_H
00002 #define _GLUCAT_INDEX_SET_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set.h : Declare a class for a set of non-zero integer indices
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036
00037 #include <boost/static_assert.hpp>
00038
00039 #include <bitset>
00040 #include <utility>
00041
00042 namespace glucat
00043 {
00044     template<const index_t LO, const index_t HI>
00045     class index_set; // forward
00046
00047     template<const index_t LO, const index_t HI>
00048     auto
00049     operator^ (const index_set<LO,HI>& lhs,
00050               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00051
00052     template<const index_t LO, const index_t HI>
00053     auto
00054     operator& (const index_set<LO,HI>& lhs,
00055               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00056
00057     template<const index_t LO, const index_t HI>
00058     auto
00059     operator| (const index_set<LO,HI>& lhs,
00060               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00061
00062     // -1 if a<b, +1 if a>b, 0 if a==b
00063     template<const index_t LO, const index_t HI>
00064     auto
00065     compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int;
00066
00067     template<const index_t LO, const index_t HI>
00068     class index_set :
00069     private std::bitset<HI-LO>
00070     {
00071     private:
00072         BOOST_STATIC_ASSERT((LO <= 0) && (0 <= HI) && (LO < HI) && \
00073             (-LO < _GLUCAT_BITS_PER_ULONG) && \
00074             (HI < _GLUCAT_BITS_PER_ULONG) && \
00075             (HI-LO <= _GLUCAT_BITS_PER_ULONG));
00076         using bitset_t = std::bitset<HI - LO>;
00077         using error_t = error<index_set>;
00078     public:
00079         using index_set_t = index_set;
00080         using index_pair_t = std::pair<index_t, index_t>;
00081
00082         static const index_t v_lo = LO;

```

```

00088     static const index_t v_hi = HI;
00089
00090     static auto classname() -> const std::string;
00092     index_set      () = default;
00094     index_set      (const bitset_t bst);
00096     index_set      (const index_t idx);
00098     index_set      (const set_value_t folded_val, const index_set_t frm, const bool prechecked = false);
00100     index_set      (const index_pair_t& range, const bool prechecked = false);
00102     index_set      (const std::string& str);
00103
00105     auto operator== (const index_set_t rhs) const -> bool;
00107     auto operator!= (const index_set_t rhs) const -> bool;
00109     auto operator~  () const -> index_set_t;
00111     auto operator^= (const index_set_t rhs) -> index_set_t&;
00113     auto operator&= (const index_set_t rhs) -> index_set_t&;
00115     auto operator|= (const index_set_t rhs) -> index_set_t&;
00117     auto operator[] (const index_t idx) const -> bool;
00119     auto test(const index_t idx) const -> bool;
00121     auto set() -> index_set_t&;
00123     auto set(const index_t idx) -> index_set_t&;
00125     auto set(const index_t idx, const int val) -> index_set_t&;
00127     auto reset() -> index_set_t&;
00129     auto reset(const index_t idx) -> index_set_t&;
00131     auto flip() -> index_set_t&;
00133     auto flip(const index_t idx) -> index_set_t&;
00135     auto count() const -> index_t;
00137     auto count_neg() const -> index_t;
00139     auto count_pos() const -> index_t;
00141     auto min() const -> index_t;
00143     auto max() const -> index_t;
00144
00145     // Functions which support Clifford algebra operations
00147     auto operator< (const index_set_t rhs) const -> bool;
00149     auto is_contiguous () const -> bool;
00151     auto fold          () const -> const index_set_t;
00153     auto fold          (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00155     auto unfold        (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00157     auto value_of_fold (const index_set_t frm) const -> set_value_t;
00159     auto sign_of_mult  (const index_set_t ist) const -> int;
00161     auto sign_of_square() const -> int;
00162
00164     auto hash_fn      () const -> size_t;
00165
00166     // Friends
00167     friend auto operator^<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00168     friend auto operator&<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00169     friend auto operator|<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00170     friend auto compare<> (const index_set_t& lhs, const index_set_t& rhs) -> int;
00171
00172     // Member reference:
00173     class reference;
00174     friend class reference;
00175
00177     class reference {
00178     friend class index_set;
00179
00180     public:
00182     reference() = delete;
00183     reference (index_set_t& ist, index_t idx);
00184     ~reference () = default;
00186     auto operator== (const reference& c_j) const -> bool;
00188     auto operator= (const bool x) -> reference&;
00190     auto operator= (const reference& c_j) -> reference&;
00192     auto operator~ () const -> bool;
00194     operator bool () const;
00196     auto flip() -> reference&;
00197
00198     private:
00199     index_set_t* m_pst;
00200     index_t      m_idx;
00201     };
00203     auto operator[] (index_t idx) -> reference;
00204 private:
00206     auto lex_less_than (const index_set_t rhs) const -> bool;
00207 };
00208
00210     _GLUCAT_CTAssert(sizeof(set_value_t) >= sizeof(std::bitset<DEFAULT_HI-DEFAULT_LO>),
00211         Default_index_set_too_big_for_value)
00212
00213     // non-members
00214
00216     template<const index_t LO, const index_t HI>
00217     auto
00218     operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&;
00219

```

```

00221 template<const index_t LO, const index_t HI>
00222 auto
00223 operator» (std::istream& s, index_set<LO,HI>& ist) -> std::istream&;
00224
00225 // Functions which support Clifford algebra operations
00227 auto sign_of_square(index_t j) -> int;
00228
00230 template<const index_t LO, const index_t HI>
00231 auto
00232 min_neg(const index_set<LO,HI>& ist) -> index_t;
00233
00235 template<const index_t LO, const index_t HI>
00236 auto
00237 max_pos(const index_set<LO,HI>& ist) -> index_t;
00238 }
00239 #endif // _GLUCAT_INDEX_SET_H

```

## 7.27 glucat/index\_set\_imp.h File Reference

```

#include "glucat/index_set.h"
#include <string>
#include <sstream>

```

Include dependency graph for index\_set\_imp.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace [glucat](#)

### Functions

- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator^](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Symmetric set difference: exclusive or.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator&](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Set intersection: and.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator|](#) (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >  
*Set union: or.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::compare](#) (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator<<](#) (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &  
*Write out index set.*
- template<const index\_t LO, const index\_t HI>  
auto [glucat::operator>>](#) (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &  
*Read in index set.*
- static auto [glucat::inverse\\_reversed\\_gray](#) (unsigned long x) -> unsigned long  
*Inverse reversed Gray code.*
- static auto [glucat::inverse\\_gray](#) (unsigned long x) -> unsigned long  
*Inverse Gray code.*

- auto `glucat::sign_of_square` (index\_t j) -> int  
*Square of generator {j}.*
- template<const index\_t LO, const index\_t HI>  
auto `glucat::min_neg` (const index\_set< LO, HI > &ist) -> index\_t  
*Minimum negative index, or 0 if none.*
- template<const index\_t LO, const index\_t HI>  
auto `glucat::max_pos` (const index\_set< LO, HI > &ist) -> index\_t  
*Maximum positive index, or 0 if none.*

## 7.28 index\_set\_imp.h

Go to the documentation of this file.

```

00001 #ifndef _GLUCAT_INDEX_SET_IMP_H
00002 #define _GLUCAT_INDEX_SET_IMP_H
00003 /*****
00004      GluCat : Generic library of universal Clifford algebra templates
00005      index_set_imp.h : Implement a class for a set of non-zero integer indices
00006                      -----
00007      begin                : Sun 2001-12-09
00008      copyright            : (C) 2001-2016 by Paul C. Leopardi
00009      *****/
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****/
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****/
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034 #include "glucat/index_set.h"
00035
00036 #include <string>
00037 #include <sstream>
00038
00039 namespace glucat
00040 {
00041     // References for algorithms:
00042     // [JA]: Joerg Arndt, "Algorithms for programmers", http://www.jjj.de/fxt/fxtbook.pdf
00043     //      Chapter 1, Bit wizardry, http://www.jjj.de/bitwizardry/bitwizardrypage.html
00044     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00045
00046     template<const index_t LO, const index_t HI>
00047     inline
00048     auto
00049     index_set<LO,HI>::
00050     classname() -> const std::string
00051     { return "index_set"; }
00052
00053     template<const index_t LO, const index_t HI>
00054     index_set<LO,HI>::
00055     index_set(const index_t idx)
00056     { this->set(idx); }
00057
00058     template<const index_t LO, const index_t HI>
00059     index_set<LO,HI>::
00060     index_set(const bitset_t bst):
00061     bitset_t(bst)
00062     { }
00063
00064     template<const index_t LO, const index_t HI>

```

```

00068 index_set<LO,HI>::
00069 index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked)
00070 {
00071     if (!prechecked && folded_val >= (set_value_t(1) << frm.count()))
00072         throw error_t("index_set(val,frm): cannot create: value gives an index set outside of frame");
00073     const index_set_t folded_frame = frm.fold();
00074     const index_t min_index = folded_frame.min();
00075     const index_t skip = min_index > 0 ? 1 : 0;
00076     const index_set_t folded_set = index_set_t(bitset_t(folded_val) << (min_index - skip - LO));
00077     *this = folded_set.unfold(frm);
00078 }
00079
00081 template<const index_t LO, const index_t HI>
00082 index_set<LO,HI>::
00083 index_set(const index_pair_t& range, const bool prechecked)
00084 {
00085     if (!prechecked && (range.first < LO || range.second > HI))
00086         throw error_t("index_set(range): cannot create: range is too large");
00087     const index_t begin_bit = (range.first < 0)
00088                             ? range.first-LO
00089                             : range.first-LO-1;
00090     const index_t end_bit = (range.second < 0)
00091                           ? range.second-LO+1
00092                           : range.second-LO;
00093     unsigned long mask = ( (end_bit == _GLUCAT_BITS_PER_ULONGLONG)
00094                         ? -1UL
00095                         : (1UL << end_bit)-1UL)
00096                       & ~((1UL << begin_bit)-1UL);
00097     *this = bitset_t(mask);
00098 }
00099
00101 template<const index_t LO, const index_t HI>
00102 index_set<LO,HI>::
00103 index_set(const std::string& str)
00104 {
00105     std::istringstream ss(str);
00106     ss >> *this;
00107     if (!ss)
00108         throw error_t("index_set_t(str): could not parse string");
00109     // Peek to see if the end of the string has been reached.
00110     ss.peek();
00111     if (!ss.eof())
00112         throw error_t("index_set_t(str): could not parse entire string");
00113 }
00114
00116 template<const index_t LO, const index_t HI>
00117 inline
00118 auto
00119 index_set<LO,HI>::
00120 operator== (const index_set_t rhs) const -> bool
00121 {
00122     const auto* pthis = static_cast<const bitset_t*>(this);
00123     return *pthis == static_cast<bitset_t>(rhs);
00124 }
00125
00127 template<const index_t LO, const index_t HI>
00128 inline
00129 auto
00130 index_set<LO,HI>::
00131 operator!= (const index_set_t rhs) const -> bool
00132 {
00133     const auto* pthis = static_cast<const bitset_t*>(this);
00134     return *pthis != static_cast<bitset_t>(rhs);
00135 }
00136
00138 template<const index_t LO, const index_t HI>
00139 inline
00140 auto
00141 index_set<LO,HI>::
00142 operator~ () const -> index_set_t
00143 { return bitset_t::operator~(); }
00144
00146 template<const index_t LO, const index_t HI>
00147 inline
00148 auto
00149 index_set<LO,HI>::
00150 operator^= (const index_set_t rhs) -> index_set_t&
00151 {
00152     bitset_t* pthis = this;
00153     *pthis ^= static_cast<bitset_t>(rhs);
00154     return *this;
00155 }
00156
00158 template<const index_t LO, const index_t HI>
00159 inline
00160 auto
00161 operator^ (const index_set<LO,HI>& lhs,

```

```

00162         const index_set<LO,HI>& rhs) -> const
00163     index_set<LO,HI>
00164     {
00165         using index_set_t = index_set<LO, HI>;
00166         using bitset_t = typename index_set_t::bitset_t;
00167         return static_cast<bitset_t>(lhs) ^ static_cast<bitset_t>(rhs);
00168     }
00169
00171     template<const index_t LO, const index_t HI>
00172     inline
00173     auto
00174     index_set<LO,HI>::
00175     operator&= (const index_set_t rhs) -> index_set_t&
00176     {
00177         bitset_t* pthis = this;
00178         *pthis &= static_cast<bitset_t>(rhs);
00179         return *this;
00180     }
00181
00183     template<const index_t LO, const index_t HI>
00184     inline
00185     auto
00186     operator& (const index_set<LO,HI>& lhs,
00187               const index_set<LO,HI>& rhs) -> const
00188     index_set<LO,HI>
00189     {
00190         using index_set_t = index_set<LO, HI>;
00191         using bitset_t = typename index_set_t::bitset_t;
00192         return static_cast<bitset_t>(lhs) & static_cast<bitset_t>(rhs);
00193     }
00194
00196     template<const index_t LO, const index_t HI>
00197     inline
00198     auto
00199     index_set<LO,HI>::
00200     operator|= (const index_set_t rhs) -> index_set_t&
00201     {
00202         bitset_t* pthis = this;
00203         *pthis |= static_cast<bitset_t>(rhs);
00204         return *this;
00205     }
00206
00208     template<const index_t LO, const index_t HI>
00209     inline
00210     auto
00211     operator| (const index_set<LO,HI>& lhs,
00212               const index_set<LO,HI>& rhs) -> const
00213     index_set<LO,HI>
00214     {
00215         using index_set_t = index_set<LO, HI>;
00216         using bitset_t = typename index_set_t::bitset_t;
00217         return static_cast<bitset_t>(lhs) | static_cast<bitset_t>(rhs);
00218     }
00219
00221     template<const index_t LO, const index_t HI>
00222     inline
00223     auto
00224     index_set<LO,HI>::
00225     operator[] (const index_t idx) -> reference
00226     { return reference(*this, idx); }
00227
00229     template<const index_t LO, const index_t HI>
00230     inline
00231     auto
00232     index_set<LO,HI>::
00233     operator[] (const index_t idx) const -> bool
00234     { return this->test(idx); }
00235
00237     template<const index_t LO, const index_t HI>
00238     inline
00239     auto
00240     index_set<LO,HI>::
00241     test(const index_t idx) const -> bool
00242     {
00243         // Reference: [JA], 1.2.1
00244         return (idx < 0)
00245             ? bool(bitset_t::to_ulong() & (1UL << (idx - LO)))
00246             : (idx > 0)
00247             ? bool(bitset_t::to_ulong() & (1UL << (idx - LO - 1)))
00248             : false;
00249     }
00250
00252     template<const index_t LO, const index_t HI>
00253     inline
00254     auto
00255     index_set<LO,HI>::
00256     set() -> index_set_t&

```

```

00257 {
00258     bitset_t::set();
00259     return *this;
00260 }
00261
00262 template<const index_t LO, const index_t HI>
00263 inline
00264 auto
00265 index_set<LO,HI>::
00266 set(index_t idx) -> index_set_t&
00267 {
00268     {
00269         if (idx > 0)
00270             bitset_t::set(idx-LO-1);
00271         else if (idx < 0)
00272             bitset_t::set(idx-LO);
00273         return *this;
00274     }
00275
00276 template<const index_t LO, const index_t HI>
00277 inline
00278 auto
00279 index_set<LO,HI>::
00280 set(const index_t idx, const int val) -> index_set_t&
00281 {
00282     {
00283         if (idx > 0)
00284             bitset_t::set(idx-LO-1, val);
00285         else if (idx < 0)
00286             bitset_t::set(idx-LO, val);
00287         return *this;
00288     }
00289
00290 template<const index_t LO, const index_t HI>
00291 inline
00292 auto
00293 index_set<LO,HI>::
00294 reset() -> index_set_t&
00295 {
00296     {
00297         bitset_t::reset();
00298         return *this;
00299     }
00300
00301 template<const index_t LO, const index_t HI>
00302 inline
00303 auto
00304 index_set<LO,HI>::
00305 reset(const index_t idx) -> index_set_t&
00306 {
00307     {
00308         if (idx > 0)
00309             bitset_t::reset(idx-LO-1);
00310         else if (idx < 0)
00311             bitset_t::reset(idx-LO);
00312         return *this;
00313     }
00314
00315 template<const index_t LO, const index_t HI>
00316 inline
00317 auto
00318 index_set<LO,HI>::
00319 flip() -> index_set<LO,HI>&
00320 {
00321     {
00322         bitset_t::flip();
00323         return *this;
00324     }
00325
00326 template<const index_t LO, const index_t HI>
00327 inline
00328 auto
00329 index_set<LO,HI>::
00330 flip(const index_t idx) -> index_set_t&
00331 {
00332     {
00333         if (idx > 0)
00334             bitset_t::flip(idx-LO-1);
00335         else if (idx < 0)
00336             bitset_t::flip(idx-LO);
00337         return *this;
00338     }
00339
00340 template<const index_t LO, const index_t HI>
00341 inline
00342 auto
00343 index_set<LO,HI>::
00344 count() const -> index_t
00345 {
00346     {
00347         unsigned long val = bitset_t::to_ulong();
00348         // Reference: [JA], 1.3
00349         if (val == 0)
00350             return 0;

```

```

00351     else
00352     {
00353         index_t result = 1;
00354         while (val &= val-1)
00355             ++result;
00356         return result;
00357     }
00358 }
00359
00360 template<const index_t LO, const index_t HI>
00361 inline
00362 auto
00363 index_set<LO,HI>::
00364 count_neg() const -> index_t
00365 {
00366     static const index_set_t lo_mask = bitset_t((1UL < -LO) - 1UL);
00367     const index_set_t neg_part = *this & lo_mask;
00368     return neg_part.count();
00369 }
00370
00371 template<const index_t LO, const index_t HI>
00372 inline
00373 auto
00374 index_set<LO,HI>::
00375 count_pos() const -> index_t
00376 {
00377     const auto* pthis = static_cast<const bitset_t*>(this);
00378     const index_set_t pos_part = *pthis > -LO;
00379     return pos_part.count();
00380 }
00381
00382 #if (_GLUCAT_BITS_PER_ULONG == 64)
00383 template<const index_t LO, const index_t HI>
00384 inline
00385 auto
00386 index_set<LO,HI>::
00387 min() const -> index_t
00388 {
00389     // Reference: [JA], 1.3
00390     unsigned long val = bitset_t::to_ulong();
00391     if (val == 0)
00392         return 0;
00393     else
00394     {
00395         val -= val & (val-1); // isolate lowest bit
00396
00397         index_t idx = 0;
00398         const index_t nbits = HI - LO;
00399
00400         if (nbits > 8)
00401         {
00402             if (val & 0xffffffff00000000ul)
00403                 idx += 32;
00404             if (val & 0xffff0000ffff0000ul)
00405                 idx += 16;
00406             if (val & 0xff00ff00ff00ff00ul)
00407                 idx += 8;
00408             if (val & 0xf0f0f0f0f0f0f0f0ul)
00409                 idx += 4;
00410             if (val & 0xcccccccccccccccul)
00411                 idx += 2;
00412             if (val & 0xaaaaaaaaaaaaaaul)
00413                 idx += 1;
00414
00415             return idx + ((idx < -LO) ? LO : LO+1);
00416         }
00417     }
00418 }
00419 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00420 template<const index_t LO, const index_t HI>
00421 inline
00422 auto
00423 index_set<LO,HI>::
00424 min() const
00425 {
00426     // Reference: [JA], 1.3
00427     unsigned long val = bitset_t::to_ulong();
00428     if (val == 0)
00429         return 0;
00430     else
00431     {
00432         val -= val & (val-1); // isolate lowest bit
00433
00434         index_t idx = 0;
00435         const index_t nbits = HI - LO;
00436         if (nbits > 8)
00437         {
00438

```



```

00442         if (val & 0xffff0000ul)
00443             idx += 16;
00444         if (val & 0xff00ff00ul)
00445             idx += 8;
00446     }
00447     if (val & 0xf0f0f0f0ul)
00448         idx += 4;
00449     if (val & 0xccccccccul)
00450         idx += 2;
00451     if (val & 0xaaaaaaaaul)
00452         idx += 1;
00453
00454     return idx + ((idx < -LO) ? LO : LO+1);
00455 }
00456 }
00457 #else
00458 template<const index_t LO, const index_t HI>
00459 auto
00460 index_set<LO,HI>::
00461 min() const -> index_t
00462 {
00463     {
00464         for (auto
00465             idx = LO;
00466             idx != 0;
00467             ++idx)
00468             if (this->test(idx))
00469                 return idx;
00470         for (auto
00471             idx = index_t(1);
00472             idx <= HI;
00473             ++idx)
00474             if (this->test(idx))
00475                 return idx;
00476         return 0;
00477     }
00478 #endif
00479
00480 #if (_GLUCAT_BITS_PER_ULONG == 64)
00481 template<const index_t LO, const index_t HI>
00482 inline
00483 auto
00484 index_set<LO,HI>::
00485 max() const -> index_t
00486 {
00487     {
00488         // Reference: [JA], 1.6
00489         auto val = bitset_t::to_ulong();
00490         if (val == 0)
00491             return 0;
00492         else
00493         {
00494             auto idx = index_t(0);
00495             const auto nbits = HI - LO;
00496             if (nbits > 8)
00497             {
00498                 if (val & 0xffffffff00000000ul)
00499                     { val >>= 32; idx += 32; }
00500                 if (val & 0x00000000ffff0000ul)
00501                     { val >>= 16; idx += 16; }
00502                 if (val & 0x0000000000ff00ul)
00503                     { val >>= 8; idx += 8; }
00504             }
00505             if (val & 0x000000000000f0ul)
00506                 { val >>= 4; idx += 4; }
00507             if (val & 0x0000000000000cul)
00508                 { val >>= 2; idx += 2; }
00509             if (val & 0x00000000000002ul)
00510                 { idx += 1; }
00511             return idx + ((idx < -LO) ? LO : LO+1);
00512         }
00513     }
00514 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00515 template<const index_t LO, const index_t HI>
00516 inline
00517 auto
00518 index_set<LO,HI>::
00519 max() const -> index_t
00520 {
00521     {
00522         // Reference: [JA], 1.6
00523         auto val = bitset_t::to_ulong();
00524         if (val == 0)
00525             return 0;
00526         else
00527         {
00528             auto idx = index_t(0);
00529             const auto nbits = HI - LO;
00530             if (nbits > 8)
00531             {

```

```

00532         if (val & 0xffff0000ul)
00533         { val >>= 16; idx += 16; }
00534         if (val & 0x0000ff00ul)
00535         { val >>= 8; idx += 8; }
00536     }
00537     if (val & 0x000000f0ul)
00538     { val >>= 4; idx += 4; }
00539     if (val & 0x0000000cul)
00540     { val >>= 2; idx += 2; }
00541     if (val & 0x00000002ul)
00542     { idx += 1; }
00543     return idx + ((idx < -LO) ? LO : LO+1);
00544 }
00545 }
00546 #else
00547 template<const index_t LO, const index_t HI>
00548 auto
00549 index_set<LO,HI>::
00550 max() const -> index_t
00551 {
00552     {
00553         for (auto
00554             idx = HI;
00555             idx != 0;
00556             --idx)
00557             if (this->test(idx))
00558                 return idx;
00559         for (auto
00560             idx = index_t(-1);
00561             idx >= LO;
00562             --idx)
00563             if (this->test(idx))
00564                 return idx;
00565         return 0;
00566     }
00567 #endif
00568
00569 // eg. {3,4,5} is less than {3,7,8}
00570 template<const index_t LO, const index_t HI>
00571 inline
00572 auto
00573 compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int
00574 {
00575     {
00576         return (a == b)
00577             ? 0
00578             : a.lex_less_than(b)
00579             ? -1
00580             : 1;
00581     }
00582
00583 // eg. {3,4,5} is less than {3,7,8}
00584 template<const index_t LO, const index_t HI>
00585 inline
00586 auto
00587 index_set<LO,HI>::
00588 lex_less_than(const index_set_t rhs) const -> bool
00589 { return bitset_t::to_ulong() < rhs.bitset_t::to_ulong(); }
00590
00591 // Order by count, then order lexicographically within the equivalence class of count.
00592 template<const index_t LO, const index_t HI>
00593 inline
00594 auto
00595 index_set<LO,HI>::
00596 operator< (const index_set_t rhs) const -> bool
00597 {
00598     {
00599         const auto this_grade = this->count();
00600         const auto rhs_grade = rhs.count();
00601         return (this_grade < rhs_grade)
00602             ? true
00603             : (this_grade > rhs_grade)
00604             ? false
00605             : this->lex_less_than(rhs);
00606     }
00607 }
00608
00609 template<const index_t LO, const index_t HI>
00610 auto
00611 operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&
00612 {
00613     {
00614         index_t i;
00615         os << '{';
00616         for (i = LO;
00617             (i <= HI) && !(ist[i]);
00618             ++i)
00619             { }
00620         if (i <= HI)
00621             os << i;
00622         for (++i;
00623             i <= HI;

```

```

00624         ++i)
00625         if (ist[i])
00626             os << ',' << i;
00627     os << '}'';
00628     return os;
00629 }
00630
00631 template<const index_t LO, const index_t HI>
00632 auto
00633 operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&
00634 {
00635     // Parsing variables.
00636     auto i = index_t(0);
00637     using index_set_t = index_set<LO,HI>;
00638     auto local_ist = index_set_t();
00639     // Parsing control variables.
00640     auto parse_index_list = true;
00641     auto expect_closing_brace = false;
00642     auto expect_index = false;
00643     // Parse an optional opening brace.
00644     auto c = s.peek();
00645     // If there is a failure or end of file, this ends parsing.
00646     if (!s.good())
00647         parse_index_list = false;
00648     else
00649     { // Check for an opening brace.
00650         expect_closing_brace = (c == int('{'));
00651         if (expect_closing_brace)
00652         { // Consume the opening brace.
00653             s.get();
00654             // The next character may be a closing brace,
00655             // indicating the empty index set.
00656             c = s.peek();
00657             if (s.good() && (c == int('}')))
00658             { // A closing brace has been parsed and is no longer expected.
00659                 expect_closing_brace = false;
00660                 // Consume the closing brace.
00661                 s.get();
00662                 // This ends parsing.
00663                 parse_index_list = false;
00664             }
00665         }
00666     }
00667     if (s.good() && parse_index_list)
00668     { // Parse an optional index list.
00669         // The index list starts with a first index.
00670         for (s >> i;
00671              !s.fail();
00672              s >> i)
00673         { // An index has been parsed. Check to see if it is in range.
00674             if ((i < LO) || (i > HI))
00675             { // An index out of range is a failure.
00676                 s.clear(std::istream::failbit);
00677                 break;
00678             }
00679             // Add the index to the index set local_ist.
00680             local_ist.set(i);
00681             // Immediately after parsing an index, an index is no longer expected.
00682             expect_index = false;
00683             // Reading the index may have resulted in an end of file condition.
00684             // If so, this ends the index list.
00685             if (s.eof())
00686                 break;
00687             // The index list continues with a comma, and
00688             // may be ended by a closing brace, if it was begun with an opening brace.
00689             // Parse a possible comma or closing brace.
00690             c = s.peek();
00691             if (!s.good())
00692                 break;
00693             // First, test for a closing brace, if expected.
00694             if (expect_closing_brace && (c == int('}')))
00695             { // Consume the closing brace.
00696                 s.get();
00697                 // Immediately after parsing the closing brace, it is no longer expected.
00698                 expect_closing_brace = false;
00699                 // A closing brace ends the index list.
00700                 break;
00701             }
00702             // Now test for a comma.
00703             if (c == int(','))
00704             { // Consume the comma.
00705                 s.get();
00706                 // A index is expected after the comma.
00707                 expect_index = true;
00708             }
00709             else
00710             { // Any other character here is a failure.

```

```

00712         s.clear(std::istream::failbit);
00713         break;
00714     }
00715 }
00716 }
00717 // If an index or a closing brace is still expected, this is a failure.
00718 if (expect_index || expect_closing_brace)
00719     s.clear(std::istream::failbit);
00720 // End of file is not a failure.
00721 if (s)
00722 { // The index set has been successfully parsed.
00723     ist = local_ist;
00724 }
00725 return s;
00726 }
00727
00728 template<const index_t LO, const index_t HI>
00729 inline
00730 auto
00731 index_set<LO,HI>::
00732 is_contiguous () const -> bool
00733 {
00734     const auto min_index = this->min();
00735     const auto max_index = this->max();
00736     return (min_index < 0 && max_index > 0)
00737         ? max_index - min_index == this->count()
00738         : (min_index == 1 || max_index == -1) &&
00739           (max_index - min_index == this->count() - 1);
00740 }
00741
00742 template<const index_t LO, const index_t HI>
00743 inline
00744 auto
00745 index_set<LO,HI>::
00746 fold() const -> const
00747 index_set<LO,HI>
00748 { return this->fold(*this, true); }
00749
00750 template<const index_t LO, const index_t HI>
00751 auto
00752 index_set<LO,HI>::
00753 fold(const index_set_t frm, const bool prechecked) const -> const
00754 index_set<LO,HI>
00755 {
00756     if (!prechecked && ((*this | frm) != frm))
00757         throw error_t("fold(frm): cannot fold from outside of frame");
00758     const auto frm_min = frm.min();
00759     const auto frm_max = frm.max();
00760     auto result = index_set_t();
00761     auto fold_idx = index_t(-1);
00762     for (auto
00763         unfold_idx = fold_idx;
00764         unfold_idx >= frm_min;
00765         --unfold_idx)
00766     if (frm.test(unfold_idx))
00767         // result.set(fold_idx--, this->test(unfold_idx));
00768     {
00769         if (this->test(unfold_idx))
00770             result.set(fold_idx);
00771         --fold_idx;
00772     }
00773     fold_idx = index_t(1);
00774     for (auto
00775         unfold_idx = fold_idx;
00776         unfold_idx <= frm_max;
00777         ++unfold_idx)
00778     if (frm.test(unfold_idx))
00779         // result.set(fold_idx++, this->test(unfold_idx));
00780     {
00781         if (this->test(unfold_idx))
00782             result.set(fold_idx);
00783         ++fold_idx;
00784     }
00785     return result;
00786 }
00787
00788 template<const index_t LO, const index_t HI>
00789 auto
00790 index_set<LO,HI>::
00791 unfold(const index_set_t frm, const bool prechecked) const -> const index_set_t
00792 {
00793     const char* msg =
00794         "unfold(frm): cannot unfold into a smaller frame";
00795     const auto frm_min = frm.min();
00796     const auto frm_max = frm.max();
00797     auto result = index_set_t();
00798     auto fold_idx = index_t(-1);

```

```

00803     for (auto
00804         unfold_idx = fold_idx;
00805         unfold_idx >= frm_min;
00806         --unfold_idx)
00807         if (frm.test(unfold_idx))
00808             if (this->test(fold_idx--))
00809                 result.set(unfold_idx);
00810     if (!prechecked && ((fold_idx+1) > this->min()))
00811         throw error_t(msg);
00812     fold_idx = index_t(1);
00813     for (auto
00814         unfold_idx = fold_idx;
00815         unfold_idx <= frm_max;
00816         ++unfold_idx)
00817         if (frm.test(unfold_idx))
00818             if (this->test(fold_idx++))
00819                 result.set(unfold_idx);
00820     if (!prechecked && ((fold_idx-1) < this->max()))
00821         throw error_t(msg);
00822     return result;
00823 }
00824
00826 template<const index_t LO, const index_t HI>
00827 inline
00828 auto
00829 index_set<LO,HI>::
00830 value_of_fold(const index_set_t frm) const -> set_value_t
00831 {
00832     const auto min_index = frm.fold().min();
00833     if (min_index == 0)
00834         return 0;
00835     else
00836     {
00837         const auto folded_set = this->fold(frm);
00838         const auto skip = min_index > 0 ? index_t(1) : index_t(0);
00839         return folded_set.bitset_t::to_ulong() >> (min_index-LO-skip);
00840     }
00841 }
00842
00844 inline
00845 static
00846 auto inverse_reversed_gray(unsigned long x) -> unsigned long
00847 {
00848     // Reference: [JA]
00849 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00850     x ^= x << 32; // for 64-bit words
00851 #endif
00852     x ^= x << 16; // reversed_gray ** 16
00853     x ^= x << 8;  // reversed_gray ** 8
00854     x ^= x << 4;  // reversed_gray ** 4
00855     x ^= x << 2;  // reversed_gray ** 2
00856     x ^= x << 1;  // reversed_gray ** 1
00857     return x;
00858 }
00859
00861 inline
00862 static
00863 auto inverse_gray(unsigned long x) -> unsigned long
00864 {
00865     // Reference: [JA]
00866 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00867     x ^= x >> 32; // for 64-bit words
00868 #endif
00869     x ^= x >> 16; // gray ** 16
00870     x ^= x >> 8;  // gray ** 8
00871     x ^= x >> 4;  // gray ** 4
00872     x ^= x >> 2;  // gray ** 2
00873     x ^= x >> 1;  // gray ** 1
00874     return x;
00875 }
00876
00878 template<const index_t LO, const index_t HI>
00879 auto
00880 index_set<LO,HI>::
00881 sign_of_mult(const index_set_t rhs) const -> int
00882 {
00883     // Implemented using Walsh functions and Gray codes.
00884     // Reference: [L] Chapter 21, 21.3
00885     // Reference: [JA]
00886     const auto uthis = this->bitset_t::to_ulong();
00887     const auto urhs  = rhs.bitset_t::to_ulong();
00888     const auto nbits = HI - LO;
00889     auto negative = 0UL;
00890     if (nbits > 8)
00891     {
00892         // Set h to be the inverse reversed Gray code of rhs.
00893         // This sets each bit of h to be the cumulative ^ of

```

```

00894     // the same and lower bits of rhs.
00895     const auto h = inverse_reversed_gray(urhs);
00896     // Set k to be the inverse Gray code of *this & h.
00897     // This sets the low bit of k to be parity(*this & h).
00898     const auto k = inverse_gray(uthis & h);
00899     // Set q to be the inverse Gray code of the positive part of *this & rhs.
00900     const auto q = inverse_gray((uthis & urhs) >> -LO);
00901     negative = k ^ q;
00902 }
00903 else
00904 {
00905     auto h = 0UL;
00906     for (auto
00907         j = index_t(0);
00908         j < -LO;
00909         ++j)
00910     {
00911         h ^= urhs >> j;
00912         negative ^= h & (uthis >> j);
00913     }
00914     for (auto
00915         j = index_t(-LO);
00916         j < nbits;
00917         ++j)
00918     {
00919         negative ^= h & (uthis >> j);
00920         h ^= urhs >> j;
00921     }
00922 }
00923 return 1 - int((negative & 1) << 1);
00924 }
00925
00926 template<const index_t LO, const index_t HI>
00927 inline
00928 auto
00929 index_set<LO,HI>::
00930 sign_of_square() const -> int
00931 {
00932     {
00933         auto result = 1 - int((this->count_neg() % 2) << 1);
00934         switch (this->count() % 4)
00935         {
00936             case 2:
00937             case 3:
00938                 result *= -1;
00939                 break;
00940             default:
00941                 break;
00942         }
00943         return result;
00944     }
00945
00946 template<const index_t LO, const index_t HI>
00947 inline
00948 auto
00949 index_set<LO,HI>::
00950 hash_fn() const -> size_t
00951 {
00952     {
00953         static const auto lo_mask = (1UL << -LO) - 1UL;
00954         const auto uthis = bitset_t::to_ulong();
00955         const auto neg_part = uthis & lo_mask;
00956         const auto pos_part = uthis >> -LO;
00957         return size_t(neg_part ^ pos_part);
00958     }
00959
00960 inline
00961 auto
00962 sign_of_square(index_t j) -> int
00963 { return (j < 0) ? -1 : 1; }
00964
00965 template<const index_t LO, const index_t HI>
00966 inline
00967 auto
00968 min_neg(const index_set<LO,HI>& ist) -> index_t
00969 { return std::min(ist.min(), 0); }
00970
00971 template<const index_t LO, const index_t HI>
00972 inline
00973 auto
00974 max_pos(const index_set<LO,HI>& ist) -> index_t
00975 { return std::max(ist.max(), 0); }
00976
00977 // index_set reference
00978
00979 template<const index_t LO, const index_t HI>
00980 inline
00981 index_set<LO,HI>::reference::
00982 reference(index_set_t& ist, index_t idx) :

```

```

00987     m_pst(&ist),
00988     m_idx(idx)
00989 { }
00990
00992 template<const index_t LO, const index_t HI>
00993 inline
00994 auto
00995 index_set<LO,HI>::reference::
00996 operator== (const reference& c_j) const -> bool
00997 { return m_pst == c_j.m_pst && m_idx == c_j.m_idx; }
00998
01000 template<const index_t LO, const index_t HI>
01001 inline
01002 auto
01003 index_set<LO,HI>::reference::
01004 operator= (bool x) -> reference&
01005 {
01006     if ( x )
01007         m_pst->set(m_idx);
01008     else
01009         m_pst->reset(m_idx);
01010     return *this;
01011 }
01012
01014 template<const index_t LO, const index_t HI>
01015 inline
01016 auto
01017 index_set<LO,HI>::reference::
01018 operator= (const reference& c_j) -> reference&
01019 {
01020     if (&c_j != this && c_j != *this)
01021     {
01022         if ( (c_j.m_pst)[c_j.m_idx] )
01023             m_pst->set(m_idx);
01024         else
01025             m_pst->reset(m_idx);
01026     }
01027     return *this;
01028 }
01029
01031 template<const index_t LO, const index_t HI>
01032 inline
01033 auto
01034 index_set<LO,HI>::reference::
01035 operator~ () const -> bool
01036 { return !(m_pst->test(m_idx)); }
01037
01039 template<const index_t LO, const index_t HI>
01040 inline
01041 index_set<LO,HI>::reference::
01042 operator bool () const
01043 { return m_pst->test(m_idx); }
01044
01046 template<const index_t LO, const index_t HI>
01047 inline
01048 auto
01049 index_set<LO,HI>::reference::
01050 flip() -> reference&
01051 {
01052     m_pst->flip(m_idx);
01053     return *this;
01054 }
01055 }
01056 #endif // _GLUCAT_INDEX_SET_IMP_H

```

## 7.29 glucat/long\_double.h File Reference

```
#include "glucat/global.h"
```

```
#include "glucat/scalar.h"
```

Include dependency graph for long\_double.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace [glucat](#)

## Variables

- static const long double `glucat::l_pi` = 3.1415926535897932384626433832795029L
- static const long double `glucat::l_ln2` = 0.6931471805599453094172321214581766L

## 7.30 long\_double.h

Go to the documentation of this file.

```

00001 #ifndef _GLUCAT_LONG_DOUBLE_H
00002 #define _GLUCAT_LONG_DOUBLE_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     long_double.h : Define std functions for long double
00006     -----
00007     begin                : 2001-12-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009 *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 namespace glucat
00038 {
00039     #if defined(__USE_GNU)
00040         static const long double l_pi    = M_PI1;
00041         static const long double l_ln2   = M_LN21;
00042     #else
00043         static const long double l_pi    = 3.1415926535897932384626433832795029L;
00044         static const long double l_ln2   = 0.6931471805599453094172321214581766L;
00045     #endif
00046
00047     template<>
00048     inline
00049     auto
00050     numeric_traits<long double>::
00051     pi() -> long double
00052     { return l_pi; }
00053
00054     template<>
00055     inline
00056     auto
00057     numeric_traits<long double>::
00058     ln_2() -> long double
00059     { return l_ln2; }
00060 }
00061 #endif // _GLUCAT_LONG_DOUBLE_H

```

## 7.31 glucat/matrix.h File Reference

```

#include <boost/numeric/ublas/fwd.hpp>
#include <complex>
#include <vector>

```

Include dependency graph for matrix.h: This graph shows which files directly or indirectly include this file:



## Classes

- struct [glucat::matrix::eig\\_genus< Matrix\\_T >](#)  
*Structure containing classification of eigenvalues.*

## Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

## Typedefs

- using [glucat::matrix::eig\\_case\\_t](#) = enum { safe\_eigs, neg\_real\_eigs, both\_eigs}  
*Classification of eigenvalues of a matrix.*

## Functions

- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T >  
auto [glucat::matrix::nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T >  
auto [glucat::matrix::isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T >  
auto [glucat::matrix::isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T >  
auto [glucat::matrix::unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↔  
expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto [glucat::matrix::sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↔  
expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*

- `template<typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of matrices.*
- `template<typename Scalar_T , typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T`  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$*
- `template<typename Matrix_T >`  
`auto glucat::matrix::norm\_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Square of Frobenius norm.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::trace (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Matrix trace.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >`  
*Eigenvalues of a matrix.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::classify\_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >`  
*Classify the eigenvalues of a matrix.*

## 7.32 matrix.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_H
00002 #define _GLUCAT_MATRIX_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   matrix.h : Declare common matrix functions
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright             : (C) 2001-2012 by Paul C. Leopardi
00009                       : uBLAS interface contributed by Joerg Walter
00010   *****/
00011
00012   This library is free software: you can redistribute it and/or modify
00013   it under the terms of the GNU Lesser General Public License as published
00014   by the Free Software Foundation, either version 3 of the License, or
00015   (at your option) any later version.
00016
00017   This library is distributed in the hope that it will be useful,
00018   but WITHOUT ANY WARRANTY; without even the implied warranty of
00019   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020   GNU Lesser General Public License for more details.
00021
00022   You should have received a copy of the GNU Lesser General Public License
00023   along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025   *****/
00026   This library is based on a prototype written by Arvind Raja and was
00027   licensed under the LGPL with permission of the author. See Arvind Raja,
00028   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029   in Ablamowicz, Lounesto and Parra (eds.)
00030   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031   *****/
00032   See also Arvind Raja's original header comments in glucat.h
00033   *****/
00034
00035 #include <boost/numeric/ublas/fwd.hpp>
00036
00037 #include <complex>
00038 #include <vector>
00039
00040 namespace glucat
00041 {
00042     namespace ublas = boost::numeric::ublas;
00043
00044     namespace matrix
00045     {
00046         template< typename LHS_T, typename RHS_T >

```

```

00048     auto
00049     kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00050     RHS_T;
00051
00052     template< typename LHS_T, typename RHS_T >
00053     auto
00054     mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00055     RHS_T;
00056
00057     template< typename LHS_T, typename RHS_T >
00058     auto
00059     nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono = true) -> const
00060     RHS_T;
00061
00062     template< typename LHS_T, typename RHS_T >
00063     auto
00064     signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00065     RHS_T;
00066
00067     template< typename Matrix_T >
00068     auto
00069     nnz(const Matrix_T& m) -> typename Matrix_T::size_type;
00070
00071     template< typename Matrix_T >
00072     auto
00073     isinf(const Matrix_T& m) -> bool;
00074
00075     template< typename Matrix_T >
00076     auto
00077     isnan(const Matrix_T& m) -> bool;
00078
00079     template< typename Matrix_T >
00080     auto
00081     unit(const typename Matrix_T::size_type n) -> const
00082     Matrix_T;
00083
00084     template< typename LHS_T, typename RHS_T >
00085     auto
00086     mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00087               const ublas::matrix_expression<RHS_T>& rhs) -> const
00088     typename RHS_T::expression_type;
00089
00090     template< typename LHS_T, typename RHS_T >
00091     auto
00092     sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00093                const ublas::matrix_expression<RHS_T>& rhs) -> const
00094     typename RHS_T::expression_type;
00095
00096     template< typename LHS_T, typename RHS_T >
00097     auto
00098     prod(const ublas::matrix_expression<LHS_T>& lhs,
00099          const ublas::matrix_expression<RHS_T>& rhs) -> const
00100     typename RHS_T::expression_type;
00101
00102     template< typename LHS_T, typename RHS_T >
00103     auto
00104     inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T;
00105
00106     template< typename Matrix_T >
00107     auto
00108     norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type;
00109
00110     template< typename Matrix_T >
00111     auto
00112     trace(const Matrix_T& val) -> typename Matrix_T::value_type;
00113
00114     template< typename Matrix_T >
00115     auto
00116     eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >;
00117
00118     using eig_case_t = enum {
00119         safe_eigs,
00120         neg_real_eigs,
00121         both_eigs};
00122
00123     template< typename Matrix_T >
00124     struct eig_genus
00125     {
00126         using Scalar_T = typename Matrix_T::value_type;
00127         bool m_is_singular = false;
00128         eig_case_t m_eig_case = safe_eigs;
00129         Scalar_T m_safe_arg = Scalar_T(0);
00130     };
00131
00132     template< typename Matrix_T >
00133     auto
00134     classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>;

```

```

00155     }
00156 }
00157
00158 #endif // _GLUCAT_MATRIX_H

```

## 7.33 glucat/matrix\_imp.h File Reference

```

#include "glucat/errors.h"
#include "glucat/scalar.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/vector.hpp>
#include <boost/numeric/ublas/vector_proxy.hpp>
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/bindings/lapack/driver/gees.hpp>
#include <boost/numeric/bindings/ublas.hpp>
#include <set>
#include <vector>

```

Include dependency graph for matrix\_imp.h: This graph shows which files directly or indirectly include this file:

## Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

## Functions

- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Kronecker tensor product of matrices - as per Matlab kron.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::mono\_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Sparse Kronecker tensor product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T >`  
`void glucat::matrix::nork\_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T`  
*Left inverse of Kronecker product.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::signed\_perm\_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::nnz (const Matrix_T &m) -> typename Matrix_T::size_type`  
*Number of non-zeros.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::isinf (const Matrix_T &m) -> bool`

- Infinite.*
- template<typename Matrix\_T >  
auto `glucat::matrix::isnan` (const Matrix\_T &m) -> bool
- Not a Number.*
- template<typename Matrix\_T >  
auto `glucat::matrix::unit` (const typename Matrix\_T::size\_type n) -> const Matrix\_T
- Unit matrix - as per Matlab eye.*
- template<typename LHS\_T , typename RHS\_T >  
auto `glucat::matrix::mono_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type
- Product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto `glucat::matrix::sparse_prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type
- Product of sparse matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto `glucat::matrix::prod` (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type
- Product of matrices.*
- template<typename Scalar\_T , typename LHS\_T , typename RHS\_T >  
auto `glucat::matrix::inner` (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T
- Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$*
- template<typename Matrix\_T >  
auto `glucat::matrix::norm_frob2` (const Matrix\_T &val) -> typename Matrix\_T::value\_type
- Square of Frobenius norm.*
- template<typename Matrix\_T >  
auto `glucat::matrix::trace` (const Matrix\_T &val) -> typename Matrix\_T::value\_type
- Matrix trace.*
- template<typename Matrix\_T >  
static auto `glucat::matrix::to_lapack` (const Matrix\_T &val) -> ublas::matrix< double, ublas::column\_major >
- Convert matrix to LAPACK format.*
- template<typename Matrix\_T >  
auto `glucat::matrix::eigenvalues` (const Matrix\_T &val) -> std::vector< std::complex< double > >
- Eigenvalues of a matrix.*
- template<typename Matrix\_T >  
auto `glucat::matrix::classify_eigenvalues` (const Matrix\_T &val) -> eig\_genus< Matrix\_T >
- Classify the eigenvalues of a matrix.*

## 7.34 matrix\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_IMP_H
00002 #define _GLUCAT_MATRIX_IMP_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  matrix_imp.h : Implement common matrix functions
00006  -----
00007  begin                : Sun 2001-12-09
00008  copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                      : uBLAS interface contributed by Joerg Walter
00010  *****/
00011
00012  This library is free software: you can redistribute it and/or modify
00013  it under the terms of the GNU Lesser General Public License as published
00014  by the Free Software Foundation, either version 3 of the License, or
00015  (at your option) any later version.
00016
00017  This library is distributed in the hope that it will be useful,
```

```

00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025     *****
00026     This library is based on a prototype written by Arvind Raja and was
00027     licensed under the LGPL with permission of the author. See Arvind Raja,
00028     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029     in Ablamowicz, Lounesto and Parra (eds.)
00030     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031     *****
00032     See also Arvind Raja's original header comments in glucat.h
00033     *****/
00034
00035 #include "glucat/errors.h"
00036 #include "glucat/scalar.h"
00037 #include "glucat/matrix.h"
00038
00039 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00040 #   pragma GCC diagnostic push
00041 #   pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00042 # endif
00043 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00044 #   include <boost/serialization/array_wrapper.hpp>
00045 # endif
00046 #include <boost/numeric/ublas/vector.hpp>
00047 #include <boost/numeric/ublas/vector_proxy.hpp>
00048 #include <boost/numeric/ublas/matrix.hpp>
00049 #include <boost/numeric/ublas/matrix_expression.hpp>
00050 #include <boost/numeric/ublas/matrix_proxy.hpp>
00051 #include <boost/numeric/ublas/matrix_sparse.hpp>
00052 #include <boost/numeric/ublas/operation.hpp>
00053 #include <boost/numeric/ublas/operation_sparse.hpp>
00054
00055 #if defined(_GLUCAT_USE_BINDINGS)
00056 # include <boost/numeric/bindings/lapack/driver/gees.hpp>
00057 # include <boost/numeric/bindings/ublas.hpp>
00058 #endif
00059
00060 #if defined(_GLUCAT_USE_BLAZE)
00061 #include <blaze/Math.h>
00062 #include <blaze/math/DynamicMatrix.h>
00063 #include <blaze/math/DynamicVector.h>
00064 #endif
00065
00066 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00067 #   pragma GCC diagnostic pop
00068 # endif
00069
00070 #include <set>
00071 #include <vector>
00072
00073 namespace glucat { namespace matrix
00074 {
00075     // References for algorithms:
00076     // [v]: C. F. van Loan and N. Pitsianis, "Approximation with Kronecker products",
00077     // in Linear Algebra for Large Scale and Real-Time Applications, Marc S. Moonen,
00078     // Gene H. Golub, and Bart L. R. Moor (eds.), 1993, pp. 293--314.
00079
00080     template< typename LHS_T, typename RHS_T >
00081     auto
00082     kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00083     RHS_T
00084     {
00085         {
00086             const auto rhs_s1 = rhs.size1();
00087             const auto rhs_s2 = rhs.size2();
00088             auto result = RHS_T(lhs.size1()*rhs_s1, lhs.size2()*rhs_s2);
00089             result.clear();
00090
00091             for (auto
00092                 lhs_it1 = lhs.begin1();
00093                 lhs_it1 != lhs.end1();
00094                 ++lhs_it1)
00095                 for (auto
00096                     lhs_it2 = lhs_it1.begin();
00097                     lhs_it2 != lhs_it1.end();
00098                     ++lhs_it2)
00099                 {
00100                     const auto start1 = rhs_s1 * lhs_it2.index1();
00101                     const auto start2 = rhs_s2 * lhs_it2.index2();
00102                     const auto& lhs_val = *lhs_it2;
00103                     for (auto
00104                         rhs_it1 = rhs.begin1();
00105                         rhs_it1 != rhs.end1();

```

```

00106         ++rhs_it1)
00107     for (auto
00108         rhs_it2 = rhs_it1.begin();
00109         rhs_it2 != rhs_it1.end();
00110         ++rhs_it2)
00111         result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00112     }
00113     return result;
00114 }
00115
00116 template< typename LHS_T, typename RHS_T >
00117 auto
00118 mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00119 RHS_T
00120 {
00121     {
00122         const auto rhs_s1 = rhs.size1();
00123         const auto rhs_s2 = rhs.size2();
00124         const auto dim = lhs.size1()*rhs_s1;
00125         auto result = RHS_T(dim, dim, dim);
00126         result.clear();
00127
00128         for (auto
00129             lhs_it1 = lhs.begin1();
00130             lhs_it1 != lhs.end1();
00131             ++lhs_it1)
00132         {
00133             const auto lhs_it2 = lhs_it1.begin();
00134             const auto start1 = rhs_s1 * lhs_it2.index1();
00135             const auto start2 = rhs_s2 * lhs_it2.index2();
00136             const auto& lhs_val = *lhs_it2;
00137             for (auto
00138                 rhs_it1 = rhs.begin1();
00139                 rhs_it1 != rhs.end1();
00140                 ++rhs_it1)
00141             {
00142                 const auto rhs_it2 = rhs_it1.begin();
00143                 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00144             }
00145         }
00146         return result;
00147     }
00148
00149 template< typename LHS_T, typename RHS_T >
00150 void
00151 nork_range(RHS_T& result,
00152            const typename LHS_T::const_iterator2 lhs_it2,
00153            const RHS_T& rhs,
00154            const typename RHS_T::size_type res_s1,
00155            const typename RHS_T::size_type res_s2)
00156 {
00157     // Definition matches [v] Section 4, Theorem 4.1.
00158     const auto start1 = res_s1 * lhs_it2.index1();
00159     const auto start2 = res_s2 * lhs_it2.index2();
00160     using ublas::range;
00161     const auto& range1 = range(start1, start1 + res_s1);
00162     const auto& range2 = range(start2, start2 + res_s2);
00163     using matrix_range_t = ublas::matrix_range<const RHS_T>;
00164     const auto& rhs_range = matrix_range_t(rhs, range1, range2);
00165     using Scalar_T = typename RHS_T::value_type;
00166     const auto lhs_val = numeric_traits<Scalar_T>::to_scalar_t(*lhs_it2);
00167     for (auto
00168         rhs_it1 = rhs_range.begin1();
00169         rhs_it1 != rhs_range.end1();
00170         ++rhs_it1)
00171     for (auto
00172         rhs_it2 = rhs_it1.begin();
00173         rhs_it2 != rhs_it1.end();
00174         ++rhs_it2)
00175         result(rhs_it2.index1(), rhs_it2.index2()) += lhs_val * *rhs_it2;
00176 }
00177
00178 template< typename LHS_T, typename RHS_T >
00179 auto
00180 nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono) -> const
00181 RHS_T
00182 {
00183     // nork(A, kron(A, B)) is close to B
00184     // Definition matches [v] Section 4, Theorem 4.1.
00185     const auto lhs_s1 = lhs.size1();
00186     const auto lhs_s2 = lhs.size2();
00187     const auto rhs_s1 = rhs.size1();
00188     const auto rhs_s2 = rhs.size2();
00189     const auto res_s1 = rhs_s1 / lhs_s1;
00190     const auto res_s2 = rhs_s2 / lhs_s2;
00191     using Scalar_T = typename RHS_T::value_type;
00192     const auto norm_frob2_lhs = norm_frob2(lhs);
00193     if (!mono)

```

```

00196     {
00197         using error_t = error<RHS_T>;
00198         if (rhs_s1 == 0)
00199             throw error_t("matrix", "nork: number of rows must not be 0");
00200         if (rhs_s2 == 0)
00201             throw error_t("matrix", "nork: number of cols must not be 0");
00202         if (res_s1 * lhs_s1 != rhs_s1)
00203             throw error_t("matrix", "nork: incompatible numbers of rows");
00204         if (res_s2 * lhs_s2 != rhs_s2)
00205             throw error_t("matrix", "nork: incompatible numbers of cols");
00206         if (norm_frob2_lhs == Scalar_T(0))
00207             throw error_t("matrix", "nork: LHS must not be 0");
00208     }
00209     auto result = RHS_T(res_s1, res_s2);
00210     result.clear();
00211     for (auto
00212         lhs_it1 = lhs.begin1();
00213         lhs_it1 != lhs.end1();
00214         ++lhs_it1)
00215         for (auto
00216             lhs_it2 = lhs_it1.begin();
00217             lhs_it2 != lhs_it1.end();
00218             ++lhs_it2)
00219             if (*lhs_it2 != Scalar_T(0))
00220                 nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00221     result /= norm_frob2_lhs;
00222     return result;
00223 }
00224
00226 template< typename LHS_T, typename RHS_T >
00227 auto
00228 signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00229 RHS_T
00230 {
00231     // signed_perm_nork(A, kron(A, B)) is close to B
00232     // Definition matches [v] Section 4, Theorem 4.1.
00233     const auto lhs_s1 = lhs.size1();
00234     const auto lhs_s2 = lhs.size2();
00235     const auto rhs_s1 = rhs.size1();
00236     const auto rhs_s2 = rhs.size2();
00237     const auto res_s1 = rhs_s1 / lhs_s1;
00238     const auto res_s2 = rhs_s2 / lhs_s2;
00239     using Scalar_T = typename RHS_T::value_type;
00240     const auto norm_frob2_lhs = Scalar_T( double(lhs_s1) );
00241     auto result = RHS_T(res_s1, res_s2);
00242     result.clear();
00243     for (auto
00244         lhs_it1 = lhs.begin1();
00245         lhs_it1 != lhs.end1();
00246         ++lhs_it1)
00247     {
00248         const auto lhs_it2 = lhs_it1.begin();
00249         nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00250     }
00251     result /= norm_frob2_lhs;
00252     return result;
00253 }
00254
00256 template< typename Matrix_T >
00257 auto
00258 nnz(const Matrix_T& m) -> typename Matrix_T::size_type
00259 {
00260     using size_t = typename Matrix_T::size_type;
00261     auto result = size_t(0);
00262     for (auto
00263         it1 = m.begin1();
00264         it1 != m.end1();
00265         ++it1)
00266         for (auto& entry : it1)
00267             if (entry != 0)
00268                 ++result;
00269     return result;
00270 }
00271
00273 template< typename Matrix_T >
00274 auto
00275 isinf(const Matrix_T& m) -> bool
00276 {
00277     using Scalar_T = typename Matrix_T::value_type;
00278     for (auto
00279         it1 = m.begin1();
00280         it1 != m.end1();
00281         ++it1)
00282         for (auto& entry : it1)
00283             if (numeric_traits<Scalar_T>::isInf(entry))
00284                 return true;
00285 }

```



```

00286     return false;
00287 }
00288
00290 template< typename Matrix_T >
00291 auto
00292 isnan(const Matrix_T& m) -> bool
00293 {
00294     using Scalar_T = typename Matrix_T::value_type;
00295     for (auto
00296         it1 = m.begin1();
00297         it1 != m.end1();
00298         ++it1)
00299         for (auto& entry : it1)
00300             if (numeric_traits<Scalar_T>::isNaN(entry))
00301                 return true;
00302
00303     return false;
00304 }
00305
00307 template< typename Matrix_T >
00308 inline
00309 auto
00310 unit(const typename Matrix_T::size_type dim) -> const
00311 Matrix_T
00312 {
00313     using Scalar_T = typename Matrix_T::value_type;
00314     return ublas::identity_matrix<Scalar_T>(dim);
00315 }
00316
00318 template< typename LHS_T, typename RHS_T >
00319 auto
00320 mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00321           const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00322 {
00323     using rhs_expression_t = const RHS_T;
00324     using matrix_row_t = typename ublas::matrix_row<rhs_expression_t>;
00325
00326     const auto dim = lhs().size1();
00327     // The following assumes that RHS_T is a sparse matrix type.
00328     auto result = RHS_T(dim, dim, dim);
00329     for (auto
00330         lhs_row = lhs().begin1();
00331         lhs_row != lhs().end1();
00332         ++lhs_row)
00333     {
00334         const auto& lhs_it = lhs_row.begin();
00335         if (lhs_it != lhs_row.end())
00336         {
00337             const auto& rhs_row = matrix_row_t(rhs(), lhs_it.index2());
00338             const auto& rhs_it = rhs_row.begin();
00339             if (rhs_it != rhs_row.end())
00340                 result(lhs_it.index1(), rhs_it.index()) = (*lhs_it) * (*rhs_it);
00341         }
00342     }
00343     return result;
00344 }
00345
00347 template< typename LHS_T, typename RHS_T >
00348 inline
00349 auto
00350 sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00351            const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00352 {
00353     using expression_t = typename RHS_T::expression_type;
00354     return ublas::sparse_prod<expression_t>(lhs(), rhs());
00355 }
00356
00358 template< typename LHS_T, typename RHS_T >
00359 inline
00360 auto
00361 prod(const ublas::matrix_expression<LHS_T>& lhs,
00362      const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00363 {
00364     const auto dim = lhs().size1();
00365     RHS_T result(dim, dim);
00366     ublas::axpy_prod(lhs, rhs, result, true);
00367     return result;
00368 }
00369
00371 template< typename Scalar_T, typename LHS_T, typename RHS_T >
00372 auto
00373 inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T
00374 {
00375     auto result = Scalar_T(0);
00376     for (auto
00377         lhs_it1 = lhs.begin1();
00378         lhs_it1 != lhs.end1();

```

```

00379         ++lhs_it1)
00380     for (auto
00381         lhs_it2 = lhs_it1.begin();
00382         lhs_it2 != lhs_it1.end();
00383         ++lhs_it2)
00384     {
00385         const auto& rhs_val = rhs(lhs_it2.index1(), lhs_it2.index2());
00386         if (rhs_val != Scalar_T(0))
00387             result += (*lhs_it2) * rhs_val;
00388     }
00389     return result / lhs.size1();
00390 }
00391
00392 template< typename Matrix_T >
00393 auto
00394 norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type
00395 {
00396     using Scalar_T = typename Matrix_T::value_type;
00397
00398     auto result = Scalar_T(0);
00399     for (auto
00400         val_it1 = val.begin1();
00401         val_it1 != val.end1();
00402         ++val_it1)
00403     for (auto& val_entry : val_it1)
00404     {
00405         if (numeric_traits<Scalar_T>::isNaN(val_entry))
00406             return numeric_traits<Scalar_T>::NaN();
00407         result += val_entry * val_entry;
00408     }
00409     return result;
00410 }
00411
00412 template< typename Matrix_T >
00413 auto
00414 trace(const Matrix_T& val) -> typename Matrix_T::value_type
00415 {
00416     using Scalar_T = typename Matrix_T::value_type;
00417
00418     auto result = Scalar_T(0);
00419     auto dim = val.size1();
00420     for (auto
00421         ndx = decltype(dim)(0);
00422         ndx != dim;
00423         ++ndx)
00424     {
00425         const Scalar_T crd = val(ndx, ndx);
00426         if (numeric_traits<Scalar_T>::isNaN(crd))
00427             return numeric_traits<Scalar_T>::NaN();
00428         result += crd;
00429     }
00430     return result;
00431 }
00432
00433 #if defined(_GLUCAT_USE_BINDINGS)
00434 template< typename Matrix_T >
00435 static
00436 auto
00437 to_lapack(const Matrix_T& val) -> ublas::matrix<double, ublas::column_major>
00438 {
00439     const auto s1 = val.size1();
00440     const auto s2 = val.size2();
00441
00442     using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00443     auto result = lapack_matrix_t(s1, s2);
00444     result.clear();
00445
00446     using Scalar_T = typename Matrix_T::value_type;
00447     using traits_t = numeric_traits<Scalar_T>;
00448
00449     for (auto
00450         val_it1 = val.begin1();
00451         val_it1 != val.end1();
00452         ++val_it1)
00453     for (auto
00454         val_it2 = val_it1.begin();
00455         val_it2 != val_it1.end();
00456         ++val_it2)
00457         result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00458
00459     return result;
00460 }
00461 #endif
00462
00463 #if defined(_GLUCAT_USE_BLAZE)
00464 template< typename Matrix_T >
00465 static

```

```

00470     auto
00471     to_blaze(const Matrix_T& val) -> blaze::DynamicMatrix<double, blaze::rowMajor>
00472     {
00473         const auto s1 = val.size1();
00474         const auto s2 = val.size2();
00475
00476         using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00477         auto result = blaze_matrix_t(s1, s2);
00478
00479         using Scalar_T = typename Matrix_T::value_type;
00480         using traits_t = numeric_traits<Scalar_T>;
00481
00482         for (auto
00483             val_it1 = val.begin1();
00484             val_it1 != val.end1();
00485             ++val_it1)
00486             for (auto
00487                 val_it2 = val_it1.begin();
00488                 val_it2 != val_it1.end();
00489                 ++val_it2)
00490                 result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00491
00492         return result;
00493     }
00494
00495 #endif
00496
00497 template< typename Matrix_T >
00498 auto
00499 eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >
00500 {
00501     using complex_t = std::complex<double>;
00502     using complex_vector_t = typename std::vector<complex_t>;
00503
00504     const auto dim = val.size1();
00505     auto lambda = complex_vector_t(dim);
00506
00507     #if defined(_GLUCAT_USE_BINDINGS)
00508     namespace lapack = boost::numeric::bindings::lapack;
00509     using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00510
00511     auto T = to_lapack(val);
00512     auto V = T;
00513     using vector_t = typename ublas::vector<double>;
00514     auto real_lambda = vector_t(dim);
00515     auto imag_lambda = vector_t(dim);
00516     fortran_int_t sdim = 0;
00517
00518     lapack::gees('N', 'N', nullptr, T, sdim, real_lambda, imag_lambda, V);
00519
00520     for (auto
00521         k = decltype(dim)(0);
00522         k != dim;
00523         ++k)
00524         lambda[k] = complex_t(real_lambda[k], imag_lambda[k]);
00525     #endif
00526 #if defined(_GLUCAT_USE_BLAZE)
00527     using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00528     using complex_t = std::complex<double>;
00529     using blaze_complex_vector_t = blaze::DynamicVector<complex_t, blaze::columnVector>;
00530
00531     auto blaze_val = to_blaze(val);
00532     auto blaze_lambda = blaze_complex_vector_t(dim);
00533     blaze::geev(blaze_val, blaze_lambda);
00534
00535     for (auto
00536         k = decltype(dim)(0);
00537         k != dim;
00538         ++k)
00539         lambda[k] = blaze_lambda[k];
00540     #endif
00541     return lambda;
00542 }
00543
00544 template< typename Matrix_T >
00545 auto
00546 classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>
00547 {
00548     using Scalar_T = typename Matrix_T::value_type;
00549     eig_genus<Matrix_T> result;
00550
00551     using complex_t = std::complex<double>;
00552     using complex_vector_t = typename std::vector<complex_t>;
00553     auto lambda = eigenvalues(val);
00554
00555     std::set<double> arg_set;
00556
00557
00558

```

```

00559     using vector_index_t = typename complex_vector_t::size_type;
00560     const auto dim = lambda.size();
00561     static const auto epsilon =
00562         std::max(std::numeric_limits<double>::epsilon(),
00563             numeric_traits<Scalar_T>::to_double(std::numeric_limits<Scalar_T>::epsilon()));
00564     static const auto zero_eig_tol = 4096.0*epsilon;
00565
00566     bool neg_real_eig_found = false;
00567     bool imag_eig_found = false;
00568     bool zero_eig_found = false;
00569
00570     for (auto
00571         k = decltype(dim)(0);
00572         k != dim;
00573         ++k)
00574     {
00575         const auto lambda_k = lambda[k];
00576         arg_set.insert(std::arg(lambda_k));
00577
00578         const auto real_lambda_k = std::real(lambda_k);
00579         const auto imag_lambda_k = std::imag(lambda_k);
00580         const auto norm_tol = 4096.0*epsilon*std::norm(lambda_k);
00581
00582         if (!neg_real_eig_found &&
00583             real_lambda_k < -epsilon &&
00584             (imag_lambda_k == 0.0 ||
00585             imag_lambda_k * imag_lambda_k < norm_tol))
00586             neg_real_eig_found = true;
00587         if (!imag_eig_found &&
00588             imag_lambda_k > epsilon &&
00589             (real_lambda_k == 0.0 ||
00590             real_lambda_k * real_lambda_k < norm_tol))
00591             imag_eig_found = true;
00592         if (!zero_eig_found &&
00593             std::norm(lambda_k) < zero_eig_tol)
00594             zero_eig_found = true;
00595     }
00596
00597     if (zero_eig_found)
00598         result.m_is_singular = true;
00599
00600     static const auto pi = numeric_traits<double>::pi();
00601     if (neg_real_eig_found)
00602     {
00603         if (imag_eig_found)
00604             result.m_eig_case = both_eigs;
00605         else
00606         {
00607             result.m_eig_case = neg_real_eigs;
00608             result.m_safe_arg = Scalar_T(-pi / 2.0);
00609         }
00610     }
00611
00612     if (result.m_eig_case == both_eigs)
00613     {
00614         auto arg_it = arg_set.begin();
00615         auto first_arg = *arg_it;
00616         auto best_arg = first_arg;
00617         auto best_diff = 0.0;
00618         auto previous_arg = first_arg;
00619         for (++arg_it;
00620             arg_it != arg_set.end();
00621             ++arg_it)
00622         {
00623             const auto arg_diff = *arg_it - previous_arg;
00624             if (arg_diff > best_diff)
00625             {
00626                 best_diff = arg_diff;
00627                 best_arg = previous_arg;
00628             }
00629             previous_arg = *arg_it;
00630         }
00631         const auto arg_diff = first_arg + 2.0*pi - previous_arg;
00632         if (arg_diff > best_diff)
00633         {
00634             best_diff = arg_diff;
00635             best_arg = previous_arg;
00636         }
00637         result.m_safe_arg = Scalar_T(pi - (best_arg + best_diff / 2.0));
00638     }
00639     return result;
00640 }
00641 }
00642
00643 #endif // _GLUCAT_MATRIX_IMP_H

```

## 7.35 glucat/matrix\_multi.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <fstream>
#include <string>
#include <utility>
#include <vector>
```

Include dependency graph for matrix\_multi.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >](#)  
*A matrix\_multi<Scalar\_T,LO,HI,Tune\_P> is a matrix approximation to a multivector.*
- struct [std::numeric\\_limits< glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P > >](#)  
*Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.*

### Namespaces

- namespace [glucat](#)
- namespace [std](#)

### Functions

- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::operator\\*](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::operator^](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::operator&](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::operator%](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::star](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T , const index\_t LO, const index\_t HI, typename Tune\_P >  
 auto [glucat::operator/](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >

*Geometric quotient.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T,`  
`LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`

*Read multivector from input.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`

*Write multivector to output.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T,`  
`LO, HI, Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T,`  
`LO, HI, Tune_P > &rhs_reframed) -> const index_set< LO, HI >`

*Find a common frame for operands of a binary operator.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix\_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix\_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T,`  
`LO, HI, Tune_P >`

*Exponential of multivector.*

## 7.36 matrix\_multi.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_MATRIX_MULTI_H
00002 #define _GLUCAT_MATRIX_MULTI_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   matrix_multi.h : Declare a class for the matrix representation of a multivector
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
```

```

00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039 #include "glucat/framed_multi.h"
00040
00041 #include <boost/numeric/ublas/fwd.hpp>
00042
00043 #include <fstream>
00044 #include <string>
00045 #include <utility>
00046 #include <vector>
00047
00048 namespace glucat
00049 {
00050     namespace ublas = boost::numeric::ublas;
00051
00052     // Forward declarations for friends
00053
00054     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00055     class framed_multi; // forward
00056
00057     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00058     class matrix_multi; // forward
00059
00060     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00061     auto
00062     operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00064
00065     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00066     auto
00067     operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00069
00070     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00071     auto
00072     operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00074
00075     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076     auto
00077     operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00079
00080     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00081     auto
00082     star (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00084
00085     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00086     auto
00087     operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00089
00090     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00091     auto
00092     operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00094
00095     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00096     auto
00097     operator> (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00100     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00101     auto
00102     operator< (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00104
00105     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00106     auto
00107     reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const

```

```

matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs,
00109     matrix_multi<Scalar_T, LO, HI, Tune_P>& lhs_reframed,
matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs_reframed) -> const index_set<LO, HI>;

00110     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111     auto
00112     sqrt(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val, const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
00113     bool prechecked) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>;

00115     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00116     auto
00117     matrix_sqrt(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val,
00118     const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
00119     const index_t level) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>;

00122     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00123     auto
00124     log(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val, const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
00125     bool prechecked) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>;

00127     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00128     auto
00129     matrix_log(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val,
00130     const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
00131     const index_t level) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>;

00134     template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
00135     typename Tune_P = tuning<> >
00136     class matrix_multi {
00137     public clifford_algebra< Scalar_T, index_set<LO, HI>, matrix_multi<Scalar_T, LO, HI, Tune_P> >
00138     {
00139     public:
00140     using multivector_t = matrix_multi;
00141     using matrix_multi_t = multivector_t;
00142     using scalar_t = Scalar_T;
00143     using tune_p = Tune_P;
00144     using index_set_t = index_set<LO, HI>;
00145     using term_t = std::pair<const index_set_t, Scalar_T>;
00146     using vector_t = std::vector<Scalar_T>;
00147     using error_t = error<multivector_t>;
00148     using framed_multi_t = framed_multi<Scalar_T, LO, HI, Tune_P>;
00149     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00150     Other_Tune_P >
00151     friend class framed_multi;
00152     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
00153     Other_Tune_P >
00154     friend class matrix_multi;
00155     private:
00156     using orientation_t = ublas::row_major;
00157     using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>;
00158     using matrix_t = ublas::matrix<Scalar_T, orientation_t>;
00159     using matrix_index_t = typename matrix_t::size_type;
00160     public:
00161     static auto classname() -> const std::string;
00162     ~matrix_multi() override = default;
00163     matrix_multi();
00164     template< typename Other_Scalar_T >
00165     matrix_multi(const matrix_multi<Other_Scalar_T, LO, HI, Tune_P>& val);
00166     template< typename Other_Scalar_T >
00167     matrix_multi(const matrix_multi<Other_Scalar_T, LO, HI, Tune_P>& val,
00168     const index_set_t frm, const bool prechecked = false);
00169     matrix_multi(const multivector_t& val,
00170     const index_set_t frm, const bool prechecked = false);
00171     matrix_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00172     matrix_multi(const index_set_t ist, const Scalar_T& crd,
00173     const index_set_t frm, const bool prechecked = false);
00174     matrix_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00175     matrix_multi(const int scr, const index_set_t frm = index_set_t());
00176     matrix_multi(const vector_t& vec,
00177     const index_set_t frm, const bool prechecked = false);
00178     matrix_multi(const std::string& str);
00179     matrix_multi(const std::string& str,
00180     const index_set_t frm, const bool prechecked = false);
00181     matrix_multi(const char* str)
00182     { *this = matrix_multi(std::string(str)); };
00183     matrix_multi(const char* str,
00184     const index_set_t frm, const bool prechecked = false)
00185     { *this = matrix_multi(std::string(str), frm, prechecked); };
00186     template< typename Other_Scalar_T >
00187     matrix_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val);
00188     template< typename Other_Scalar_T >
00189     matrix_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val,
00190     const index_set_t frm, const bool prechecked = false);
00191     auto fast_matrix_multi(const index_set_t frm) const -> const matrix_multi_t;
00192     template< typename Other_Scalar_T >

```



```

00213     auto fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00214
00215 private:
00216     template< typename Matrix_T >
00217     matrix_multi(const Matrix_T& mtx, const index_set_t frm);
00220     matrix_multi(const matrix_t& mtx, const index_set_t frm);
00222     auto basis_element(const index_set<LO,HI>& ist) const -> const basis_matrix_t;
00223
00224 public:
00225     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00226
00228     auto operator= (const multivector_t& rhs) -> multivector_t&;
00229
00231     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t;
00232
00233     // Friend declarations
00234
00235     friend auto
00236     operator* <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00237     friend auto
00238     operator^ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00239     friend auto
00240     operator& <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00241     friend auto
00242     operator% <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00243     friend auto
00244     star <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> Scalar_T;
00245     friend auto
00246     operator/ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00247     friend auto
00248     operator| <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00249
00250     friend auto
00251     operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00252     friend auto
00253     operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00254     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00255     friend auto
00256     reframe (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs, const
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs,
00257     matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs_reframed,
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs_reframed) -> const
index_set<Other_LO,Other_HI>;
00258     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00259     friend auto
00260     matrix_sqrt (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00262     const index_t level)
-> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00264     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00265     friend auto
00266     matrix_log(const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00268     const index_t level)
-> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00270
00272     auto operator+= (const term_t& rhs) -> multivector_t&;
00273
00274 private:
00275     // Data members
00276
00278     index_set_t      m_frame;
00280     matrix_t         m_matrix;
00281 };
00282
00283 // Non-members
00284
00286 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00287 auto
00288 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00289
00290 }
00291
00292 namespace std
00293 {
00295     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00296     struct numeric_limits< glucat::matrix_multi<Scalar_T,LO,HI,Tune_P> > :
00297     public numeric_limits<Scalar_T>
00298     { };
00299 }
00300 #endif // _GLUCAT_MATRIX_MULTI_H

```

## 7.37 glucat/matrix\_multi\_imp.h File Reference

```
#include "glucat/matrix_multi.h"
#include "glucat/scalar.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/ublas/triangular.hpp>
#include <boost/numeric/ublas/lu.hpp>
#include <boost/numeric/ublas/io.hpp>
#include <fstream>
#include <iomanip>
#include <array>
#include <iostream>
```

Include dependency graph for matrix\_multi\_imp.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::basis\\_table< Scalar\\_T, LO, HI, Matrix\\_T >](#)  
*Table of basis elements used as a cache by basis\_element()*
- struct [pade::pade\\_sqrt\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_numer< float >](#)
- struct [pade::pade\\_sqrt\\_denom< float >](#)
- struct [pade::pade\\_sqrt\\_numer< long double >](#)
- struct [pade::pade\\_sqrt\\_denom< long double >](#)
- struct [pade::pade\\_sqrt\\_numer< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_numer< qd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< qd\\_real >](#)
- struct [pade::pade\\_log\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade::pade\\_log\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade::pade\\_log\\_numer< float >](#)
- struct [pade::pade\\_log\\_denom< float >](#)
- struct [pade::pade\\_log\\_numer< long double >](#)
- struct [pade::pade\\_log\\_denom< long double >](#)
- struct [pade::pade\\_log\\_numer< dd\\_real >](#)
- struct [pade::pade\\_log\\_denom< dd\\_real >](#)
- struct [pade::pade\\_log\\_numer< qd\\_real >](#)
- struct [pade::pade\\_log\\_denom< qd\\_real >](#)

## Namespaces

- namespace [glucat](#)
- namespace [pade](#)

## Functions

- auto [glucat::offset\\_level](#) (const index\_t p, const index\_t q) -> index\_t  
*Determine the log2 dim corresponding to signature p, q.*
- template<typename Matrix\_Index\_T, const index\_t LO, const index\_t HI>  
static auto [glucat::folded\\_dim](#) (const index\_set< LO, HI > &sub) -> Matrix\_Index\_T  
*Determine the matrix dimension of the fold of a subalgebra.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::reframe](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs, matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs\_reframed, matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs\_reframed) -> const index\_set< LO, HI >  
*Find a common frame for operands of a binary operator.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator\\*](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator^](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator&](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator%](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::star](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator/](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator|](#) (const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &lhs, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const matrix\_multi< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator<<](#) (std::ostream &os, const matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T, const index\_t LO, const index\_t HI, typename Tune\_P >  
auto [glucat::operator>>](#) (std::istream &s, matrix\_multi< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*

- `template<typename Multivector_T , typename Matrix_T , typename Basis_Matrix_T >`  
`static auto glucat::fast (const Matrix_T &X, index_t level) -> Multivector_T`  
*Inverse generalized Fast Fourier Transform.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P , const size_t Size>`  
`static auto glucat::pade\_approx (const std::array< Scalar_T, Size > &numer, const std::array< Scalar_T, Size > &denom, const matrix_multi< Scalar_T, LO, HI, Tune_P > &X) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Pade' approximation.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static void glucat::db\_step (matrix_multi< Scalar_T, LO, HI, Tune_P > &M, matrix_multi< Scalar_T, LO, HI, Tune_P > &Y)`  
*Single step of product form of Denman-Beavers square root iteration.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static auto glucat::db\_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 4)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Product form of Denman-Beavers square root iteration.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static auto glucat::cr\_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_Y_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 1)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Cyclic reduction square root iteration.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix\_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static auto glucat::pade\_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Pade' approximation of log.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`static auto glucat::cascade\_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Incomplete square root cascade and Pade' approximation of log.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix\_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*

## 7.38 `matrix_multi_imp.h`

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_MULTI_IMP_H
00002 #define _GLUCAT_MATRIX_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi_imp.h : Implement the matrix representation of a multivector
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/matrix_multi.h"
00035
00036 #include "glucat/scalar.h"
00037 #include "glucat/generation.h"
00038 #include "glucat/matrix.h"
00039
00040 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00041 # pragma GCC diagnostic push
00042 # pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00043 # endif
00044 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00045 # include <boost/serialization/array_wrapper.hpp>
00046 # endif
00047 #include <boost/numeric/ublas/matrix.hpp>
00048 #include <boost/numeric/ublas/matrix_expression.hpp>
00049 #include <boost/numeric/ublas/matrix_proxy.hpp>
00050 #include <boost/numeric/ublas/matrix_sparse.hpp>
00051 #include <boost/numeric/ublas/operation.hpp>
00052 #include <boost/numeric/ublas/operation_sparse.hpp>
00053 #include <boost/numeric/ublas/triangular.hpp>
00054 #include <boost/numeric/ublas/lu.hpp>
00055 #include <boost/numeric/ublas/io.hpp>
00056 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00057 # pragma GCC diagnostic pop
00058 # endif
00059
00060 #include <fstream>
00061 #include <iomanip>
00062 #include <array>
00063 #include <iostream>
00064
00065 namespace glucat
00066 {
00067     // References for algorithms:
00068     // [CHKL]:
00069     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00070     // [MB]: Beatrice Meini, "The Matrix Square Root From a New Functional Perspective:
00071     // Theoretical Results and Computational Issues", SIAM Journal on
00072     // Matrix Analysis and Applications 26(2):362-376, 2004.
00073     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00074
00075     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076     auto
00077     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00078     classname() -> const std::string
00079     { return "matrix_multi"; }
00080
00081
00082     // Reference: [P] Table 15.27, p 133
00083     inline
00084     auto
00085     offset_level(const index_t p, const index_t q) -> index_t
00086     {
00087         // Offsets between the log2 of the matrix dimension for the current signature
00088         // and that of the real superalgebra

```

```

00090     static const std::array<int, 8> offset_log2_dim = {0, 1, 0, 1, 1, 2, 1, 1};
00091     const index_t bott = pos_mod(p-q, 8);
00092     return (p+q)/2 + offset_log2_dim[bott];
00093 }
00094
00096 // Reference: [P] Table 15.27, p 133
00097 template< typename Matrix_Index_T, const index_t LO, const index_t HI >
00098 inline
00099 static
00100 auto
00101 folded_dim( const index_set<LO,HI>& sub ) -> Matrix_Index_T
00102 { return 1 « offset_level(sub.count_pos(), sub.count_neg()); }
00103
00105 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00106 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00107 matrix_multi()
00108 : m_frame( index_set_t() ),
00109   m_matrix( matrix_t( 1, 1 ) )
00110 { this->m_matrix.clear(); }
00111
00113 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00114 template< typename Other_Scalar_T >
00115 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00116 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00117 : m_frame( val.m_frame ), m_matrix( val.m_matrix.size1(), val.m_matrix.size2() )
00118 {
00119     this->m_matrix.clear();
00120     for (auto
00121         val_it1 = val.m_matrix.begin1();
00122         val_it1 != val.m_matrix.end1();
00123         ++val_it1)
00124         for (auto
00125             val_it2 = val_it1.begin();
00126             val_it2 != val_it1.end();
00127             ++val_it2)
00128             this->m_matrix(val_it2.index1(), val_it2.index2()) =
00129             numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00130 }
00132 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00133 template< typename Other_Scalar_T >
00134 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00135 matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val, const index_set_t frm, const bool
00136 prechecked)
00137 : m_frame( frm )
00138 {
00139     if (frm != val.m_frame)
00140         *this = multivector_t(framed_multi_t(val), frm);
00141     else
00142     {
00143         const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00144         this->m_matrix.resize(dim, dim, false);
00145         this->m_matrix.clear();
00146         for (auto
00147             val_it1 = val.m_matrix.begin1();
00148             val_it1 != val.m_matrix.end1();
00149             ++val_it1)
00150             for (auto
00151                 val_it2 = val_it1.begin();
00152                 val_it2 != val_it1.end();
00153                 ++val_it2)
00154                 this->m_matrix(val_it2.index1(), val_it2.index2()) =
00155                 numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00156     }
00157 }
00158
00159 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00160 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00161 matrix_multi(const multivector_t& val, const index_set_t frm, const bool prechecked)
00162 : m_frame( frm )
00163 {
00164     if (frm != val.m_frame)
00165         *this = multivector_t(framed_multi_t(val), frm);
00166     else
00167         this->m_matrix = val.m_matrix;
00168 }
00170 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00171 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00172 matrix_multi(const index_set_t ist, const Scalar_T& crd)
00173 : m_frame( ist )
00174 {
00175     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00176     this->m_matrix.resize(dim, dim, false);
00177     this->m_matrix.clear();
00178     *this += term_t(ist, crd);
00179 }

```

```

00180
00182     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00183     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00184     matrix_multi(const index_set_t ist, const Scalar_T& crd, const index_set_t frm, const bool
prechecked)
00185     : m_frame( frm )
00186     {
00187         if (!prechecked && (ist | frm) != frm)
00188             throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00189         const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00190         this->m_matrix.resize(dim, dim, false);
00191         this->m_matrix.clear();
00192         *this += term_t(ist, crd);
00193     }
00194
00196     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00197     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00198     matrix_multi(const Scalar_T& scr, const index_set_t frm)
00199     : m_frame( frm )
00200     {
00201         const auto dim = folded_dim<matrix_index_t>(frm);
00202         this->m_matrix.resize(dim, dim, false);
00203         this->m_matrix.clear();
00204         *this += term_t(index_set_t(), scr);
00205     }
00206
00208     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00209     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00210     matrix_multi(const int scr, const index_set_t frm)
00211     { *this = multivector_t(Scalar_T(scr), frm); }
00212
00214     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00216     matrix_multi(const vector_t& vec,
00217                 const index_set_t frm, const bool prechecked)
00218     : m_frame( frm )
00219     {
00220         if (!prechecked && index_t(vec.size()) != frm.count())
00221             throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00222         const auto dim = folded_dim<matrix_index_t>(frm);
00223         this->m_matrix.resize(dim, dim, false);
00224         this->m_matrix.clear();
00225         auto idx = frm.min();
00226         const auto frm_end = frm.max()+1;
00227         for (auto& crd : vec)
00228         {
00229             *this += term_t(index_set_t(idx), crd);
00230             for (
00231                 ++idx;
00232                 idx != frm_end && !frm[idx];
00233                 ++idx)
00234             ;
00235         }
00236     }
00237
00239     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00240     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00241     matrix_multi(const std::string& str)
00242     { *this = framed_multi_t(str); }
00243
00245     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00246     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00247     matrix_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00248     { *this = multivector_t(framed_multi_t(str), frm, prechecked); }
00249
00251     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00252     template< typename Other_Scalar_T >
00253     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00254     matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00255     : m_frame( val.frame() )
00256     {
00257         if (val.size() >= Tune_P::fast_size_threshold)
00258             try
00259             {
00260                 *this = val.template fast_matrix_multi<Scalar_T>(this->m_frame);
00261                 return;
00262             }
00263             catch (const glucat_error& e)
00264             { }
00265         const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00266         this->m_matrix.resize(dim, dim, false);
00267         this->m_matrix.clear();
00268
00269         using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00270         for (auto& val_term : val)
00271             *this += val_term;
00272     }

```

```

00273
00275 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00276 template< typename Other_Scalar_T >
00277 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00278 matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& framed_val, const index_set_t frm,
00279 const bool prechecked)
00280 {
00281     const auto val = framed_val.truncated();
00282     const auto our_frame = val.frame() | frm;
00283     if (val.size() >= Tune_P::fast_size_threshold)
00284     {
00285         *this = val.template fast_matrix_multi<Scalar_T>(our_frame);
00286         return;
00287     }
00288     catch (const glucat_error& e)
00289     { }
00290     this->m_frame = our_frame;
00291     const auto dim = folded_dim<matrix_index_t>(our_frame);
00292     this->m_matrix.resize(dim, dim, false);
00293     this->m_matrix.clear();
00294
00295     using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00296     for (auto& val_term : val)
00297         *this += val_term;
00298 }
00299
00301 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00302 template< typename Matrix_T >
00303 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00304 matrix_multi(const Matrix_T& mtx, const index_set_t frm)
00305 : m_frame( frm ), m_matrix( mtx.size1(), mtx.size2() )
00306 {
00307     this->m_matrix.clear();
00308
00309     for (auto
00310         mtx_it1 = mtx.begin1();
00311         mtx_it1 != mtx.end1();
00312         ++mtx_it1)
00313     {
00314         for (auto
00315             mtx_it2 = mtx_it1.begin();
00316             mtx_it2 != mtx_it1.end();
00317             ++mtx_it2)
00318             this->m_matrix(mtx_it2.index1(), mtx_it2.index2()) =
00319             numeric_traits<Scalar_T>::to_scalar_t(*mtx_it2);
00320     }
00321
00322 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00323 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00324 matrix_multi(const matrix_t& mtx, const index_set_t frm)
00325 : m_frame( frm ), m_matrix( mtx )
00326 { }
00327
00328 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00329 auto
00330 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00331 operator= (const multivector_t& rhs) -> multivector_t&
00332 {
00333     // Check for assignment to self
00334     if (this == &rhs)
00335         return *this;
00336     this->m_frame = rhs.m_frame;
00337     this->m_matrix = rhs.m_matrix;
00338     return *this;
00339 }
00340
00342 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00343 inline
00344 auto
00345 reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00346         matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00347         matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
00348         matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>
00349 {
00350     using index_set_t = index_set<LO, HI>;
00351     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00352     using framed_multi_t = typename multivector_t::framed_multi_t;
00353     // Determine the initial common frame
00354     index_set_t our_frame = lhs.m_frame | rhs.m_frame;
00355     framed_multi_t framed_lhs;
00356     framed_multi_t framed_rhs;
00357     if ((lhs.m_frame != our_frame) || (rhs.m_frame != our_frame))
00358     {
00359         // The common frame may expand as a result of the transform to framed_multi_t
00360         framed_lhs = framed_multi_t(lhs);
00361         framed_rhs = framed_multi_t(rhs);
00362         our_frame |= framed_lhs.frame() | framed_rhs.frame();
00363     }

```



```

00361     }
00362     // Do the reframing only where necessary
00363     if (lhs.m_frame != our_frame)
00364         lhs_reframed = multivector_t(framed_lhs, our_frame, true);
00365     if (rhs.m_frame != our_frame)
00366         rhs_reframed = multivector_t(framed_rhs, our_frame, true);
00367     return our_frame;
00368 }
00369
00370 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00371 auto
00372 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00373 operator== (const multivector_t& rhs) const -> bool
00374 {
00375     // Ensure that there is no aliasing
00376     if (this == &rhs)
00377         return true;
00378
00379     // Operate only within a common frame
00380     multivector_t lhs_reframed;
00381     multivector_t rhs_reframed;
00382     const index_set_t our_frame = reframe(*this, rhs, lhs_reframed, rhs_reframed);
00383     const multivector_t& lhs_ref = (this->m_frame == our_frame)
00384         ? *this
00385         : lhs_reframed;
00386     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00387         ? rhs
00388         : rhs_reframed;
00389
00390     return ublas::norm_inf(lhs_ref.m_matrix - rhs_ref.m_matrix) == 0;
00391 }
00392
00393 // Test for equality of multivector and scalar
00394 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00395 inline
00396 auto
00397 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00398 operator== (const Scalar_T& scr) const -> bool
00399 {
00400     if (scr != Scalar_T(0))
00401         return *this == multivector_t(framed_multi_t(scr), this->m_frame, true);
00402     else if (ublas::norm_inf(this->m_matrix) != 0)
00403         return false;
00404     else
00405     {
00406         const matrix_index_t dim = this->m_matrix.size1();
00407         return !(dim == 1 && this->isnan());
00408     }
00409 }
00410
00411 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00412 inline
00413 auto
00414 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00415 operator+= (const Scalar_T& scr) -> multivector_t&
00416 { return *this += term_t(index_set_t(), scr); }
00417
00418 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00419 inline
00420 auto
00421 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00422 operator+= (const multivector_t& rhs) -> multivector_t&
00423 {
00424     // Ensure that there is no aliasing
00425     if (this == &rhs)
00426         return *this *= Scalar_T(2);
00427
00428     // Operate only within a common frame
00429     multivector_t rhs_reframed;
00430     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00431     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00432         ? rhs
00433         : rhs_reframed;
00434
00435     noalias(this->m_matrix) += rhs_ref.m_matrix;
00436     return *this;
00437 }
00438
00439 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00440 inline
00441 auto
00442 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00443 operator-= (const Scalar_T& scr) -> multivector_t&
00444 { return *this += term_t(index_set_t(), -scr); }
00445
00446 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00447 inline

```

```

00453 auto
00454 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00455 operator-= (const multivector_t& rhs) -> multivector_t&
00456 {
00457     // Ensure that there is no aliasing
00458     if (this == &rhs)
00459         return *this = Scalar_T(0);
00460
00461     // Operate only within a common frame
00462     multivector_t rhs_reframed;
00463     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00464     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00465         ? rhs
00466         : rhs_reframed;
00467
00468     noalias(this->m_matrix) -= rhs_ref.m_matrix;
00469     return *this;
00470 }
00471
00472 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00473 inline
00474 auto
00475 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00476 operator- () const -> const multivector_t
00477 { return multivector_t(-(this->m_matrix), this->m_frame); }
00478
00479 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00480 inline
00481 auto
00482 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00483 operator*= (const Scalar_T& scr) -> multivector_t&
00484 { // multiply coordinates of all terms by scalar
00485
00486     using traits_t = numeric_traits<Scalar_T>;
00487     if (traits_t::isNan_or_isInf(scr) || this->isnan())
00488         return *this = traits_t::NaN();
00489     if (scr == Scalar_T(0))
00490         *this = Scalar_T(0);
00491     else
00492         this->m_matrix *= scr;
00493     return *this;
00494 }
00495
00496 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00497 inline
00498 auto
00499 operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00500 matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00501 {
00502     {
00503         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00504         using index_set_t = typename multivector_t::index_set_t;
00505
00506         if (lhs.isnan() || rhs.isnan())
00507             return numeric_traits<Scalar_T>::NaN();
00508
00509         // Operate only within a common frame
00510         multivector_t lhs_reframed;
00511         multivector_t rhs_reframed;
00512         const index_set_t our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00513         const multivector_t& lhs_ref = (lhs.m_frame == our_frame)
00514             ? lhs
00515             : lhs_reframed;
00516         const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00517             ? rhs
00518             : rhs_reframed;
00519
00520         using matrix_t = typename multivector_t::matrix_t;
00521         using matrix_index_t = typename matrix_t::size_type;
00522
00523         const matrix_index_t dim = lhs_ref.m_matrix.size1();
00524         multivector_t result = multivector_t(matrix_t(dim, dim), our_frame);
00525         result.m_matrix.clear();
00526         ublas::axpy_prod(lhs_ref.m_matrix, rhs_ref.m_matrix, result.m_matrix, true);
00527         return result;
00528     }
00529 }
00530
00531 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00532 inline
00533 auto
00534 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00535 operator*= (const multivector_t& rhs) -> multivector_t&
00536 { return *this = *this * rhs; }
00537
00538 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00539 inline
00540 auto
00541 operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const

```

```

matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
00544 {
00545     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
00546     using framed_multi_t = typename multivector_t::framed_multi_t;
00547     return framed_multi_t(lhs) ^ framed_multi_t(rhs);
00548 }
00549
00551 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00552 inline
00553 auto
00554 matrix_multi<Scalar_T, LO, HI, Tune_P>::
00555 operator^= (const multivector_t& rhs) -> multivector_t&
00556 { return *this = *this ^ rhs; }
00557
00559 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00560 inline
00561 auto
00562 operator& (const matrix_multi<Scalar_T, LO, HI, Tune_P>& lhs, const
matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
00563 {
00564     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
00565     using framed_multi_t = typename multivector_t::framed_multi_t;
00566     return framed_multi_t(lhs) & framed_multi_t(rhs);
00567 }
00568
00570 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00571 inline
00572 auto
00573 matrix_multi<Scalar_T, LO, HI, Tune_P>::
00574 operator&= (const multivector_t& rhs) -> multivector_t&
00575 { return *this = *this & rhs; }
00576
00578 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00579 inline
00580 auto
00581 operator% (const matrix_multi<Scalar_T, LO, HI, Tune_P>& lhs, const
matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
00582 {
00583     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
00584     using framed_multi_t = typename multivector_t::framed_multi_t;
00585     return framed_multi_t(lhs) % framed_multi_t(rhs);
00586 }
00587
00589 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00590 inline
00591 auto
00592 matrix_multi<Scalar_T, LO, HI, Tune_P>::
00593 operator%= (const multivector_t& rhs) -> multivector_t&
00594 { return *this = *this % rhs; }
00595
00597 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00598 inline
00599 auto
00600 star(const matrix_multi<Scalar_T, LO, HI, Tune_P>& lhs, const matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs)
-> Scalar_T
00601 { return (lhs * rhs).scalar(); }
00602
00604 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00605 inline
00606 auto
00607 matrix_multi<Scalar_T, LO, HI, Tune_P>::
00608 operator/= (const Scalar_T& scr) -> multivector_t&
00609 { return *this *= Scalar_T(1)/scr; }
00610
00612 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00613 auto
00614 operator/ (const matrix_multi<Scalar_T, LO, HI, Tune_P>& lhs, const
matrix_multi<Scalar_T, LO, HI, Tune_P>& rhs) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
00615 {
00616     using traits_t = numeric_traits<Scalar_T>;
00617
00618     if (lhs.isnan() || rhs.isnan())
00619         return traits_t::NaN();
00620
00621     if (rhs == Scalar_T(0))
00622         return traits_t::NaN();
00623
00624     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
00625     using index_set_t = typename multivector_t::index_set_t;
00626
00627     // Operate only within a common frame
00628     multivector_t lhs_reframed;
00629     multivector_t rhs_reframed;
00630     const auto our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00631     const auto& lhs_ref = (lhs.m_frame == our_frame)
00632         ? lhs
00633         : lhs_reframed;

```

```

00634     const auto& rhs_ref = (rhs.m_frame == our_frame)
00635     ? rhs
00636     : rhs_reframed;
00637
00638     // Solve result == lhs_ref/rhs_ref <=> result*rhs_ref == lhs_ref
00639     // We now solve X == B/A
00640     // (where X == result, B == lhs_ref.m_matrix and A == rhs_ref.m_matrix)
00641     // X == B/A <=> X*A == B <=> AT*XT == BT
00642     // So, we solve AT*XT == BT
00643
00644     using matrix_t = typename multivector_t::matrix_t;
00645     using matrix_index_t = typename matrix_t::size_type;
00646
00647     const auto& AT = matrix_t(ublas::trans(rhs_ref.m_matrix));
00648     auto LU = AT;
00649
00650     using permutation_t = ublas::permutation_matrix<matrix_index_t>;
00651
00652     auto pvector = permutation_t(AT.size());
00653     if (!ublas::lu_factorize(LU, pvector))
00654     {
00655         const auto& BT = matrix_t(ublas::trans(lhs_ref.m_matrix));
00656         auto XT = BT;
00657         ublas::lu_substitute(LU, pvector, XT);
00658         if (matrix::isnan(XT))
00659             return traits_t::NaN();
00660
00661         // Iterative refinement.
00662         // Reference: Nicholas J. Higham, "Accuracy and Stability of Numerical Algorithms",
00663         // SIAM, 1996, ISBN 0-89871-355-2, Chapter 11
00664         if (Tune_P::div_max_steps > 0)
00665         {
00666             // matrix_t R = ublas::prod(AT, XT) - BT;
00667             auto R = matrix_t(-BT);
00668             ublas::axpy_prod(AT, XT, R, false);
00669             if (matrix::isnan(R))
00670                 return traits_t::NaN();
00671
00672             auto nr = Scalar_T(ublas::norm_inf(R));
00673             if (nr != Scalar_T(0) && !traits_t::isNaN_or_isInf(nr))
00674             {
00675                 auto XTnew = XT;
00676                 auto nrold = nr + Scalar_T(1);
00677                 for (auto
00678                     step = 0;
00679                     step != Tune_P::div_max_steps &&
00680                     nr < nrold &&
00681                     nr != Scalar_T(0) &&
00682                     nr == nr;
00683                     ++step)
00684                 {
00685                     nrold = nr;
00686                     if (step != 0)
00687                         XT = XTnew;
00688                     auto& D = R;
00689                     ublas::lu_substitute(LU, pvector, D);
00690                     XTnew -= D;
00691                     // noalias(R) = ublas::prod(AT, XTnew) - BT;
00692                     R = -BT;
00693                     ublas::axpy_prod(AT, XTnew, R, false);
00694                     nr = ublas::norm_inf(R);
00695                 }
00696             }
00697         }
00698         return multivector_t(ublas::trans(XT), our_frame);
00699     }
00700     else
00701         // AT is singular. Return NaN
00702         return traits_t::NaN();
00703 }
00704
00706 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00707 inline
00708 auto
00709 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00710 operator/=(const multivector_t& rhs) -> multivector_t&
00711 { return *this = *this / rhs; }
00712
00714 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00715 inline
00716 auto
00717 operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00718 { return rhs * lhs / rhs.involute(); }
00719
00721 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00722 inline

```

```

00723     auto
00724     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00725     operator|= (const multivector_t& rhs) -> multivector_t&
00726     { return *this = rhs * *this / rhs.involute(); }
00727
00729     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00730     inline
00731     auto
00732     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00733     inv() const -> const multivector_t
00734     { return multivector_t(Scalar_T(1), this->m_frame) / *this; }
00735
00737     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00738     inline
00739     auto
00740     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00741     pow(int m) const -> const multivector_t
00742     { return glucat::pow(*this, m); }
00743
00745     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00746     auto
00747     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00748     outer_pow(int m) const -> const multivector_t
00749     {
00750         if (m < 0)
00751             throw error_t("outer_pow(m): negative exponent");
00752         framed_multi_t a = *this;
00753         return a.outer_pow(m);
00754     }
00755
00757     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00758     inline
00759     auto
00760     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00761     grade() const -> index_t
00762     { return framed_multi_t(*this).grade(); }
00763
00765     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00766     inline
00767     auto
00768     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00769     frame() const -> const index_set_t
00770     { return this->m_frame; }
00771
00773     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00774     inline
00775     auto
00776     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00777     operator[] (const index_set_t ist) const -> Scalar_T
00778     {
00779         // Use matrix inner product only if ist is in frame
00780         if ( (ist | this->m_frame) == this->m_frame)
00781             return matrix::inner<Scalar_T>(this->basis_element(ist), this->m_matrix);
00782         else
00783             return Scalar_T(0);
00784     }
00785
00787     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788     inline
00789     auto
00790     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00791     operator() (index_t grade) const -> const multivector_t
00792     {
00793         if ((grade < 0) || (grade > HI-LO))
00794             return 0;
00795         else
00796             return (framed_multi_t(*this))(grade);
00797     }
00798
00800     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00801     inline
00802     auto
00803     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00804     scalar() const -> Scalar_T
00805     {
00806         const matrix_index_t dim = this->m_matrix.size1();
00807         return matrix::trace(this->m_matrix) / Scalar_T( double(dim) );
00808     }
00809
00811     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00812     inline
00813     auto
00814     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00815     pure() const -> const multivector_t
00816     { return *this - this->scalar(); }
00817
00819     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```

```

00820 inline
00821 auto
00822 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00823 even() const -> const multivector_t
00824 { return framed_multi_t(*this).even(); }
00825
00826 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00827 inline
00828 auto
00829 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00830 odd() const -> const multivector_t
00831 { return framed_multi_t(*this).odd(); }
00832
00833 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00834 auto
00835 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00836 vector_part() const -> const vector_t
00837 { return this->vector_part(this->frame(), true); }
00838
00839 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840 auto
00841 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00842 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00843 {
00844     if (!prechecked && (this->frame() | frm) != frm)
00845         throw error_t("vector_part(frm): value is outside of requested frame");
00846     vector_t result;
00847     // If we need to enlarge the frame we may as well use a framed_multi_t
00848     if (this->frame() != frm)
00849         return framed_multi_t(*this).vector_part(frm, true);
00850
00851     const auto begin_index = frm.min();
00852     const auto end_index = frm.max()+1;
00853     for (auto
00854         idx = begin_index;
00855         idx != end_index;
00856         ++idx)
00857     {
00858         if (frm[idx])
00859             // Frame may contain indices which do not correspond to a grade 1 term but
00860             // frame cannot omit any index corresponding to a grade 1 term
00861             result.push_back(
00862                 matrix::inner<Scalar_T>(this->basis_element(index_set_t(idx)),
00863                     this->m_matrix));
00864     }
00865     return result;
00866 }
00867
00868 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00869 inline
00870 auto
00871 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00872 involute() const -> const multivector_t
00873 { return framed_multi_t(*this).involute(); }
00874
00875 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00876 inline
00877 auto
00878 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00879 reverse() const -> const multivector_t
00880 { return framed_multi_t(*this).reverse(); }
00881
00882 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00883 inline
00884 auto
00885 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00886 conj() const -> const multivector_t
00887 { return framed_multi_t(*this).conj(); }
00888
00889 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00890 inline
00891 auto
00892 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00893 quad() const -> Scalar_T
00894 { // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
00895     // Arvind Raja ref: "old clical: quadfunction(p:pter):pterm in file compmod.pas"
00896     return framed_multi_t(*this).quad();
00897 }
00898
00899 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00900 inline
00901 auto
00902 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00903 norm() const -> Scalar_T
00904 {
00905     const matrix_index_t dim = this->m_matrix.size1();
00906     return matrix::norm_frob2(this->m_matrix) / Scalar_T( double(dim) );
00907 }
00908
00909
00910
00911
00912
00913
00914

```

```

00916     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00917     inline
00918     auto
00919     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00920     max_abs() const -> Scalar_T
00921     { return framed_multi_t(*this).max_abs(); }
00922
00923     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00924     inline
00925     auto
00926     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00927     random(const index_set<LO,HI> frm, Scalar_T fill) -> const multivector_t
00928     {
00929         return framed_multi<Scalar_T,LO,HI,Tune_P>::random(frm, fill);
00930     }
00931
00932     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00933     inline
00934     void
00935     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00936     write(const std::string& msg) const
00937     { framed_multi_t(*this).write(msg); }
00938
00939     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00940     inline
00941     void
00942     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00943     write(std::ofstream& ofile, const std::string& msg) const
00944     {
00945         if (!ofile)
00946             throw error_t("write(ofile,msg): cannot write to output file");
00947         framed_multi_t(*this).write(ofile, msg);
00948     }
00949
00950     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00951     inline
00952     auto
00953     operator<< (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
00954     {
00955         os << typename matrix_multi<Scalar_T,LO,HI,Tune_P>::framed_multi_t(val);
00956         return os;
00957     }
00958
00959     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00960     inline
00961     auto
00962     operator>> (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&
00963     { // Input looks like 1.0-2.0{1,2}+3.2{3,4}
00964         framed_multi<Scalar_T,LO,HI,Tune_P> local;
00965         s >> local;
00966         // If s.bad() then we have a corrupt input
00967         // otherwise we are fine and can copy the resulting matrix_multi
00968         if (!s.bad())
00969             val = local;
00970         return s;
00971     }
00972
00973     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00974     inline
00975     auto
00976     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00977     isinf() const -> bool
00978     {
00979         if (std::numeric_limits<Scalar_T>::has_infinity)
00980             return matrix::isinf(this->m_matrix);
00981         else
00982             return false;
00983     }
00984
00985     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00986     inline
00987     auto
00988     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00989     isnan() const -> bool
00990     {
00991         if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
00992             return matrix::isnan(this->m_matrix);
00993         else
00994             return false;
00995     }
00996
00997     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00998     inline
00999     auto
01000     matrix_multi<Scalar_T,LO,HI,Tune_P>::
01001     truncated(const Scalar_T& limit) const -> const multivector_t
01002     { return framed_multi_t(*this).truncated(limit); }
01003
01004
01005
01006
01007
01008
01009
01010

```

```

01012 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01013 inline
01014 auto
01015 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01016 operator+= (const term_t& term) -> multivector_t&
01017 {
01018     if (term.second != Scalar_T(0))
01019         this->m_matrix.plus_assign(matrix_t(this->basis_element(term.first)) * term.second);
01020     return *this;
01021 }
01022
01023 template< typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T >
01024 static
01025 auto
01026 fast(const Matrix_T& X, index_t level) -> Multivector_T
01027 {
01028     using framed_multi_t = Multivector_T;
01029
01030     using index_set_t = typename framed_multi_t::index_set_t;
01031     using Scalar_T = typename framed_multi_t::scalar_t;
01032     using matrix_t = Matrix_T;
01033     using basis_matrix_t = Basis_Matrix_T;
01034     using basis_scalar_t = typename basis_matrix_t::value_type;
01035     using traits_t = numeric_traits<Scalar_T>;
01036
01037     if (level == 0)
01038         return framed_multi_t(traits_t::to_scalar_t(X(0,0)));
01039
01040     if (ublas::norm_inf(X) == 0)
01041         return Scalar_T(0);
01042
01043     const basis_matrix_t& I = matrix::unit<basis_matrix_t>(2);
01044     basis_matrix_t J(2,2,2);
01045     J.clear();
01046     J(0,1) = basis_scalar_t(-1);
01047     J(1,0) = basis_scalar_t(1);
01048     basis_matrix_t K = J;
01049     K(0,1) = basis_scalar_t(1);
01050     basis_matrix_t JK = I;
01051     JK(0,0) = basis_scalar_t(-1);
01052
01053     using matrix::signed_perm_nork;
01054     const index_set_t ist_mn = index_set_t(-level);
01055     const index_set_t ist_pn = index_set_t(level);
01056     const index_set_t ist_mnpn = ist_mn | ist_pn;
01057     if (level == 1)
01058     {
01059         using term_t = typename framed_multi_t::term_t;
01060         const Scalar_T i_x = traits_t::to_scalar_t(signed_perm_nork(I, X)(0, 0));
01061         const Scalar_T j_x = traits_t::to_scalar_t(signed_perm_nork(J, X)(0, 0));
01062         const Scalar_T k_x = traits_t::to_scalar_t(signed_perm_nork(K, X)(0, 0));
01063         const Scalar_T jk_x = traits_t::to_scalar_t(signed_perm_nork(JK,X)(0, 0));
01064         framed_multi_t
01065             result = i_x;
01066         result += term_t(ist_mn, j_x); // j_x * mn;
01067         result += term_t(ist_pn, k_x); // k_x * pn;
01068         return result += term_t(ist_mnpn, jk_x); // jk_x * mnpn;
01069     }
01070     else
01071     {
01072         const framed_multi_t& mn = framed_multi_t(ist_mn);
01073         const framed_multi_t& pn = framed_multi_t(ist_pn);
01074         const framed_multi_t& mnpn = framed_multi_t(ist_mnpn);
01075         const framed_multi_t& i_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01076             (signed_perm_nork(I, X), level-1);
01077         const framed_multi_t& j_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01078             (signed_perm_nork(J, X), level-1);
01079         const framed_multi_t& k_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01080             (signed_perm_nork(K, X), level-1);
01081         const framed_multi_t& jk_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01082             (signed_perm_nork(JK,X), level-1);
01083         framed_multi_t
01084             result = i_x.even() - jk_x.odd();
01085         result += (j_x.even() - k_x.odd()) * mn;
01086         result += (k_x.even() - j_x.odd()) * pn;
01087         return result += (jk_x.even() - i_x.odd()) * mnpn;
01088     }
01089 }
01090
01091 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01092 inline
01093 auto
01094 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01095 fast_matrix_multi(const index_set_t frm) const -> const multivector_t
01096 {
01097     if (this->m_frame == frm)
01098         return *this;
01099 }
01100

```



```

01101     else
01102         return (this->template fast_framed_multi<Scalar_T>()).template fast_matrix_multi<Scalar_T>(frm);
01103     }
01104
01106     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01107     template<typename Other_Scalar_T>
01108     auto
01109     matrix_multi<Scalar_T,LO,HI,Tune_P>::
01110     fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>
01111     {
01112         // Determine the amount of off-centering needed
01113         index_t p = this->m_frame.count_pos();
01114         index_t q = this->m_frame.count_neg();
01115
01116         const index_t bott = pos_mod(p-q, 8);
01117         p += std::max(gen::offset_to_super[bott],index_t(0));
01118         q -= std::min(gen::offset_to_super[bott],index_t(0));
01119
01120         const index_t orig_p = p;
01121         const index_t orig_q = q;
01122         while (p-q > 4)
01123             { p -= 4; q += 4; }
01124         while (p-q < -3)
01125             { p += 4; q -= 4; }
01126         if (p-q > 1)
01127         {
01128             index_t old_p = p;
01129             p = q+1;
01130             q = old_p-1;
01131         }
01132         const index_t level = (p+q)/2;
01133
01134         // Do the inverse fast transform
01135         using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
01136         framed_multi_t val = fast<framed_multi_t, matrix_t, basis_matrix_t>(this->m_matrix, level);
01137
01138         // Off-centre val
01139         switch (pos_mod(orig_p-orig_q, 8))
01140         {
01141             case 2:
01142             case 3:
01143             case 4:
01144                 val.centre_qpl_pml(p, q);
01145                 break;
01146             default:
01147                 break;
01148         }
01149         if (orig_p-orig_q > 4)
01150             while (p != orig_p)
01151                 val.centre_pp4_qm4(p, q);
01152         if (orig_p-orig_q < -3)
01153             while (p != orig_p)
01154                 val.centre_pm4_qp4(p, q);
01155
01156         // Return unfolded val
01157         return val.unfold(this->m_frame);
01158     }
01159
01161     template< typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T >
01162     class basis_table :
01163     public std::map< std::pair< const index_set<LO,HI>, const index_set<LO,HI> >,
01164                     Matrix_T* >
01165     {
01166     public:
01167         static auto basis() -> basis_table& { static basis_table b; return b;}
01168     private:
01169         friend class friend_for_private_destructor;
01170         // Enforce singleton
01171         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
01172         basis_table() = default;
01173         ~basis_table() = default;
01174     public:
01175         basis_table(const basis_table&) = delete;
01176         auto operator= (const basis_table&) -> basis_table& = delete;
01177     };
01178
01184     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01185     auto
01186     matrix_multi<Scalar_T,LO,HI,Tune_P>::
01187     basis_element(const index_set_t& ist) const -> const basis_matrix_t
01188     {
01189         using index_set_pair_t = std::pair<const index_set_t, const index_set_t>;
01190         const auto& unfolded_pair = index_set_pair_t(ist, this->m_frame);
01191
01192         using basis_table_t = basis_table<Scalar_T, LO, HI, basis_matrix_t>;
01193         auto& basis_cache = basis_table_t::basis();
01194     }

```

```

01195     const auto frame_count = this->m_frame.count();
01196     const auto use_cache = frame_count <= index_t(Tune_P::basis_max_count);
01197
01198     if (use_cache)
01199     {
01200         const auto basis_it = basis_cache.find(unfolded_pair);
01201         if (basis_it != basis_cache.end())
01202             return *(basis_it->second);
01203     }
01204     const auto folded_set = ist.fold(this->m_frame);
01205     const auto folded_frame = this->m_frame.fold();
01206     const auto& folded_pair = index_set_pair_t(folded_set, folded_frame);
01207     using basis_pair_t = std::pair<const index_set_pair_t, basis_matrix_t*>;
01208     if (use_cache)
01209     {
01210         const auto basis_it = basis_cache.find(folded_pair);
01211         if (basis_it != basis_cache.end())
01212         {
01213             auto* result_ptr = basis_it->second;
01214             basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01215             return *result_ptr;
01216         }
01217     }
01218     const auto folded_max = folded_frame.max();
01219     const auto folded_min = folded_frame.min();
01220     const auto p = std::max(folded_max, index_t(0));
01221     const auto q = std::max(index_t(-folded_min), index_t(0));
01222     const auto* e = (gen::generator_table<basis_matrix_t>::generator())(p, q);
01223     const auto dim = matrix_index_t(1) << offset_level(p, q);
01224     auto result = matrix::unit<basis_matrix_t>(dim);
01225     for (auto
01226         k = folded_min;
01227         k <= folded_max;
01228         ++k)
01229         if (folded_set[k])
01230             result = matrix::mono_prod(result, e[k]);
01231     if (use_cache)
01232     {
01233         auto* result_ptr = new basis_matrix_t(result);
01234         basis_cache.insert(basis_pair_t(folded_pair, result_ptr));
01235         basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01236     }
01237     return result;
01238 }
01239
01241 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size
01242 >
01243 static
01244 auto
01245 pade_approx(
01246     const std::array<Scalar_T, Size>& numer,
01247     const std::array<Scalar_T, Size>& denom,
01248     const matrix_multi<Scalar_T, LO, HI, Tune_P>& X) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
01249 {
01250     // Pade' approximation
01251     // Reference: [GW], Section 4.3, pp318-322
01252     // Reference: [GL], Section 11.3, p572-576.
01253
01254     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
01255     using traits_t = numeric_traits<Scalar_T>;
01256
01257     if (X.isnan())
01258         return traits_t::NaN();
01259
01260     // Array size is assumed to be even
01261     const auto nbr_even_powers = Size/2 - 1;
01262
01263     // Create an array of even powers
01264     auto XX = std::vector<multivector_t>(nbr_even_powers);
01265     XX[0] = X * X;
01266     XX[1] = XX[0] * XX[0];
01267     for (auto
01268         k = size_t(2);
01269         k != nbr_even_powers;
01270         ++k)
01271         XX[k] = XX[k-2] * XX[1];
01272
01273     // Calculate numerator N and denominator D
01274     auto N = multivector_t(numer[1]);
01275     for (auto
01276         k = size_t(0);
01277         k != nbr_even_powers;
01278         ++k)
01279         N += XX[k] * numer[2*k + 3];
01280     N *= X;
01281     N += numer[0];

```

```

01282     for (auto
01283         k = size_t(0);
01284         k != nbr_even_powers;
01285         ++k)
01286         N += XX[k] * numer[2*k + 2];
01287     auto D = multivector_t(denom[1]);
01288     for (auto
01289         k = size_t(0);
01290         k != nbr_even_powers;
01291         ++k)
01292         D += XX[k] * denom[2*k + 3];
01293     D *= X;
01294     D += denom[0];
01295     for (auto
01296         k = size_t(0);
01297         k != nbr_even_powers;
01298         ++k)
01299         D += XX[k] * denom[2*k + 2];
01300     return N / D;
01301 }
01302
01303 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01304 inline
01305 static
01306 void
01307 db_step(matrix_multi<Scalar_T,LO,HI,Tune_P>& M, matrix_multi<Scalar_T,LO,HI,Tune_P>& Y)
01308 {
01309     // Reference: [CHKL]
01310     const auto& invM = inv(M);
01311     M = ((M + invM)/Scalar_T(2) + Scalar_T(1)) / Scalar_T(2);
01312     Y *= (invM + Scalar_T(1)) / Scalar_T(2);
01313 }
01314
01315 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01316 static
01317 auto
01318 db_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01319         Scalar_T norm_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) -> const
01320 matrix_multi<Scalar_T,LO,HI,Tune_P>
01321 {
01322     // Reference: [CHKL]
01323     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01324
01325     if (val == Scalar_T(0))
01326         return val;
01327
01328     static const auto sqrt_max_steps = Tune_P::db_sqrt_max_steps;
01329     auto M = val;
01330     auto Y = val;
01331
01332     for (auto
01333         step = 0;
01334         step != sqrt_max_steps && norm(M - Scalar_T(1)) > norm_tol;
01335         ++step)
01336     {
01337         if (Y.isnan())
01338             return numeric_traits<Scalar_T>::NaN();
01339         db_step(M, Y);
01340     }
01341     return Y;
01342 }
01343
01344 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01345 static
01346 auto
01347 cr_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01348         Scalar_T norm_Y_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) -> const
01349 matrix_multi<Scalar_T,LO,HI,Tune_P>
01350 {
01351     // Reference: [MB]
01352     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01353
01354     if (val == Scalar_T(0))
01355         return val;
01356
01357     static const auto sqrt_max_steps = Tune_P::cr_sqrt_max_steps;
01358     auto Z = Scalar_T(2) * (Scalar_T(1) + val);
01359     auto Y = Scalar_T(1) - val;
01360     using traits_t = numeric_traits<Scalar_T>;
01361     auto norm_Y = norm(Y);
01362     for (auto
01363         step = 0;
01364         step != sqrt_max_steps && norm_Y > norm_Y_tol;
01365         ++step)
01366     {
01367         const auto old_norm_Y = norm_Y;
01368         Y = (-Y / Z) * Y;
01369     }

```

```

01370     norm_Y = norm(Y);
01371     if (Y.isnan() || (norm_Y > old_norm_Y * Scalar_T(2)))
01372         return numeric_traits<Scalar_T>::NaN();
01373
01374     Z += Y * Scalar_T(2);
01375 }
01376 return Z / Scalar_T(4);
01377 }
01378 }
01379
01380 namespace pade {
01381     // Reference: [Z], Padel
01382     template< typename Scalar_T >
01383     struct pade_sqrt_numer
01384     {
01385         using array = std::array<Scalar_T, 14>;
01386         static const array numer;
01387     };
01388     template< typename Scalar_T >
01389     const typename pade_sqrt_numer<Scalar_T>::array pade_sqrt_numer<Scalar_T>::numer =
01390     {
01391         {
01392             1.0,          27.0/4.0,          81.0/4.0,          2277.0/64.0,
01393             10395.0/256.0,  32319.0/1024.0,   8721.0/512.0,   26163.0/4096.0,
01394             53703.0/32768.0, 36465.0/131072.0, 3861.0/131072.0, 7371.0/4194304.0,
01395             819.0/16777216.0, 27.0/67108864.0
01396         };
01397     };
01398     // Reference: [Z], Padel
01399     template< typename Scalar_T >
01400     struct pade_sqrt_denom
01401     {
01402         using array = std::array<Scalar_T, 14>;
01403         static const array denom;
01404     };
01405     template< typename Scalar_T >
01406     const typename pade_sqrt_denom<Scalar_T>::array pade_sqrt_denom<Scalar_T>::denom =
01407     {
01408         {
01409             1.0,          25.0/4.0,          69.0/4.0,          1771.0/64.0,
01410             7315.0/256.0,  20349.0/1024.0,   4845.0/512.0,   12597.0/4096.0,
01411             21879.0/32768.0, 12155.0/131072.0, 1001.0/131072.0, 1365.0/4194304.0,
01412             91.0/16777216.0, 1.0/67108864.0
01413         };
01414     };
01415     template< >
01416     struct pade_sqrt_numer<float>
01417     {
01418         using array = std::array<float, 10>;
01419         static const array numer;
01420     };
01421     const typename pade_sqrt_numer<float>::array pade_sqrt_numer<float>::numer =
01422     {
01423         {
01424             1.0,          19.0/4.0,          19.0/2.0,          665.0/64.0,
01425             1729.0/256.0,  2717.0/1024.0,   627.0/1024.0,   627.0/8192.0,
01426             285.0/65536.0, 19.0/262144.0
01427         };
01428     };
01429     template< >
01430     struct pade_sqrt_denom<float>
01431     {
01432         using array = std::array<float, 10>;
01433         static const array denom;
01434     };
01435     const typename pade_sqrt_denom<float>::array pade_sqrt_denom<float>::denom =
01436     {
01437         {
01438             1.0,          17.0/4.0,          15.0/2.0,          455.0/64.0,
01439             1001.0/256.0,  1287.0/1024.0,   231.0/1024.0,   165.0/8192.0,
01440             45.0/65536.0, 1.0/262144.0
01441         };
01442     };
01443     template< >
01444     struct pade_sqrt_numer<long double>
01445     {
01446         using array = std::array<long double, 18>;
01447         static const array numer;
01448     };
01449     const typename pade_sqrt_numer<long double>::array pade_sqrt_numer<long double>::numer =
01450     {
01451         {
01452             1.0L,          35.0L/4.0L,          35.0L,          5425.0L/64.0L,
01453             35525.0L/256.0L, 166257.0L/1024.0L, 143325.0L/1024.0L, 740025.0L/8192.0L,
01454             2877875.0L/65536.0L, 4206125.0L/262144.0L, 572033.0L/131072.0L, 1820105.0L/2097152.0L,
01455             1028755.0L/8388608.0L, 395675.0L/33554432.0L, 24225.0L/33554432.0L, 6783.0L/268435456.0L,
01456             1785.0L/4294967296.0L, 35.0L/17179869184.0L
01457         };
01458     };
01459     template< >
01460     struct pade_sqrt_denom<long double>
01461     {
01462         using array = std::array<long double, 18>;
01463         static const array denom;
01464     };

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```

01459     };
01460     const typename pade_sqrt_denom<long double>::array pade_sqrt_denom<long double>::denom =
01461     {
01462         1.0L,                33.0L/4.0L,                31.0L,                4495.0L/64.0L,
01463         27405.0L/256.0L,      118755.0L/1024.0L,      94185.0L/1024.0L,      444015.0L/8192.0L,
01464         1562275.0L/65536.0L,  2042975.0L/262144.0L,  245157.0L/131072.0L,  676039.0L/2097152.0L,
01465         323323.0L/8388608.0L, 101745.0L/33554432.0L,  4845.0L/33554432.0L,  969.0L/268435456.0L,
01466         153.0L/4294967296.0L, 1.0L/17179869184.0L
01467     };
01468
01469 #if defined(_GLUCAT_USE_QD)
01470     template< >
01471     struct pade_sqrt_numer<dd_real>
01472     {
01473         using array = std::array<dd_real, 22>;
01474         static const array number;
01475     };
01476     const typename pade_sqrt_numer<dd_real>::array pade_sqrt_numer<dd_real>::number =
01477     {
01478         dd_real("1"),                dd_real("43")/dd_real("4"),
01479         dd_real("215")/dd_real("4"),  dd_real("10621")/dd_real("64"),
01480         dd_real("90687")/dd_real("256"), dd_real("567987")/dd_real("1024"),
01481         dd_real("168861")/dd_real("256"), dd_real("1246355")/dd_real("2048"),
01482         dd_real("7228859")/dd_real("16384"), dd_real("16583853")/dd_real("65536"),
01483         dd_real("7538115")/dd_real("65536"), dd_real("173376645")/dd_real("4194304"),
01484         dd_real("195747825")/dd_real("16777216"), dd_real("171655785")/dd_real("67108864"),
01485         dd_real("14375115")/dd_real("33554432"), dd_real("14375115")/dd_real("268435456"),
01486         dd_real("20764055")/dd_real("4294967296"), dd_real("5167525")/dd_real("17179869184"),
01487         dd_real("206701")/dd_real("17179869184"), dd_real("76153")/dd_real("274877906944"),
01488         dd_real("3311")/dd_real("1099511627776"), dd_real("43")/dd_real("4398046511104")
01489     };
01490     template< >
01491     struct pade_sqrt_denom<dd_real>
01492     {
01493         using array = std::array<dd_real, 22>;
01494         static const array denom;
01495     };
01496     const typename pade_sqrt_denom<dd_real>::array pade_sqrt_denom<dd_real>::denom =
01497     {
01498         dd_real("1"),                dd_real("41")/dd_real("4"),
01499         dd_real("195")/dd_real("4"),  dd_real("9139")/dd_real("64"),
01500         dd_real("73815")/dd_real("256"), dd_real("435897")/dd_real("1024"),
01501         dd_real("121737")/dd_real("256"), dd_real("840565")/dd_real("2048"),
01502         dd_real("4539051")/dd_real("16384"), dd_real("9641775")/dd_real("65536"),
01503         dd_real("4032015")/dd_real("65536"), dd_real("84672315")/dd_real("4194304"),
01504         dd_real("86493225")/dd_real("16777216"), dd_real("67863915")/dd_real("67108864"),
01505         dd_real("5014575")/dd_real("33554432"), dd_real("4345965")/dd_real("268435456"),
01506         dd_real("5311735")/dd_real("4294967296"), dd_real("1081575")/dd_real("17179869184"),
01507         dd_real("33649")/dd_real("17179869184"), dd_real("8855")/dd_real("274877906944"),
01508         dd_real("231")/dd_real("1099511627776"), dd_real("1")/dd_real("4398046511104")
01509     };
01510
01511     template< >
01512     struct pade_sqrt_numer<qd_real>
01513     {
01514         using array = std::array<qd_real, 34>;
01515         static const array number;
01516     };
01517     const typename pade_sqrt_numer<qd_real>::array pade_sqrt_numer<qd_real>::number =
01518     {
01519         qd_real("1"),                qd_real("67")/qd_real("4"),
01520         qd_real("134"),                qd_real("43617")/qd_real("64"),
01521         qd_real("633485")/qd_real("256"), qd_real("6992857")/qd_real("1024"),
01522         qd_real("15246721")/qd_real("1024"), qd_real("215632197")/qd_real("8192"),
01523         qd_real("2518145487")/qd_real("65536"),
01524         qd_real("12301285425")/qd_real("262144"),
01525         qd_real("6344873535")/qd_real("131072"),
01526         qd_real("89075432355")/qd_real("2097152"),
01527         qd_real("267226297065")/qd_real("8388608"),
01528         qd_real("687479618945")/qd_real("33554432"),
01529         qd_real("379874182975")/qd_real("33554432"),
01530         qd_real("1443521895305")/qd_real("268435456"),
01531         qd_real("9425348845815")/qd_real("4294967296"),
01532         qd_real("13195488384141")/qd_real("17179869184"),
01533         qd_real("987417498133")/qd_real("4294967296"),
01534         qd_real("8055248011085")/qd_real("137438953472"),
01535         qd_real("6958363175533")/qd_real("549755813888"),
01536         qd_real("5056698705201")/qd_real("219902325552"),
01537         qd_real("766166470485")/qd_real("219902325552"),
01538         qd_real("766166470485")/qd_real("17592186044416"),
01539         qd_real("623623871325")/qd_real("140737488355328"),
01540         qd_real("203123203803")/qd_real("562949953421312"),
01541         qd_real("6478601247")/qd_real("281474976710656"),
01542         qd_real("5038912081")/qd_real("4503599627370496"),
01543         qd_real("719844583")/qd_real("18014398509481984"),
01544         qd_real("71853815")/qd_real("72057594037927936"),
01545         qd_real("1165197")/qd_real("72057594037927936"),

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    qd_real("87703")/qd_real("576460752303423488"),
01535     qd_real("12529")/qd_real("18446744073709551616"),
    qd_real("67")/qd_real("73786976294838206464")
01536 };
01537 template< >
01538 struct pade_sqrt_denom<qd_real>
01539 {
01540     using array = std::array<qd_real, 34>;
01541     static const array denom;
01542 };
01543 const typename pade_sqrt_denom<qd_real>::array pade_sqrt_denom<qd_real>::denom =
01544 {
01545     qd_real("1"),
01546     qd_real("126"),
01547     qd_real("557845")/qd_real("256"),
01548     qd_real("12515965")/qd_real("1024"),
01549     qd_real("1916797311")/qd_real("65536"),
    qd_real("8996462475")/qd_real("262144"),
01550     qd_real("4450881435")/qd_real("131072"),
    qd_real("59826782925")/qd_real("2097152"),
01551     qd_real("171503444385")/qd_real("8388608"),
    qd_real("420696483235")/qd_real("33554432"),
01552     qd_real("221120793075")/qd_real("33554432"),
    qd_real("797168807855")/qd_real("268435456"),
01553     qd_real("4923689695575")/qd_real("4294967296"),
    qd_real("6499270398159")/qd_real("17179869184"),
01554     qd_real("456864812569")/qd_real("4294967296"),
    qd_real("3486599885395")/qd_real("137438953472"),
01555     qd_real("2804116503573")/qd_real("549755813888"),
    qd_real("1886827875075")/qd_real("2199023255552"),
01556     qd_real("263012370465")/qd_real("2199023255552"),
    qd_real("240141729555")/qd_real("17592186044416"),
01557     qd_real("176848560525")/qd_real("140737488355328"),
    qd_real("51538723353")/qd_real("562949953421312"),
01558     qd_real("1450433115")/qd_real("281474976710656"),
    qd_real("977699359")/qd_real("4503599627370496"),
01559     qd_real("118183439")/qd_real("18014398509481984"),
    qd_real("9652005")/qd_real("72057594037927936"),
01560     qd_real("121737")/qd_real("72057594037927936"),
    qd_real("6545")/qd_real("576460752303423488"),
01561     qd_real("561")/qd_real("18446744073709551616"),
    qd_real("1")/qd_real("73786976294838206464")
01562 };
01563 #endif
01564 }
01565
01566 namespace glucat
01567 {
01568     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01569     auto
01570     matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01571     const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01572     const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01573     {
01574         // Reference: [GW], Section 4.3, pp318-322
01575         // Reference: [GL], Section 11.3, p572-576
01576         // Reference: [Z], Padel
01577
01578         using traits_t = numeric_traits<Scalar_T>;
01579
01580         if (val.isnan())
01581             return traits_t::NaN();
01582
01583         const auto scr_val = val.scalar();
01584         if (val == scr_val)
01585         {
01586             if (scr_val < Scalar_T(0))
01587                 return i * traits_t::sqrt(-scr_val);
01588             else
01589                 return traits_t::sqrt(scr_val);
01590         }
01591
01592         // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01593         const auto scale =
01594             (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < Scalar_T(1))
01595             ? scr_val
01596             : (scr_val < Scalar_T(0))
01597             ? -abs(val)
01598             : abs(val);
01599         const auto sqrt_scale = traits_t::sqrt(traits_t::abs(scale));
01600         if (traits_t::isNaN_or_isInf(sqrt_scale))
01601             return traits_t::NaN();
01602
01603         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01604         auto rescale = multivector_t(sqrt_scale);
01605         if (scale < Scalar_T(0))
01606             rescale = i * sqrt_scale;

```

```

01608
01609     const auto& unitval = val / scale;
01610     static const auto max_norm = Scalar_T(1.0/4.0);
01611     auto use_approx_sqrt = true;
01612     auto use_cr_sqrt = false;
01613     auto scaled_result = multivector_t();
01614 #if defined(_GLUCAT_USE_EIGENVALUES)
01615     static const auto sqrt_2 = traits_t::sqrt(Scalar_T(2));
01616     if (level == 0)
01617     {
01618         using matrix_t = typename multivector_t::matrix_t;
01619
01620         // What kind of eigenvalues does the matrix contain?
01621         const auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
01622         const index_t next_level =
01623             (genus.m_is_singular)
01624             ? level
01625             : level + 1;
01626         switch (genus.m_eig_case)
01627         {
01628             case matrix::neg_real_eigs:
01629                 scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01630                 use_approx_sqrt = false;
01631                 break;
01632             case matrix::both_eigs:
01633             {
01634                 const auto safe_arg = genus.m_safe_arg;
01635                 scaled_result = matrix_sqrt(exp(i*safe_arg) * unitval, i, next_level) * exp(-i*safe_arg /
Scalar_T(2));
01636             }
01637             use_approx_sqrt = false;
01638             break;
01639             default:
01640                 break;
01641         }
01642         use_cr_sqrt = genus.m_is_singular;
01643     }
01644 #endif
01645     if (use_approx_sqrt)
01646     {
01647         scaled_result =
01648             (norm(unitval - Scalar_T(1)) < max_norm)
01649             // Pade' approximation of square root
01650             ? pade_approx(pade::pade_sqrt_numer<Scalar_T>::numer,
01651                 pade::pade_sqrt_denom<Scalar_T>::denom,
01652                 unitval - Scalar_T(1))
01653             // Product form of Denman-Beavers square root iteration
01654             : (use_cr_sqrt)
01655             ? cr_sqrt(unitval)
01656             : db_sqrt(unitval);
01657     }
01658     return (scaled_result.isnan() ||
01659         !approx_equal(pow(scaled_result, 2), unitval))
01660         ? traits_t::NaN()
01661         : scaled_result * rescale;
01662 }
01663
01664 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01665 auto
01666 sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01667 bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01668 {
01669     // Reference: [GW], Section 4.3, pp318-322
01670     // Reference: [GL], Section 11.3, p572-576
01671     // Reference: [Z], Padel
01672
01673     using traits_t = numeric_traits<Scalar_T>;
01674
01675     if (val.isnan())
01676         return traits_t::NaN();
01677
01678     check_complex(val, i, prechecked);
01679
01680     switch (Tune_P::function_precision)
01681     {
01682     case precision_demoted:
01683     {
01684         using demoted_scalar_t = typename traits_t::demoted::type;
01685         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
01686
01687         const auto& demoted_val = demoted_multivector_t(val);
01688         const auto& demoted_i = demoted_multivector_t(i);
01689
01690         return matrix_sqrt(demoted_val, demoted_i, 0);
01691     }
01692     case precision_promoted:

```

```

01694     {
01695         using promoted_scalar_t = typename traits_t::promoted::type;
01696         using promoted_multivector_t = matrix_multi<promoted_scalar_t, LO, HI, Tune_P>;
01697
01698         const auto& promoted_val = promoted_multivector_t(val);
01699         const auto& promoted_i = promoted_multivector_t(i);
01700
01701         return matrix_sqrt(promoted_val, promoted_i, 0);
01702     }
01703     break;
01704     default:
01705         return matrix_sqrt(val, i, 0);
01706     }
01707 }
01708 }
01709
01710 namespace pade {
01711     // Reference: [Z], Padel
01712     template< typename Scalar_T >
01713     struct pade_log_number
01714     {
01715         using array = std::array<Scalar_T, 14>;
01716         static const array number;
01717     };
01718     template< typename Scalar_T >
01719     const typename pade_log_number<Scalar_T>::array pade_log_number<Scalar_T>::number =
01720     {
01721         {
01722             0.0, 1.0, 6.0, 4741.0/300.0,
01723             1441.0/60.0, 107091.0/4600.0, 8638.0/575.0, 263111.0/40250.0,
01724             153081.0/80500.0, 395243.0/1101240.0, 28549.0/688275.0, 605453.0/228813200.0,
01725             785633.0/10296594000.0, 1145993.0/1873980108000.0
01726         };
01727     };
01728     // Reference: [Z], Padel
01729     template< typename Scalar_T >
01730     struct pade_log_denom
01731     {
01732         using array = std::array<Scalar_T, 14>;
01733         static const array denom;
01734     };
01735     template< typename Scalar_T >
01736     const typename pade_log_denom<Scalar_T>::array pade_log_denom<Scalar_T>::denom =
01737     {
01738         {
01739             1.0, 13.0/2.0, 468.0/25.0, 1573.0/50.0,
01740             1573.0/46.0, 11583.0/460.0, 10296.0/805.0, 2574.0/575.0,
01741             11583.0/10925.0, 143.0/874.0, 572.0/37145.0, 117.0/148580.0,
01742             13.0/742900.0, 1.0/10400600.0
01743         };
01744     };
01745     template< >
01746     struct pade_log_number<float>
01747     {
01748         using array = std::array<float, 10>;
01749         static const array number;
01750     };
01751     const typename pade_log_number<float>::array pade_log_number<float>::number =
01752     {
01753         {
01754             0.0, 1.0, 4.0, 1337.0/204.0,
01755             385.0/68.0, 1879.0/680.0, 193.0/255.0, 197.0/1820.0,
01756             419.0/61880.0, 7129.0/61261200.0
01757         };
01758     };
01759     template< >
01760     struct pade_log_denom<float>
01761     {
01762         using array = std::array<float, 10>;
01763         static const array denom;
01764     };
01765     const typename pade_log_denom<float>::array pade_log_denom<float>::denom =
01766     {
01767         {
01768             1.0, 9.0/2.0, 144.0/17.0, 147.0/17.0,
01769             441.0/85.0, 63.0/34.0, 84.0/221.0, 9.0/221.0,
01770             9.0/4862.0, 1.0/48620.0
01771         };
01772     };
01773     template< >
01774     struct pade_log_number<long double>
01775     {
01776         using array = std::array<long double, 18>;
01777         static const array number;
01778     };
01779     const typename pade_log_number<long double>::array pade_log_number<long double>::number =
01780     {
01781         {
01782             0.0L, 1.0L, 8.0L,
01783             3835.0L/132.0L, 11363807.0L/122760.0L, 162981.0L/1705.0L,
01784             8365.0L/132.0L, 9036157.0L/125860.0L, 44211925.0L/2718576.0L, 4149566.0L/849555.0L,
01785             18009875.0L/453096.0L,

```



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16973929.0L/16020180.0L,
01781 172459.0L/1068012.0L, 116317061.0L/7025382936.0L, 19679783.0L/18441630207.0L,
23763863.0L/614721006900.0L,
01782 50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
01783 };
01784 template< >
01785 struct pade_log_denom<long double>
01786 {
01787     using array = std::array<long double, 18>;
01788     static const array denom;
01789 };
01790 const typename pade_log_denom<long double>::array pade_log_denom<long double>::denom =
01791 {
01792     1.0L, 17.0L/2.0L, 1088.0L/33.0L,
01793     850.0L/11.0L, 41650.0L/341.0L, 140777.0L/1023.0L, 1126216.0L/9889.0L,
01794     63206.0L/899.0L, 790075.0L/24273.0L, 60775.0L/5394.0L, 38896.0L/13485.0L,
01795     21658.0L/40455.0L, 21658.0L/310155.0L, 4165.0L/682341.0L, 680.0L/2047023.0L,
01796     34.0L/3411705.0L, 17.0L/129644790.0L, 1.0L/2333606220
01797 };
01798 #if defined(_GLUCAT_USE_QD)
01799 template< >
01800 struct pade_log_number<dd_real>
01801 {
01802     using array = std::array<dd_real, 22>;
01803     static const array number;
01804 };
01805 const typename pade_log_number<dd_real>::array pade_log_number<dd_real>::number =
01806 {
01807     dd_real("0"), dd_real("1"),
01808     dd_real("10"), dd_real("22781")/dd_real("492"),
01809     dd_real("21603")/dd_real("164"), dd_real("5492649")/dd_real("21320"),
01810     dd_real("978724")/dd_real("2665"), dd_real("4191605")/dd_real("10619"),
01811     dd_real("12874933")/dd_real("39442"), dd_real("11473457")/dd_real("54612"),
01812     dd_real("2406734")/dd_real("22755"), dd_real("166770367")/dd_real("4004880"),
01813     dd_real("30653165")/dd_real("2402928"), dd_real("647746389")/dd_real("215195552"),
01814     dd_real("25346331")/dd_real("47074027"), dd_real("278270613")/dd_real("3900419380"),
01815     dd_real("105689791")/dd_real("15601677520"), dd_real("606046475")/dd_real("1379188292768"),
01816     dd_real("969715")/dd_real("53502994116"), dd_real("11098301")/dd_real("26204577562592"),
01817     dd_real("118999")/dd_real("26204577562592"), dd_real("18858053")/dd_real("1392249205900512960")
01818 };
01819 template< >
01820 struct pade_log_denom<dd_real>
01821 {
01822     using array = std::array<dd_real, 22>;
01823     static const array denom;
01824 };
01825 const typename pade_log_denom<dd_real>::array pade_log_denom<dd_real>::denom =
01826 {
01827     dd_real("1"), dd_real("21")/dd_real("2"),
01828     dd_real("2100")/dd_real("41"), dd_real("12635")/dd_real("82"),
01829     dd_real("341145")/dd_real("1066"), dd_real("1037799")/dd_real("2132"),
01830     dd_real("11069856")/dd_real("19721"), dd_real("9883800")/dd_real("19721"),
01831     dd_real("6918660")/dd_real("19721"), dd_real("293930")/dd_real("1517"),
01832     dd_real("1410864")/dd_real("16687"), dd_real("88179")/dd_real("3034"),
01833     dd_real("734825")/dd_real("94054"), dd_real("305235")/dd_real("188108"),
01834     dd_real("348840")/dd_real("1363783"), dd_real("40698")/dd_real("1363783"),
01835     dd_real("6783")/dd_real("2727566"), dd_real("9975")/dd_real("70916716"),
01836     dd_real("266")/dd_real("53187537"), dd_real("7")/dd_real("70916716"),
01837     dd_real("7")/dd_real("8155422340"), dd_real("1")/dd_real("538257874440")
01838 };
01839
01840 template< >
01841 struct pade_log_number<qd_real>
01842 {
01843     using array = std::array<qd_real, 34>;
01844     static const array number;
01845 };
01846 const typename pade_log_number<qd_real>::array pade_log_number<qd_real>::number =
01847 {
01848     qd_real("0"), qd_real("1"),
01849     qd_real("16"), qd_real("95201")/qd_real("780"),
01850     qd_real("30721")/qd_real("52"), qd_real("7416257")/qd_real("3640"),
01851     qd_real("1039099")/qd_real("195"), qd_real("6097772319")/qd_real("555100"),
01852     qd_real("1564058073")/qd_real("85400"), qd_real("30404640205")/qd_real("1209264"),
01853     qd_real("725351278")/qd_real("25193"), qd_real("4092322670789")/qd_real("147429436"),
01854     qd_real("4559713849589")/qd_real("201040140"), qd_real("5049361751189")/qd_real("320023080"),
01855     qd_real("74979677195")/qd_real("8000577"),

```

```

    qd_real("16569850691873")/qd_real("3481514244"),
01856     qd_real("1065906022369")/qd_real("515779888"),
    qd_real("335956770855841")/qd_real("438412904800"),
01857     qd_real("1462444287585964")/qd_real("6041877844275"),
    qd_real("397242326339851")/qd_real("6122436215532"),
01858     qd_real("64211291334131")/qd_real("4373168725380"),
    qd_real("142322343550859")/qd_real("51080680851480"),
01859     qd_real("154355972958659")/qd_real("351179680853925"),
    qd_real("167483568676259")/qd_real("2937139148960100"),
01860     qd_real("4230788929433")/qd_real("704913395750424"),
    qd_real("197968763176019")/qd_real("392923948371995600"),
01861     qd_real("10537522306718")/qd_real("319250708052246425"),
    qd_real("236648286272519")/qd_real("144249197475035425500"),
01862     qd_real("260715545088119")/qd_real("4375558990076074573500"),
    qd_real("289596255666839")/qd_real("192874640282553367199880"),
01863     qd_real("8802625510547")/qd_real("361639950529787563499775"),
    qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
01864     qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
    qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
01865 };
01866 template< >
01867 struct pade_log_denom<qd_real>
01868 {
01869     using array = std::array<qd_real, 34>;
01870     static const array denom;
01871 };
01872 const typename pade_log_denom<qd_real>::array pade_log_denom<qd_real>::denom =
01873 {
01874     qd_real("1"),
    qd_real("33")/qd_real("2"),
01875     qd_real("8448")/qd_real("65"),
    qd_real("42284")/qd_real("65"),
01876     qd_real("211420")/qd_real("91"),
    qd_real("573562")/qd_real("91"),
01877     qd_real("32119472")/qd_real("2379"),
    qd_real("92917044")/qd_real("3965"),
01878     qd_real("603960786")/qd_real("17995"),
    qd_real("144626625")/qd_real("3599"),
01879     qd_real("2776831200")/qd_real("68381"),
    qd_real("16692542100")/qd_real("478667"),
01880     qd_real("12241197540")/qd_real("478667"),
    qd_real("1098569010")/qd_real("68381"),
01881     qd_real("31387686000")/qd_real("3624193"),
    qd_real("9939433900")/qd_real("2479711"),
01882     qd_real("67091178825")/qd_real("42155087"),
    qd_real("2683647153")/qd_real("4959422"),
01883     qd_real("19083713088")/qd_real("121505839"),
    qd_real("4708152900")/qd_real("121505839"),
01884     qd_real("941630580")/qd_real("116546417"),
    qd_real("88704330")/qd_real("62755763"),
01885     qd_real("12902448")/qd_real("62755763"),
    qd_real("1542684")/qd_real("62755763"),
01886     qd_real("6427850")/qd_real("2698497809"),
    qd_real("3471039")/qd_real("18889484663"),
01887     qd_real("8544096")/qd_real("774468871183"),
    qd_real("39556")/qd_real("79027435835"),
01888     qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
01889     qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
01890     qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
01891 };
01892 #endif
01893 }
01894
01895 namespace glucat{
01896     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01897     static
01898     auto
01899     pade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
01900     matrix_multi<Scalar_T,LO,HI,Tune_P>
01901     {
01902         // Reference: [GW], Section 4.3, pp318-322
01903         // Reference: [CHKL]
01904         // Reference: [GL], Section 11.3, p572-576
01905         // Reference: [Z], Padel
01906
01907         using traits_t = numeric_traits<Scalar_T>;
01908         if (val == Scalar_T(0) || val.isnan())
01909             return traits_t::NaN();
01910         else
01911             return pade_approx(pade::pade_log_numer<Scalar_T>::numer,
01912                                pade::pade_log_denom<Scalar_T>::denom,
01913                                val - Scalar_T(1));
01914     }
01915 }

```

```

01917     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01918     static
01919     auto
01920     cascade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01921     {
01922         // Reference: [CHKL]
01923         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01924         using traits_t = numeric_traits<Scalar_T>;
01925         if (val == Scalar_T(0) || val.isnan())
01926             return traits_t::NaN();
01927
01928         using limits_t = std::numeric_limits<Scalar_T>;
01929         static const auto epsilon = limits_t::epsilon();
01930         static const auto max_inner_norm = traits_t::pow(epsilon, 2);
01931         static const auto max_outer_norm = Scalar_T(6.0/limits_t::digits);
01932         auto Y = val;
01933         auto E = multivector_t(Scalar_T(0));
01934         Scalar_T norm_Y_1;
01935         auto pow_2_outer_step = Scalar_T(1);
01936         auto pow_4_outer_step = Scalar_T(1);
01937         int outer_step;
01938         for (outer_step = 0, norm_Y_1 = norm(Y - Scalar_T(1));
01939             outer_step != Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm;
01940             ++outer_step, norm_Y_1 = norm(Y - Scalar_T(1)))
01941         {
01942             if (Y == Scalar_T(0) || Y.isnan())
01943                 return traits_t::NaN();
01944
01945             // Incomplete product form of Denman-Beavers square root iteration
01946             auto M = Y;
01947             for (auto
01948                 inner_step = 0;
01949                 inner_step != Tune_P::log_max_inner_steps &&
01950                 norm(M - Scalar_T(1)) * pow_4_outer_step > max_inner_norm;
01951                 ++inner_step)
01952                 db_step(M, Y);
01953
01954             E += (M - Scalar_T(1)) * pow_2_outer_step;
01955             pow_2_outer_step *= Scalar_T(2);
01956             pow_4_outer_step *= Scalar_T(4);
01957         }
01958         if (outer_step == Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm)
01959             return traits_t::NaN();
01960         else
01961             return pade_log(Y) * pow_2_outer_step - E;
01962     }
01963
01964     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01965     auto
01966     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01967               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01968               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01969     {
01970         // Scaled incomplete square root cascade and scaled Pade' approximation of log
01971         // Reference: [CHKL]
01972
01973         using traits_t = numeric_traits<Scalar_T>;
01974         if (val == Scalar_T(0) || val.isnan())
01975             return traits_t::NaN();
01976
01977         static const auto pi = traits_t::pi();
01978         const auto scr_val = val.scalar();
01979         if (val == scr_val)
01980         {
01981             if (scr_val < Scalar_T(0))
01982                 return i * pi + traits_t::log(-scr_val);
01983             else
01984                 return traits_t::log(scr_val);
01985         }
01986
01987         // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01988         const auto max_norm = Scalar_T(1.0/9.0);
01989         const auto scale =
01990             (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < max_norm)
01991             ? scr_val
01992             : (scr_val < Scalar_T(0))
01993               ? -abs(val)
01994               : abs(val);
01995         if (scale == Scalar_T(0))
01996             return traits_t::NaN();
01997
01998         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01999         const auto log_scale = traits_t::log(traits_t::abs(scale));
02000         auto rescale = multivector_t(log_scale);
02001         if (scale < Scalar_T(0))
02002             rescale = i * pi + log_scale;

```

```

02004     const auto unitval = val/scale;
02005     if (inv(unitval).isnan())
02006         return traits_t::NaN();
02007
02008 #if defined(_GLUCAT_USE_EIGENVALUES)
02009     auto scaled_result = multivector_t();
02010     if (level == 0)
02011     {
02012         using matrix_t = typename multivector_t::matrix_t;
02013
02014         // What kind of eigenvalues does the matrix contain?
02015         auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
02016         switch (genus.m_eig_case)
02017         {
02018             case matrix::neg_real_eigs:
02019                 scaled_result = matrix_log(-i * unitval, i, level + 1) + i * pi/Scalar_T(2);
02020                 break;
02021             case matrix::both_eigs:
02022             {
02023                 const Scalar_T safe_arg = genus.m_safe_arg;
02024                 scaled_result = matrix_log(exp(i*safe_arg) * unitval, i, level + 1) - i * safe_arg;
02025             }
02026             break;
02027             default:
02028                 scaled_result = cascade_log(unitval);
02029                 break;
02030         }
02031     }
02032     else
02033         scaled_result = cascade_log(unitval);
02034 #else
02035     auto scaled_result = cascade_log(unitval);
02036 #endif
02037     return (scaled_result.isnan()
02038         ? traits_t::NaN()
02039         : scaled_result + rescale;
02040     }
02041
02042 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02043 auto
02044 log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
02045 bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02046 {
02047     using traits_t = numeric_traits<Scalar_T>;
02048
02049     if (val == Scalar_T(0) || val.isnan())
02050         return traits_t::NaN();
02051
02052     check_complex(val, i, prechecked);
02053
02054     switch (Tune_P::function_precision)
02055     {
02056     case precision_demoted:
02057     {
02058         using demoted_scalar_t = typename traits_t::demoted::type;
02059         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02060
02061         const auto& demoted_val = demoted_multivector_t(val);
02062         const auto& demoted_i = demoted_multivector_t(i);
02063
02064         return matrix_log(demoted_val, demoted_i, 0);
02065     }
02066     break;
02067     case precision_promoted:
02068     {
02069         using promoted_scalar_t = typename traits_t::promoted::type;
02070         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02071
02072         const auto& promoted_val = promoted_multivector_t(val);
02073         const auto& promoted_i = promoted_multivector_t(i);
02074
02075         return matrix_log(promoted_val, promoted_i, 0);
02076     }
02077     break;
02078     default:
02079         return matrix_log(val, i, 0);
02080     }
02081 }
02082
02083 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02084 auto
02085 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02086 {
02087     using traits_t = numeric_traits<Scalar_T>;
02088     if (val.isnan())
02089         return traits_t::NaN();
02090
02091

```

```

02092     const auto scr_val = val.scalar();
02093     if (val == scr_val)
02094         return traits_t::exp(scr_val);
02095
02096     switch (Tune_P::function_precision)
02097     {
02098     case precision_demoted:
02099     {
02100         using demoted_scalar_t = typename traits_t::demoted::type;
02101         using demoted_multivector_t = matrix_multi<demoted_scalar_t, LO, HI, Tune_P>;
02102
02103         const auto& demoted_val = demoted_multivector_t(val);
02104         return clifford_exp(demoted_val);
02105     }
02106     break;
02107     case precision_promoted:
02108     {
02109         using promoted_scalar_t = typename traits_t::promoted::type;
02110         using promoted_multivector_t = matrix_multi<promoted_scalar_t, LO, HI, Tune_P>;
02111
02112         const auto& promoted_val = promoted_multivector_t(val);
02113         return clifford_exp(promoted_val);
02114     }
02115     break;
02116     default:
02117         return clifford_exp(val);
02118     }
02119 }
02120 }
02121 #endif // _GLUCAT_MATRIX_MULTI_IMP_H

```

## 7.39 glucat/portability.h File Reference

```
#include <boost/version.hpp>
```

```
#include <cmath>
```

Include dependency graph for portability.h: This graph shows which files directly or indirectly include this file:

### Macros

- `#define _GLUCAT_ISNAN(x) (x != x)`
- `#define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))`
- `#define UBLAS_ABS abs`
- `#define UBLAS_SQRT sqrt`

### 7.39.1 Macro Definition Documentation

#### 7.39.1.1 \_GLUCAT\_ISINF

```

#define _GLUCAT_ISINF(
    x ) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))

```

Definition at line 43 of file [portability.h](#).

### 7.39.1.2 \_GLUCAT\_ISNAN

```
#define _GLUCAT_ISNAN(
    x ) (x != x)
```

Definition at line 42 of file [portability.h](#).

### 7.39.1.3 UBLAS\_ABS

```
#define UBLAS_ABS abs
```

Definition at line 51 of file [portability.h](#).

### 7.39.1.4 UBLAS\_SQRT

```
#define UBLAS_SQRT sqrt
```

Definition at line 52 of file [portability.h](#).

## 7.40 portability.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_PORTABILITY_H
00002 #define _GLUCAT_PORTABILITY_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     portability.h : Work around non-standard compilers and libraries
00006     -----
00007     begin                : Sun 2001-08-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include <boost/version.hpp>
00035 #include <cmath>
00036
00037 // Workaround for isnan and isinf
00038 #if __cplusplus > 199711L
00039 # define _GLUCAT_ISNAN(x) (std::isnan(x))
```

```

00040 # define _GLUCAT_ISINF(x) (std::isinf(x))
00041 #else
00042 # define _GLUCAT_ISNAN(x) (x != x)
00043 # define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
00044 #endif
00045
00046 // Workaround for abs and sqrt
00047 #if BOOST_VERSION >= 103400
00048 # define UBLAS_ABS type_abs
00049 # define UBLAS_SQRT type_sqrt
00050 #else
00051 # define UBLAS_ABS abs
00052 # define UBLAS_SQRT sqrt
00053 #endif
00054
00055 // Use with Cygwin gcc to obtain __WORDSIZE
00056 #if defined(HAVE_BITS_WORDSIZE_H)
00057 # include <bits/wordsize.h>
00058 #endif
00059
00060 #endif // _GLUCAT_PORTABILITY_H

```

## 7.41 glucat/promotion.h File Reference

```

#include "glucat/global.h"
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <cfloat>
#include <limits>
#include <qd/qd_real.h>

```

Include dependency graph for promotion.h: This graph shows which files directly or indirectly include this file:

### Classes

- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

### Namespaces

- namespace [glucat](#)

## 7.42 promotion.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_PROMOTION_H
00002 #define _GLUCAT_PROMOTION_H
00003 /*****
00004      GluCat : Generic library of universal Clifford algebra templates
00005      promotion.h : Define promotion and demotion for specific scalar types
00006      -----
00007      begin                : 2021-11-13
00008      copyright             : (C) 2021 by Paul C. Leopardi
00009      *****/
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,

```

```

00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036 #include "glucat/qd.h"
00037
00038 #include <cfloat>
00039 #include <limits>
00040
00041 #if defined(_GLUCAT_USE_QD)
00042 # include <qd/qd_real.h>
00043 #endif
00044
00045 namespace glucat
00046 {
00047     // Reference: [AA], 2.4, p. 30-31
00048
00049     #if !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00050     # if DBL_MANT_DIG < LDBL_MANT_DIG
00051     template<>
00052     struct
00053     numeric_traits<double>::
00054     promoted {using type = long double;};
00055
00056     template<>
00057     struct
00058     numeric_traits<long double>::
00059     demoted {using type = double;};
00060
00061     # else
00062     template<>
00063     struct
00064     numeric_traits<double>::
00065     promoted {using type = double;};
00066
00067     template<>
00068     struct
00069     numeric_traits<long double>::
00070     demoted {using type = float;};
00071
00072     # endif // DBL_MANT_DIG < LDBL_MANT_DIG
00073
00074     template<>
00075     struct
00076     numeric_traits<long double>::
00077     promoted {using type = long double;};
00078
00079     #else
00080     # if (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00081     template<>
00082     struct
00083     numeric_traits<double>::
00084     promoted {using type = long double;};
00085
00086     template<>
00087     struct
00088     numeric_traits<long double>::
00089     demoted {using type = double;};
00090
00091     template<>
00092     struct
00093     numeric_traits<long double>::
00094     promoted {using type = dd_real;};
00095
00096     template<>
00097     struct
00098     numeric_traits<dd_real>::
00099     demoted {using type = double;};
00100
00101     template<>
00102     struct
00103     numeric_traits<dd_real>::
00104     promoted {using type = dd_real;};
00105
00106     template<>
00107     struct
00108     numeric_traits<dd_real>::
00109     demoted {using type = dd_real;};
00110
00111     template<>
00112     struct
00113     numeric_traits<dd_real>::

```



```

00114     demoted {using type = long double;};
00115
00117     template<>
00118     struct
00119     numeric_traits<dd_real>::
00120     promoted {using type = qd_real;};
00121
00123     template<>
00124     struct
00125     numeric_traits<qd_real>::
00126     demoted {using type = dd_real;};
00127
00128 # elif (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00129
00131     template<>
00132     struct
00133     numeric_traits<double>::
00134     promoted {using type = dd_real;};
00135
00137     template<>
00138     struct
00139     numeric_traits<long double>::
00140     demoted {using type = float;};
00141
00143     template<>
00144     struct
00145     numeric_traits<long double>::
00146     promoted {using type = dd_real;};
00147
00149     template<>
00150     struct
00151     numeric_traits<dd_real>::
00152     demoted {using type = double;};
00153
00155     template<>
00156     struct
00157     numeric_traits<dd_real>::
00158     promoted {using type = qd_real;};
00159
00161     template<>
00162     struct
00163     numeric_traits<qd_real>::
00164     demoted {using type = dd_real;};
00165
00166 # else
00167
00169     template<>
00170     struct
00171     numeric_traits<double>::
00172     promoted {using type = dd_real;};
00173
00175     template<>
00176     struct
00177     numeric_traits<dd_real>::
00178     demoted {using type = double;};
00179
00181     template<>
00182     struct
00183     numeric_traits<dd_real>::
00184     promoted {using type = long double;};
00185
00187     template<>
00188     struct
00189     numeric_traits<long double>::
00190     demoted {using type = dd_real;};
00191
00193     template<>
00194     struct
00195     numeric_traits<long double>::
00196     promoted {using type = qd_real;};
00197
00199     template<>
00200     struct
00201     numeric_traits<qd_real>::
00202     demoted {using type = long double;};
00203
00204 # endif // (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00205
00207     template<>
00208     struct
00209     numeric_traits<qd_real>::
00210     promoted {using type = qd_real;};
00211
00212 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00213
00214 } // namespace glucat
00215

```

```
00216 #endif // _GLUCAT_PROMOTION_H
```

## 7.43 glucat/qd.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
#include <qd/qd_real.h>
```

Include dependency graph for qd.h: This graph shows which files directly or indirectly include this file:

## Namespaces

- namespace [glucat](#)

## 7.44 qd.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_QD_H
00002 #define _GLUCAT_QD_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   qd.h : Define functions for dd_real and qd_real as scalar_t
00006   -----
00007   begin           : 2010-03-23
00008   copyright       : (C) 2010-2016 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030   *****/
00031   See also Arvind Raja's original header comments and references in glucat.h
00032   *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 #if defined(_GLUCAT_USE_QD)
00038 # include <qd/qd_real.h>
00039 #endif
00040
00041 namespace glucat
00042 {
00043     // Reference: [AA], 2.4, p. 30-31
00044
00045     #if defined(_GLUCAT_USE_QD) && defined(QD_API)
00046     # define _GLUCAT_QD_F(_T, _F) \
00047         template<> \
00048         inline \
00049         auto \
00050         numeric_traits<_T>:: \
00051         _F(const _T& val) -> _T \
00052         { return ::_F(val); }
00053     #endif
00054 }
```

```

00058     template<>
00059     inline
00060     auto
00061     numeric_traits<dd_real>::
00062     isNaN(const dd_real& val) -> bool
00063     { return val.isnan(); }
00064
00066     template<>
00067     inline
00068     auto
00069     numeric_traits<dd_real>::
00070     isInf(const dd_real& val) -> bool
00071     { return val.isinf(); }
00072
00074     template<>
00075     inline
00076     auto
00077     numeric_traits<dd_real>::
00078     isNaN_or_isInf(const dd_real& val) -> bool
00079     { return val.isnan() || val.isinf(); }
00080
00082     template<>
00083     inline
00084     auto
00085     numeric_traits<dd_real>::
00086     to_int(const dd_real& val) -> int
00087     { return ::to_int(val); }
00088
00090     template<>
00091     inline
00092     auto
00093     numeric_traits<dd_real>::
00094     to_double(const dd_real& val) -> double
00095     { return ::to_double(val); }
00096
00098     template<>
00099     inline
00100     auto
00101     numeric_traits<dd_real>::
00102     fmod(const dd_real& lhs, const dd_real& rhs) -> dd_real
00103     { return ::fmod(lhs, rhs); }
00104
00106     template<>
00107     inline
00108     auto
00109     numeric_traits<dd_real>::
00110     pow(const dd_real& val, int n) -> dd_real
00111     {
00112         if (val == dd_real(0))
00113         {
00114             return
00115                 (n < 0)
00116                 ? NaN()
00117                 : (n == 0)
00118                 ? dd_real(1)
00119                 : dd_real(0);
00120         }
00121         auto result = dd_real(1);
00122         auto power =
00123             (n < 0)
00124             ? dd_real(1)/val
00125             : val;
00126         for (auto
00127             k = std::abs(n);
00128             k != 0;
00129             k /= 2)
00130         {
00131             if (k % 2)
00132                 result *= power;
00133             power *= power;
00134         }
00135         return result;
00136     }
00137
00139     template<>
00140     inline
00141     auto
00142     numeric_traits<dd_real>::
00143     pi() -> dd_real
00144     { return dd_real::_pi; }
00145
00147     template<>
00148     inline
00149     auto
00150     numeric_traits<dd_real>::
00151     ln_2() -> dd_real
00152     { return dd_real::_log2; }

```

```

00153
00155     _GLUCAT_QD_F(dd_real, exp)
00156
00157
00158     _GLUCAT_QD_F(dd_real, log)
00159
00160
00161     _GLUCAT_QD_F(dd_real, cos)
00162
00163
00164     _GLUCAT_QD_F(dd_real, acos)
00165
00166
00167     _GLUCAT_QD_F(dd_real, cosh)
00168
00169
00170     _GLUCAT_QD_F(dd_real, sin)
00171
00172
00173     _GLUCAT_QD_F(dd_real, asin)
00174
00175
00176     _GLUCAT_QD_F(dd_real, sinh)
00177
00178
00179     _GLUCAT_QD_F(dd_real, tan)
00180
00181
00182     _GLUCAT_QD_F(dd_real, atan)
00183
00184
00185     _GLUCAT_QD_F(dd_real, tanh)
00186
00187
00188     template<>
00189     inline
00190     auto
00191     numeric_traits<qd_real>::
00192     isNaN(const qd_real& val) -> bool
00193     { return val.isnan(); }
00194
00196     template<>
00197     inline
00198     auto
00199     numeric_traits<qd_real>::
00200     isInf(const qd_real& val) -> bool
00201     { return val.isinf(); }
00202
00204     template<>
00205     inline
00206     auto
00207     numeric_traits<qd_real>::
00208     isNaN_or_isInf(const qd_real& val) -> bool
00209     { return val.isnan() || val.isinf(); }
00210
00212     template<>
00213     inline
00214     auto
00215     numeric_traits<qd_real>::
00216     to_int(const qd_real& val) -> int
00217     { return ::to_int(val); }
00218
00220     template<>
00221     inline
00222     auto
00223     numeric_traits<qd_real>::
00224     to_double(const qd_real& val) -> double
00225     { return ::to_double(val); }
00226
00228     template<>
00229     inline
00230     auto
00231     numeric_traits<qd_real>::
00232     fmod(const qd_real& lhs, const qd_real& rhs) -> qd_real
00233     { return ::fmod(lhs, rhs); }
00234
00236     template<>
00237     inline
00238     auto
00239     numeric_traits<qd_real>::
00240     pow(const qd_real& val, int n) -> qd_real
00241     {
00242         if (val == qd_real(0))
00243         {
00244             return
00245                 (n < 0)
00246                 ? NaN()

```

```

00247         : (n == 0)
00248         ? qd_real(1)
00249         : qd_real(0);
00250     }
00251     auto result = qd_real(1);
00252     auto power =
00253     (n < 0)
00254     ? qd_real(1)/val
00255     : val;
00256     for (auto
00257         k = std::abs(n);
00258         k != 0;
00259         k /= 2)
00260     {
00261         if (k % 2)
00262             result *= power;
00263         power *= power;
00264     }
00265     return result;
00266 }
00267
00269 template<>
00270 inline
00271 auto
00272 numeric_traits<qd_real>::
00273 pi() -> qd_real
00274 { return qd_real::_pi; }
00275
00277 template<>
00278 inline
00279 auto
00280 numeric_traits<qd_real>::
00281 ln_2() -> qd_real
00282 { return qd_real::_log2; }
00283
00285 _GLUCAT_QD_F(qd_real, exp)
00286
00287 _GLUCAT_QD_F(qd_real, log)
00288
00289 _GLUCAT_QD_F(qd_real, cos)
00290
00291 _GLUCAT_QD_F(qd_real, cos)
00292
00293 _GLUCAT_QD_F(qd_real, acos)
00294
00295 _GLUCAT_QD_F(qd_real, cosh)
00296
00297 _GLUCAT_QD_F(qd_real, cosh)
00298
00299 _GLUCAT_QD_F(qd_real, sin)
00300
00301 _GLUCAT_QD_F(qd_real, sin)
00302
00303 _GLUCAT_QD_F(qd_real, asin)
00304
00305 _GLUCAT_QD_F(qd_real, sinh)
00306
00307 _GLUCAT_QD_F(qd_real, sinh)
00308
00309 _GLUCAT_QD_F(qd_real, tan)
00310
00311 _GLUCAT_QD_F(qd_real, atan)
00312
00313 _GLUCAT_QD_F(qd_real, tanh)
00314
00315 _GLUCAT_QD_F(qd_real, tanh)
00316
00317 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00318
00319 } // namespace glucat
00320
00321 #endif // _GLUCAT_QD_H

```

## 7.45 glucat/random.h File Reference

#include <random>

Include dependency graph for random.h: This graph shows which files directly or indirectly include this file:

## Classes

- class `glucat::random_generator< Scalar_T >`  
Random number generator with single instance per `Scalar_T`.

## Namespaces

- namespace `glucat`

## 7.46 random.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_RANDOM_H
00002 #define _GLUCAT_RANDOM_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     random.h : Random number generator with single instance per Scalar_T
00006                -----
00007     begin                : 2010-03-28
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include <random>
00035
00036 namespace glucat
00037 {
00038     // Enforce singleton
00039     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00040     template< typename Scalar_T >
00041     class random_generator
00042     {
00043     private:
00044         friend class friend_for_private_destructor;
00045     public:
00046         static auto generator() -> random_generator& { static random_generator g; return g; }
00047         random_generator(const random_generator&) = delete;
00048         auto operator= (const random_generator&) -> random_generator& = delete;
00049     private:
00050         static const unsigned long seed = 19590921UL;
00051
00052         std::mt19937 uint_gen;
00053         std::uniform_real_distribution<double> uniform_dist;
00054         std::normal_distribution<double> normal_dist;
00055
00056         random_generator() :
00057             uint_gen(), uniform_dist(0.0, 1.0), normal_dist(0.0, 1.0)
00058         { this->uint_gen.seed(seed); }
00059
00060         ~random_generator() = default;
00061     public:
00062         auto uniform() -> Scalar_T

```

```

00069     { return Scalar_T(this->uniform_dist(this->uint_gen)); }
00070     auto normal() -> Scalar_T
00071     { return Scalar_T(this->normal_dist(this->uint_gen)); }
00072 };
00073 }
00074
00075 #endif // _GLUCAT_RANDOM_H

```

## 7.47 glucat/scalar.h File Reference

```

#include "glucat/portability.h"
#include "glucat/global.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::numeric\\_traits< Scalar\\_T >](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

### Namespaces

- namespace [glucat](#)

### Functions

- template<typename Scalar\_T >  
auto [glucat::log2](#) (const Scalar\_T &x) -> Scalar\_T  
*Log base 2 of scalar.*

## 7.48 scalar.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_H
00002 #define _GLUCAT_SCALAR_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar.h : Define functions for scalar_t
00006     -----
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,

```

```

00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/portability.h"
00035 #include "glucat/global.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045     template< typename Scalar_T >
00046     class numeric_traits
00047     {
00048     private:
00049         inline
00050         static
00051         auto
00052         isInf(const Scalar_T& val, bool_to_type<false>) -> bool
00053         { return false; }
00054
00055         inline
00056         static
00057         auto
00058         isInf(const Scalar_T& val, bool_to_type<true>) -> bool
00059         { return _GLUCAT_ISINF(val); }
00060
00061         inline
00062         static
00063         auto
00064         isNaN(const Scalar_T& val, bool_to_type<false>) -> bool
00065         { return false; }
00066
00067         inline
00068         static
00069         auto
00070         isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00071         { return _GLUCAT_ISNAN(val); }
00072
00073     public:
00074         inline
00075         static
00076         auto
00077         isInf(const Scalar_T& val) -> bool
00078         {
00079             return isInf(val,
00080                 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00081         }
00082
00083         inline
00084         static
00085         auto
00086         isNaN(const Scalar_T& val) -> bool
00087         {
00088             return isNaN(val,
00089                 bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() );
00090         }
00091
00092         inline
00093         static
00094         auto
00095         isNaN_or_isInf(const Scalar_T& val) -> bool
00096         {
00097             return isNaN(val,
00098                 bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() )
00099                 || isInf(val,
00100                     bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00101         }
00102
00103         inline

```



```

00113     static
00114     auto
00115     NaN() -> Scalar_T
00116     {
00117         return std::numeric_limits<Scalar_T>::has_quiet_NaN
00118             ? std::numeric_limits<Scalar_T>::quiet_NaN()
00119             : Scalar_T(std::log(0.0));
00120     }
00121
00122     inline
00123     static
00124     auto
00125     to_int(const Scalar_T& val) -> int
00126     { return static_cast<int>(val); }
00127
00128     inline
00129     static
00130     auto
00131     to_double(const Scalar_T& val) -> double
00132     { return static_cast<double>(val); }
00133
00134     template <typename Other_Scalar_T >
00135     inline
00136     static
00137     auto
00138     to_scalar_t(const Other_Scalar_T& val) -> Scalar_T
00139     { return static_cast<Scalar_T>(val); }
00140
00141     struct promoted {using type = double;};
00142
00143     struct demoted {using type = float;};
00144
00145     inline
00146     static
00147     auto
00148     fmod(const Scalar_T& lhs, const Scalar_T& rhs) -> Scalar_T
00149     { return std::fmod(lhs, rhs); }
00150
00151     inline
00152     static
00153     auto
00154     conj(const Scalar_T& val) -> Scalar_T
00155     { return val; }
00156
00157     inline
00158     static
00159     auto
00160     real(const Scalar_T& val) -> Scalar_T
00161     { return val; }
00162
00163     inline
00164     static
00165     auto
00166     imag(const Scalar_T& val) -> Scalar_T
00167     { return Scalar_T(0); }
00168
00169     inline
00170     static
00171     auto
00172     abs(const Scalar_T& val) -> Scalar_T
00173     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_ABS(val); }
00174
00175     inline
00176     static
00177     auto
00178     pi() -> Scalar_T
00179     { return Scalar_T(3.14159265358979323); }
00180
00181     inline
00182     static
00183     auto
00184     ln_2() -> Scalar_T
00185     { return Scalar_T(0.693147180559945309); }
00186
00187     inline
00188     static
00189     auto
00190     pow(const Scalar_T& val, int n) -> Scalar_T
00191     { return std::pow(val, n); }
00192
00193     inline
00194     static
00195     auto
00196     sqrt(const Scalar_T& val) -> Scalar_T
00197     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_SQRT(val); }
00198
00199     inline

```

```

00215     static
00216     auto
00217     exp(const Scalar_T& val) -> Scalar_T
00218     { return std::exp(val); }
00219
00221     inline
00222     static
00223     auto
00224     log(const Scalar_T& val) -> Scalar_T
00225     { return std::log(val); }
00226
00228     inline
00229     static
00230     auto
00231     log2(const Scalar_T& val) -> Scalar_T
00232     { return log(val)/ln_2(); }
00233
00235     inline
00236     static
00237     auto
00238     cos(const Scalar_T& val) -> Scalar_T
00239     { return std::cos(val); }
00240
00242     inline
00243     static
00244     auto
00245     acos(const Scalar_T& val) -> Scalar_T
00246     { return std::acos(val); }
00247
00249     inline
00250     static
00251     auto
00252     cosh(const Scalar_T& val) -> Scalar_T
00253     { return std::cosh(val); }
00254
00256     inline
00257     static
00258     auto
00259     sin(const Scalar_T& val) -> Scalar_T
00260     { return std::sin(val); }
00261
00263     inline
00264     static
00265     auto
00266     asin(const Scalar_T& val) -> Scalar_T
00267     { return std::asin(val); }
00268
00270     inline
00271     static
00272     auto
00273     sinh(const Scalar_T& val) -> Scalar_T
00274     { return std::sinh(val); }
00275
00277     inline
00278     static
00279     auto
00280     tan(const Scalar_T& val) -> Scalar_T
00281     { return std::tan(val); }
00282
00284     inline
00285     static
00286     auto
00287     atan(const Scalar_T& val) -> Scalar_T
00288     { return std::atan(val); }
00289
00291     inline
00292     static
00293     auto
00294     tanh(const Scalar_T& val) -> Scalar_T
00295     { return std::tanh(val); }
00296
00297 };
00298
00300 template< typename Scalar_T >
00301 inline
00302 auto
00303 log2(const Scalar_T& x) -> Scalar_T
00304 { return numeric_traits<Scalar_T>::log2(x); }
00305 }
00306
00307 #endif // _GLUCAT_SCALAR_H

```

## 7.49 glucat/scalar\_imp.h File Reference

```
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>
```

Include dependency graph for scalar\_imp.h: This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace [glucat](#)

### Functions

- `template<typename Scalar_T>`  
`auto glucat::to\_promote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::promoted::type`  
*Cast to promote.*
- `template<typename Scalar_T>`  
`auto glucat::to\_demote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::demoted::type`  
*Cast to demote.*

## 7.50 scalar\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_SCALAR_IMP_H
00002 #define _GLUCAT_SCALAR_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar_imp.h : Define functions for scalar_t
00006     -----
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2014 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/scalar.h"
00035 #include "glucat/qd.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
```

```

00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045
00046     template< >
00047     template< typename Other_Scalar_T >
00048     inline
00049     auto
00050     numeric_traits<float>::
00051     to_scalar_t(const Other_Scalar_T& val) -> float
00052     { return static_cast<float>(numeric_traits<Other_Scalar_T>::to_double(val)); }
00053
00054     template< >
00055     template< typename Other_Scalar_T >
00056     inline
00057     auto
00058     numeric_traits<double>::
00059     to_scalar_t(const Other_Scalar_T& val) -> double
00060     { return numeric_traits<Other_Scalar_T>::to_double(val); }
00061
00062 #if defined(_GLUCAT_USE_QD)
00063     template< >
00064     template< >
00065     inline
00066     auto
00067     numeric_traits<long double>::
00068     to_scalar_t(const dd_real& val) -> long double
00069     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00070
00071     template< >
00072     template< >
00073     inline
00074     auto
00075     numeric_traits<long double>::
00076     to_scalar_t(const qd_real& val) -> long double
00077     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00078
00079     template< >
00080     template< >
00081     inline
00082     auto
00083     numeric_traits<dd_real>::
00084     to_scalar_t(const long double& val) -> dd_real
00085     { return {double(val), double(val - static_cast<long double>(double(val)))}; }
00086
00087     template< >
00088     template< >
00089     inline
00090     auto
00091     numeric_traits<dd_real>::
00092     to_scalar_t(const qd_real& val) -> dd_real
00093     { return {val.x[0], val.x[1]}; }
00094
00095     template< >
00096     template< >
00097     inline
00098     auto
00099     numeric_traits<qd_real>::
00100     to_scalar_t(const long double& val) -> qd_real
00101     { return {double(val), double(val - static_cast<long double>(double(val))), 0.0, 0.0}; }
00102
00103     template< >
00104     template< >
00105     inline
00106     auto
00107     numeric_traits<qd_real>::
00108     to_scalar_t(const dd_real& val) -> qd_real
00109     { return {val.x[0], val.x[1], 0.0, 0.0}; }
00110 #endif
00111
00112     template< typename Scalar_T >
00113     inline
00114     auto
00115     to_promote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::promoted::type
00116     {
00117         using promoted_scalar_t = typename numeric_traits<Scalar_T>::promoted::type;
00118         return numeric_traits<promoted_scalar_t>::to_scalar_t(val);
00119     }
00120
00121     template< typename Scalar_T >
00122     inline
00123     auto
00124     to_demote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::demoted::type
00125     {
00126         using demoted_scalar_t = typename numeric_traits<Scalar_T>::demoted::type;
00127         return numeric_traits<demoted_scalar_t>::to_scalar_t(val);
00128     }

```

```

00139     }
00140 }
00141
00142 #endif // _GLUCAT_SCALAR_IMP_H

```

## 7.51 glucat/tuning.h File Reference

This graph shows which files directly or indirectly include this file:

### Functions

- [\\_GLUCAT\\_CTAssert](#) (std::numeric\_limits< unsigned int >::radix==2, CannotSetThresholds) namespace glucat

### 7.51.1 Function Documentation

#### 7.51.1.1 \_GLUCAT\_CTAssert()

```

_GLUCAT_CTAssert (
    std::numeric_limits< unsigned int >::radix == 2,
    CannotSetThresholds )

```

Base class for policies

Precision policy

Tuning policy

Minimum index count needed to invoke matrix multiplication algorithm

Maximum steps of iterative refinement in division algorithm

Maximum number of steps in cyclic reduction square root iteration

Maximum number of steps in Denman-Beavers square root iteration

Maximum number of incomplete square roots in cascade log algorithm

Maximum number of steps in incomplete square root within cascade log algorithm

Maximum index count of folded frames in basis cache

Minimum map size needed to invoke generalized FFT

Minimum matrix dimension needed to invoke inverse generalized FFT

Minimum size needed for to invoke faster products algorithms

Denominator of proportion of different bits allowed in approximate equality

Extra number of different bits allowed in approximate equality

Precision used for exp, log and sqrt functions

Definition at line 35 of file [tuning.h](#).

## 7.52 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TUNING_H
00002 #define GLUCAT_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Policy classes to control tuning
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 // If radix of int is not 2, we can't easily set thresholds
00035 _GLUCAT_CTAssert(std::numeric_limits<unsigned int>::radix == 2, CannotSetThresholds)
00036
00037 namespace glucat
00038 {
00039     struct policy{};
00040
00041     enum precision_t
00042     {
00043         precision_demoted,
00044         precision_same,
00045         precision_promoted
00046     };
00047
00048 // Tuning policy default constants
00049
00050 const unsigned int Tuning_Default_Mult_Matrix_Threshold = 8;
00051 const unsigned int Tuning_Default_Div_Max_Steps = 4;
00052 const unsigned int Tuning_Default_CR_Sqrt_Max_Steps = 256;
00053 const unsigned int Tuning_Default_DB_Sqrt_Max_Steps = 256;
00054 const unsigned int Tuning_Default_Log_Max_Outer_Steps = 256;
00055 const unsigned int Tuning_Default_Log_Max_Inner_Steps = 32;
00056 const unsigned int Tuning_Default_Basis_Max_Count = 12;
00057 const unsigned int Tuning_Default_Fast_Size_Threshold = 1 << 6;
00058 const unsigned int Tuning_Default_Inv_Fast_Dim_Threshold = 1 << 3;
00059 const unsigned int Tuning_Default_Products_Size_Threshold = 1 << 22;
00060 const unsigned int Tuning_Default_Denom_Different_Bits = 8;
00061 const unsigned int Tuning_Default_Extra_Different_Bits = 8;
00062 const precision_t Tuning_Default_Function_Precision = precision_same;
00063
00064 template
00065 <
00066     unsigned int Mult_Matrix_Threshold = Tuning_Default_Mult_Matrix_Threshold,
00067     unsigned int Div_Max_Steps = Tuning_Default_Div_Max_Steps,
00068     unsigned int CR_Sqrt_Max_Steps = Tuning_Default_CR_Sqrt_Max_Steps,
00069     unsigned int DB_Sqrt_Max_Steps = Tuning_Default_DB_Sqrt_Max_Steps,
00070     unsigned int Log_Max_Outer_Steps = Tuning_Default_Log_Max_Outer_Steps,
00071     unsigned int Log_Max_Inner_Steps = Tuning_Default_Log_Max_Inner_Steps,
00072     unsigned int Basis_Max_Count = Tuning_Default_Basis_Max_Count,
00073     unsigned int Fast_Size_Threshold = Tuning_Default_Fast_Size_Threshold,
00074     unsigned int Inv_Fast_Dim_Threshold = Tuning_Default_Inv_Fast_Dim_Threshold,
00075     unsigned int Products_Size_Threshold = Tuning_Default_Products_Size_Threshold,
00076     unsigned int Denom_Different_Bits = Tuning_Default_Denom_Different_Bits,
00077     unsigned int Extra_Different_Bits = Tuning_Default_Extra_Different_Bits,
00078     precision_t Function_Precision = Tuning_Default_Function_Precision
00079 >
00080 struct tuning : policy
00081 {
00082     using tune_p = tuning
00083 <

```

```

00086     Mult_Matrix_Threshold,
00087     Div_Max_Steps,
00088     CR_Sqrt_Max_Steps,
00089     DB_Sqrt_Max_Steps,
00090     Log_Max_Outer_Steps,
00091     Log_Max_Inner_Steps,
00092     Basis_Max_Count,
00093     Fast_Size_Threshold,
00094     Inv_Fast_Dim_Threshold,
00095     Products_Size_Threshold,
00096     Denom_Different_Bits,
00097     Extra_Different_Bits,
00098     Function_Precision
00099 >;
00100 // Tuning for multiplication
00102 enum { mult_matrix_threshold = Mult_Matrix_Threshold };
00103 // Tuning for division
00105 enum { div_max_steps = Div_Max_Steps };
00106 // Tuning for sqrt
00108 enum { cr_sqrt_max_steps = CR_Sqrt_Max_Steps };
00110 enum { db_sqrt_max_steps = DB_Sqrt_Max_Steps };
00111 // Tuning for log
00113 enum { log_max_outer_steps = Log_Max_Outer_Steps };
00115 enum { log_max_inner_steps = Log_Max_Inner_Steps };
00116 // Tuning for basis cache
00118 enum { basis_max_count = Basis_Max_Count };
00119 // Tuning for FFT
00121 enum { fast_size_threshold = Fast_Size_Threshold };
00123 enum { inv_fast_dim_threshold = Inv_Fast_Dim_Threshold };
00124 // Tuning for products (other than geometric product)
00126 enum { products_size_threshold = Products_Size_Threshold };
00127 // Tuning for precision of exp, log and sqrt functions
00129 enum { denom_different_bits = Denom_Different_Bits };
00131 enum { extra_different_bits = Extra_Different_Bits };
00133 static const precision_t function_precision = Function_Precision;
00134 };
00135
00136 using tuning_demoted = tuning
00137 <
00138     Tuning_Default_Mult_Matrix_Threshold,
00139     Tuning_Default_Div_Max_Steps,
00140     Tuning_Default_CR_Sqrt_Max_Steps,
00141     Tuning_Default_DB_Sqrt_Max_Steps,
00142     Tuning_Default_Log_Max_Outer_Steps,
00143     Tuning_Default_Log_Max_Inner_Steps,
00144     Tuning_Default_Basis_Max_Count,
00145     Tuning_Default_Fast_Size_Threshold,
00146     Tuning_Default_Inv_Fast_Dim_Threshold,
00147     Tuning_Default_Products_Size_Threshold,
00148     Tuning_Default_Denom_Different_Bits,
00149     Tuning_Default_Extra_Different_Bits,
00150     precision_demoted
00151 >;
00152
00153 using tuning_promoted = tuning
00154 <
00155     Tuning_Default_Mult_Matrix_Threshold,
00156     Tuning_Default_Div_Max_Steps,
00157     Tuning_Default_CR_Sqrt_Max_Steps,
00158     Tuning_Default_DB_Sqrt_Max_Steps,
00159     Tuning_Default_Log_Max_Outer_Steps,
00160     Tuning_Default_Log_Max_Inner_Steps,
00161     Tuning_Default_Basis_Max_Count,
00162     Tuning_Default_Fast_Size_Threshold,
00163     Tuning_Default_Inv_Fast_Dim_Threshold,
00164     Tuning_Default_Products_Size_Threshold,
00165     Tuning_Default_Denom_Different_Bits,
00166     Tuning_Default_Extra_Different_Bits,
00167     precision_promoted
00168 >;
00169 }
00170
00171 #endif // GLUCAT_TUNING_H

```

## 7.53 test/tuning.h File Reference

This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace [glucat](#)

## Typedefs

- using [glucat::tuning\\_slow](#) = tuning< Tuning\_Slow\_Mult\_Matrix\_Threshold, Tuning\_Default\_Div\_Max\_Steps, Tuning\_Default\_CR\_Sqrt\_Max\_Steps, Tuning\_Default\_DB\_Sqrt\_Max\_Steps, Tuning\_Default\_Log\_Max\_↵\_Outer\_Steps, Tuning\_Default\_Log\_Max\_Inner\_Steps, Tuning\_Slow\_Basis\_Max\_Count, Tuning\_Slow\_↵\_Fast\_Size\_Threshold, Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold, Tuning\_Slow\_Products\_Size\_Threshold, Tuning\_Default\_Denom\_Different\_Bits, Tuning\_Default\_Extra\_Different\_Bits, Tuning\_Default\_Function\_↵\_Precision >
- using [glucat::tuning\\_naive](#) = tuning< Tuning\_Naive\_Mult\_Matrix\_Threshold, Tuning\_Default\_Div\_Max\_↵\_Steps, Tuning\_Default\_CR\_Sqrt\_Max\_Steps, Tuning\_Default\_DB\_Sqrt\_Max\_Steps, Tuning\_Default\_Log\_↵\_Max\_Outer\_Steps, Tuning\_Default\_Log\_Max\_Inner\_Steps, Tuning\_Naive\_Basis\_Max\_Count, Tuning\_↵\_Naive\_Fast\_Size\_Threshold, Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold, Tuning\_Default\_Products\_Size\_↵\_Threshold, Tuning\_Default\_Denom\_Different\_Bits, Tuning\_Default\_Extra\_Different\_Bits, Tuning\_Default\_↵\_Function\_Precision >
- using [glucat::tuning\\_fast](#) = tuning< Tuning\_Fast\_Mult\_Matrix\_Threshold, Tuning\_Fast\_Div\_Max\_Steps, Tuning\_Fast\_CR\_Sqrt\_Max\_Steps, Tuning\_Fast\_DB\_Sqrt\_Max\_Steps, Tuning\_Fast\_Log\_Max\_Outer\_↵\_Steps, Tuning\_Fast\_Log\_Max\_Inner\_Steps, Tuning\_Fast\_Basis\_Max\_Count, Tuning\_Fast\_Fast\_Size\_↵\_Threshold, Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold, Tuning\_Fast\_Products\_Size\_Threshold, Tuning\_↵\_Default\_Denom\_Different\_Bits, Tuning\_Default\_Extra\_Different\_Bits, Tuning\_Default\_Function\_Precision >

## Variables

- const unsigned int [glucat::Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [glucat::Tuning\\_Max\\_Threshold](#) = 1 << Tuning\_Int\_Digits
- const unsigned int [glucat::Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [glucat::Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = Tuning\_Max\_Threshold
- const unsigned int [glucat::Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [glucat::Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [glucat::Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 7.54 tuning.h

Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TEST_TUNING_H
00002 #define GLUCAT_TEST_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Class definitions to control test tuning
00006     -----
00007     begin                               : Sun 2001-12-09
```



```

00008      copyright          : (C) 2001-2021 by Paul C. Leopardi
00009      *****
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034 namespace glucat
00035 {
00036     const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits;
00037     const unsigned int Tuning_Max_Threshold = 1 « Tuning_Int_Digits;
00038
00039     // Specific tuning policy constants and tuning policies
00040
00041     const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold;
00042     const unsigned int Tuning_Slow_Basis_Max_Count = 0;
00043     const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold;
00044     const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00045     const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold;
00046
00047     using tuning_slow = tuning
00048     <
00049         Tuning_Slow_Mult_Matrix_Threshold,
00050         Tuning_Default_Div_Max_Steps,
00051         Tuning_Default_CR_Sqrt_Max_Steps,
00052         Tuning_Default_DB_Sqrt_Max_Steps,
00053         Tuning_Default_Log_Max_Outer_Steps,
00054         Tuning_Default_Log_Max_Inner_Steps,
00055         Tuning_Slow_Basis_Max_Count,
00056         Tuning_Slow_Fast_Size_Threshold,
00057         Tuning_Slow_Inv_Fast_Dim_Threshold,
00058         Tuning_Slow_Products_Size_Threshold,
00059         Tuning_Default_Denom_Different_Bits,
00060         Tuning_Default_Extra_Different_Bits,
00061         Tuning_Default_Function_Precision
00062     >;
00063
00064     const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0;
00065     const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold;
00066     const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold;
00067     const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00068
00069     using tuning_naive = tuning
00070     <
00071         Tuning_Naive_Mult_Matrix_Threshold,
00072         Tuning_Default_Div_Max_Steps,
00073         Tuning_Default_CR_Sqrt_Max_Steps,
00074         Tuning_Default_DB_Sqrt_Max_Steps,
00075         Tuning_Default_Log_Max_Outer_Steps,
00076         Tuning_Default_Log_Max_Inner_Steps,
00077         Tuning_Naive_Basis_Max_Count,
00078         Tuning_Naive_Fast_Size_Threshold,
00079         Tuning_Naive_Inv_Fast_Dim_Threshold,
00080         Tuning_Default_Products_Size_Threshold,
00081         Tuning_Default_Denom_Different_Bits,
00082         Tuning_Default_Extra_Different_Bits,
00083         Tuning_Default_Function_Precision
00084     >;
00085
00086     const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0;
00087     const unsigned int Tuning_Fast_Div_Max_Steps = 0;
00088     const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256;
00089     const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256;
00090     const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16;
00091     const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8;
00092     const unsigned int Tuning_Fast_Basis_Max_Count = 1;
00093     const unsigned int Tuning_Fast_Fast_Size_Threshold = 0;
00094     const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0;

```

```

00095     const unsigned int Tuning_Fast_Products_Size_Threshold =      0;
00096
00097     using tuning_fast = tuning
00098     <
00099         Tuning_Fast_Mult_Matrix_Threshold,
00100         Tuning_Fast_Div_Max_Steps,
00101         Tuning_Fast_CR_Sqrt_Max_Steps,
00102         Tuning_Fast_DB_Sqrt_Max_Steps,
00103         Tuning_Fast_Log_Max_Outer_Steps,
00104         Tuning_Fast_Log_Max_Inner_Steps,
00105         Tuning_Fast_Basis_Max_Count,
00106         Tuning_Fast_Fast_Size_Threshold,
00107         Tuning_Fast_Inv_Fast_Dim_Threshold,
00108         Tuning_Fast_Products_Size_Threshold,
00109         Tuning_Default_Denom_Different_Bits,
00110         Tuning_Default_Extra_Different_Bits,
00111         Tuning_Default_Function_Precision
00112     >;
00113 }
00114 #endif // GLUCAT_TEST_TUNING_H

```

## 7.55 pyclical/glucat.pxd File Reference

### Namespaces

- namespace [glucat](#)

## 7.56 glucat.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #     Generic library of universal Clifford algebra templates
00006 #
00007 # glucat.pxd: Basic Cython definitions
00008 #     corresponding to C++ definitions from PyClical.h.
00009 # Kept as a separate module from PyClical.pxd to avoid namespace clashes.
00010 #
00011 #     copyright           : (C) 2008-2012 by Paul C. Leopardi
00012 #
00013 #     This library is free software: you can redistribute it and/or modify
00014 #     it under the terms of the GNU Lesser General Public License as published
00015 #     by the Free Software Foundation, either version 3 of the License, or
00016 #     (at your option) any later version.
00017 #
00018 #     This library is distributed in the hope that it will be useful,
00019 #     but WITHOUT ANY WARRANTY; without even the implied warranty of
00020 #     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00021 #     GNU Lesser General Public License for more details.
00022 #
00023 #     You should have received a copy of the GNU Lesser General Public License
00024 #     along with this library. If not, see <http://www.gnu.org/licenses/>.
00025
00026 from libcpp.vector cimport vector
00027
00028 cdef extern from "PyClical.h":
00029
00030     cdef cppclass String:
00031         char* c_str()
00032
00033     cdef cppclass IndexSet:
00034         IndexSet ()
00035         IndexSet (IndexSet Ist) except+
00036         IndexSet (int idx) except+
00037         IndexSet (char* str) except+
00038         inline bint operator==(IndexSet Rhs)
00039         inline bint operator!=(IndexSet Rhs)
00040         inline bint operator<(IndexSet Rhs)
00041         inline IndexSet invert "operator~"()
00042         inline bint getitem "operator[]"(int idx)
00043         inline IndexSet set()
00044         inline IndexSet set(int idx) except+

```

```

00045     inline IndexSet set(int idx, int val) except+
00046     inline IndexSet reset()
00047     inline IndexSet reset(int idx) except+
00048     int count()
00049     int count_pos()
00050     int count_neg()
00051     int min()
00052     int max()
00053     int sign_of_mult(IndexSet Rhs)
00054     int sign_of_square()
00055     int hash_fn()
00056
00057     int compare(IndexSet Lhs, IndexSet Rhs)
00058     int min_neg(IndexSet Ist)
00059     int max_pos(IndexSet Ist)
00060
00061     ctypedef double scalar_t
00062
00063     cdef cppclass Clifford:
00064         Clifford ()
00065         Clifford (Clifford Clf) except+
00066         Clifford (Clifford Clf, IndexSet ist) except+
00067         Clifford (scalar_t scr) except+
00068         Clifford (char* str) except+
00069         Clifford (IndexSet ist, scalar_t scr) except+
00070         Clifford (vector[scalar_t] vec, IndexSet ist) except+
00071         bint operator==(Clifford Rhs)
00072         bint operator!=(Clifford Rhs)
00073         Clifford neg "operator-"()
00074         scalar_t getitem "operator[]"(IndexSet Ist)
00075         Clifford call "operator()(int grade)
00076         scalar_t scalar()
00077         Clifford pure()
00078         Clifford even()
00079         Clifford odd()
00080         vector[scalar_t] vector_part()
00081         vector[scalar_t] vector_part(IndexSet frm) except+
00082         Clifford involute()
00083         Clifford reverse()
00084         Clifford conj()
00085         Clifford random(IndexSet Ist, scalar_t fill)
00086         scalar_t norm()
00087         scalar_t quad()
00088         IndexSet frame()
00089         scalar_t max_abs()
00090         Clifford inv()
00091         Clifford pow(int m)
00092         Clifford outer_pow(int m)
00093         Clifford truncated(scalar_t limit)
00094         bint isinf()
00095         bint isnan()
00096         void write(char* msg)
00097
00098         scalar_t error_squared_tol(Clifford Clf)
00099         scalar_t error_squared(Clifford Lhs, Clifford Rhs, scalar_t threshold)
00100         bint approx_equal(Clifford Lhs, Clifford Rhs, scalar_t threshold, scalar_t tol)
00101         scalar_t scalar(Clifford Clf)
00102         scalar_t real(Clifford Clf)
00103         scalar_t imag(Clifford Clf)
00104         Clifford pure(Clifford Clf)
00105         Clifford even(Clifford Clf)
00106         Clifford odd(Clifford Clf)
00107         Clifford involute(Clifford Clf)
00108         Clifford reverse(Clifford Clf)
00109         Clifford conj(Clifford Clf)
00110         scalar_t norm(Clifford Clf)
00111         scalar_t abs(Clifford Clf)
00112         scalar_t max_abs(Clifford Clf)
00113         scalar_t quad(Clifford Clf)
00114         Clifford inv(Clifford Clf)
00115         Clifford pow(Clifford Clf, int m)
00116         Clifford outer_pow(Clifford Clf, int m)
00117
00118         Clifford complexifier(Clifford Clf)
00119         Clifford sqrt(Clifford Clf, Clifford I) except+
00120         Clifford sqrt(Clifford Clf)
00121         Clifford exp(Clifford Clf)
00122         Clifford log(Clifford Clf, Clifford I) except+
00123         Clifford log(Clifford Clf)
00124         Clifford cos(Clifford Clf, Clifford I) except+
00125         Clifford cos(Clifford Clf)
00126         Clifford acos(Clifford Clf, Clifford I) except+
00127         Clifford acos(Clifford Clf)
00128         Clifford cosh(Clifford Clf)
00129         Clifford acosh(Clifford Clf, Clifford I) except+
00130         Clifford acosh(Clifford Clf)
00131         Clifford sin(Clifford Clf, Clifford I) except+

```

```

00132 Clifford sin(Clifford Clf)
00133 Clifford asin(Clifford Clf, Clifford I) except+
00134 Clifford asin(Clifford Clf)
00135 Clifford sinh(Clifford Clf)
00136 Clifford asinh(Clifford Clf, Clifford I) except+
00137 Clifford asinh(Clifford Clf)
00138 Clifford tan(Clifford Clf, Clifford I) except+
00139 Clifford tan(Clifford Clf)
00140 Clifford atan(Clifford Clf, Clifford I) except+
00141 Clifford atan(Clifford Clf)
00142 Clifford tanh(Clifford Clf)
00143 Clifford atanh(Clifford Clf, Clifford I) except+
00144 Clifford atanh(Clifford Clf)
00145
00146 cdef extern from "PyClical.h" namespace "cga3":
00147 Clifford agc3(Clifford Clf)
00148 Clifford cga3(Clifford Clf)
00149 Clifford cga3std(Clifford Clf)

```

## 7.57 pyclical/PyClical.h File Reference

```

#include "glucat/glucat_config.h"
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include <iostream>
#include <sstream>
#include <iomanip>
#include <limits>

```

Include dependency graph for PyClical.h:

## 7.58 PyClical.h

[Go to the documentation of this file.](#)

```

00001 /*****
00002 GluCat : Generic library of universal Clifford algebra templates
00003 PyClical.h : C++ definitions needed by PyClical
00004 -----
00005 copyright : (C) 2008-2021 by Paul C. Leopardi
00006 *****/
00007
00008 This library is free software: you can redistribute it and/or modify
00009 it under the terms of the GNU Lesser General Public License as published
00010 by the Free Software Foundation, either version 3 of the License, or
00011 (at your option) any later version.
00012
00013 This library is distributed in the hope that it will be useful,
00014 but WITHOUT ANY WARRANTY; without even the implied warranty of
00015 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00016 GNU Lesser General Public License for more details.
00017
00018 You should have received a copy of the GNU Lesser General Public License
00019 along with this library. If not, see <http://www.gnu.org/licenses/>.
00020
00021 *****/
00022 This library is based on a prototype written by Arvind Raja and was
00023 licensed under the LGPL with permission of the author. See Arvind Raja,
00024 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00025 in Ablamowicz, Lounesto and Parra (eds.)
00026 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00027 *****/
00028 See also Arvind Raja's original header comments in glucat/glucat.h
00029 *****/
00030 // References for algorithms:
00031 // [DL]:
00032 // C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00033
00034 #include "glucat/glucat_config.h"
00035 #include "glucat/glucat.h"
00036 #include "glucat/glucat_imp.h"
00037 #include <iostream>
00038 #include <sstream>
00039 #include <iomanip>

```

```

00040 #include <limits>
00041
00044 template <typename Scalar_T>
00045 inline PyObject* PyFloat_FromDouble(Scalar_T v)
00046 { return ::PyFloat_FromDouble(glucat::numeric_traits<Scalar_T>::to_double(v)); }
00047
00048
00049 // String representations for use by PyClical Python classes.
00050
00051 using String = std::string;
00052
00053 String glucat_package_version = GLUCAT_PACKAGE_VERSION;
00054
00056 template<typename Index_Set_T>
00057 inline String index_set_to_repr(const Index_Set_T& ist)
00058 {
00059     std::ostringstream os;
00060     os << "index_set(" << ist << ")";
00061     return os.str();
00062 }
00063
00065 template<typename Index_Set_T>
00066 inline String index_set_to_str(const Index_Set_T& ist)
00067 {
00068     std::ostringstream os;
00069     os << ist;
00070     return os.str();
00071 }
00072
00074 template<typename Multivector_T>
00075 inline String clifford_to_repr(const Multivector_T& mv)
00076 {
00077     using scalar_t = typename Multivector_T::scalar_t;
00078     std::ostringstream os;
00079     os << std::setprecision(std::numeric_limits<scalar_t>::digits10 + 1);
00080     os << "clifford(\"" << mv << "\")";
00081     return os.str();
00082 }
00083
00085 template<typename Multivector_T>
00086 inline String clifford_to_str(const Multivector_T& mv)
00087 {
00088     using scalar_t = typename Multivector_T::scalar_t;
00089     std::ostringstream os;
00090     if (abs(mv) < std::numeric_limits<scalar_t>::epsilon())
00091         os << 0.0;
00092     else
00093         os << std::setprecision(4) << mv.truncated(scalar_t(1.0e-4));
00094     return os.str();
00095 }
00096
00097
00099 namespace cga3
00100 {
00102     template<typename Multivector_T>
00103     inline Multivector_T cga3(const Multivector_T& x)
00104     {
00105         using cl = Multivector_T;
00106         using ist = typename cl::index_set_t;
00107         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00108
00109         return (cl(ist(4)) - x) * ninf3 * (x - cl(ist(4)));
00110     }
00111
00113     template<typename Multivector_T>
00114     inline Multivector_T cga3std(const Multivector_T& X)
00115     {
00116         using cl = Multivector_T;
00117         using ist = typename cl::index_set_t;
00118         using scalar_t = typename cl::scalar_t;
00119         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00120
00121         return scalar_t(-2.0) * X / (X & ninf3);
00122     }
00123
00125     template<typename Multivector_T>
00126     inline Multivector_T agc3(const Multivector_T& X)
00127     {
00128         using cl = Multivector_T;
00129         using ist = typename cl::index_set_t;
00130         using scalar_t = typename cl::scalar_t;
00131
00132         const cl& cga3stdX = cga3std(X);
00133         return (cl(ist(1))*cga3stdX[ist(1)] +
00134             cl(ist(2))*cga3stdX[ist(2)] +
00135             cl(ist(3))*cga3stdX[ist(3)]) / scalar_t(2.0);
00136     }

```

```

00137 }
00138
00139
00140 // Specifications of the IndexSet and Clifford C++ classes for use with PyClical.
00141
00142 using namespace glucat;
00143 const index_t lo_ndx = DEFAULT_LO;
00144 const index_t hi_ndx = DEFAULT_HI;
00145 using IndexSet = index_set<lo_ndx, hi_ndx>;
00146
00147 using scalar_t = double;
00148 using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>;
00149
00150 const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon();
00151
00152 // Do not warn about unused values. This affects clang++ as well as g++.
00153
00154 #pragma GCC diagnostic ignored "-Wunused-value"
00155
00156 #if defined(__clang__)
00157 // Do not warn about unused functions. The affects clang++ only.
00158
00159 # pragma clang diagnostic ignored "-Wunused-function"
00160
00161 // Do not warn about unneeded internal declarations. The affects clang++ only.
00162
00163 # pragma clang diagnostic ignored "-Wunneeded-internal-declaration"
00164 #endif

```

## 7.59 pyclical/PyClical.pxd File Reference

### Namespaces

- namespace [PyClical](#)

## 7.60 PyClical.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #         Generic library of universal Clifford algebra templates
00006 #
00007 # PyClical.pxd: Basic Cython definitions for PyClical
00008 #         corresponding to C++ definitions from PyClical.h.
00009 #
00010 #         copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #         This library is free software: you can redistribute it and/or modify
00013 #         it under the terms of the GNU Lesser General Public License as published
00014 #         by the Free Software Foundation, either version 3 of the License, or
00015 #         (at your option) any later version.
00016 #
00017 #         This library is distributed in the hope that it will be useful,
00018 #         but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #         MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #         GNU Lesser General Public License for more details.
00021 #
00022 #         You should have received a copy of the GNU Lesser General Public License
00023 #         along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 cimport glucat
00026 from glucat cimport IndexSet, String, Clifford, scalar_t, vector
00027 from libcpp.string cimport string
00028
00029 cdef extern from "PyClical.h":
00030     string glucat_package_version
00031
00032     IndexSet operator&(IndexSet Lhs, IndexSet Rhs)
00033     IndexSet operator|(IndexSet Lhs, IndexSet Rhs)
00034     IndexSet operator^(IndexSet Lhs, IndexSet Rhs)
00035
00036     string index_set_to_repr(IndexSet& Ist)

```

```

00037     string index_set_to_str(IndexSet& Ist)
00038
00039     Clifford operator+(Clifford Lhs, Clifford Rhs)
00040     Clifford operator-(Clifford Lhs, Clifford Rhs)
00041     Clifford operator*(Clifford Lhs, Clifford Rhs)
00042     Clifford operator&(Clifford Lhs, Clifford Rhs)
00043     Clifford operator%(Clifford Lhs, Clifford Rhs)
00044     Clifford operator^(Clifford Lhs, Clifford Rhs)
00045     Clifford operator/(Clifford Lhs, Clifford Rhs)
00046     Clifford operator|(Clifford Lhs, Clifford Rhs)
00047
00048     string clifford_to_repr(Clifford& Clf)
00049     string clifford_to_str(Clifford& Clf)
00050
00051     const scalar_t epsilon

```

## 7.61 pyclical/PyClical.pyx File Reference

### Classes

- class [PyClical.index\\_set](#)
- class [PyClical.clifford](#)

### Namespaces

- namespace [PyClical](#)

### Functions

- def [PyClical.index\\_set\\_hidden\\_doctests](#) ()
- def [PyClical.clifford\\_hidden\\_doctests](#) ()
- def [PyClical.e](#) (obj)
- def [PyClical.istpq](#) (p, q)
- def [PyClical.\\_test](#) ()

### Variables

- [PyClical.\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [PyClical.lhs](#)
- [PyClical.rhs](#)
- [PyClical.threshold](#) = error\_squared\_tol(rhs) if threshold is None else threshold
- [PyClical.None](#)
- [PyClical.tol](#) = error\_squared\_tol(rhs) if tol is None else tol
- [PyClical.obj](#)
- [PyClical.i](#)
- [PyClical.ixt](#)
- [PyClical.fill](#)
- [PyClical.scalar\\_epsilon](#) = [epsilon](#)
- float [PyClical.pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [PyClical.tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [PyClical.cl](#) = [clifford](#)
- [PyClical.ist](#) = [index\\_set](#)
- def [PyClical.ninf3](#) = e(4) + e(-1)
- def [PyClical.nbar3](#) = e(4) - e(-1)

## 7.62 PyClicl.pyx

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 # distutils: language = c++
00004 #
00005 # PyClicl: Python interface to GluCat:
00006 #         Generic library of universal Clifford algebra templates
00007 #
00008 # PyClicl.pyx: Cython definitions visible from Python.
00009 #
00010 #     copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #     This library is free software: you can redistribute it and/or modify
00013 #     it under the terms of the GNU Lesser General Public License as published
00014 #     by the Free Software Foundation, either version 3 of the License, or
00015 #     (at your option) any later version.
00016 #
00017 #     This library is distributed in the hope that it will be useful,
00018 #     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #     GNU Lesser General Public License for more details.
00021 #
00022 #     You should have received a copy of the GNU Lesser General Public License
00023 #     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 # References for definitions:
00026 # [DL]:
00027 # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00028 #
00029 import math
00030 import numbers
00031 import collections
00032
00033 from PyClicl cimport *
00034
00035 __version__ = str(glucat_package_version, 'utf-8')
00036
00037 # Forward reference
00038 cdef class index_set
00039
00040 cdef inline IndexSet toIndexSet(obj):
00041     """
00042     Return the C++ IndexSet instance wrapped by index_set(obj).
00043     """
00044     return index_set(obj).instance[0]
00045
00046 cdef class index_set:
00047     """
00048     Python class index_set wraps C++ class IndexSet.
00049     """
00050     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
00051
00052     cdef inline wrap(index_set self, IndexSet other):
00053         """
00054         Wrap an instance of the C++ class IndexSet.
00055         """
00056         self.instance[0] = other
00057         return self
00058
00059     cdef inline IndexSet unwrap(index_set self):
00060         """
00061         Return the wrapped C++ IndexSet instance.
00062         """
00063         return self.instance[0]
00064
00065     cpdef copy(index_set self):
00066         """
00067         Copy this index_set object.
00068         """
00069         >> s=index_set(1); t=s.copy(); print(t)
00070         {1}
00071         """
00072         return index_set(self)
00073
00074     def __cinit__(self, other = 0):
00075         """
00076         Construct an object of type index_set.
00077         """
00078         >> print(index_set(1))
00079         {1}
00080         >> print(index_set({1,2}))
00081         {1,2}
00082         >> print(index_set(index_set({1,2})))

```



```

00083         {1,2}
00084     >> print(index_set({1,2}))
00085     {1,2}
00086     >> print(index_set({1,2,1}))
00087     {1,2}
00088     >> print(index_set("{1,2,1}"))
00089     {1,2}
00090     >> print(index_set(""))
00091     {}
00092     """
00093     error_msg_prefix = "Cannot initialize index_set object from"
00094     if isinstance(other, index_set):
00095         self.instance = new IndexSet((<index_set>other).unwrap())
00096     elif isinstance(other, numbers.Integral):
00097         self.instance = new IndexSet(<int>other)
00098     elif isinstance(other, (set, frozenset)):
00099         try:
00100             self.instance = new IndexSet()
00101             for idx in other:
00102                 self[idx] = True
00103         except IndexError:
00104             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
00105         except (RuntimeError, TypeError):
00106             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
00107     elif isinstance(other, str):
00108         try:
00109             bother = other.encode("UTF-8")
00110             self.instance = new IndexSet(<char *>bother)
00111         except RuntimeError:
00112             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00113     else:
00114         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00115
00116 def __dealloc__(self):
00117     """
00118     Clean up by deallocating the instance of C++ class IndexSet.
00119     """
00120     del self.instance
00121
00122 def __richcmp__(lhs, rhs, int op):
00123     """
00124     Compare two objects of class index_set.
00125
00126     >> index_set(1) == index_set({1})
00127     True
00128     >> index_set({1}) != index_set({1})
00129     False
00130     >> index_set({1}) != index_set({2})
00131     True
00132     >> index_set({1}) == index_set({2})
00133     False
00134     >> index_set({1}) < index_set({2})
00135     True
00136     >> index_set({1}) <= index_set({2})
00137     True
00138     >> index_set({1}) > index_set({2})
00139     False
00140     >> index_set({1}) >= index_set({2})
00141     False
00142     """
00143     if (lhs is None) or (rhs is None):
00144         eq = bool(lhs is rhs)
00145         if op == 2: # ==
00146             return eq
00147         elif op == 3: # !=
00148             return not eq
00149         else:
00150             if op == 0: # <
00151                 return False
00152             elif op == 1: # <=
00153                 return eq
00154             elif op == 4: # >
00155                 return False
00156             elif op == 5: # >=
00157                 return eq
00158             else:
00159                 return NotImplemented
00160     else:
00161         eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
00162         if op == 2: # ==
00163             return eq
00164         elif op == 3: # !=
00165             return not eq
00166         else:
00167             lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
00168             if op == 0: # <
00169                 return lt

```

```

00170         elif op == 1: # <=
00171             return lt or eq
00172         elif op == 4: # >
00173             return not (lt or eq)
00174         elif op == 5: # >=
00175             return not lt
00176         else:
00177             return NotImplemented
00178
00179     def __setitem__(self, idx, val):
00180         """
00181         Set the value of an index_set object at index idx to value val.
00182
00183         >> s=index_set({1}); s[2] = True; print(s)
00184         {1,2}
00185         >> s=index_set({1,2}); s[1] = False; print(s)
00186         {2}
00187         """
00188         self.instance.set(idx, val)
00189         return
00190
00191     def __getitem__(self, idx):
00192         """
00193         Get the value of an index_set object at an index.
00194
00195         >> index_set({1})[1]
00196         True
00197         >> index_set({1})[2]
00198         False
00199         >> index_set({2})[-1]
00200         False
00201         >> index_set({2})[1]
00202         False
00203         >> index_set({2})[2]
00204         True
00205         >> index_set({2})[33]
00206         False
00207         """
00208         return self.instance.getitem(idx)
00209
00210     def __contains__(self, idx):
00211         """
00212         Check that an index_set object contains the index idx: idx in self.
00213
00214         >> 1 in index_set({1})
00215         True
00216         >> 2 in index_set({1})
00217         False
00218         >> -1 in index_set({2})
00219         False
00220         >> 1 in index_set({2})
00221         False
00222         >> 2 in index_set({2})
00223         True
00224         >> 33 in index_set({2})
00225         False
00226         """
00227         return self.instance.getitem(idx)
00228
00229     def __iter__(self):
00230         """
00231         Iterate over the indices of an index_set.
00232
00233         >> for i in index_set({-3,4,7}):print(i, end=",")
00234         -3,4,7,
00235         """
00236         for idx in range(self.min(), self.max()+1):
00237             if idx in self:
00238                 yield idx
00239
00240     def __invert__(self):
00241         """
00242         Set complement: not.
00243
00244         >>
00245         print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))
00246         {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}
00247         """
00248         return index_set().wrap( self.instance.invert() )
00249
00250     def __xor__(lhs, rhs):
00251         """
00252         Symmetric set difference: exclusive or.
00253
00254         >> print(index_set({1}) ^ index_set({2}))
00255         {1,2}

```

```

00255         >> print(index_set({1,2}) ^ index_set({2}))
00256         {1}
00257         """
00258         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
00259
00260     def __ixor__(self, rhs):
00261         """
00262         Symmetric set difference: exclusive or.
00263
00264         >> x = index_set({1}); x ^= index_set({2}); print(x)
00265         {1,2}
00266         >> x = index_set({1,2}); x ^= index_set({2}); print(x)
00267         {1}
00268         """
00269         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
00270
00271     def __and__(lhs, rhs):
00272         """
00273         Set intersection: and.
00274
00275         >> print(index_set({1}) & index_set({2}))
00276         {}
00277         >> print(index_set({1,2}) & index_set({2}))
00278         {2}
00279         """
00280         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
00281
00282     def __iand__(self, rhs):
00283         """
00284         Set intersection: and.
00285
00286         >> x = index_set({1}); x &= index_set({2}); print(x)
00287         {}
00288         >> x = index_set({1,2}); x &= index_set({2}); print(x)
00289         {2}
00290         """
00291         return self.wrap( self.unwrap() & toIndexSet(rhs) )
00292
00293     def __or__(lhs, rhs):
00294         """
00295         Set union: or.
00296
00297         >> print(index_set({1}) | index_set({2}))
00298         {1,2}
00299         >> print(index_set({1,2}) | index_set({2}))
00300         {1,2}
00301         """
00302         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
00303
00304     def __ior__(self, rhs):
00305         """
00306         Set union: or.
00307
00308         >> x = index_set({1}); x |= index_set({2}); print(x)
00309         {1,2}
00310         >> x = index_set({1,2}); x |= index_set({2}); print(x)
00311         {1,2}
00312         """
00313         return self.wrap( self.unwrap() | toIndexSet(rhs) )
00314
00315     def count(self):
00316         """
00317         Cardinality: Number of indices included in set.
00318
00319         >> index_set({-1,1,2}).count()
00320         3
00321         """
00322         return self.instance.count()
00323
00324     def count_neg(self):
00325         """
00326         Number of negative indices included in set.
00327
00328         >> index_set({-1,1,2}).count_neg()
00329         1
00330         """
00331         return self.instance.count_neg()
00332
00333     def count_pos(self):
00334         """
00335         Number of positive indices included in set.
00336
00337         >> index_set({-1,1,2}).count_pos()
00338         2
00339         """
00340         return self.instance.count_pos()
00341

```

```

00342     def min(self):
00343         """
00344         Minimum member.
00345
00346         >> index_set({-1,1,2}).min()
00347         -1
00348         """
00349         return self.instance.min()
00350
00351     def max(self):
00352         """
00353         Maximum member.
00354
00355         >> index_set({-1,1,2}).max()
00356         2
00357         """
00358         return self.instance.max()
00359
00360     def hash_fn(self):
00361         """
00362         Hash function.
00363         """
00364         return self.instance.hash_fn()
00365
00366     def sign_of_mult(self, rhs):
00367         """
00368         Sign of geometric product of two Clifford basis elements.
00369
00370         >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
00371         1
00372         """
00373         return self.instance.sign_of_mult(toIndexSet(rhs))
00374
00375     def sign_of_square(self):
00376         """
00377         Sign of geometric square of a Clifford basis element.
00378
00379         >> s = index_set({1,2}); s.sign_of_square()
00380         -1
00381         """
00382         return self.instance.sign_of_square()
00383
00384     def __repr__(self):
00385         """
00386         The "official" string representation of self.
00387
00388         >> index_set({1,2}).__repr__()
00389         'index_set({1,2})'
00390         >> repr(index_set({1,2}))
00391         'index_set({1,2})'
00392         """
00393         return index_set_to_repr( self.unwrap() ).decode()
00394
00395     def __str__(self):
00396         """
00397         The "informal" string representation of self.
00398
00399         >> index_set({1,2}).__str__()
00400         '{1,2}'
00401         >> str(index_set({1,2}))
00402         '{1,2}'
00403         """
00404         return index_set_to_str( self.unwrap() ).decode()
00405
00406 def index_set_hidden_doctests():
00407     """
00408     Tests for functions that Doctest cannot see.
00409
00410     For index_set.__cinit__: Construct index_set.
00411
00412     >> print(index_set(1))
00413     {1}
00414     >> print(index_set({1,2}))
00415     {1,2}
00416     >> print(index_set(index_set({1,2})))
00417     {1,2}
00418     >> print(index_set({1,2}))
00419     {1,2}
00420     >> print(index_set({1,2,1}))
00421     {1,2}
00422     >> print(index_set({1,2,1}))
00423     {1,2}
00424     >> print(index_set(""))
00425     {}
00426     >> print(index_set("{}"))
00427     Traceback (most recent call last):
00428     ...

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```

00429     ValueError: Cannot initialize index_set object from invalid string '{'.
00430     >> print(index_set("{1}"))
00431     Traceback (most recent call last):
00432     ...
00433     ValueError: Cannot initialize index_set object from invalid string '{1'.
00434     >> print(index_set("{1,2,100}"))
00435     Traceback (most recent call last):
00436     ...
00437     ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
00438     >> print(index_set({1,2,100}))
00439     Traceback (most recent call last):
00440     ...
00441     IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
00442     >> print(index_set([1,2]))
00443     Traceback (most recent call last):
00444     ...
00445     TypeError: Cannot initialize index_set object from <class 'list'>.
00446
00447     For index_set.__richcmp__: Compare two objects of class index_set.
00448
00449     >> index_set(1) == index_set({1})
00450     True
00451     >> index_set({1}) != index_set({1})
00452     False
00453     >> index_set({1}) != index_set({2})
00454     True
00455     >> index_set({1}) == index_set({2})
00456     False
00457     >> index_set({1}) < index_set({2})
00458     True
00459     >> index_set({1}) <= index_set({2})
00460     True
00461     >> index_set({1}) > index_set({2})
00462     False
00463     >> index_set({1}) >= index_set({2})
00464     False
00465     >> None == index_set({1,2})
00466     False
00467     >> None != index_set({1,2})
00468     True
00469     >> None < index_set({1,2})
00470     False
00471     >> None <= index_set({1,2})
00472     False
00473     >> None > index_set({1,2})
00474     False
00475     >> None >= index_set({1,2})
00476     False
00477     >> index_set({1,2}) == None
00478     False
00479     >> index_set({1,2}) != None
00480     True
00481     >> index_set({1,2}) < None
00482     False
00483     >> index_set({1,2}) <= None
00484     False
00485     >> index_set({1,2}) > None
00486     False
00487     >> index_set({1,2}) >= None
00488     False
00489     """
00490     return
00491
00492 cpdef inline compare(lhs,rhs):
00493     """
00494     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
00495     -1 if a<b, +1 if a>b, 0 if a==b.
00496
00497     >> compare(index_set({1,2}),index_set({-1,3}))
00498     -1
00499     >> compare(index_set({-1,4}),index_set({-1,3}))
00500     1
00501     """
00502     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
00503
00504 cpdef inline min_neg(obj):
00505     """
00506     Minimum negative index, or 0 if none.
00507
00508     >> min_neg(index_set({1,2}))
00509     0
00510     """
00511     return glucat.min_neg( toIndexSet(obj) )
00512
00513 cpdef inline max_pos(obj):
00514     """
00515     Maximum positive index, or 0 if none.

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00516
00517     >> max_pos(index_set({1,2}))
00518     2
00519     """
00520     return glucat.max_pos( toIndexSet(obj) )
00521
00522 cdef inline vector[scalar_t] list_to_vector(lst):
00523     """
00524     Create a C++ std::vector[scalar_t] from an iterable Python object.
00525     """
00526     cdef vector[scalar_t] v
00527     for s in lst:
00528         v.push_back(<scalar_t>s)
00529     return v
00530
00531 # Forward reference.
00532 cdef class clifford
00533
00534 cdef inline Clifford toClifford(obj):
00535     return clifford(obj).instance[0]
00536
00537 cdef class clifford:
00538     """
00539     Python class clifford wraps C++ class Clifford.
00540     """
00541     cdef Clifford *instance # Wrapped instance of C++ class Clifford.
00542
00543     cdef inline wrap(clifford self, Clifford other):
00544         """
00545         Wrap an instance of the C++ class Clifford.
00546         """
00547         self.instance[0] = other
00548         return self
00549
00550     cdef inline Clifford unwrap(clifford self):
00551         """
00552         Return the wrapped C++ Clifford instance.
00553         """
00554         return self.instance[0]
00555
00556     cpdef copy(clifford self):
00557         """
00558         Copy this clifford object.
00559
00560         >> x=clifford("1{2}"); y=x.copy(); print(y)
00561         {2}
00562         """
00563         return clifford(self)
00564
00565     def __cinit__(self, other = 0, ixt = None):
00566         """
00567         Construct an object of type clifford.
00568
00569         >> print(clifford(2))
00570         2
00571         >> print(clifford(2.0))
00572         2
00573         >> print(clifford(1.0e-1))
00574         0.1
00575         >> print(clifford("2"))
00576         2
00577         >> print(clifford("2{1,2,3}"))
00578         2{1,2,3}
00579         >> print(clifford(clifford("2{1,2,3}")))
00580         2{1,2,3}
00581         >> print(clifford("-{1}"))
00582         -{1}
00583         >> print(clifford(2, index_set({1,2})))
00584         2{1,2}
00585         >> print(clifford([2,3], index_set({1,2})))
00586         2{1}+3{2}
00587         """
00588         error_msg_prefix = "Cannot initialize clifford object from"
00589         if ixt is None:
00590             try:
00591                 if isinstance(other, clifford):
00592                     self.instance = new Clifford((<clifford>other).unwrap())
00593                 elif isinstance(other, index_set):
00594                     self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
00595                 elif isinstance(other, numbers.Real):
00596                     self.instance = new Clifford(<scalar_t>other)
00597                 elif isinstance(other, str):
00598                     try:
00599                         bother = other.encode("UTF-8")
00600                         self.instance = new Clifford(<char *>bother)
00601                     except RuntimeError:
00602                         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")

```

```

00603         else:
00604             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00605     except RuntimeError as err:
00606         raise ValueError(error_msg_prefix + " " + str(type(other))
00607                           + " value " + repr(other) + ":"
00608                           + "\n\t" + str(err))
00609     elif isinstance(ixt, index_set):
00610         if isinstance(other, numbers.Real):
00611             self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
00612         elif isinstance(other, collections.abc.Sequence):
00613             self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
00614         else:
00615             raise TypeError(error_msg_prefix + " (" + str(type(other))
00616                             + ", " + repr(ixt) + ").")
00617     else:
00618         raise TypeError(error_msg_prefix + " (" + str(type(other))
00619                         + ", " + str(type(ixt)) + ").")
00620
00621 def __dealloc__(self):
00622     """
00623     Clean up by deallocating the instance of C++ class Clifford.
00624     """
00625     del self.instance
00626
00627 def __contains__(self, x):
00628     """
00629     Not applicable.
00630
00631     >> x=clifford(index_set({-3,4,7})); -3 in x
00632     Traceback (most recent call last):
00633     ...
00634     TypeError: Not applicable.
00635     """
00636     raise TypeError("Not applicable.")
00637
00638 def __iter__(self):
00639     """
00640     Not applicable.
00641
00642     >> for a in clifford(index_set({-3,4,7})):print(a, end=",")
00643     Traceback (most recent call last):
00644     ...
00645     TypeError: Not applicable.
00646     """
00647     raise TypeError("Not applicable.")
00648
00649 def reframe(self, ixt):
00650     """
00651     Put self into a larger frame, containing the union of self.frame() and index set ixt.
00652     This can be used to make multiplication faster, by multiplying within a common frame.
00653
00654     >> clifford("2+3{1}").reframe(index_set({1,2,3}))
00655     clifford("2+3{1}")
00656     >> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() ==
(s|t);
00657     True
00658     """
00659     error_msg_prefix = "Cannot reframe"
00660     if isinstance(ixt, index_set):
00661         try:
00662             result = clifford()
00663             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
00664         except RuntimeError as err:
00665             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
00666                             + str(ixt) + ":"
00667                             + "\n\t" + str(err))
00668     else:
00669         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
00670     return result
00671
00672 def __richcmp__(lhs, rhs, int op):
00673     """
00674     Compare objects of type clifford.
00675
00676     >> clifford("{1}") == clifford("1{1}")
00677     True
00678     >> clifford("{1}") != clifford("1.0{1}")
00679     False
00680     >> clifford("{1}") != clifford("1.0")
00681     True
00682     >> clifford("{1,2}") == None
00683     False
00684     >> clifford("{1,2}") != None
00685     True
00686     >> None == clifford("{1,2}")
00687     False
00688     >> None != clifford("{1,2}")

```

```

00689         True
00690         """
00691         if op == 2: # ==
00692             if (lhs is None) or (rhs is None):
00693                 return bool(lhs is rhs)
00694             else:
00695                 return bool( toClifford(lhs) == toClifford(rhs) )
00696         elif op == 3: # !=
00697             if (lhs is None) or (rhs is None):
00698                 return not bool(lhs is rhs)
00699             else:
00700                 return bool( toClifford(lhs) != toClifford(rhs) )
00701         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
00702             raise TypeError("This comparison operator is not implemented for "
00703                             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
00704         else:
00705             return NotImplemented
00706
00707     def __getitem__(self, ixt):
00708         """
00709         Subscripting: map from index set to scalar coordinate.
00710
00711         >>> clifford("{1}") [index_set(1)]
00712         1.0
00713         >>> clifford("{1}") [index_set({1})]
00714         1.0
00715         >>> clifford("{1}") [index_set({1,2})]
00716         0.0
00717         >>> clifford("2{1,2}") [index_set({1,2})]
00718         2.0
00719         """
00720         return self.instance.getitem(toIndexSet(ixt))
00721
00722     def __neg__(self):
00723         """
00724         Unary -.
00725
00726         >>> print(-clifford("{1}"))
00727         -{1}
00728         """
00729         return clifford().wrap( self.instance.neg() )
00730
00731     def __pos__(self):
00732         """
00733         Unary +.
00734
00735         >>> print(+clifford("{1}"))
00736         {1}
00737         """
00738         return clifford(self)
00739
00740     def __add__(lhs, rhs):
00741         """
00742         Geometric sum.
00743
00744         >>> print(clifford(1) + clifford("{2}"))
00745         1+{2}
00746         >>> print(clifford("{1}") + clifford("{2}"))
00747         {1}+{2}
00748         """
00749         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
00750
00751     def __iadd__(self, rhs):
00752         """
00753         Geometric sum.
00754
00755         >>> x = clifford(1); x += clifford("{2}"); print(x)
00756         1+{2}
00757         """
00758         return self.wrap( self.unwrap() + toClifford(rhs) )
00759
00760     def __sub__(lhs, rhs):
00761         """
00762         Geometric difference.
00763
00764         >>> print(clifford(1) - clifford("{2}"))
00765         1-{2}
00766         >>> print(clifford("{1}") - clifford("{2}"))
00767         {1}-{2}
00768         """
00769         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
00770
00771     def __isub__(self, rhs):
00772         """
00773         Geometric difference.
00774
00775         >>> x = clifford(1); x -= clifford("{2}"); print(x)

```



```

00776         1-{2}
00777         """
00778         return self.wrap( self.unwrap() - toClifford(rhs) )
00779
00780     def __mul__(lhs, rhs):
00781         """
00782         Geometric product.
00783
00784         >> print(clifford("{1}") * clifford("{2}"))
00785         {1,2}
00786         >> print(clifford(2) * clifford("{2}"))
00787         2{2}
00788         >> print(clifford("{1}") * clifford("{1,2}"))
00789         {2}
00790         """
00791         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
00792
00793     def __imul__(self, rhs):
00794         """
00795         Geometric product.
00796
00797         >> x = clifford(2); x *= clifford("{2}"); print(x)
00798         2{2}
00799         >> x = clifford("{1}"); x *= clifford("{2}"); print(x)
00800         {1,2}
00801         >> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
00802         {2}
00803         """
00804         return self.wrap( self.unwrap() * toClifford(rhs) )
00805
00806     def __mod__(lhs, rhs):
00807         """
00808         Contraction.
00809
00810         >> print(clifford("{1}") % clifford("{2}"))
00811         0
00812         >> print(clifford(2) % clifford("{2}"))
00813         2{2}
00814         >> print(clifford("{1}") % clifford("{1}"))
00815         1
00816         >> print(clifford("{1}") % clifford("{1,2}"))
00817         {2}
00818         """
00819         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
00820
00821     def __imod__(self, rhs):
00822         """
00823         Contraction.
00824
00825         >> x = clifford("{1}"); x %= clifford("{2}"); print(x)
00826         0
00827         >> x = clifford(2); x %= clifford("{2}"); print(x)
00828         2{2}
00829         >> x = clifford("{1}"); x %= clifford("{1}"); print(x)
00830         1
00831         >> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
00832         {2}
00833         """
00834         return self.wrap( self.unwrap() % toClifford(rhs) )
00835
00836     def __and__(lhs, rhs):
00837         """
00838         Inner product.
00839
00840         >> print(clifford("{1}") & clifford("{2}"))
00841         0
00842         >> print(clifford(2) & clifford("{2}"))
00843         0
00844         >> print(clifford("{1}") & clifford("{1}"))
00845         1
00846         >> print(clifford("{1}") & clifford("{1,2}"))
00847         {2}
00848         """
00849         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
00850
00851     def __iand__(self, rhs):
00852         """
00853         Inner product.
00854
00855         >> x = clifford("{1}"); x &= clifford("{2}"); print(x)
00856         0
00857         >> x = clifford(2); x &= clifford("{2}"); print(x)
00858         0
00859         >> x = clifford("{1}"); x &= clifford("{1}"); print(x)
00860         1
00861         >> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
00862         {2}

```

```

00863         """
00864         return self.wrap( self.unwrap() & toClifford(rhs) )
00865
00866     def __xor__(lhs, rhs):
00867         """
00868         Outer product.
00869
00870         >> print(clifford("{1}") ^ clifford("{2}"))
00871         {1,2}
00872         >> print(clifford(2) ^ clifford("{2}"))
00873         2{2}
00874         >> print(clifford("{1}") ^ clifford("{1}"))
00875         0
00876         >> print(clifford("{1}") ^ clifford("{1,2}"))
00877         0
00878         """
00879         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
00880
00881     def __ixor__(self, rhs):
00882         """
00883         Outer product.
00884
00885         >> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
00886         {1,2}
00887         >> x = clifford(2); x ^= clifford("{2}"); print(x)
00888         2{2}
00889         >> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
00890         0
00891         >> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
00892         0
00893         """
00894         return self.wrap( self.unwrap() ^ toClifford(rhs) )
00895
00896     def __truediv__(lhs, rhs):
00897         """
00898         Geometric quotient.
00899
00900         >> print(clifford("{1}") / clifford("{2}"))
00901         {1,2}
00902         >> print(clifford(2) / clifford("{2}"))
00903         2{2}
00904         >> print(clifford("{1}") / clifford("{1}"))
00905         1
00906         >> print(clifford("{1}") / clifford("{1,2}"))
00907         -{2}
00908         """
00909         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
00910
00911     def __idiv__(self, rhs):
00912         """
00913         Geometric quotient.
00914
00915         >> x = clifford("{1}"); x /= clifford("{2}"); print(x)
00916         {1,2}
00917         >> x = clifford(2); x /= clifford("{2}"); print(x)
00918         2{2}
00919         >> x = clifford("{1}"); x /= clifford("{1}"); print(x)
00920         1
00921         >> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
00922         -{2}
00923         """
00924         return self.wrap( self.unwrap() / toClifford(rhs) )
00925
00926     def inv(self):
00927         """
00928         Geometric multiplicative inverse.
00929
00930         >> x = clifford("{1}"); print(x.inv())
00931         {1}
00932         >> x = clifford(2); print(x.inv())
00933         0.5
00934         >> x = clifford("{1,2}"); print(x.inv())
00935         -{1,2}
00936         """
00937         return clifford().wrap( self.instance.inv() )
00938
00939     def __or__(lhs, rhs):
00940         """
00941         Transform left hand side, using right hand side as a transformation.
00942
00943         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
00944         -{1}
00945         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
00946         -{1}
00947         """
00948         return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
00949

```

```

00950     def __ior__(self, rhs):
00951         """
00952         Transform left hand side, using right hand side as a transformation.
00953
00954         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
00955         -{1}
00956         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
00957         -{1}
00958         """
00959         return self.wrap( self.unwrap() | toClifford(rhs) )
00960
00961     def __pow__(self, m, dummy):
00962         """
00963         Power: self to the m.
00964
00965         >> x=clifford("{1}"); print(x ** 2)
00966         1
00967         >> x=clifford("2"); print(x ** 2)
00968         4
00969         >> x=clifford("2+{1}"); print(x ** 0)
00970         1
00971         >> x=clifford("2+{1}"); print(x ** 1)
00972         2+{1}
00973         >> x=clifford("2+{1}"); print(x ** 2)
00974         5+4{1}
00975         >> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
00976         1
00977         """
00978         return pow(self, m)
00979
00980     def pow(self, m):
00981         """
00982         Power: self to the m.
00983
00984         >> x=clifford("{1}"); print(x.pow(2))
00985         1
00986         >> x=clifford("2"); print(x.pow(2))
00987         4
00988         >> x=clifford("2+{1}"); print(x.pow(0))
00989         1
00990         >> x=clifford("2+{1}"); print(x.pow(1))
00991         2+{1}
00992         >> x=clifford("2+{1}"); print(x.pow(2))
00993         5+4{1}
00994         >> print(clifford("1+{1}+{1,2}").pow(3))
00995         1+3{1}+3{1,2}
00996         >> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
00997         1
00998         """
00999         if isinstance(m, numbers.Integral):
01000             return clifford().wrap( self.instance.pow(m) )
01001         else:
01002             return exp(m * log(self))
01003
01004     def outer_pow(self, m):
01005         """
01006         Outer product power.
01007
01008         >> x=clifford("2+{1}"); print(x.outer_pow(0))
01009         1
01010         >> x=clifford("2+{1}"); print(x.outer_pow(1))
01011         2+{1}
01012         >> x=clifford("2+{1}"); print(x.outer_pow(2))
01013         4+4{1}
01014         >> print(clifford("1+{1}+{1,2}").outer_pow(3))
01015         1+3{1}+3{1,2}
01016
01017         """
01018         return clifford().wrap( self.instance.outer_pow(m) )
01019
01020     def __call__(self, grade):
01021         """
01022         Pure grade-vector part.
01023
01024         >> print(clifford("{1}") (1))
01025         {1}
01026         >> print(clifford("{1}") (0))
01027         0
01028         >> print(clifford("1+{1}+{1,2}") (0))
01029         1
01030         >> print(clifford("1+{1}+{1,2}") (1))
01031         {1}
01032         >> print(clifford("1+{1}+{1,2}") (2))
01033         {1,2}
01034         >> print(clifford("1+{1}+{1,2}") (3))
01035         0
01036         """

```

```

01037         return clifford().wrap( self.instance.call(grade) )
01038
01039 def scalar(self):
01040     """
01041     Scalar part.
01042
01043     >> clifford("1+{1}+{1,2}").scalar()
01044     1.0
01045     >> clifford("{1,2}").scalar()
01046     0.0
01047     """
01048     return self.instance.scalar()
01049
01050 def pure(self):
01051     """
01052     Pure part.
01053
01054     >> print(clifford("1+{1}+{1,2}").pure())
01055     {1}+{1,2}
01056     >> print(clifford("{1,2}").pure())
01057     {1,2}
01058     """
01059     return clifford().wrap( self.instance.pure() )
01060
01061 def even(self):
01062     """
01063     Even part of multivector, sum of even grade terms.
01064
01065     >> print(clifford("1+{1}+{1,2}").even())
01066     1+{1,2}
01067     """
01068     return clifford().wrap( self.instance.even() )
01069
01070 def odd(self):
01071     """
01072     Odd part of multivector, sum of odd grade terms.
01073
01074     >> print(clifford("1+{1}+{1,2}").odd())
01075     {1}
01076     """
01077     return clifford().wrap( self.instance.odd() )
01078
01079 def vector_part(self, frm = None):
01080     """
01081     Vector part of multivector, as a Python list, with respect to frm.
01082
01083     >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
01084     [2.0, 3.0]
01085     >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
01086     [0.0, 2.0, 3.0]
01087     """
01088     error_msg_prefix = "Cannot take vector part of "
01089     cdef vector[scalar_t] vec
01090     cdef int n
01091     cdef int i
01092     try:
01093         if frm is None:
01094             vec = self.instance.vector_part()
01095         else:
01096             vec = self.instance.vector_part((<index_set>frm).unwrap())
01097             n = vec.size()
01098             lst = [0.0]*n
01099             for i in xrange(n):
01100                 lst[i] = vec[i]
01101             return lst
01102     except RuntimeError as err:
01103         raise ValueError(error_msg_prefix + str(self) + " using invalid "
01104             + repr(frm) + " as frame:\n\t"
01105             + str(err))
01106
01107 def involute(self):
01108     """
01109     Main involution, each {i} is replaced by -{i} in each term,
01110     eg. clifford("{1}") -> -clifford("{1}").
01111
01112     >> print(clifford("{1}").involute())
01113     -{1}
01114     >> print((clifford("{2}") * clifford("{1}")).involute())
01115     -{1,2}
01116     >> print((clifford("{1}") * clifford("{2}")).involute())
01117     {1,2}
01118     >> print(clifford("1+{1}+{1,2}").involute())
01119     1-{1}+{1,2}
01120     """
01121     return clifford().wrap( self.instance.involute() )
01122
01123 def reverse(self):

```

```

01124         """
01125         Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
01126
01127         >> print(clifford("{1}").reverse())
01128         {1}
01129         >> print((clifford("{2}") * clifford("{1}")).reverse())
01130         {1,2}
01131         >> print((clifford("{1}") * clifford("{2}")).reverse())
01132         -{1,2}
01133         >> print(clifford("1+{1}+{1,2}").reverse())
01134         1+{1}-{1,2}
01135         """
01136         return clifford().wrap( self.instance.reverse() )
01137
01138     def conj(self):
01139         """
01140         Conjugation, reverse o involute == involute o reverse.
01141
01142         >> print((clifford("{1}")).conj())
01143         -{1}
01144         >> print((clifford("{2}") * clifford("{1}")).conj())
01145         {1,2}
01146         >> print((clifford("{1}") * clifford("{2}")).conj())
01147         -{1,2}
01148         >> print(clifford("1+{1}+{1,2}").conj())
01149         1-{1}-{1,2}
01150         """
01151         return clifford().wrap( self.instance.conj() )
01152
01153     def quad(self):
01154         """
01155         Quadratic form == (rev(x)*x)(0).
01156
01157         >> print(clifford("1+{1}+{1,2}").quad())
01158         3.0
01159         >> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
01160         2.0
01161         """
01162         return self.instance.quad()
01163
01164     def norm(self):
01165         """
01166         Norm == sum of squares of coordinates.
01167
01168         >> clifford("1+{1}+{1,2}").norm()
01169         3.0
01170         >> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
01171         4.0
01172         """
01173         return self.instance.norm()
01174
01175     def abs(self):
01176         """
01177         Absolute value: square root of norm.
01178
01179         >> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
01180         2.0
01181         """
01182         return glucat.abs( self.unwrap() )
01183
01184     def max_abs(self):
01185         """
01186         Maximum of absolute values of components of multivector: multivector infinity norm.
01187
01188         >> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
01189         1.0
01190         >> clifford("3+2{1}+{1,2}").max_abs()
01191         3.0
01192         """
01193         return self.instance.max_abs()
01194
01195     def truncated(self, limit):
01196         """
01197         Remove all terms of self with relative size smaller than limit.
01198
01199         >> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
01200         clifford("100000000")
01201         >> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
01202         clifford("10000+{1}")
01203         """
01204         return clifford().wrap( self.instance.truncated(limit) )
01205
01206     def isinf(self):
01207         """
01208         Check if a multivector contains any infinite values.
01209
01210         >> clifford().isinf()

```

```

01211         False
01212         """
01213         return self.instance.isnan()
01214
01215     def isnan(self):
01216         """
01217         Check if a multivector contains any IEEE NaN values.
01218
01219         >> clifford().isnan()
01220         False
01221         """
01222         return self.instance.isnan()
01223
01224     def frame(self):
01225         """
01226         Subalgebra generated by all generators of terms of given multivector.
01227
01228         >> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
01229         {-2,-1,1,2,7}
01230         >> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
01231         <class 'PyClical.index_set'>
01232         """
01233         return index_set().wrap( self.instance.frame() )
01234
01235     def __repr__(self):
01236         """
01237         The "official" string representation of self.
01238
01239         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
01240         'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
01241         """
01242         return clifford_to_repr( self.unwrap() ).decode()
01243
01244     def __str__(self):
01245         """
01246         The "informal" string representation of self.
01247
01248         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
01249         '1+3{-1}+2{1,2}+4{-2,7}'
01250         """
01251         return clifford_to_str( self.unwrap() ).decode()
01252
01253     def clifford_hidden_doctests():
01254         """
01255         Tests for functions that Doctest cannot see.
01256
01257         For clifford.__cinit__: Construct an object of type clifford.
01258
01259         >> print(clifford(2))
01260         2
01261         >> print(clifford(2.0))
01262         2
01263         >> print(clifford(1.0e-1))
01264         0.1
01265         >> print(clifford("2"))
01266         2
01267         >> print(clifford("2{1,2,3}"))
01268         2{1,2,3}
01269         >> print(clifford(clifford("2{1,2,3}")))
01270         2{1,2,3}
01271         >> print(clifford("-{1}"))
01272         -{1}
01273         >> print(clifford(2, index_set({1,2})))
01274         2{1,2}
01275         >> print(clifford([2,3], index_set({1,2})))
01276         2{1}+3{2}
01277         >> print(clifford([1,2]))
01278         Traceback (most recent call last):
01279         ...
01280         TypeError: Cannot initialize clifford object from <class 'list'>.
01281         >> print(clifford(None))
01282         Traceback (most recent call last):
01283         ...
01284         TypeError: Cannot initialize clifford object from <class 'NoneType'>.
01285         >> print(clifford(None, [1,2]))
01286         Traceback (most recent call last):
01287         ...
01288         TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
01289         >> print(clifford([1,2], [1,2]))
01290         Traceback (most recent call last):
01291         ...
01292         TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
01293         >> print(clifford(""))
01294         Traceback (most recent call last):
01295         ...
01296         ValueError: Cannot initialize clifford object from invalid string ".
01297         >> print(clifford("{}"))

```

```

01298     Traceback (most recent call last):
01299         ...
01300     ValueError: Cannot initialize clifford object from invalid string '{'.
01301     >> print(clifford("{1}"))
01302     Traceback (most recent call last):
01303         ...
01304     ValueError: Cannot initialize clifford object from invalid string '{1'.
01305     >> print(clifford("{1}"))
01306     Traceback (most recent call last):
01307         ...
01308     ValueError: Cannot initialize clifford object from invalid string '+'.
01309     >> print(clifford("{1}"))
01310     Traceback (most recent call last):
01311         ...
01312     ValueError: Cannot initialize clifford object from invalid string '-'.
01313     >> print(clifford("{1}"))
01314     Traceback (most recent call last):
01315         ...
01316     ValueError: Cannot initialize clifford object from invalid string '{1}+'.
01317
01318     For clifford.__richcmp__: Compare objects of type clifford.
01319
01320     >> clifford("{1}") == clifford("1{1}")
01321     True
01322     >> clifford("{1}") != clifford("1.0{1}")
01323     False
01324     >> clifford("{1}") != clifford("1.0")
01325     True
01326     >> clifford("{1,2}") == None
01327     False
01328     >> clifford("{1,2}") != None
01329     True
01330     >> None == clifford("{1,2}")
01331     False
01332     >> None != clifford("{1,2}")
01333     True
01334     """
01335     return
01336
01337 cpdef inline error_squared_tol(obj):
01338     """
01339     Quadratic norm error tolerance relative to a specific multivector.
01340
01341     >> print(error_squared_tol(clifford("{1}")) * 3.0 - error_squared_tol(clifford("1{1}-2{2}+3{3}")))
01342     0.0
01343     """
01344     return glucat.error_squared_tol(toClifford(obj))
01345
01346 cpdef inline error_squared(lhs, rhs, threshold):
01347     """
01348     Relative or absolute error using the quadratic norm.
01349
01350     >> err2=scalar_epsilon*scalar_epsilon
01351
01352     >> print(error_squared(clifford("{1}"), clifford("1{1}"), err2))
01353     0.0
01354     >> print(error_squared(clifford("1{1}-3{2}+4{3}"), clifford("{1}"), err2))
01355     25.0
01356     """
01357     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
01358
01359 cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
01360     """
01361     Test for approximate equality of multivectors.
01362
01363     >> err2=scalar_epsilon*scalar_epsilon
01364
01365     >> print(approx_equal(clifford("{1}"), clifford("1{1}")))
01366     True
01367     >> print(approx_equal(clifford("1{1}-3{2}+4{3}"), clifford("{1}")))
01368     False
01369     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+0.001"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01370     False
01371     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+1.0e-30"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01372     True
01373     """
01374     threshold = error_squared_tol(rhs) if threshold is None else threshold
01375     tol = error_squared_tol(rhs) if tol is None else tol
01376     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold, <scalar_t>tol)
01377
01378 cpdef inline inv(obj):
01379     """
01380     Geometric multiplicative inverse.
01381
01382     >> print(inv(clifford("{1}")))
01383     {1}
01384     >> print(inv(clifford("{-1}")))

```

```

01385     -{-1}
01386     >> print(inv(clifford("{-2,-1}")))
01387     -{-2,-1}
01388     >> print(inv(clifford("{-1}+{1}")))
01389     nan
01390     """
01391     return clifford(obj).inv()
01392
01393 cpdef inline scalar(obj):
01394     """
01395     Scalar part.
01396
01397     >> scalar(clifford("1+{1}+{1,2}"))
01398     1.0
01399     >> scalar(clifford("{1,2}"))
01400     0.0
01401     """
01402     return clifford(obj).scalar()
01403
01404 cpdef inline real(obj):
01405     """
01406     Real part: synonym for scalar part.
01407
01408     >> real(clifford("1+{1}+{1,2}"))
01409     1.0
01410     >> real(clifford("{1,2}"))
01411     0.0
01412     """
01413     return clifford(obj).scalar()
01414
01415 cpdef inline imag(obj):
01416     """
01417     Imaginary part: deprecated (always 0).
01418
01419     >> imag(clifford("1+{1}+{1,2}"))
01420     0.0
01421     >> imag(clifford("{1,2}"))
01422     0.0
01423     """
01424     return 0.0
01425
01426 cpdef inline pure(obj):
01427     """
01428     Pure part
01429
01430     >> print(pure(clifford("1+{1}+{1,2}")))
01431     {1}+{1,2}
01432     >> print(pure(clifford("{1,2}")))
01433     {1,2}
01434     """
01435     return clifford(obj).pure()
01436
01437 cpdef inline even(obj):
01438     """
01439     Even part of multivector, sum of even grade terms.
01440
01441     >> print(even(clifford("1+{1}+{1,2}")))
01442     1+{1,2}
01443     """
01444     return clifford(obj).even()
01445
01446 cpdef inline odd(obj):
01447     """
01448     Odd part of multivector, sum of odd grade terms.
01449
01450     >> print(odd(clifford("1+{1}+{1,2}")))
01451     {1}
01452     """
01453     return clifford(obj).odd()
01454
01455 cpdef inline involute(obj):
01456     """
01457     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
01458
01459     >> print(involute(clifford("{1}")))
01460     -{1}
01461     >> print(involute(clifford("{2}") * clifford("{1}")))
01462     -{1,2}
01463     >> print(involute(clifford("{1}") * clifford("{2}")))
01464     {1,2}
01465     >> print(involute(clifford("1+{1}+{1,2}")))
01466     1-{1}+{1,2}
01467     """
01468     return clifford(obj).involute()
01469
01470 cpdef inline reverse(obj):
01471     """

```



```

01472     Reversion, eg. {1}*{2} -> {2}*{1}
01473
01474     >> print(reverse(clifford("{1}")))
01475     {1}
01476     >> print(reverse(clifford("{2}") * clifford("{1}")))
01477     {1,2}
01478     >> print(reverse(clifford("{1}") * clifford("{2}")))
01479     -{1,2}
01480     >> print(reverse(clifford("1+{1}+{1,2}")))
01481     1+{1}-{1,2}
01482     """
01483     return clifford(obj).reverse()
01484
01485 cpdef inline conj(obj):
01486     """
01487     Conjugation, reverse o involute == involute o reverse.
01488
01489     >> print(conj(clifford("{1}")))
01490     -{1}
01491     >> print(conj(clifford("{2}") * clifford("{1}")))
01492     {1,2}
01493     >> print(conj(clifford("{1}") * clifford("{2}")))
01494     -{1,2}
01495     >> print(conj(clifford("1+{1}+{1,2}")))
01496     1-{1}-{1,2}
01497     """
01498     return clifford(obj).conj()
01499
01500 cpdef inline quad(obj):
01501     """
01502     Quadratic form == (rev(x)*x)(0).
01503
01504     >> print(quad(clifford("1+{1}+{1,2}")))
01505     3.0
01506     >> print(quad(clifford("1+{-1}+{1,2}+{1,2,3}")))
01507     2.0
01508     """
01509     return clifford(obj).quad()
01510
01511 cpdef inline norm(obj):
01512     """
01513     norm == sum of squares of coordinates.
01514
01515     >> norm(clifford("1+{1}+{1,2}"))
01516     3.0
01517     >> norm(clifford("1+{-1}+{1,2}+{1,2,3}"))
01518     4.0
01519     """
01520     return clifford(obj).norm()
01521
01522 cpdef inline abs(obj):
01523     """
01524     Absolute value of multivector: multivector 2-norm.
01525
01526     >> abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01527     2.0
01528     """
01529     return glucat.abs(toClifford(obj))
01530
01531 cpdef inline max_abs(obj):
01532     """
01533     Maximum absolute value of coordinates multivector: multivector infinity-norm.
01534
01535     >> max_abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01536     1.0
01537     >> max_abs(clifford("3+2{1}+{1,2}"))
01538     3.0
01539     """
01540     return glucat.max_abs(toClifford(obj))
01541
01542 cpdef inline pow(obj, m):
01543     """
01544     Integer power of multivector: obj to the m.
01545
01546     >> x=clifford("{1}"); print(pow(x,2))
01547     1
01548     >> x=clifford("2"); print(pow(x,2))
01549     4
01550     >> x=clifford("2+{1}"); print(pow(x,0))
01551     1
01552     >> x=clifford("2+{1}"); print(pow(x,1))
01553     2+{1}
01554     >> x=clifford("2+{1}"); print(pow(x,2))
01555     5+4{1}
01556     >> print(pow(clifford("1+{1}+{1,2}"),3))
01557     1+3{1}+3{1,2}

```

```

01559     >> i=clifford("{1,2}"); print(exp(pi/2) * pow(i, i))
01560     1
01561     """
01562     try:
01563         math.pow(obj, m)
01564     except:
01565         return clifford(obj).pow(m)
01566
01567 cpdef inline outer_pow(obj, m):
01568     """
01569     Outer product power of multivector.
01570
01571     >> print(outer_pow(clifford("1+{1}+{1,2}"),3))
01572     1+3{1}+3{1,2}
01573     """
01574     return clifford(obj).outer_pow(m)
01575
01576 cpdef inline complexifier(obj):
01577     """
01578     Square root of -1 which commutes with all members of the frame of the given multivector.
01579
01580     >> print(complexifier(clifford(index_set({1})))
01581     {1,2,3}
01582     >> print(complexifier(clifford(index_set({-1})))
01583     {-1}
01584     >> print(complexifier(index_set({1})))
01585     {1,2,3}
01586     >> print(complexifier(index_set({-1})))
01587     {-1}
01588     """
01589     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
01590
01591 cpdef inline sqrt(obj, i = None):
01592     """
01593     Square root of multivector with optional complexifier.
01594
01595     >> print(sqrt(-1))
01596     {-1}
01597     >> print(sqrt(clifford("2{-1}")))
01598     1+{-1}
01599     >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)
01600     {1,2,3}
01601     -1
01602     >> j=sqrt(-1,"{1,2,3}"); print(j); print(j*j)
01603     {1,2,3}
01604     -1
01605     """
01606     if not (i is None):
01607         return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
01608     else:
01609         try:
01610             return math.sqrt(obj)
01611         except:
01612             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
01613
01614 cpdef inline exp(obj):
01615     """
01616     Exponential of multivector.
01617
01618     >> x=clifford("{1,2}") * pi/4; print(exp(x))
01619     0.7071+0.7071{1,2}
01620     >> x=clifford("{1,2}") * pi/2; print(exp(x))
01621     {1,2}
01622     """
01623     try:
01624         return math.exp(obj)
01625     except:
01626         return clifford().wrap( glucat.exp(toClifford(obj)) )
01627
01628 cpdef inline log(obj,i = None):
01629     """
01630     Natural logarithm of multivector with optional complexifier.
01631
01632     >> x=clifford("{-1}"); print((log(x,"{-1}") * 2/pi))
01633     {-1}
01634     >> x=clifford("{1,2}"); print((log(x,"{1,2,3}") * 2/pi))
01635     {1,2}
01636     >> x=clifford("{1,2}"); print((log(x) * 2/pi))
01637     {1,2}
01638     >> x=clifford("{1,2}"); print((log(x,"{1,2}") * 2/pi))
01639     Traceback (most recent call last):
01640     ...
01641     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
01642     """
01643     if not (i is None):
01644         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
01645     else:

```

```

01646         try:
01647             return math.log(obj)
01648         except:
01649             return clifford().wrap( glucat.log(toClifford(obj)) )
01650
01651 cpdef inline cos(obj,i = None):
01652     """
01653     Cosine of multivector with optional complexifier.
01654
01655     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01656     {1,2}
01657     >> x=clifford("{1,2}"); print(cos(acos(x)))
01658     {1,2}
01659     """
01660     if not (i is None):
01661         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
01662     else:
01663         try:
01664             return math.cos(obj)
01665         except:
01666             return clifford().wrap( glucat.cos(toClifford(obj)) )
01667
01668 cpdef inline acos(obj,i = None):
01669     """
01670     Inverse cosine of multivector with optional complexifier.
01671
01672     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01673     {1,2}
01674     >> x=clifford("{1,2}"); print(cos(acos(x),"{-1,1,2,3,4}"))
01675     {1,2}
01676     >> print(acos(0) / pi)
01677     0.5
01678     >> x=clifford("{1,2}"); print(cos(acos(x)))
01679     {1,2}
01680     """
01681     if not (i is None):
01682         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
01683     else:
01684         try:
01685             return math.acos(obj)
01686         except:
01687             return clifford().wrap( glucat.acos(toClifford(obj)) )
01688
01689 cpdef inline cosh(obj):
01690     """
01691     Hyperbolic cosine of multivector.
01692
01693     >> x=clifford("{1,2}") * pi; print(cosh(x))
01694     -1
01695     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01696     {1,2,3}
01697     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01698     {1,2}
01699     """
01700     try:
01701         return math.cosh(obj)
01702     except:
01703         return clifford().wrap( glucat.cosh(toClifford(obj)) )
01704
01705 cpdef inline acosh(obj,i = None):
01706     """
01707     Inverse hyperbolic cosine of multivector with optional complexifier.
01708
01709     >> print(acosh(0,"{-2,-1,1}"))
01710     1.571{-2,-1,1}
01711     >> x=clifford("{1,2,3}"); print(cosh(acosh(x),"{-1,1,2,3,4}"))
01712     {1,2,3}
01713     >> print(acosh(0))
01714     1.571{-1}
01715     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01716     {1,2,3}
01717     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01718     {1,2}
01719     """
01720     if not (i is None):
01721         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
01722     else:
01723         try:
01724             return math.acosh(obj)
01725         except:
01726             return clifford().wrap( glucat.acosh(toClifford(obj)) )
01727
01728 cpdef inline sin(obj,i = None):
01729     """
01730     Sine of multivector with optional complexifier.
01731
01732     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))

```

```

01733     {-1}
01734     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01735     {-1}
01736     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01737     {1,2,3}
01738     """
01739     if not (i is None):
01740         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
01741     else:
01742         try:
01743             return math.sin(obj)
01744         except:
01745             return clifford().wrap( glucat.sin(toClifford(obj)) )
01746
01747 cpdef inline asin(obj,i = None):
01748     """
01749     Inverse sine of multivector with optional complexifier.
01750
01751     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),s))
01752     {-1}
01753     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01754     {-1}
01755     >> print(asin(1) / pi)
01756     0.5
01757     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01758     {1,2,3}
01759     """
01760     if not (i is None):
01761         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
01762     else:
01763         try:
01764             return math.asin(obj)
01765         except:
01766             return clifford().wrap( glucat.asin(toClifford(obj)) )
01767
01768 cpdef inline sinh(obj):
01769     """
01770     Hyperbolic sine of multivector.
01771
01772     >> x=clifford("{1,2}") * pi/2; print(sinh(x))
01773     {1,2}
01774     >> x=clifford("{1,2}") * pi/6; print(sinh(x))
01775     0.5{1,2}
01776     """
01777     try:
01778         return math.sinh(obj)
01779     except:
01780         return clifford().wrap( glucat.sinh(toClifford(obj)) )
01781
01782 cpdef inline asinh(obj,i = None):
01783     """
01784     Inverse hyperbolic sine of multivector with optional complexifier.
01785
01786     >> x=clifford("{1,2}"); print(asinh(x,"{1,2,3}") * 2/pi)
01787     {1,2}
01788     >> x=clifford("{1,2}"); print(asinh(x) * 2/pi)
01789     {1,2}
01790     >> x=clifford("{1,2}") / 2; print(asinh(x) * 6/pi)
01791     {1,2}
01792     """
01793     if not (i is None):
01794         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
01795     else:
01796         try:
01797             return math.asinh(obj)
01798         except:
01799             return clifford().wrap( glucat.asinh(toClifford(obj)) )
01800
01801 cpdef inline tan(obj,i = None):
01802     """
01803     Tangent of multivector with optional complexifier.
01804
01805     >> x=clifford("{1,2}"); print(tan(x,"{1,2,3}"))
01806     0.7616{1,2}
01807     >> x=clifford("{1,2}"); print(tan(x))
01808     0.7616{1,2}
01809     """
01810     if not (i is None):
01811         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
01812     else:
01813         try:
01814             return math.tan(obj)
01815         except:
01816             return clifford().wrap( glucat.tan(toClifford(obj)) )
01817
01818 cpdef inline atan(obj,i = None):
01819     """

```

```

01820     Inverse tangent of multivector with optional complexifier.
01821
01822     >> s=index_set({1,2,3}); x=clifford("{1}"); print(tan(atan(x,s),s))
01823     {1}
01824     >> x=clifford("{1}"); print(tan(atan(x)))
01825     {1}
01826     """
01827     if not (i is None):
01828         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
01829     else:
01830         try:
01831             return math.atan(obj)
01832         except:
01833             return clifford().wrap( glucat.atan(toClifford(obj)) )
01834
01835 cpdef inline tanh(obj):
01836     """
01837     Hyperbolic tangent of multivector.
01838
01839     >> x=clifford("{1,2}") * pi/4; print(tanh(x))
01840     {1,2}
01841     """
01842     try:
01843         return math.tanh(obj)
01844     except:
01845         return clifford().wrap( glucat.tanh(toClifford(obj)) )
01846
01847 cpdef inline atanh(obj,i = None):
01848     """
01849     Inverse hyperbolic tangent of multivector with optional complexifier.
01850
01851     >> s=index_set({1,2,3}); x=clifford("{1,2}"); print(tanh(atanh(x,s)))
01852     {1,2}
01853     >> x=clifford("{1,2}"); print(tanh(atanh(x)))
01854     {1,2}
01855     """
01856     if not (i is None):
01857         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
01858     else:
01859         try:
01860             return math.atanh(obj)
01861         except:
01862             return clifford().wrap( glucat.atanh(toClifford(obj)) )
01863
01864 cpdef inline random_clifford(index_set ixt, fill = 1.0):
01865     """
01866     Random multivector within a frame.
01867
01868     >> print(random_clifford(index_set({-3,-1,2})).frame())
01869     {-3,-1,2}
01870     """
01871     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
01872
01873 cpdef inline cga3(obj):
01874     """
01875     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
    definition.
01876
01877     >> x=clifford("2{1}+9{2}+{3}"); print(cga3(x))
01878     87{-1}+4{1}+18{2}+2{3}+85{4}
01879     """
01880     return clifford().wrap( glucat.cga3(toClifford(obj)) )
01881
01882 cpdef inline cga3std(obj):
01883     """
01884     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
01885
01886     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x)))
01887     87{-1}+4{1}+18{2}+2{3}+85{4}
01888     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x))-cga3(x))
01889     0
01890     """
01891     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
01892
01893 cpdef inline agc3(obj):
01894     """
01895     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
01896
01897     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x)))
01898     2{1}+9{2}+{3}
01899     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x))-x)
01900     0
01901     """
01902     return clifford().wrap( glucat.agc3(toClifford(obj)) )
01903
01904 # Some abbreviations.
01905 scalar_epsilon = epsilon

```

```

01906
01907 pi = atan(clifford(1.0)) * 4.0
01908 tau = atan(clifford(1.0)) * 8.0
01909
01910 cl = clifford
01911 """
01912 Abbreviation for clifford.
01913
01914 >> print(cl(2))
01915 2
01916 >> print(cl(2.0))
01917 2
01918 >> print(cl(5.0e-1))
01919 0.5
01920 >> print(cl("2"))
01921 2
01922 >> print(cl("2{1,2,3}"))
01923 2{1,2,3}
01924 >> print(cl(cl("2{1,2,3}")))
01925 2{1,2,3}
01926 """
01927
01928 ist = index_set
01929 """
01930 Abbreviation for index_set.
01931
01932 >> print(ist("{1,2,3}"))
01933 {1,2,3}
01934 """
01935
01936 def e(obj):
01937     """
01938     Abbreviation for clifford(index_set(obj)).
01939
01940     >> print(e(1))
01941     {1}
01942     >> print(e(-1))
01943     {-1}
01944     >> print(e(0))
01945     1
01946     """
01947     return clifford(index_set(obj))
01948
01949 def istpq(p, q):
01950     """
01951     Abbreviation for index_set({-q,...p}).
01952
01953     >> print(istpq(2,3))
01954     {-3,-2,-1,1,2}
01955     """
01956     return index_set(set(range(-q,p+1)))
01957
01958 ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
01959 nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
01960
01961 # Doctest interface.
01962 def _test():
01963     import PyClical, doctest
01964     return doctest.testmod(PyClical)
01965
01966 if __name__ == "__main__":
01967     _test()

```

## 7.63 pyclical/PyClical\_nocython.cpp File Reference

```
#include "Python.h"
```

Include dependency graph for PyClical\_nocython.cpp:

### Macros

- #define `PY_SSIZE_T_CLEAN`

### 7.63.1 Macro Definition Documentation

## 7.63.1.1 PY\_SSIZE\_T\_CLEAN

```
#define PY_SSIZE_T_CLEAN
```

Definition at line 23 of file [PyClical\\_nocython.cpp](#).

## 7.64 PyClical\_nocython.cpp

[Go to the documentation of this file.](#)

```
00001 /* Generated by Cython 0.29.28 */
00002
00003 /* BEGIN: Cython Metadata
00004 {
00005     "distutils": {
00006         "depends": [
00007             "PyClical.h"
00008         ],
00009         "include_dirs": [
00010             "."
00011         ],
00012         "language": "c++",
00013         "name": "PyClical",
00014         "sources": [
00015             "PyClical.pyx"
00016         ]
00017     },
00018     "module_name": "PyClical"
00019 }
00020 END: Cython Metadata */
00021
00022 #ifndef PY_SSIZE_T_CLEAN
00023 #define PY_SSIZE_T_CLEAN
00024 #endif /* PY_SSIZE_T_CLEAN */
00025 #include "Python.h"
00026 #ifndef Py_PYTHON_H
00027     #error Python headers needed to compile C extensions, please install development version of
        Python.
00028 #elif PY_VERSION_HEX < 0x02060000 || (0x03000000 <= PY_VERSION_HEX && PY_VERSION_HEX < 0x03030000)
00029     #error Cython requires Python 2.6+ or Python 3.3+.
00030 #else
00031 #define CYTHON_ABI "0_29_28"
00032 #define CYTHON_HEX_VERSION 0x001D1CF0
00033 #define CYTHON_FUTURE_DIVISION 1
00034 #include <stddef.h>
00035 #ifndef offsetof
00036     #define offsetof(type, member) ( (size_t) & ((type*)0) -> member )
00037 #endif
00038 #if !defined(WIN32) && !defined(MS_WINDOWS)
00039     #ifndef __stdcall
00040         #define __stdcall
00041     #endif
00042     #ifndef __cdecl
00043         #define __cdecl
00044     #endif
00045     #ifndef __fastcall
00046         #define __fastcall
00047     #endif
00048 #endif
00049 #ifndef DL_IMPORT
00050     #define DL_IMPORT(t) t
00051 #endif
00052 #ifndef DL_EXPORT
00053     #define DL_EXPORT(t) t
00054 #endif
00055 #define __PYX_COMMA ,
00056 #ifndef HAVE_LONG_LONG
00057     #if PY_VERSION_HEX >= 0x02070000
00058         #define HAVE_LONG_LONG
00059     #endif
00060 #endif
00061 #ifndef PY_LONG_LONG
00062     #define PY_LONG_LONG LONG_LONG
00063 #endif
00064 #ifndef Py_HUGE_VAL
00065     #define Py_HUGE_VAL HUGE_VAL
00066 #endif
00067 #ifndef PYPY_VERSION
00068     #define CYTHON_COMPILING_IN_PYPY 1
```

```
00069 #define CYTHON_COMPILING_IN_PYSTON 0
00070 #define CYTHON_COMPILING_IN_CPYTHON 0
00071 #undef CYTHON_USE_TYPE_SLOTS
00072 #define CYTHON_USE_TYPE_SLOTS 0
00073 #undef CYTHON_USE_PYTYPE_LOOKUP
00074 #define CYTHON_USE_PYTYPE_LOOKUP 0
00075 #if PY_VERSION_HEX < 0x03050000
00076     #undef CYTHON_USE_ASYNC_SLOTS
00077     #define CYTHON_USE_ASYNC_SLOTS 0
00078 #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00079     #define CYTHON_USE_ASYNC_SLOTS 1
00080 #endif
00081 #undef CYTHON_USE_PYLIST_INTERNALS
00082 #define CYTHON_USE_PYLIST_INTERNALS 0
00083 #undef CYTHON_USE_UNICODE_INTERNALS
00084 #define CYTHON_USE_UNICODE_INTERNALS 0
00085 #undef CYTHON_USE_UNICODE_WRITER
00086 #define CYTHON_USE_UNICODE_WRITER 0
00087 #undef CYTHON_USE_PYLONG_INTERNALS
00088 #define CYTHON_USE_PYLONG_INTERNALS 0
00089 #undef CYTHON_AVOID_BORROWED_REFS
00090 #define CYTHON_AVOID_BORROWED_REFS 1
00091 #undef CYTHON_ASSUME_SAFE_MACROS
00092 #define CYTHON_ASSUME_SAFE_MACROS 0
00093 #undef CYTHON_UNPACK_METHODS
00094 #define CYTHON_UNPACK_METHODS 0
00095 #undef CYTHON_FAST_THREAD_STATE
00096 #define CYTHON_FAST_THREAD_STATE 0
00097 #undef CYTHON_FAST_PYCALL
00098 #define CYTHON_FAST_PYCALL 0
00099 #undef CYTHON_PEP489_MULTI_PHASE_INIT
00100 #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00101 #undef CYTHON_USE_TP_FINALIZE
00102 #define CYTHON_USE_TP_FINALIZE 0
00103 #undef CYTHON_USE_DICT_VERSIONS
00104 #define CYTHON_USE_DICT_VERSIONS 0
00105 #undef CYTHON_USE_EXC_INFO_STACK
00106 #define CYTHON_USE_EXC_INFO_STACK 0
00107 #elif defined(PYSTON_VERSION)
00108     #define CYTHON_COMPILING_IN_PYPY 0
00109     #define CYTHON_COMPILING_IN_PYSTON 1
00110     #define CYTHON_COMPILING_IN_CPYTHON 0
00111     #ifndef CYTHON_USE_TYPE_SLOTS
00112         #define CYTHON_USE_TYPE_SLOTS 1
00113     #endif
00114     #undef CYTHON_USE_PYTYPE_LOOKUP
00115     #define CYTHON_USE_PYTYPE_LOOKUP 0
00116     #undef CYTHON_USE_ASYNC_SLOTS
00117     #define CYTHON_USE_ASYNC_SLOTS 0
00118     #undef CYTHON_USE_PYLIST_INTERNALS
00119     #define CYTHON_USE_PYLIST_INTERNALS 0
00120     #ifndef CYTHON_USE_UNICODE_INTERNALS
00121         #define CYTHON_USE_UNICODE_INTERNALS 1
00122     #endif
00123     #undef CYTHON_USE_UNICODE_WRITER
00124     #define CYTHON_USE_UNICODE_WRITER 0
00125     #undef CYTHON_USE_PYLONG_INTERNALS
00126     #define CYTHON_USE_PYLONG_INTERNALS 0
00127     #ifndef CYTHON_AVOID_BORROWED_REFS
00128         #define CYTHON_AVOID_BORROWED_REFS 0
00129     #endif
00130     #ifndef CYTHON_ASSUME_SAFE_MACROS
00131         #define CYTHON_ASSUME_SAFE_MACROS 1
00132     #endif
00133     #ifndef CYTHON_UNPACK_METHODS
00134         #define CYTHON_UNPACK_METHODS 1
00135     #endif
00136     #undef CYTHON_FAST_THREAD_STATE
00137     #define CYTHON_FAST_THREAD_STATE 0
00138     #undef CYTHON_FAST_PYCALL
00139     #define CYTHON_FAST_PYCALL 0
00140     #undef CYTHON_PEP489_MULTI_PHASE_INIT
00141     #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00142     #undef CYTHON_USE_TP_FINALIZE
00143     #define CYTHON_USE_TP_FINALIZE 0
00144     #undef CYTHON_USE_DICT_VERSIONS
00145     #define CYTHON_USE_DICT_VERSIONS 0
00146     #undef CYTHON_USE_EXC_INFO_STACK
00147     #define CYTHON_USE_EXC_INFO_STACK 0
00148 #else
00149     #define CYTHON_COMPILING_IN_PYPY 0
00150     #define CYTHON_COMPILING_IN_PYSTON 0
00151     #define CYTHON_COMPILING_IN_CPYTHON 1
00152     #ifndef CYTHON_USE_TYPE_SLOTS
00153         #define CYTHON_USE_TYPE_SLOTS 1
00154     #endif
00155     #if PY_VERSION_HEX < 0x02070000
```



```

00156     #undef CYTHON_USE_PYTYPE_LOOKUP
00157     #define CYTHON_USE_PYTYPE_LOOKUP 0
00158     #elif !defined(CYTHON_USE_PYTYPE_LOOKUP)
00159     #define CYTHON_USE_PYTYPE_LOOKUP 1
00160     #endif
00161     #if PY_MAJOR_VERSION < 3
00162     #undef CYTHON_USE_ASYNC_SLOTS
00163     #define CYTHON_USE_ASYNC_SLOTS 0
00164     #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00165     #define CYTHON_USE_ASYNC_SLOTS 1
00166     #endif
00167     #if PY_VERSION_HEX < 0x02070000
00168     #undef CYTHON_USE_PYLONG_INTERNALS
00169     #define CYTHON_USE_PYLONG_INTERNALS 0
00170     #elif !defined(CYTHON_USE_PYLONG_INTERNALS)
00171     #define CYTHON_USE_PYLONG_INTERNALS 1
00172     #endif
00173     #ifndef CYTHON_USE_PYLIST_INTERNALS
00174     #define CYTHON_USE_PYLIST_INTERNALS 1
00175     #endif
00176     #ifndef CYTHON_USE_UNICODE_INTERNALS
00177     #define CYTHON_USE_UNICODE_INTERNALS 1
00178     #endif
00179     #if PY_VERSION_HEX < 0x030300F0 || PY_VERSION_HEX >= 0x030B00A2
00180     #undef CYTHON_USE_UNICODE_WRITER
00181     #define CYTHON_USE_UNICODE_WRITER 0
00182     #elif !defined(CYTHON_USE_UNICODE_WRITER)
00183     #define CYTHON_USE_UNICODE_WRITER 1
00184     #endif
00185     #ifndef CYTHON_AVOID_BORROWED_REFS
00186     #define CYTHON_AVOID_BORROWED_REFS 0
00187     #endif
00188     #ifndef CYTHON_ASSUME_SAFE_MACROS
00189     #define CYTHON_ASSUME_SAFE_MACROS 1
00190     #endif
00191     #ifndef CYTHON_UNPACK_METHODS
00192     #define CYTHON_UNPACK_METHODS 1
00193     #endif
00194     #if PY_VERSION_HEX >= 0x030B00A4
00195     #undef CYTHON_FAST_THREAD_STATE
00196     #define CYTHON_FAST_THREAD_STATE 0
00197     #elif !defined(CYTHON_FAST_THREAD_STATE)
00198     #define CYTHON_FAST_THREAD_STATE 1
00199     #endif
00200     #ifndef CYTHON_FAST_PYCALL
00201     #define CYTHON_FAST_PYCALL (PY_VERSION_HEX < 0x030B00A1)
00202     #endif
00203     #ifndef CYTHON_PEP489_MULTI_PHASE_INIT
00204     #define CYTHON_PEP489_MULTI_PHASE_INIT (PY_VERSION_HEX >= 0x03050000)
00205     #endif
00206     #ifndef CYTHON_USE_TP_FINALIZE
00207     #define CYTHON_USE_TP_FINALIZE (PY_VERSION_HEX >= 0x030400a1)
00208     #endif
00209     #ifndef CYTHON_USE_DICT_VERSIONS
00210     #define CYTHON_USE_DICT_VERSIONS (PY_VERSION_HEX >= 0x030600B1)
00211     #endif
00212     #if PY_VERSION_HEX >= 0x030B00A4
00213     #undef CYTHON_USE_EXC_INFO_STACK
00214     #define CYTHON_USE_EXC_INFO_STACK 0
00215     #elif !defined(CYTHON_USE_EXC_INFO_STACK)
00216     #define CYTHON_USE_EXC_INFO_STACK (PY_VERSION_HEX >= 0x030700A3)
00217     #endif
00218 #endif
00219 #if !defined(CYTHON_FAST_PYCCALL)
00220 #define CYTHON_FAST_PYCCALL (CYTHON_FAST_PYCALL && PY_VERSION_HEX >= 0x030600B1)
00221 #endif
00222 #if CYTHON_USE_PYLONG_INTERNALS
00223     #if PY_MAJOR_VERSION < 3
00224     #include "longintrepr.h"
00225     #endif
00226     #undef SHIFT
00227     #undef BASE
00228     #undef MASK
00229     #ifdef SIZEOF_VOID_P
00230     enum { __pyx_check_sizeof_voidp = 1 / (int) (SIZEOF_VOID_P == sizeof(void*)) };
00231     #endif
00232 #endif
00233 #ifndef __has_attribute
00234     #define __has_attribute(x) 0
00235 #endif
00236 #ifndef __has_cpp_attribute
00237     #define __has_cpp_attribute(x) 0
00238 #endif
00239 #ifndef CYTHON_RESTRICT
00240     #if defined(__GNUC__)
00241     #define CYTHON_RESTRICT __restrict
00242     #elif defined(_MSC_VER) && _MSC_VER >= 1400

```

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00243     #define CYTHON_RESTRICT __restrict
00244     #elif defined (__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00245     #define CYTHON_RESTRICT restrict
00246     #else
00247     #define CYTHON_RESTRICT
00248     #endif
00249 #endif
00250 #ifndef CYTHON_UNUSED
00251 # if defined(__GNUC__)
00252 #   if !(defined(__cplusplus)) || (__GNUC__ > 3 || (__GNUC__ == 3 && __GNUC_MINOR__ >= 4))
00253 #     define CYTHON_UNUSED __attribute__ ((__unused__))
00254 #   else
00255 #     define CYTHON_UNUSED
00256 #   endif
00257 # elif defined(__ICC) || (defined(__INTEL_COMPILER) && !defined(_MSC_VER))
00258 #   define CYTHON_UNUSED __attribute__ ((__unused__))
00259 # else
00260 #   define CYTHON_UNUSED
00261 # endif
00262 #endif
00263 #ifndef CYTHON_MAYBE_UNUSED_VAR
00264 # if defined(__cplusplus)
00265     template<class T> void CYTHON_MAYBE_UNUSED_VAR( const T& ) { }
00266 # else
00267 #   define CYTHON_MAYBE_UNUSED_VAR(x) (void)(x)
00268 # endif
00269 #endif
00270 #ifndef CYTHON_NCP_UNUSED
00271 # if CYTHON_COMPILING_IN_CPYTHON
00272 #   define CYTHON_NCP_UNUSED
00273 # else
00274 #   define CYTHON_NCP_UNUSED CYTHON_UNUSED
00275 # endif
00276 #endif
00277 #define __Pyx_void_to_None(void_result) ((void)(void_result), Py_INCREF(Py_None), Py_None)
00278 #ifndef _MSC_VER
00279     #ifndef _MSC_STDINT_H_
00280         #if _MSC_VER < 1300
00281             typedef unsigned char    uint8_t;
00282             typedef unsigned int      uint32_t;
00283         #else
00284             typedef unsigned __int8   uint8_t;
00285             typedef unsigned __int32   uint32_t;
00286         #endif
00287     #endif
00288 #else
00289     #include <stdint.h>
00290 #endif
00291 #ifndef CYTHON_FALLTHROUGH
00292     #if defined(__cplusplus) && __cplusplus >= 201103L
00293         #if __has_cpp_attribute(fallthrough)
00294             #define CYTHON_FALLTHROUGH [[fallthrough]]
00295         #elif __has_cpp_attribute(clang::fallthrough)
00296             #define CYTHON_FALLTHROUGH [[clang::fallthrough]]
00297         #elif __has_cpp_attribute(gnu::fallthrough)
00298             #define CYTHON_FALLTHROUGH [[gnu::fallthrough]]
00299         #endif
00300     #endif
00301     #ifndef CYTHON_FALLTHROUGH
00302         #if __has_attribute(fallthrough)
00303             #define CYTHON_FALLTHROUGH __attribute__((fallthrough))
00304         #else
00305             #define CYTHON_FALLTHROUGH
00306         #endif
00307     #endif
00308     #if defined(__clang__) && defined(__apple_build_version__)
00309         #if __apple_build_version__ < 7000000
00310             #undef CYTHON_FALLTHROUGH
00311             #define CYTHON_FALLTHROUGH
00312         #endif
00313     #endif
00314 #endif
00315
00316 #ifndef __cplusplus
00317     #error "Cython files generated with the C++ option must be compiled with a C++ compiler."
00318 #endif
00319 #ifndef CYTHON_INLINE
00320     #if defined(__clang__)
00321         #define CYTHON_INLINE __inline__ __attribute__ ((__unused__))
00322     #else
00323         #define CYTHON_INLINE inline
00324     #endif
00325 #endif
00326 template<typename T>
00327 void __Pyx_call_destructor(T& x) {
00328     x.~T();
00329 }

```

```

00330 template<typename T>
00331 class __Pyx_FakeReference {
00332 public:
00333     __Pyx_FakeReference() : ptr(NULL) { }
00334     __Pyx_FakeReference(const T& ref) : ptr(const_cast<T*>(&ref)) { }
00335     T *operator->() { return ptr; }
00336     T *operator&() { return ptr; }
00337     operator T&() { return *ptr; }
00338     template<typename U> bool operator ==(U other) { return *ptr == other; }
00339     template<typename U> bool operator !=(U other) { return *ptr != other; }
00340 private:
00341     T *ptr;
00342 };
00343
00344 #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x02070600 && !defined(Py_OptimizeFlag)
00345     #define Py_OptimizeFlag 0
00346 #endif
00347 #define __PYX_BUILD_PY_SSIZE_T "n"
00348 #define CYTHON_FORMAT_SSIZE_T "z"
00349 #if PY_MAJOR_VERSION < 3
00350     #define __Pyx_BUILTIN_MODULE_NAME "__builtin__"
00351     #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00352         PyCode_New(a+k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00353     #define __Pyx_DefaultClassType PyClass_Type
00354 #else
00355     #define __Pyx_BUILTIN_MODULE_NAME "builtins"
00356     #define __Pyx_DefaultClassType PyType_Type
00357 #if PY_VERSION_HEX >= 0x030B00A1
00358     static CYTHON_INLINE PyCodeObject* __Pyx_PyCode_New(int a, int k, int l, int s, int f,
00359         PyObject *code, PyObject *c, PyObject *n, PyObject
00360 *v,
00361         PyObject *fv, PyObject *cell, PyObject *fn,
00362         PyObject *name, int fline, PyObject *lnos) {
00363     PyObject *kwds=NULL, *argcount=NULL, *posonlyargcount=NULL, *kwnonlyargcount=NULL;
00364     PyObject *nlocals=NULL, *stacksize=NULL, *flags=NULL, *replace=NULL, *call_result=NULL,
00365     *empty=NULL;
00366     const char *fn_cstr=NULL;
00367     const char *name_cstr=NULL;
00368     PyCodeObject* co=NULL;
00369     PyObject *type, *value, *traceback;
00370     PyErr_Fetch(&type, &value, &traceback);
00371     if (!(kwds=PyDict_New())) goto end;
00372     if (!(argcount=PyLong_FromLong(a))) goto end;
00373     if (!(posonlyargcount=PyLong_FromLong(0))) goto end;
00374     if (PyDict_SetItemString(kwds, "co_argcount", argcount) != 0) goto end;
00375     if (PyDict_SetItemString(kwds, "co_posonlyargcount", posonlyargcount) != 0) goto end;
00376     if (!(kwnonlyargcount=PyLong_FromLong(k))) goto end;
00377     if (PyDict_SetItemString(kwds, "co_kwnonlyargcount", kwnonlyargcount) != 0) goto end;
00378     if (!(nlocals=PyLong_FromLong(l))) goto end;
00379     if (PyDict_SetItemString(kwds, "co_nlocals", nlocals) != 0) goto end;
00380     if (!(stacksize=PyLong_FromLong(s))) goto end;
00381     if (PyDict_SetItemString(kwds, "co_stacksize", stacksize) != 0) goto end;
00382     if (!(flags=PyLong_FromLong(f))) goto end;
00383     if (PyDict_SetItemString(kwds, "co_flags", flags) != 0) goto end;
00384     if (PyDict_SetItemString(kwds, "co_code", code) != 0) goto end;
00385     if (PyDict_SetItemString(kwds, "co_consts", c) != 0) goto end;
00386     if (PyDict_SetItemString(kwds, "co_names", n) != 0) goto end;
00387     if (PyDict_SetItemString(kwds, "co_varnames", v) != 0) goto end;
00388     if (PyDict_SetItemString(kwds, "co_freevars", fv) != 0) goto end;
00389     if (PyDict_SetItemString(kwds, "co_cellvars", cell) != 0) goto end;
00390     if (PyDict_SetItemString(kwds, "co_linetable", lnos) != 0) goto end;
00391     if (!(fn_cstr=PyUnicode_AsUTF8AndSize(fn, NULL))) goto end;
00392     if (!(name_cstr=PyUnicode_AsUTF8AndSize(name, NULL))) goto end;
00393     if (!(co = PyCode_NewEmpty(fn_cstr, name_cstr, fline))) goto end;
00394     if (!(replace = PyObject_GetAttrString((PyObject*)co, "replace"))) goto cleanup_code_too;
00395     if (!(empty = PyTuple_New(0))) goto cleanup_code_too; // unfortunately __pyx_empty_tuple isn't
available here
00396     if (!(call_result = PyObject_Call(replace, empty, kwds))) goto cleanup_code_too;
00397     Py_XDECREF((PyObject*)co);
00398     co = (PyCodeObject*)call_result;
00399     call_result = NULL;
00400     if (0) {
00401         cleanup_code_too:
00402         Py_XDECREF((PyObject*)co);
00403         co = NULL;
00404     }
00405     end:
00406     Py_XDECREF(kwds);
00407     Py_XDECREF(argcount);
00408     Py_XDECREF(posonlyargcount);
00409     Py_XDECREF(kwnonlyargcount);
00410     Py_XDECREF(nlocals);
00411     Py_XDECREF(stacksize);
00412     Py_XDECREF(replace);
00413     Py_XDECREF(call_result);
00414     Py_XDECREF(empty);
00415     if (type) {

```

```

00414         PyErr_Restore(type, value, traceback);
00415     }
00416     return co;
00417 }
00418 #else
00419 #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00420     PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00421 #endif
00422 #define __Pyx_DefaultClassType PyType_Type
00423 #endif
00424 #ifndef Py_TPFLAGS_CHECKTYPES
00425 #define Py_TPFLAGS_CHECKTYPES 0
00426 #endif
00427 #ifndef Py_TPFLAGS_HAVE_INDEX
00428 #define Py_TPFLAGS_HAVE_INDEX 0
00429 #endif
00430 #ifndef Py_TPFLAGS_HAVE_NEWBUFFER
00431 #define Py_TPFLAGS_HAVE_NEWBUFFER 0
00432 #endif
00433 #ifndef Py_TPFLAGS_HAVE_FINALIZE
00434 #define Py_TPFLAGS_HAVE_FINALIZE 0
00435 #endif
00436 #ifndef METH_STACKLESS
00437 #define METH_STACKLESS 0
00438 #endif
00439 #if PY_VERSION_HEX <= 0x030700A3 || !defined(METH_FASTCALL)
00440     #ifndef METH_FASTCALL
00441         #define METH_FASTCALL 0x80
00442     #endif
00443     typedef PyObject *(*__Pyx_PyCFunctionFast) (PyObject *self, PyObject *const *args, Py_ssize_t
nargs);
00444     typedef PyObject *(*__Pyx_PyCFunctionFastWithKeywords) (PyObject *self, PyObject *const *args,
Py_ssize_t nargs, PyObject *kwnames);
00445 #else
00446     #define __Pyx_PyCFunctionFast _PyCFunctionFast
00447     #define __Pyx_PyCFunctionFastWithKeywords _PyCFunctionFastWithKeywords
00448 #endif
00449 #if CYTHON_FAST_PYCCALL
00450 #define __Pyx_PyFastCFunction_Check(func)\
00451     ((PyCFunction_Check(func) && (METH_FASTCALL == (PyCFunction_GET_FLAGS(func) & ~(METH_CLASS |
METH_STATIC | METH_COEXIST | METH_KEYWORDS | METH_STACKLESS))))
00452 #else
00453 #define __Pyx_PyFastCFunction_Check(func) 0
00454 #endif
00455 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Malloc)
00456     #define PyObject_Malloc(s)    PyMem_Malloc(s)
00457     #define PyObject_Free(p)      PyMem_Free(p)
00458     #define PyObject_Realloc(p)   PyMem_Realloc(p)
00459 #endif
00460 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX < 0x030400A1
00461     #define PyMem_RawMalloc(n)     PyMem_Malloc(n)
00462     #define PyMem_RawRealloc(p, n) PyMem_Realloc(p, n)
00463     #define PyMem_RawFree(p)       PyMem_Free(p)
00464 #endif
00465 #if CYTHON_COMPILING_IN_PYSTON
00466     #define __Pyx_PyCode_HasFreeVars(co) PyCode_HasFreeVars(co)
00467     #define __Pyx_PyFrame_SetLineNumber(frame, lineno) PyFrame_SetLineNumber(frame, lineno)
00468 #else
00469     #define __Pyx_PyCode_HasFreeVars(co) (PyCode_GetNumFree(co) > 0)
00470     #define __Pyx_PyFrame_SetLineNumber(frame, lineno) (frame)->f_lineno = (lineno)
00471 #endif
00472 #if !CYTHON_FAST_THREAD_STATE || PY_VERSION_HEX < 0x02070000
00473     #define __Pyx_PyThreadState_Current PyThreadState_GET()
00474     #elif PY_VERSION_HEX >= 0x03060000
00475     #define __Pyx_PyThreadState_Current _PyThreadState_UncheckedGet()
00476     #elif PY_VERSION_HEX >= 0x03000000
00477     #define __Pyx_PyThreadState_Current PyThreadState_GET()
00478 #else
00479     #define __Pyx_PyThreadState_Current _PyThreadState_Current
00480 #endif
00481 #if PY_VERSION_HEX < 0x030700A2 && !defined(PyThread_tss_create) && !defined(Py_tss_NEEDS_INIT)
00482     #include "pthread.h"
00483     #define Py_tss_NEEDS_INIT 0
00484     typedef int Py_tss_t;
00485     static CYTHON_INLINE int PyThread_tss_create(Py_tss_t *key) {
00486         *key = PyThread_create_key();
00487         return 0;
00488     }
00489     static CYTHON_INLINE Py_tss_t * PyThread_tss_alloc(void) {
00490         Py_tss_t *key = (Py_tss_t *)PyObject_Malloc(sizeof(Py_tss_t));
00491         *key = Py_tss_NEEDS_INIT;
00492         return key;
00493     }
00494     static CYTHON_INLINE void PyThread_tss_free(Py_tss_t *key) {
00495         PyObject_Free(key);
00496     }
00497     static CYTHON_INLINE int PyThread_tss_is_created(Py_tss_t *key) {

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00499     return *key != Py_tss_NEEDS_INIT;
00500 }
00501 static CYTHON_INLINE void PyThread_tss_delete(Py_tss_t *key) {
00502     PyThread_delete_key(*key);
00503     *key = Py_tss_NEEDS_INIT;
00504 }
00505 static CYTHON_INLINE int PyThread_tss_set(Py_tss_t *key, void *value) {
00506     return PyThread_set_key_value(*key, value);
00507 }
00508 static CYTHON_INLINE void * PyThread_tss_get(Py_tss_t *key) {
00509     return PyThread_get_key_value(*key);
00510 }
00511 #endif
00512 #if CYTHON_COMPILING_IN_CPYTHON || defined(_PyDict_NewPresized)
00513 #define __Pyx_PyDict_NewPresized(n)  ((n <= 8) ? PyDict_New() : _PyDict_NewPresized(n))
00514 #else
00515 #define __Pyx_PyDict_NewPresized(n)  PyDict_New()
00516 #endif
00517 #if PY_MAJOR_VERSION >= 3 || CYTHON_FUTURE_DIVISION
00518     #define __Pyx_PyNumber_Divide(x,y)      PyNumber_TrueDivide(x,y)
00519     #define __Pyx_PyNumber_InPlaceDivide(x,y)  PyNumber_InPlaceTrueDivide(x,y)
00520 #else
00521     #define __Pyx_PyNumber_Divide(x,y)      PyNumber_Divide(x,y)
00522     #define __Pyx_PyNumber_InPlaceDivide(x,y)  PyNumber_InPlaceDivide(x,y)
00523 #endif
00524 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1 && CYTHON_USE_UNICODE_INTERNALS
00525 #define __Pyx_PyDict_GetItemStr(dict, name)  _PyDict_GetItem_KnownHash(dict, name, ((PyASCIIObject *)
    name)->hash)
00526 #else
00527 #define __Pyx_PyDict_GetItemStr(dict, name)  PyDict_GetItem(dict, name)
00528 #endif
00529 #if PY_VERSION_HEX > 0x03030000 && defined(PyUnicode_KIND)
00530     #define CYTHON_PEP393_ENABLED 1
00531     #if defined(PyUnicode_IS_READY)
00532     #define __Pyx_PyUnicode_READY(op)       (likely(PyUnicode_IS_READY(op)) ?\
00533         0 : _PyUnicode_Ready((PyObject *) (op)))
00534     #else
00535     #define __Pyx_PyUnicode_READY(op)       (0)
00536     #endif
00537     #define __Pyx_PyUnicode_GET_LENGTH(u)   PyUnicode_GET_LENGTH(u)
00538     #define __Pyx_PyUnicode_READ_CHAR(u, i) PyUnicode_READ_CHAR(u, i)
00539     #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u)   PyUnicode_MAX_CHAR_VALUE(u)
00540     #define __Pyx_PyUnicode_KIND(u)        PyUnicode_KIND(u)
00541     #define __Pyx_PyUnicode_DATA(u)        PyUnicode_DATA(u)
00542     #define __Pyx_PyUnicode_READ(k, d, i)  PyUnicode_READ(k, d, i)
00543     #define __Pyx_PyUnicode_WRITE(k, d, i, ch)  PyUnicode_WRITE(k, d, i, ch)
00544     #if defined(PyUnicode_IS_READY) && defined(PyUnicode_GET_SIZE)
00545     #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03090000
00546     #define __Pyx_PyUnicode_IS_TRUE(u)      (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : ((PyCompactUnicodeObject *) (u))->wstr_length))
00547     #else
00548     #define __Pyx_PyUnicode_IS_TRUE(u)      (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : PyUnicode_GET_SIZE(u)))
00549     #endif
00550     #else
00551     #define __Pyx_PyUnicode_IS_TRUE(u)      (0 != PyUnicode_GET_LENGTH(u))
00552     #endif
00553 #else
00554     #define CYTHON_PEP393_ENABLED 0
00555     #define PyUnicode_1BYTE_KIND 1
00556     #define PyUnicode_2BYTE_KIND 2
00557     #define PyUnicode_4BYTE_KIND 4
00558     #define __Pyx_PyUnicode_READY(op)       (0)
00559     #define __Pyx_PyUnicode_GET_LENGTH(u)   PyUnicode_GET_SIZE(u)
00560     #define __Pyx_PyUnicode_READ_CHAR(u, i) ((Py_UCS4) PyUnicode_AS_UNICODE(u)[i])
00561     #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u)   ((sizeof(Py_UNICODE) == 2) ? 65535 : 1114111)
00562     #define __Pyx_PyUnicode_KIND(u)        (sizeof(Py_UNICODE))
00563     #define __Pyx_PyUnicode_DATA(u)        ((void*)PyUnicode_AS_UNICODE(u))
00564     #define __Pyx_PyUnicode_READ(k, d, i)  ((void)(k), (Py_UCS4) (((Py_UNICODE*)d)[i]))
00565     #define __Pyx_PyUnicode_WRITE(k, d, i, ch)  (((void)(k)), ((Py_UNICODE*)d)[i] = ch)
00566     #define __Pyx_PyUnicode_IS_TRUE(u)      (0 != PyUnicode_GET_SIZE(u))
00567 #endif
00568 #if CYTHON_COMPILING_IN_PYPY
00569     #define __Pyx_PyUnicode_Concat(a, b)      PyNumber_Add(a, b)
00570     #define __Pyx_PyUnicode_ConcatSafe(a, b)  PyNumber_Add(a, b)
00571 #else
00572     #define __Pyx_PyUnicode_Concat(a, b)      PyUnicode_Concat(a, b)
00573     #define __Pyx_PyUnicode_ConcatSafe(a, b)  ((unlikely((a) == Py_None) || unlikely((b) == Py_None)) ?\
PyNumber_Add(a, b) : __Pyx_PyUnicode_Concat(a, b))
00574 #endif
00575 #if CYTHON_COMPILING_IN_PYPY && !defined(PyUnicode_Contains)
00576     #define PyUnicode_Contains(u, s)  PySequence_Contains(u, s)
00577 #endif
00578 #if CYTHON_COMPILING_IN_PYPY && !defined(PyByteArray_Check)
00579     #define PyByteArray_Check(obj)  PyObject_TypeCheck(obj, &PyByteArray_Type)
00580 #endif
00581 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Format)

```

```

00583 #define PyObject_Format(obj, fmt) PyObject_CallMethod(obj, "__format__", "O", fmt)
00584 #endif
00585 #define __Pyx_PyString_FormatSafe(a, b) ((unlikely((a) == Py_None || (PyString_Check(b) &&
!PyString_CheckExact(b)))) ? PyNumber_Remainder(a, b) : __Pyx_PyString_Format(a, b))
00586 #define __Pyx_PyUnicode_FormatSafe(a, b) ((unlikely((a) == Py_None || (PyUnicode_Check(b) &&
!PyUnicode_CheckExact(b)))) ? PyNumber_Remainder(a, b) : PyUnicode_Format(a, b))
00587 #if PY_MAJOR_VERSION >= 3
00588 #define __Pyx_PyString_Format(a, b) PyUnicode_Format(a, b)
00589 #else
00590 #define __Pyx_PyString_Format(a, b) PyString_Format(a, b)
00591 #endif
00592 #if PY_MAJOR_VERSION < 3 && !defined(PyObject_ASCII)
00593 #define PyObject_ASCII(o) PyObject_Repr(o)
00594 #endif
00595 #if PY_MAJOR_VERSION >= 3
00596 #define PyBaseString_Type PyUnicode_Type
00597 #define PyStringObject PyUnicodeObject
00598 #define PyString_Type PyUnicode_Type
00599 #define PyString_Check PyUnicode_Check
00600 #define PyString_CheckExact PyUnicode_CheckExact
00601 #ifndef PyObject_Unicode
00602 #define PyObject_Unicode PyObject_Str
00603 #endif
00604 #endif
00605 #if PY_MAJOR_VERSION >= 3
00606 #define __Pyx_PyBaseString_Check(obj) PyUnicode_Check(obj)
00607 #define __Pyx_PyBaseString_CheckExact(obj) PyUnicode_CheckExact(obj)
00608 #else
00609 #define __Pyx_PyBaseString_Check(obj) (PyString_Check(obj) || PyUnicode_Check(obj))
00610 #define __Pyx_PyBaseString_CheckExact(obj) (PyString_CheckExact(obj) || PyUnicode_CheckExact(obj))
00611 #endif
00612 #ifndef PySet_CheckExact
00613 #define PySet_CheckExact(obj) (Py_TYPE(obj) == &PySet_Type)
00614 #endif
00615 #if PY_VERSION_HEX >= 0x030900A4
00616 #define __Pyx_SET_REFCNT(obj, refcnt) Py_SET_REFCNT(obj, refcnt)
00617 #define __Pyx_SET_SIZE(obj, size) Py_SET_SIZE(obj, size)
00618 #else
00619 #define __Pyx_SET_REFCNT(obj, refcnt) Py_REFCNT(obj) = (refcnt)
00620 #define __Pyx_SET_SIZE(obj, size) Py_SIZE(obj) = (size)
00621 #endif
00622 #if CYTHON_ASSUME_SAFE_MACROS
00623 #define __Pyx_PySequence_SIZE(seq) Py_SIZE(seq)
00624 #else
00625 #define __Pyx_PySequence_SIZE(seq) PySequence_Size(seq)
00626 #endif
00627 #if PY_MAJOR_VERSION >= 3
00628 #define PyIntObject PyLongObject
00629 #define PyInt_Type PyLong_Type
00630 #define PyInt_Check(op) PyLong_Check(op)
00631 #define PyInt_CheckExact(op) PyLong_CheckExact(op)
00632 #define PyInt_FromString PyLong_FromString
00633 #define PyInt_FromUnicode PyLong_FromUnicode
00634 #define PyInt_FromLong PyLong_FromLong
00635 #define PyInt_FromSize_t PyLong_FromSize_t
00636 #define PyInt_FromSsize_t PyLong_FromSsize_t
00637 #define PyInt_AsLong PyLong_AsLong
00638 #define PyInt_AS_LONG PyLong_AS_LONG
00639 #define PyInt_AsSsize_t PyLong_AsSsize_t
00640 #define PyInt_AsUnsignedLongMask PyLong_AsUnsignedLongMask
00641 #define PyInt_AsUnsignedLongLongMask PyLong_AsUnsignedLongLongMask
00642 #define PyNumber_Int PyNumber_Long
00643 #endif
00644 #if PY_MAJOR_VERSION >= 3
00645 #define PyBoolObject PyLongObject
00646 #endif
00647 #if PY_MAJOR_VERSION >= 3 && CYTHON_COMPILING_IN_PYPY
00648 #ifndef PyUnicode_InternFromString
00649 #define PyUnicode_InternFromString(s) PyUnicode_FromString(s)
00650 #endif
00651 #endif
00652 #if PY_VERSION_HEX < 0x030200A4
00653 typedef long Py_hash_t;
00654 #define __Pyx_PyInt_FromHash_t PyInt_FromLong
00655 #define __Pyx_PyInt_AsHash_t __Pyx_PyIndex_AsHash_t
00656 #else
00657 #define __Pyx_PyInt_FromHash_t PyInt_FromSsize_t
00658 #define __Pyx_PyInt_AsHash_t __Pyx_PyIndex_AsSsize_t
00659 #endif
00660 #if PY_MAJOR_VERSION >= 3
00661 #define __Pyx_PyMethod_New(func, self, klass) ((self) ? ((void)(klass), PyMethod_New(func, self)) :
__Pyx_NewRef(func))
00662 #else
00663 #define __Pyx_PyMethod_New(func, self, klass) PyMethod_New(func, self, klass)
00664 #endif
00665 #if CYTHON_USE_ASYNC_SLOTS
00666 #if PY_VERSION_HEX >= 0x030500B1

```

```

00667     #define __Pyx_PyAsyncMethodsStruct PyAsyncMethods
00668     #define __Pyx_PyType_AsAsync(obj) (Py_TYPE(obj)->tp_as_async)
00669     #else
00670     #define __Pyx_PyType_AsAsync(obj) ((__Pyx_PyAsyncMethodsStruct*) (Py_TYPE(obj)->tp_reserved))
00671     #endif
00672     #else
00673     #define __Pyx_PyType_AsAsync(obj) NULL
00674     #endif
00675     #ifndef __Pyx_PyAsyncMethodsStruct
00676     typedef struct {
00677         unaryfunc am_await;
00678         unaryfunc am_aiter;
00679         unaryfunc am_anext;
00680     } __Pyx_PyAsyncMethodsStruct;
00681     #endif
00682
00683     #if defined(WIN32) || defined(MS_WINDOWS)
00684     #define _USE_MATH_DEFINES
00685     #endif
00686     #include <math.h>
00687     #ifndef NAN
00688     #define __PYX_NAN() ((float) NAN)
00689     #else
00690     static CYTHON_INLINE float __PYX_NAN() {
00691         float value;
00692         memset(&value, 0xFF, sizeof(value));
00693         return value;
00694     }
00695     #endif
00696     #if defined(__CYGWIN__) && defined(_LDBL_EQ_DBL)
00697     #define __Pyx_trunc1 trunc
00698     #else
00699     #define __Pyx_trunc1 trunc1
00700     #endif
00701
00702     #define __PYX_MARK_ERR_POS(f_index, lineno) \
00703     { __pyx_filename = __pyx_f[f_index]; (void)__pyx_filename; __pyx_lineno = lineno; \
      (void)__pyx_lineno; __pyx_clineno = __LINE__; (void)__pyx_clineno; }
00704     #define __PYX_ERR(f_index, lineno, Ln_error) \
00705     { __PYX_MARK_ERR_POS(f_index, lineno) goto Ln_error; }
00706
00707     #ifndef __PYX_EXTERN_C
00708     #ifdef __cplusplus
00709     #define __PYX_EXTERN_C extern "C"
00710     #else
00711     #define __PYX_EXTERN_C extern
00712     #endif
00713     #endif
00714
00715     #define __PYX_HAVE__PyClical
00716     #define __PYX_HAVE_API__PyClical
00717     /* Early includes */
00718     #include "ios"
00719     #include "new"
00720     #include "stdexcept"
00721     #include "typeinfo"
00722     #include <vector>
00723     #include "PyClical.h"
00724     #include <string.h>
00725     #include <string>
00726     #ifdef _OPENMP
00727     #include <omp.h>
00728     #endif /* _OPENMP */
00729
00730     #if defined(PYREX_WITHOUT_ASSERTIONS) && !defined(CYTHON_WITHOUT_ASSERTIONS)
00731     #define CYTHON_WITHOUT_ASSERTIONS
00732     #endif
00733
00734     typedef struct {PyObject **p; const char *s; const Py_ssize_t n; const char* encoding;
00735         const char is_unicode; const char is_str; const char intern; } __Pyx_StringTabEntry;
00736
00737     #define __PYX_DEFAULT_STRING_ENCODING_IS_ASCII 0
00738     #define __PYX_DEFAULT_STRING_ENCODING_IS_UTF8 0
00739     #define __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT (PY_MAJOR_VERSION >= 3 &&
      __PYX_DEFAULT_STRING_ENCODING_IS_UTF8)
00740     #define __PYX_DEFAULT_STRING_ENCODING ""
00741     #define __Pyx_PyObject_FromString __Pyx_PyBytes_FromString
00742     #define __Pyx_PyObject_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00743     #define __Pyx_uchar_cast(c) ((unsigned char)c)
00744     #define __Pyx_long_cast(x) ((long)x)
00745     #define __Pyx_fits_Py_ssize_t(v, type, is_signed) (\
00746         (sizeof(type) < sizeof(Py_ssize_t)) ||\
00747         (sizeof(type) > sizeof(Py_ssize_t) &&\
00748          likely(v < (type)PY_SSIZE_T_MAX ||\
00749               v == (type)PY_SSIZE_T_MAX) &&\
00750         (!is_signed || likely(v > (type)PY_SSIZE_T_MIN ||\
00751                               v == (type)PY_SSIZE_T_MIN))) ||\

```



```

00752     (sizeof(type) == sizeof(Py_ssize_t) &&\
00753     (is_signed || likely(v < (type)PY_SSIZE_T_MAX ||\
00754     v == (type)PY_SSIZE_T_MAX))) )
00755 static CYTHON_INLINE int __Pyx_is_valid_index(Py_ssize_t i, Py_ssize_t limit) {
00756     return (size_t) i < (size_t) limit;
00757 }
00758 #if defined (__cplusplus) && __cplusplus >= 201103L
00759     #include <cstdlib>
00760     #define __Pyx_sst_abs(value) std::abs(value)
00761 #elif SIZEOF_INT >= SIZEOF_SIZE_T
00762     #define __Pyx_sst_abs(value) abs(value)
00763 #elif SIZEOF_LONG >= SIZEOF_SIZE_T
00764     #define __Pyx_sst_abs(value) labs(value)
00765 #elif defined (_MSC_VER)
00766     #define __Pyx_sst_abs(value) ((Py_ssize_t)_abs64(value))
00767 #elif defined (__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00768     #define __Pyx_sst_abs(value) llabs(value)
00769 #elif defined (__GNUC__)
00770     #define __Pyx_sst_abs(value) __builtin_llabs(value)
00771 #else
00772     #define __Pyx_sst_abs(value) ((value<0) ? -value : value)
00773 #endif
00774 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject*);
00775 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject*, Py_ssize_t* length);
00776 #define __Pyx_PyByteArray_FromString(s) PyByteArray_FromStringAndSize((const char*)s, strlen((const
    char*)s))
00777 #define __Pyx_PyByteArray_FromStringAndSize(s, l) PyByteArray_FromStringAndSize((const char*)s, l)
00778 #define __Pyx_PyBytes_FromString PyBytes_FromString
00779 #define __Pyx_PyBytes_FromStringAndSize PyBytes_FromStringAndSize
00780 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char*);
00781 #if PY_MAJOR_VERSION < 3
00782     #define __Pyx_PyStr_FromString __Pyx_PyBytes_FromString
00783     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00784 #else
00785     #define __Pyx_PyStr_FromString __Pyx_PyUnicode_FromString
00786     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyUnicode_FromStringAndSize
00787 #endif
00788 #define __Pyx_PyBytes_AsWritableString(s) ((char*) PyBytes_AS_STRING(s))
00789 #define __Pyx_PyBytes_AsWritableSString(s) ((signed char*) PyBytes_AS_STRING(s))
00790 #define __Pyx_PyBytes_AsWritableUString(s) ((unsigned char*) PyBytes_AS_STRING(s))
00791 #define __Pyx_PyBytes_AsString(s) ((const char*) PyBytes_AS_STRING(s))
00792 #define __Pyx_PyBytes_AsSString(s) ((const signed char*) PyBytes_AS_STRING(s))
00793 #define __Pyx_PyBytes_AsUString(s) ((const unsigned char*) PyBytes_AS_STRING(s))
00794 #define __Pyx_PyObject_AsWritableString(s) ((char*) __Pyx_PyObject_AsString(s))
00795 #define __Pyx_PyObject_AsWritableSString(s) ((signed char*) __Pyx_PyObject_AsString(s))
00796 #define __Pyx_PyObject_AsWritableUString(s) ((unsigned char*) __Pyx_PyObject_AsString(s))
00797 #define __Pyx_PyObject_AsSString(s) ((const signed char*) __Pyx_PyObject_AsString(s))
00798 #define __Pyx_PyObject_AsUString(s) ((const unsigned char*) __Pyx_PyObject_AsString(s))
00799 #define __Pyx_PyObject_FromCString(s) __Pyx_PyObject_FromString((const char*)s)
00800 #define __Pyx_PyBytes_FromCString(s) __Pyx_PyBytes_FromString((const char*)s)
00801 #define __Pyx_PyByteArray_FromCString(s) __Pyx_PyByteArray_FromString((const char*)s)
00802 #define __Pyx_PyStr_FromCString(s) __Pyx_PyStr_FromString((const char*)s)
00803 #define __Pyx_PyUnicode_FromCString(s) __Pyx_PyUnicode_FromString((const char*)s)
00804 static CYTHON_INLINE size_t __Pyx_Py_UNICODE_strlen(const Py_UNICODE *u) {
00805     const Py_UNICODE *u_end = u;
00806     while (*u_end++);
00807     return (size_t)(u_end - u - 1);
00808 }
00809 #define __Pyx_PyUnicode_FromUnicode(u) PyUnicode_FromUnicode(u, __Pyx_Py_UNICODE_strlen(u))
00810 #define __Pyx_PyUnicode_FromUnicodeAndLength PyUnicode_FromUnicode
00811 #define __Pyx_PyUnicode_AsUnicode PyUnicode_AsUnicode
00812 #define __Pyx_NewRef(obj) (Py_INCREF(obj), obj)
00813 #define __Pyx_Owned_Py_None(b) __Pyx_NewRef(Py_None)
00814 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b);
00815 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject*);
00816 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject*);
00817 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x);
00818 #define __Pyx_PySequence_Tuple(obj)\
    (likely(PyTuple_CheckExact(obj)) ? __Pyx_NewRef(obj) : PySequence_Tuple(obj))
00820 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject*);
00821 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t);
00822 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject*);
00823 #if CYTHON_ASSUME_SAFE_MACROS
00824 #define __pyx_PyFloat_AsDouble(x) (PyFloat_CheckExact(x) ? PyFloat_AS_DOUBLE(x) : PyFloat_AsDouble(x))
00825 #else
00826 #define __pyx_PyFloat_AsDouble(x) PyFloat_AsDouble(x)
00827 #endif
00828 #define __pyx_PyFloat_AsFloat(x) ((float) __pyx_PyFloat_AsDouble(x))
00829 #if PY_MAJOR_VERSION >= 3
00830 #define __Pyx_PyNumber_Int(x) (PyLong_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Long(x))
00831 #else
00832 #define __Pyx_PyNumber_Int(x) (PyInt_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Int(x))
00833 #endif
00834 #define __Pyx_PyNumber_Float(x) (PyFloat_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Float(x))
00835 #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
00836 static int __Pyx_sys_getdefaultencoding_not_ascii;
00837 static int __Pyx_init_sys_getdefaultencoding_params(void) {

```



```

00838     PyObject* sys;
00839     PyObject* default_encoding = NULL;
00840     PyObject* ascii_chars_u = NULL;
00841     PyObject* ascii_chars_b = NULL;
00842     const char* default_encoding_c;
00843     sys = PyImport_ImportModule("sys");
00844     if (!sys) goto bad;
00845     default_encoding = PyObject_CallMethod(sys, (char*) "getdefaultencoding", NULL);
00846     Py_DECREF(sys);
00847     if (!default_encoding) goto bad;
00848     default_encoding_c = PyBytes_AsString(default_encoding);
00849     if (!default_encoding_c) goto bad;
00850     if (strcmp(default_encoding_c, "ascii") == 0) {
00851         __Pyx_sys_getdefaultencoding_not_ascii = 0;
00852     } else {
00853         char ascii_chars[128];
00854         int c;
00855         for (c = 0; c < 128; c++) {
00856             ascii_chars[c] = c;
00857         }
00858         __Pyx_sys_getdefaultencoding_not_ascii = 1;
00859         ascii_chars_u = PyUnicode_DecodeASCII(ascii_chars, 128, NULL);
00860         if (!ascii_chars_u) goto bad;
00861         ascii_chars_b = PyUnicode_AsEncodedString(ascii_chars_u, default_encoding_c, NULL);
00862         if (!ascii_chars_b || !PyBytes_Check(ascii_chars_b) || memcmp(ascii_chars,
PyBytes_AS_STRING(ascii_chars_b), 128) != 0) {
00863             PyErr_Format(
00864                 PyExc_ValueError,
00865                 "This module compiled with c_string_encoding=ascii, but default encoding '%.200s' is
not a superset of ascii.",
00866                 default_encoding_c);
00867             goto bad;
00868         }
00869         Py_DECREF(ascii_chars_u);
00870         Py_DECREF(ascii_chars_b);
00871     }
00872     Py_DECREF(default_encoding);
00873     return 0;
00874 bad:
00875     Py_XDECREF(default_encoding);
00876     Py_XDECREF(ascii_chars_u);
00877     Py_XDECREF(ascii_chars_b);
00878     return -1;
00879 }
00880 #endif
00881 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT && PY_MAJOR_VERSION >= 3
00882 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_DecodeUTF8(c_str, size, NULL)
00883 #else
00884 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_Decode(c_str, size,
__PYX_DEFAULT_STRING_ENCODING, NULL)
00885 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
00886 static char* __PYX_DEFAULT_STRING_ENCODING;
00887 static int __Pyx_init_sys_getdefaultencoding_params(void) {
00888     PyObject* sys;
00889     PyObject* default_encoding = NULL;
00890     char* default_encoding_c;
00891     sys = PyImport_ImportModule("sys");
00892     if (!sys) goto bad;
00893     default_encoding = PyObject_CallMethod(sys, (char*) (const char*) "getdefaultencoding", NULL);
00894     Py_DECREF(sys);
00895     if (!default_encoding) goto bad;
00896     default_encoding_c = PyBytes_AsString(default_encoding);
00897     if (!default_encoding_c) goto bad;
00898     __PYX_DEFAULT_STRING_ENCODING = (char*) malloc(strlen(default_encoding_c) + 1);
00899     if (!__PYX_DEFAULT_STRING_ENCODING) goto bad;
00900     strcpy(__PYX_DEFAULT_STRING_ENCODING, default_encoding_c);
00901     Py_DECREF(default_encoding);
00902     return 0;
00903 bad:
00904     Py_XDECREF(default_encoding);
00905     return -1;
00906 }
00907 #endif
00908 #endif
00909
00910
00911 /* Test for GCC > 2.95 */
00912 #if defined(__GNUC__) && (__GNUC__ > 2 || (__GNUC__ == 2 && (__GNUC_MINOR__ > 95)))
00913 #define likely(x) __builtin_expect(!!(x), 1)
00914 #define unlikely(x) __builtin_expect(!!(x), 0)
00915 #else /* !__GNUC__ or GCC < 2.95 */
00916 #define likely(x) (x)
00917 #define unlikely(x) (x)
00918 #endif /* __GNUC__ */
00919 static CYTHON_INLINE void __Pyx_pretend_to_initialize(void* ptr) { (void)ptr; }
00920
00921 static PyObject * __pyx_m = NULL;

```

```

00922 static PyObject * __pyx_d;
00923 static PyObject * __pyx_b;
00924 static PyObject * __pyx_cython_runtime = NULL;
00925 static PyObject * __pyx_empty_tuple;
00926 static PyObject * __pyx_empty_bytes;
00927 static PyObject * __pyx_empty_unicode;
00928 static int __pyx_lineno;
00929 static int __pyx_clineno = 0;
00930 static const char * __pyx_cfilenm= __FILE__;
00931 static const char * __pyx_filename;
00932
00933
00934 static const char * __pyx_f[] = {
00935     "PyClicl.pyx",
00936     "stringsource",
00937 };
00938
00939 /*--- Type declarations ---*/
00940 struct __pyx_obj_8PyClicl_index_set;
00941 struct __pyx_obj_8PyClicl_clifford;
00942 struct __pyx_obj_8PyClicl__pyx_scope_struct____iter__;
00943 struct __pyx_opt_args_8PyClicl_approx_equal;
00944 struct __pyx_opt_args_8PyClicl_sqrt;
00945 struct __pyx_opt_args_8PyClicl_log;
00946 struct __pyx_opt_args_8PyClicl_cos;
00947 struct __pyx_opt_args_8PyClicl_acos;
00948 struct __pyx_opt_args_8PyClicl_acosh;
00949 struct __pyx_opt_args_8PyClicl_sin;
00950 struct __pyx_opt_args_8PyClicl_asin;
00951 struct __pyx_opt_args_8PyClicl_asinh;
00952 struct __pyx_opt_args_8PyClicl_tan;
00953 struct __pyx_opt_args_8PyClicl_atan;
00954 struct __pyx_opt_args_8PyClicl_atanh;
00955 struct __pyx_opt_args_8PyClicl_random_clifford;
00956
00957 /* "PyClicl.pyx":1359
00958 *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
00959 *
00960 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):           # <<<<<<<<
00961 *     """
00962 *     Test for approximate equality of multivectors.
00963 */
00964 struct __pyx_opt_args_8PyClicl_approx_equal {
00965     int __pyx_n;
00966     PyObject *threshold;
00967     PyObject *tol;
00968 };
00969
00970 /* "PyClicl.pyx":1591
00971 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
00972 *
00973 * cpdef inline sqrt(obj, i = None):           # <<<<<<<<
00974 *     """
00975 *     Square root of multivector with optional complexifier.
00976 */
00977 struct __pyx_opt_args_8PyClicl_sqrt {
00978     int __pyx_n;
00979     PyObject *i;
00980 };
00981
00982 /* "PyClicl.pyx":1628
00983 *     return clifford().wrap( glucat.exp(toClifford(obj)) )
00984 *
00985 * cpdef inline log(obj,i = None):           # <<<<<<<<
00986 *     """
00987 *     Natural logarithm of multivector with optional complexifier.
00988 */
00989 struct __pyx_opt_args_8PyClicl_log {
00990     int __pyx_n;
00991     PyObject *i;
00992 };
00993
00994 /* "PyClicl.pyx":1651
00995 *     return clifford().wrap( glucat.log(toClifford(obj)) )
00996 *
00997 * cpdef inline cos(obj,i = None):           # <<<<<<<<
00998 *     """
00999 *     Cosine of multivector with optional complexifier.
01000 */
01001 struct __pyx_opt_args_8PyClicl_cos {
01002     int __pyx_n;
01003     PyObject *i;
01004 };
01005
01006 /* "PyClicl.pyx":1668
01007 *     return clifford().wrap( glucat.cos(toClifford(obj)) )
01008 *

```

```

01009 * cpdef inline acos(obj,i = None):          # ««««««««
01010 *      """
01011 *      Inverse cosine of multivector with optional complexifier.
01012 */
01013 struct __pyx_opt_args_8PyClical_acos {
01014     int __pyx_n;
01015     PyObject *i;
01016 };
01017
01018 /* "PyClical.pyx":1705
01019 *      return clifford().wrap( glucat.cosh(toClifford(obj)) )
01020 *
01021 * cpdef inline acosh(obj,i = None):          # ««««««««
01022 *      """
01023 *      Inverse hyperbolic cosine of multivector with optional complexifier.
01024 */
01025 struct __pyx_opt_args_8PyClical_acosh {
01026     int __pyx_n;
01027     PyObject *i;
01028 };
01029
01030 /* "PyClical.pyx":1728
01031 *      return clifford().wrap( glucat.acosh(toClifford(obj)) )
01032 *
01033 * cpdef inline sin(obj,i = None):           # ««««««««
01034 *      """
01035 *      Sine of multivector with optional complexifier.
01036 */
01037 struct __pyx_opt_args_8PyClical_sin {
01038     int __pyx_n;
01039     PyObject *i;
01040 };
01041
01042 /* "PyClical.pyx":1747
01043 *      return clifford().wrap( glucat.sin(toClifford(obj)) )
01044 *
01045 * cpdef inline asin(obj,i = None):          # ««««««««
01046 *      """
01047 *      Inverse sine of multivector with optional complexifier.
01048 */
01049 struct __pyx_opt_args_8PyClical_asin {
01050     int __pyx_n;
01051     PyObject *i;
01052 };
01053
01054 /* "PyClical.pyx":1782
01055 *      return clifford().wrap( glucat.sinh(toClifford(obj)) )
01056 *
01057 * cpdef inline asinh(obj,i = None):         # ««««««««
01058 *      """
01059 *      Inverse hyperbolic sine of multivector with optional complexifier.
01060 */
01061 struct __pyx_opt_args_8PyClical_asinh {
01062     int __pyx_n;
01063     PyObject *i;
01064 };
01065
01066 /* "PyClical.pyx":1801
01067 *      return clifford().wrap( glucat.asinh(toClifford(obj)) )
01068 *
01069 * cpdef inline tan(obj,i = None):           # ««««««««
01070 *      """
01071 *      Tangent of multivector with optional complexifier.
01072 */
01073 struct __pyx_opt_args_8PyClical_tan {
01074     int __pyx_n;
01075     PyObject *i;
01076 };
01077
01078 /* "PyClical.pyx":1818
01079 *      return clifford().wrap( glucat.tan(toClifford(obj)) )
01080 *
01081 * cpdef inline atan(obj,i = None):          # ««««««««
01082 *      """
01083 *      Inverse tangent of multivector with optional complexifier.
01084 */
01085 struct __pyx_opt_args_8PyClical_atan {
01086     int __pyx_n;
01087     PyObject *i;
01088 };
01089
01090 /* "PyClical.pyx":1847
01091 *      return clifford().wrap( glucat.tanh(toClifford(obj)) )
01092 *
01093 * cpdef inline atanh(obj,i = None):         # ««««««««
01094 *      """
01095 *      Inverse hyperbolic tangent of multivector with optional complexifier.

```

```

01096 */
01097 struct __pyx_opt_args_8PyClical_atanh {
01098     int __pyx_n;
01099     PyObject *i;
01100 };
01101
01102 /* "PyClical.pyx":1864
01103 *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
01104 *
01105 * cdef inline random_clifford(index_set ixt, fill = 1.0):           # ««««««««
01106 *     """
01107 *         Random multivector within a frame.
01108 */
01109 struct __pyx_opt_args_8PyClical_random_clifford {
01110     int __pyx_n;
01111     PyObject *fill;
01112 };
01113
01114 /* "PyClical.pyx":38
01115 *
01116 * # Forward reference
01117 * cdef class index_set           # ««««««««
01118 *
01119 * cdef inline IndexSet toIndexSet(obj):
01120 */
01121 struct __pyx_obj_8PyClical_index_set {
01122     PyObject_HEAD
01123     struct __pyx_vtabstruct_8PyClical_index_set *__pyx_vtab;
01124     IndexSet *instance;
01125 };
01126
01127
01128 /* "PyClical.pyx":532
01129 *
01130 * # Forward reference.
01131 * cdef class clifford           # ««««««««
01132 *
01133 * cdef inline Clifford toClifford(obj):
01134 */
01135 struct __pyx_obj_8PyClical_clifford {
01136     PyObject_HEAD
01137     struct __pyx_vtabstruct_8PyClical_clifford *__pyx_vtab;
01138     Clifford *instance;
01139 };
01140
01141
01142 /* "PyClical.pyx":229
01143 *         return self.instance.getitem(idx)
01144 *
01145 *     def __iter__(self):           # ««««««««
01146 *         """
01147 *         Iterate over the indices of an index_set.
01148 */
01149 struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ {
01150     PyObject_HEAD
01151     PyObject *__pyx_v_idx;
01152     struct __pyx_obj_8PyClical_index_set *__pyx_v_self;
01153     PyObject *__pyx_t_0;
01154     Py_ssize_t __pyx_t_1;
01155     PyObject *(*__pyx_t_2)(PyObject *);
01156 };
01157
01158
01159
01160 /* "PyClical.pyx":46
01161 *         return index_set(obj).instance[0]
01162 *
01163 * cdef class index_set:           # ««««««««
01164 *     """
01165 *         Python class index_set wraps C++ class IndexSet.
01166 */
01167
01168 struct __pyx_vtabstruct_8PyClical_index_set {
01169     PyObject *(*wrap)(struct __pyx_obj_8PyClical_index_set *, IndexSet);
01170     IndexSet *(*unwrap)(struct __pyx_obj_8PyClical_index_set *);
01171     PyObject *(*copy)(struct __pyx_obj_8PyClical_index_set *, int __pyx_skip_dispatch);
01172 };
01173 static struct __pyx_vtabstruct_8PyClical_index_set *__pyx_vtabptr_8PyClical_index_set;
01174 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
01175 *, IndexSet);
01176 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
01177 *);
01178
01179 /* "PyClical.pyx":537
01180 *         return clifford(obj).instance[0]

```

```

01181 * cdef class clifford:                # «««««««
01182 *     """
01183 *     Python class clifford wraps C++ class Clifford.
01184 */
01185
01186 struct __pyx_vtabstruct_8PyClical_clifford {
01187     PyObject *(*wrap)(struct __pyx_obj_8PyClical_clifford *, Clifford);
01188     Clifford (*unwrap)(struct __pyx_obj_8PyClical_clifford *);
01189     PyObject *(*copy)(struct __pyx_obj_8PyClical_clifford *, int __pyx_skip_dispatch);
01190 };
01191 static struct __pyx_vtabstruct_8PyClical_clifford *__pyx_vtabptr_8PyClical_clifford;
01192 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct __pyx_obj_8PyClical_clifford *,
01193     Clifford);
01194 static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct __pyx_obj_8PyClical_clifford
01195     *);
01196
01197 /* --- Runtime support code (head) --- */
01198 /* Refnanny.proto */
01199 #ifndef CYTHON_REFNANNY
01200 #define CYTHON_REFNANNY 0
01201 #endif
01202 #if CYTHON_REFNANNY
01203     typedef struct {
01204         void (*INCRREF)(void*, PyObject*, int);
01205         void (*DECREF)(void*, PyObject*, int);
01206         void (*GOTREF)(void*, PyObject*, int);
01207         void (*GIVEREF)(void*, PyObject*, int);
01208         void* (*SetupContext)(const char*, int, const char*);
01209         void (*FinishContext)(void**);
01210     } __Pyx_RefNannyAPIStruct;
01211     static __Pyx_RefNannyAPIStruct *__Pyx_RefNanny = NULL;
01212     static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname);
01213     #define __Pyx_RefNannyDeclarations void *__pyx_refnanny = NULL;
01214 #ifdef WITH_THREAD
01215     #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01216         if (acquire_gil) {\
01217             PyGILState_STATE __pyx_gilstate_save = PyGILState_Ensure();\
01218             __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01219             PyGILState_Release(__pyx_gilstate_save);\
01220         } else {\
01221             __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01222         }
01223 #else
01224     #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01225         __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__)
01226 #endif
01227     #define __Pyx_RefNannyFinishContext()\
01228         __Pyx_RefNanny->FinishContext(&__pyx_refnanny)
01229     #define __Pyx_INCREF(r) __Pyx_RefNanny->INCRREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01230     #define __Pyx_DECREF(r) __Pyx_RefNanny->DECREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01231     #define __Pyx_GOTREF(r) __Pyx_RefNanny->GOTREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01232     #define __Pyx_GIVEREF(r) __Pyx_RefNanny->GIVEREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01233     #define __Pyx_XINCRREF(r) do { if ((r) != NULL) {__Pyx_INCREF(r); } } while(0)
01234     #define __Pyx_XDECREF(r) do { if ((r) != NULL) {__Pyx_DECREF(r); } } while(0)
01235     #define __Pyx_XGOTREF(r) do { if ((r) != NULL) {__Pyx_GOTREF(r); } } while(0)
01236     #define __Pyx_XGIVEREF(r) do { if ((r) != NULL) {__Pyx_GIVEREF(r); } } while(0)
01237 #else
01238     #define __Pyx_RefNannyDeclarations
01239     #define __Pyx_RefNannySetupContext(name, acquire_gil)
01240     #define __Pyx_RefNannyFinishContext()
01241     #define __Pyx_INCREF(r) Py_INCREF(r)
01242     #define __Pyx_DECREF(r) Py_DECREF(r)
01243     #define __Pyx_GOTREF(r)
01244     #define __Pyx_GIVEREF(r)
01245     #define __Pyx_XINCRREF(r) Py_XINCRREF(r)
01246     #define __Pyx_XDECREF(r) Py_XDECREF(r)
01247     #define __Pyx_XGOTREF(r)
01248     #define __Pyx_XGIVEREF(r)
01249 #endif
01250 #define __Pyx_XDECREF_SET(r, v) do {\
01251     PyObject *tmp = (PyObject *) r;\
01252     r = v; __Pyx_XDECREF(tmp);\
01253 } while (0)
01254 #define __Pyx_DECREF_SET(r, v) do {\
01255     PyObject *tmp = (PyObject *) r;\
01256     r = v; __Pyx_DECREF(tmp);\
01257 } while (0)
01258 #define __Pyx_CLEAR(r) do { PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp); } while(0)
01259 #define __Pyx_XCLEAR(r) do { if ((r) != NULL) {PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp); } } while(0)
01260
01261 /* PyObjectGetAttrStr.proto */
01262 #if CYTHON_USE_TYPE_SLOTS
01263 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name);
01264 #else
01265 #define __Pyx_PyObject_GetAttrStr(o,n) PyObject_GetAttr(o,n)
01266

```

```

01264 #endif
01265
01266 /* GetBuiltinName.proto */
01267 static PyObject *__Pyx_GetBuiltinName(PyObject *name);
01268
01269 /* PyCFunctionFastCall.proto */
01270 #if CYTHON_FAST_PYCCALL
01271 static CYTHON_INLINE PyObject *__Pyx_PyCFunction_FastCall(PyObject *func, PyObject **args, Py_ssize_t
    nargs);
01272 #else
01273 #define __Pyx_PyCFunction_FastCall(func, args, nargs) (assert(0), NULL)
01274 #endif
01275
01276 /* PyFunctionFastCall.proto */
01277 #if CYTHON_FAST_PYCALL
01278 #define __Pyx_PyFunction_FastCall(func, args, nargs)\
    __Pyx_PyFunction_FastCallDict((func), (args), (nargs), NULL)
01279 #if 1 || PY_VERSION_HEX < 0x030600B1
01280 static PyObject *__Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs,
    PyObject *kwargs);
01281 #else
01282 #define __Pyx_PyFunction_FastCallDict(func, args, nargs, kwargs) _PyFunction_FastCallDict(func, args,
    nargs, kwargs)
01283 #endif
01284 #define __Pyx_BUILD_ASSERT_EXPR(cond)\
    (sizeof(char [1 - 2*!(cond)]) - 1)
01285 #ifndef Py_MEMBER_SIZE
01286 #define Py_MEMBER_SIZE(type, member) sizeof(((type *)0)->member)
01287 #endif
01288 #if CYTHON_FAST_PYCALL
01289 static size_t __pyx_pyframe_localsplus_offset = 0;
01290 #include "frameobject.h"
01291 #define __Pxy_PyFrame_Initialize_Offsets()\
    ((void)__Pyx_BUILD_ASSERT_EXPR(sizeof(PyFrameObject) == offsetof(PyFrameObject, f_localsplus) +
    Py_MEMBER_SIZE(PyFrameObject, f_localsplus)),\
    (void)(__pyx_pyframe_localsplus_offset = ((size_t)PyFrame_Type.tp_basicsize) -
    Py_MEMBER_SIZE(PyFrameObject, f_localsplus)))
01292 #define __Pyx_PyFrame_GetLocalsplus(frame)\
    (assert(__pyx_pyframe_localsplus_offset), (PyObject *)((char *) (frame)) +
    __pyx_pyframe_localsplus_offset)
01293 #endif // CYTHON_FAST_PYCALL
01294 #endif
01295
01300
01301 /* PyObjectCall.proto */
01302 #if CYTHON_COMPILING_IN_CPYTHON
01303 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw);
01304 #else
01305 #define __Pyx_PyObject_Call(func, arg, kw) PyObject_Call(func, arg, kw)
01306 #endif
01307
01308 /* PyObjectCallMethO.proto */
01309 #if CYTHON_COMPILING_IN_CPYTHON
01310 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMethO(PyObject *func, PyObject *arg);
01311 #endif
01312
01313 /* PyObjectCallOneArg.proto */
01314 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg);
01315
01316 /* PyThreadStateGet.proto */
01317 #if CYTHON_FAST_THREAD_STATE
01318 #define __Pyx_PyThreadState_declare PyThreadState *__pyx_tstate;
01319 #define __Pyx_PyThreadState_assign __pyx_tstate = __Pyx_PyThreadState_Current;
01320 #define __Pyx_PyErr_Occurred() __pyx_tstate->curexc_type
01321 #else
01322 #define __Pyx_PyThreadState_declare
01323 #define __Pyx_PyThreadState_assign
01324 #define __Pyx_PyErr_Occurred() PyErr_Occurred()
01325 #endif
01326
01327 /* PyErrFetchRestore.proto */
01328 #if CYTHON_FAST_THREAD_STATE
01329 #define __Pyx_PyErr_Clear() __Pyx_ErrRestore(NULL, NULL, NULL)
01330 #define __Pyx_ErrRestoreWithState(type, value, tb) __Pyx_ErrRestoreInState(PyThreadState_GET(), type,
    value, tb)
01331 #define __Pyx_ErrFetchWithState(type, value, tb) __Pyx_ErrFetchInState(PyThreadState_GET(), type,
    value, tb)
01332 #define __Pyx_ErrRestore(type, value, tb) __Pyx_ErrRestoreInState(__pyx_tstate, type, value, tb)
01333 #define __Pyx_ErrFetch(type, value, tb) __Pyx_ErrFetchInState(__pyx_tstate, type, value, tb)
01334 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
    *value, PyObject *tb);
01335 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
    **value, PyObject **tb);
01336 #if CYTHON_COMPILING_IN_CPYTHON
01337 #define __Pyx_PyErr_SetNone(exc) (Py_INCREF(exc), __Pyx_ErrRestore((exc), NULL, NULL))
01338 #else
01339 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01340 #endif

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```

01341 #else
01342 #define __Pyx_PyErr_Clear() PyErr_Clear()
01343 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01344 #define __Pyx_ErrRestoreWithState(type, value, tb) PyErr_Restore(type, value, tb)
01345 #define __Pyx_ErrFetchWithState(type, value, tb) PyErr_Fetch(type, value, tb)
01346 #define __Pyx_ErrRestoreInState(tstate, type, value, tb) PyErr_Restore(type, value, tb)
01347 #define __Pyx_ErrFetchInState(tstate, type, value, tb) PyErr_Fetch(type, value, tb)
01348 #define __Pyx_ErrRestore(type, value, tb) PyErr_Restore(type, value, tb)
01349 #define __Pyx_ErrFetch(type, value, tb) PyErr_Fetch(type, value, tb)
01350 #endif
01351
01352 /* WriteUnraisableException.proto */
01353 static void __Pyx_WriteUnraisable(const char *name, int clineno,
01354                                 int lineno, const char *filename,
01355                                 int full_traceback, int nogil);
01356
01357 /* PyDictVersioning.proto */
01358 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
01359 #define __PYX_DICT_VERSION_INIT ((PY_UINT64_T) -1)
01360 #define __PYX_GET_DICT_VERSION(dict) (((PyDictObject*)(dict))->ma_version_tag)
01361 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)\
01362     (version_var) = __PYX_GET_DICT_VERSION(dict);\
01363     (cache_var) = (value);
01364 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) {\
01365     static PY_UINT64_T __pyx_dict_version = 0;\
01366     static PyObject *__pyx_dict_cached_value = NULL;\
01367     if (likely(__PYX_GET_DICT_VERSION(DICT) == __pyx_dict_version)) {\
01368         (VAR) = __pyx_dict_cached_value;\
01369     } else {\
01370         (VAR) = __pyx_dict_cached_value = (LOOKUP);\
01371         __pyx_dict_version = __PYX_GET_DICT_VERSION(DICT);\
01372     }\
01373 }
01374 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj);
01375 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj);
01376 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
01377 PY_UINT64_T obj_dict_version);
01378 #else
01379 #define __PYX_GET_DICT_VERSION(dict) (0)
01380 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)
01381 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) (VAR) = (LOOKUP);
01382 #endif
01383
01384 /* PyObjectCallNoArg.proto */
01385 #if CYTHON_COMPILING_IN_CPYTHON
01386 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func);
01387 #else
01388 #define __Pyx_PyObject_CallNoArg(func) __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL)
01389 #endif
01390
01391 /* RaiseDoubleKeywords.proto */
01392 static void __Pyx_RaiseDoubleKeywordsError(const char* func_name, PyObject* kw_name);
01393
01394 /* ParseKeywords.proto */
01395 static int __Pyx_ParseOptionalKeywords(PyObject *kwds, PyObject **argnames[],\
01396 PyObject *kwds2, PyObject *values[], Py_ssize_t num_pos_args,\
01397 const char* function_name);
01398
01399 /* RaiseArgTupleInvalid.proto */
01400 static void __Pyx_RaiseArgtupleInvalid(const char* func_name, int exact,\
01401 Py_ssize_t num_min, Py_ssize_t num_max, Py_ssize_t num_found);
01402
01403 /* GetModuleGlobalName.proto */
01404 #if CYTHON_USE_DICT_VERSIONS
01405 #define __Pyx_GetModuleGlobalName(var, name) {\
01406     static PY_UINT64_T __pyx_dict_version = 0;\
01407     static PyObject *__pyx_dict_cached_value = NULL;\
01408     (var) = (likely(__pyx_dict_version == __PYX_GET_DICT_VERSION(__pyx_d))) ?\
01409         (likely(__pyx_dict_cached_value) ? __Pyx_NewRef(__pyx_dict_cached_value) :\
01410         __Pyx_GetBuiltinName(name)) :\
01411         __Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01412 }
01413 #define __Pyx_GetModuleGlobalNameUncached(var, name) {\
01414     PY_UINT64_T __pyx_dict_version;\
01415     PyObject *__pyx_dict_cached_value;\
01416     (var) = __Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01417 }
01418 static PyObject *__Pyx_GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
01419 **dict_cached_value);
01420 #else
01421 #define __Pyx_GetModuleGlobalName(var, name) (var) = __Pyx_GetModuleGlobalName(name)
01422 #define __Pyx_GetModuleGlobalNameUncached(var, name) (var) = __Pyx_GetModuleGlobalName(name)
01423 static CYTHON_INLINE PyObject * __Pyx_GetModuleGlobalName(PyObject *name);
01424 #endif
01425
01426 /* GetTopmostException.proto */
01427 #if CYTHON_USE_EXC_INFO_STACK

```

```

01425 static _PyErr_StackItem * __Pyx_PyErr_GetTopmostException(PyThreadState *tstate);
01426 #endif
01427
01428 /* SaveResetException.proto */
01429 #if CYTHON_FAST_THREAD_STATE
01430 #define __Pyx_ExceptionSave(type, value, tb) __Pyx_ExceptionSave(__pyx_tstate, type, value, tb)
01431 static CYTHON_INLINE void __Pyx_ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
    **value, PyObject **tb);
01432 #define __Pyx_ExceptionReset(type, value, tb) __Pyx_ExceptionReset(__pyx_tstate, type, value, tb)
01433 static CYTHON_INLINE void __Pyx_ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
    *value, PyObject *tb);
01434 #else
01435 #define __Pyx_ExceptionSave(type, value, tb) PyErr_GetExcInfo(type, value, tb)
01436 #define __Pyx_ExceptionReset(type, value, tb) PyErr_SetExcInfo(type, value, tb)
01437 #endif
01438
01439 /* PyErrExceptionMatches.proto */
01440 #if CYTHON_FAST_THREAD_STATE
01441 #define __Pyx_PyErr_ExceptionMatches(err) __Pyx_PyErr_ExceptionMatchesInState(__pyx_tstate, err)
01442 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatchesInState(PyThreadState* tstate, PyObject* err);
01443 #else
01444 #define __Pyx_PyErr_ExceptionMatches(err) PyErr_ExceptionMatches(err)
01445 #endif
01446
01447 /* GetException.proto */
01448 #if CYTHON_FAST_THREAD_STATE
01449 #define __Pyx_GetException(type, value, tb) __Pyx_GetException(__pyx_tstate, type, value, tb)
01450 static int __Pyx_GetException(PyThreadState *tstate, PyObject **type, PyObject **value, PyObject
    **tb);
01451 #else
01452 static int __Pyx_GetException(PyObject **type, PyObject **value, PyObject **tb);
01453 #endif
01454
01455 /* RaiseException.proto */
01456 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb, PyObject *cause);
01457
01458 /* PyObjectCall2Args.proto */
01459 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject*
    arg2);
01460
01461 /* PyIntBinop.proto */
01462 #if !CYTHON_COMPILING_IN_PYPY
01463 static PyObject* __Pyx_PyInt_AddObjC(PyObject *op1, PyObject *op2, long intval, int inplace, int
    zerodivision_check);
01464 #else
01465 #define __Pyx_PyInt_AddObjC(op1, op2, intval, inplace, zerodivision_check)\
    (inplace ? PyNumber_InPlaceAdd(op1, op2) : PyNumber_Add(op1, op2))
01466 #endif
01467
01468
01469 /* PySequenceContains.proto */
01470 static CYTHON_INLINE int __Pyx_PySequence_ContainsTF(PyObject* item, PyObject* seq, int eq) {
01471     int result = PySequence_Contains(seq, item);
01472     return unlikely(result < 0) ? result : (result == (eq == Py_EQ));
01473 }
01474
01475 /* IncludeCppStringH.proto */
01476 #include <string>
01477
01478 /* decode_c_string_utf16.proto */
01479 static CYTHON_INLINE PyObject * __Pyx_PyUnicode_DecodeUTF16(const char *s, Py_ssize_t size, const char
    *errors) {
01480     int byteorder = 0;
01481     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01482 }
01483 static CYTHON_INLINE PyObject * __Pyx_PyUnicode_DecodeUTF16LE(const char *s, Py_ssize_t size, const
    char *errors) {
01484     int byteorder = -1;
01485     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01486 }
01487 static CYTHON_INLINE PyObject * __Pyx_PyUnicode_DecodeUTF16BE(const char *s, Py_ssize_t size, const
    char *errors) {
01488     int byteorder = 1;
01489     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01490 }
01491
01492 /* decode_c_bytes.proto */
01493 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
01494     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
01495     const char* encoding, const char* errors,
01496     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors));
01497
01498 /* decode_cpp_string.proto */
01499 static CYTHON_INLINE PyObject* __Pyx_decode_cpp_string(
01500     std::string cppstring, Py_ssize_t start, Py_ssize_t stop,
01501     const char* encoding, const char* errors,
01502     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors)) {
01503     return __Pyx_decode_c_bytes(

```



```

01504         cppstring.data(), cppstring.size(), start, stop, encoding, errors, decode_func);
01505     }
01506
01507     /* SwapException.proto */
01508     #if CYTHON_FAST_THREAD_STATE
01509     #define __Pyx_ExceptionSwap(type, value, tb) __Pyx_ExceptionSwap(__pyx_tstate, type, value, tb)
01510     static CYTHON_INLINE void __Pyx_ExceptionSwap(PyThreadState *tstate, PyObject **type, PyObject
        **value, PyObject **tb);
01511     #else
01512     static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject **type, PyObject **value, PyObject **tb);
01513     #endif
01514
01515     /* SetItemInt.proto */
01516     #define __Pyx_SetItemInt(o, i, v, type, is_signed, to_py_func, is_list, wraparound, boundscheck)\
01517         (__Pyx_fits_Py_ssize_t(i, type, is_signed) ?\
01518         __Pyx_SetItemInt_Fast(o, (Py_ssize_t)i, v, is_list, wraparound, boundscheck) :\
01519         (is_list ? (PyErr_SetString(PyExc_IndexError, "list assignment index out of range"), -1) :\
01520         __Pyx_SetItemInt_Generic(o, to_py_func(i), v)))
01521     static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v);
01522     static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v,
01523         int is_list, int wraparound, int boundscheck);
01524
01525     /* ArgTypeTest.proto */
01526     #define __Pyx_ArgTypeTest(obj, type, none_allowed, name, exact)\
01527         ((likely((Py_TYPE(obj) == type) | (none_allowed && (obj == Py_None)))) ? 1 :\
01528         __Pyx_ArgTypeTest(obj, type, name, exact))
01529     static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact);
01530
01531     /* Import.proto */
01532     static PyObject *__Pyx_Import(PyObject *name, PyObject *from_list, int level);
01533
01534     /* IncludeStringH.proto */
01535     #include <string.h>
01536
01537     /* PyObject_GenericGetAttrNoDict.proto */
01538     #if CYTHON_USE_PYTYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01539     static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject*
        attr_name);
01540     #else
01541     #define __Pyx_PyObject_GenericGetAttrNoDict PyObject_GenericGetAttr
01542     #endif
01543
01544     /* PyObject_GenericGetAttr.proto */
01545     #if CYTHON_USE_PYTYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01546     static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name);
01547     #else
01548     #define __Pyx_PyObject_GenericGetAttr PyObject_GenericGetAttr
01549     #endif
01550
01551     /* SetVTable.proto */
01552     static int __Pyx_SetVtable(PyObject *dict, void *vtable);
01553
01554     /* PyObjectGetAttrStrNoError.proto */
01555     static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name);
01556
01557     /* SetupReduce.proto */
01558     static int __Pyx_setup_reduce(PyObject* type_obj);
01559
01560     /* BytesEquals.proto */
01561     static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals);
01562
01563     /* UnicodeEquals.proto */
01564     static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals);
01565
01566     /* CLineInTraceback.proto */
01567     #ifdef CYTHON_CLINE_IN_TRACEBACK
01568     #define __Pyx_CLineForTraceback(tstate, c_line) (((CYTHON_CLINE_IN_TRACEBACK)) ? c_line : 0)
01569     #else
01570     static int __Pyx_CLineForTraceback(PyThreadState *tstate, int c_line);
01571     #endif
01572
01573     /* CodeObjectCache.proto */
01574     typedef struct {
01575         PyCodeObject* code_object;
01576         int code_line;
01577     } __Pyx_CodeObjectCacheEntry;
01578     struct __Pyx_CodeObjectCache {
01579         int count;
01580         int max_count;
01581         __Pyx_CodeObjectCacheEntry* entries;
01582     };
01583     static struct __Pyx_CodeObjectCache __pyx_code_cache = {0,0,NULL};
01584     static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line);
01585     static PyCodeObject* __pyx_find_code_object(int code_line);
01586     static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object);
01587
01588     /* AddTraceback.proto */

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```

01589 static void __Pyx_AddTraceback(const char *funcname, int c_line,
01590                               int py_line, const char *filename);
01591
01592 /* GCCDiagnostics.proto */
01593 #if defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 6))
01594 #define __Pyx_HAS_GCC_DIAGNOSTIC
01595 #endif
01596
01597 /* CppExceptionConversion.proto */
01598 #ifndef __Pyx_CppExn2PyErr
01599 #include <new>
01600 #include <typeinfo>
01601 #include <stdexcept>
01602 #include <ios>
01603 static void __Pyx_CppExn2PyErr() {
01604     try {
01605         if (PyErr_Occurred())
01606             ; // let the latest Python exn pass through and ignore the current one
01607         else
01608             throw;
01609     } catch (const std::bad_alloc& exn) {
01610         PyErr_SetString(PyExc_MemoryError, exn.what());
01611     } catch (const std::bad_cast& exn) {
01612         PyErr_SetString(PyExc_TypeError, exn.what());
01613     } catch (const std::bad_typeid& exn) {
01614         PyErr_SetString(PyExc_TypeError, exn.what());
01615     } catch (const std::domain_error& exn) {
01616         PyErr_SetString(PyExc_ValueError, exn.what());
01617     } catch (const std::invalid_argument& exn) {
01618         PyErr_SetString(PyExc_ValueError, exn.what());
01619     } catch (const std::ios_base::failure& exn) {
01620         PyErr_SetString(PyExc_IOError, exn.what());
01621     } catch (const std::out_of_range& exn) {
01622         PyErr_SetString(PyExc_IndexError, exn.what());
01623     } catch (const std::overflow_error& exn) {
01624         PyErr_SetString(PyExc_OverflowError, exn.what());
01625     } catch (const std::range_error& exn) {
01626         PyErr_SetString(PyExc_ArithmeticError, exn.what());
01627     } catch (const std::underflow_error& exn) {
01628         PyErr_SetString(PyExc_ArithmeticError, exn.what());
01629     } catch (const std::exception& exn) {
01630         PyErr_SetString(PyExc_RuntimeError, exn.what());
01631     }
01632     catch (...)
01633     {
01634         PyErr_SetString(PyExc_RuntimeError, "Unknown exception");
01635     }
01636 }
01637 #endif
01638
01639 /* CIntFromPy.proto */
01640 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *);
01641
01642 /* CIntToPy.proto */
01643 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value);
01644
01645 /* CIntToPy.proto */
01646 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value);
01647
01648 /* CIntFromPy.proto */
01649 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *);
01650
01651 /* FastTypeChecks.proto */
01652 #if CYTHON_COMPILING_IN_CPYTHON
01653 #define __Pyx_TypeCheck(obj, type) __Pyx_IsSubtype(Py_TYPE(obj), (PyTypeObject *)type)
01654 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b);
01655 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject *type);
01656 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *type1, PyObject
    *type2);
01657 #else
01658 #define __Pyx_TypeCheck(obj, type) PyObject_TypeCheck(obj, (PyTypeObject *)type)
01659 #define __Pyx_PyErr_GivenExceptionMatches(err, type) PyErr_GivenExceptionMatches(err, type)
01660 #define __Pyx_PyErr_GivenExceptionMatches2(err, type1, type2) (PyErr_GivenExceptionMatches(err, type1)
    || PyErr_GivenExceptionMatches(err, type2))
01661 #endif
01662 #define __Pyx_PyException_Check(obj) __Pyx_TypeCheck(obj, PyExc_Exception)
01663
01664 /* FetchCommonType.proto */
01665 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type);
01666
01667 /* PyObjectGetMethod.proto */
01668 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method);
01669
01670 /* PyObjectCallMethod1.proto */
01671 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg);
01672
01673 /* CoroutineBase.proto */

```

```

01674 typedef PyObject *(*__pyx_coroutine_body_t)(PyObject *, PyThreadState *, PyObject *);
01675 #if CYTHON_USE_EXC_INFO_STACK
01676 #define __Pyx_ExcInfoStruct __PyErr_StackItem
01677 #else
01678 typedef struct {
01679     PyObject *exc_type;
01680     PyObject *exc_value;
01681     PyObject *exc_traceback;
01682 } __Pyx_ExcInfoStruct;
01683 #endif
01684 typedef struct {
01685     PyObject_HEAD
01686     __pyx_coroutine_body_t body;
01687     PyObject *closure;
01688     __Pyx_ExcInfoStruct gi_exc_state;
01689     PyObject *gi_weakreflist;
01690     PyObject *classobj;
01691     PyObject *yieldfrom;
01692     PyObject *gi_name;
01693     PyObject *gi_qualname;
01694     PyObject *gi_modulename;
01695     PyObject *gi_code;
01696     PyObject *gi_frame;
01697     int resume_label;
01698     char is_running;
01699 } __pyx_CoroutineObject;
01700 static __pyx_CoroutineObject *__Pyx__Coroutine_New(
01701     PyTypeObject *type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
01702     PyObject *name, PyObject *qualname, PyObject *module_name);
01703 static __pyx_CoroutineObject *__Pyx__Coroutine_NewInit(
01704     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
01705         *closure, PyObject *name, PyObject *qualname, PyObject *module_name);
01706 static CYTHON_INLINE void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *self);
01707 static int __Pyx_Coroutine_clear(PyObject *self);
01708 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value);
01709 static PyObject *__Pyx_Coroutine_Close(PyObject *self);
01710 static PyObject *__Pyx_Coroutine_Throw(PyObject *gen, PyObject *args);
01711 #if CYTHON_USE_EXC_INFO_STACK
01712 #define __Pyx_Coroutine_SwapException(self)
01713 #define __Pyx_Coroutine_ResetAndClearException(self)
01714     __Pyx_Coroutine_ExceptionClear(&(self)->gi_exc_state)
01715 #else
01716 #define __Pyx_Coroutine_SwapException(self) {\
01717     __Pyx_ExceptionSwap(&(self)->gi_exc_state.exc_type, &(self)->gi_exc_state.exc_value,\
01718         &(self)->gi_exc_state.exc_traceback);\
01719     __Pyx_Coroutine_ResetFrameBackpointer(&(self)->gi_exc_state);\
01720 }
01721 #define __Pyx_Coroutine_ResetAndClearException(self) {\
01722     __Pyx_ExceptionReset((self)->gi_exc_state.exc_type, (self)->gi_exc_state.exc_value,\
01723         (self)->gi_exc_state.exc_traceback);\
01724     (self)->gi_exc_state.exc_type = (self)->gi_exc_state.exc_value =\
01725     (self)->gi_exc_state.exc_traceback = NULL;\
01726 }
01727 #endif
01728 #if CYTHON_FAST_THREAD_STATE
01729 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01730     __Pyx_PyGen_FetchStopIterationValue(__pyx_tstate, pvalue)
01731 #else
01732 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01733     __Pyx_PyGen_FetchStopIterationValue(__Pyx_PyThreadState_Current, pvalue)
01734 #endif
01735 static int __Pyx_PyGen_FetchStopIterationValue(PyThreadState *tstate, PyObject **pvalue);
01736 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state);
01737 /* PatchModuleWithCoroutine.proto */
01738 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code);
01739 /* PatchGeneratorABC.proto */
01740 static int __Pyx_patch_abc(void);
01741 /* Generator.proto */
01742 #define __Pyx_Generator_USED
01743 static PyTypeObject *__pyx_GeneratorType = 0;
01744 #define __Pyx_Generator_CheckExact(obj) (Py_TYPE(obj) == __pyx_GeneratorType)
01745 #define __Pyx_Generator_New(body, code, closure, name, qualname, module_name)\
01746     __Pyx__Coroutine_New(__pyx_GeneratorType, body, code, closure, name, qualname, module_name)
01747 static PyObject *__Pyx_Generator_Next(PyObject *self);
01748 static int __pyx_Generator_init(void);
01749 /* CheckBinaryVersion.proto */
01750 static int __Pyx_check_binary_version(void);
01751 /* InitStrings.proto */
01752 static int __Pyx_InitStrings(__Pyx_StringTabEntry *t);
01753 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set

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```

    *__pyx_v_self, IndexSet __pyx_v_other); /* proto*/
01756 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self); /* proto*/
01757 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set *__pyx_v_self,
    int __pyx_skip_dispatch); /* proto*/
01758 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, Clifford __pyx_v_other); /* proto*/
01759 static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto*/
01760 static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
    int __pyx_skip_dispatch); /* proto*/
01761
01762 /* Module declarations from 'libcpp.vector' */
01763
01764 /* Module declarations from 'glucat' */
01765
01766 /* Module declarations from 'libc.string' */
01767
01768 /* Module declarations from 'libcpp.string' */
01769
01770 /* Module declarations from 'PyClical' */
01771 static PyTypeObject *__pyx_ptype_8PyClical_index_set = 0;
01772 static PyTypeObject *__pyx_ptype_8PyClical_clifford = 0;
01773 static PyTypeObject *__pyx_ptype_8PyClical__pyx_scope_struct__iter__ = 0;
01774 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_toIndexSet(PyObject *) /*proto*/
01775 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_compare(PyObject *, PyObject *, int
    __pyx_skip_dispatch); /*proto*/
01776 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_min_neg(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01777 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_pos(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01778 static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClical_list_to_vector(PyObject *) /*proto*/
01779 static CYTHON_INLINE Clifford __pyx_f_8PyClical_toClifford(PyObject *) /*proto*/
01780 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared_tol(PyObject *, int
    __pyx_skip_dispatch); /*proto*/
01781 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared(PyObject *, PyObject *, PyObject *, int
    __pyx_skip_dispatch); /*proto*/
01782 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_approx_equal(PyObject *, PyObject *, int
    __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_approx_equal *__pyx_optional_args) /*proto*/
01783 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_inv(PyObject *, int __pyx_skip_dispatch); /*proto*/
01784 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_scalar(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01785 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_real(PyObject *, int __pyx_skip_dispatch); /*proto*/
01786 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_imag(PyObject *, int __pyx_skip_dispatch); /*proto*/
01787 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pure(PyObject *, int __pyx_skip_dispatch); /*proto*/
01788 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_even(PyObject *, int __pyx_skip_dispatch); /*proto*/
01789 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_odd(PyObject *, int __pyx_skip_dispatch); /*proto*/
01790 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_involute(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01791 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_reverse(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01792 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_conj(PyObject *, int __pyx_skip_dispatch); /*proto*/
01793 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_quad(PyObject *, int __pyx_skip_dispatch); /*proto*/
01794 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_norm(PyObject *, int __pyx_skip_dispatch); /*proto*/
01795 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_abs(PyObject *, int __pyx_skip_dispatch); /*proto*/
01796 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_abs(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01797 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pow(PyObject *, PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01798 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_outer_pow(PyObject *, PyObject *, int
    __pyx_skip_dispatch); /*proto*/
01799 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_complexifier(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01800 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sqrt(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_sqrt *__pyx_optional_args) /*proto*/
01801 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_exp(PyObject *, int __pyx_skip_dispatch); /*proto*/
01802 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_log(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_log *__pyx_optional_args) /*proto*/
01803 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cos(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_cos *__pyx_optional_args) /*proto*/
01804 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_acos(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_acos *__pyx_optional_args) /*proto*/
01805 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cosh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01806 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_acosh(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_acosh *__pyx_optional_args) /*proto*/
01807 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sin(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_sin *__pyx_optional_args) /*proto*/
01808 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asin(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_asin *__pyx_optional_args) /*proto*/
01809 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asinh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01810 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sinh(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_asinh *__pyx_optional_args) /*proto*/
01811 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tan(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_tan *__pyx_optional_args) /*proto*/
01812 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_atan(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_atan *__pyx_optional_args) /*proto*/
01813 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tanh(PyObject *, int __pyx_skip_dispatch); /*proto*/

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01814 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_atanh(PyObject *, int __pyx_skip_dispatch, struct
    __pyx_opt_args_8PyClical_atanh *__pyx_optional_args); /*proto*/
01815 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_random_clifford(struct __pyx_obj_8PyClical_index_set
    *, int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_random_clifford *__pyx_optional_args);
    /*proto*/
01816 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cga3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01817 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cga3std(PyObject *, int __pyx_skip_dispatch);
    /*proto*/
01818 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_agc3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01819 static CYTHON_INLINE PyObject *__pyx_convert_PyObject_string_to_py_std__in_string(std::string const
    &); /*proto*/
01820 static CYTHON_INLINE PyObject *__pyx_convert_PyUnicode_string_to_py_std__in_string(std::string const
    &); /*proto*/
01821 static CYTHON_INLINE PyObject *__pyx_convert_PyStr_string_to_py_std__in_string(std::string const &);
    /*proto*/
01822 static CYTHON_INLINE PyObject *__pyx_convert_PyBytes_string_to_py_std__in_string(std::string const &);
    /*proto*/
01823 static CYTHON_INLINE PyObject *__pyx_convert_PyByteArray_string_to_py_std__in_string(std::string const
    &); /*proto*/
01824 #define __Pyx_MODULE_NAME "PyClical"
01825 extern int __pyx_module_is_main_PyClical;
01826 int __pyx_module_is_main_PyClical = 0;
01827
01828 /* Implementation of 'PyClical' */
01829 static PyObject *__pyx_builtin_IndexError;
01830 static PyObject *__pyx_builtin_RuntimeError;
01831 static PyObject *__pyx_builtin_TypeError;
01832 static PyObject *__pyx_builtin_ValueError;
01833 static PyObject *__pyx_builtin_NotImplemented;
01834 static PyObject *__pyx_builtin_range;
01835 static PyObject *__pyx_builtin_xrange;
01836 static const char __pyx_k_[] = ".";
01837 static const char __pyx_k_e[] = "e";
01838 static const char __pyx_k_i[] = "i";
01839 static const char __pyx_k_m[] = "m";
01840 static const char __pyx_k_p[] = "p";
01841 static const char __pyx_k_q[] = "q";
01842 static const char __pyx_k_2[] = " ";
01843 static const char __pyx_k_5[] = ":";
01844 static const char __pyx_k_6[] = "\n\t";
01845 static const char __pyx_k_7[] = "(";
01846 static const char __pyx_k_8[] = ", ";
01847 static const char __pyx_k_9[] = ").";
01848 static const char __pyx_k_cl[] = "cl";
01849 static const char __pyx_k_pi[] = "pi";
01850 static const char __pyx_k_abc[] = "abc";
01851 static const char __pyx_k_cos[] = "cos";
01852 static const char __pyx_k_exp[] = "exp";
01853 static const char __pyx_k_frm[] = "frm";
01854 static const char __pyx_k_inv[] = "inv";
01855 static const char __pyx_k_ist[] = "ist";
01856 static const char __pyx_k_ixt[] = "ixt";
01857 static const char __pyx_k_lhs[] = "lhs";
01858 static const char __pyx_k_log[] = "log";
01859 static const char __pyx_k_max[] = "max";
01860 static const char __pyx_k_min[] = "min";
01861 static const char __pyx_k_obj[] = "obj";
01862 static const char __pyx_k_odd[] = "odd";
01863 static const char __pyx_k_pow[] = "pow";
01864 static const char __pyx_k_rhs[] = "rhs";
01865 static const char __pyx_k_sin[] = "sin";
01866 static const char __pyx_k_tan[] = "tan";
01867 static const char __pyx_k_tau[] = "tau";
01868 static const char __pyx_k_tol[] = "tol";
01869 static const char __pyx_k_Real[] = "Real";
01870 static const char __pyx_k_acos[] = "acos";
01871 static const char __pyx_k_args[] = "args";
01872 static const char __pyx_k_asin[] = "asin";
01873 static const char __pyx_k_atan[] = "atan";
01874 static const char __pyx_k_conj[] = "conj";
01875 static const char __pyx_k_copy[] = "copy";
01876 static const char __pyx_k_cosh[] = "cosh";
01877 static const char __pyx_k_even[] = "even";
01878 static const char __pyx_k_fill[] = "fill";
01879 static const char __pyx_k_from[] = " from ";
01880 static const char __pyx_k_iter[] = " _iter_ ";
01881 static const char __pyx_k_main[] = " _main_ ";
01882 static const char __pyx_k_math[] = "math";
01883 static const char __pyx_k_name[] = " _name_ ";
01884 static const char __pyx_k_norm[] = "norm";
01885 static const char __pyx_k_pure[] = "pure";
01886 static const char __pyx_k_quad[] = "quad";
01887 static const char __pyx_k_send[] = "send";
01888 static const char __pyx_k_sinh[] = "sinh";
01889 static const char __pyx_k_sqrt[] = "sqrt";
01890 static const char __pyx_k_tanh[] = "tanh";
01891 static const char __pyx_k_test[] = " _test_ ";

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01892 static const char __pyx_k_UTF_8[] = "UTF-8";
01893 static const char __pyx_k_acosh[] = "acosh";
01894 static const char __pyx_k_asinh[] = "asinh";
01895 static const char __pyx_k_atanh[] = "atanh";
01896 static const char __pyx_k_close[] = "close";
01897 static const char __pyx_k_grade[] = "grade";
01898 static const char __pyx_k_istpq[] = "istpq";
01899 static const char __pyx_k_nbar3[] = "nbar3";
01900 static const char __pyx_k_ninf3[] = "ninf3";
01901 static const char __pyx_k_other[] = "other";
01902 static const char __pyx_k_range[] = "range";
01903 static const char __pyx_k_throw[] = "throw";
01904 static const char __pyx_k_using[] = " using (";
01905 static const char __pyx_k_utf_8[] = "utf-8";
01906 static const char __pyx_k_value[] = " value ";
01907 static const char __pyx_k_encode[] = "encode";
01908 static const char __pyx_k_import[] = "__import__";
01909 static const char __pyx_k_reduce[] = "__reduce__";
01910 static const char __pyx_k_scalar[] = "scalar";
01911 static const char __pyx_k_test_2[] = "__test__";
01912 static const char __pyx_k_xrange[] = "xrange";
01913 static const char __pyx_k_doctest[] = "doctest";
01914 static const char __pyx_k_invalid[] = " invalid ";
01915 static const char __pyx_k_numbers[] = "numbers";
01916 static const char __pyx_k_reverse[] = "reverse";
01917 static const char __pyx_k_testmod[] = "testmod";
01918 static const char __pyx_k_version[] = "__version__";
01919 static const char __pyx_k_Integral[] = "Integral";
01920 static const char __pyx_k_PyClical[] = "PyClical";
01921 static const char __pyx_k_Sequence[] = "Sequence";
01922 static const char __pyx_k_as_frame[] = " as frame:\n\t";
01923 static const char __pyx_k_clifford[] = "clifford";
01924 static const char __pyx_k_getstate[] = "__getstate__";
01925 static const char __pyx_k_involute[] = "involute";
01926 static const char __pyx_k_setstate[] = "__setstate__";
01927 static const char __pyx_k_to_frame[] = " to frame ";
01928 static const char __pyx_k_TypeError[] = "TypeError";
01929 static const char __pyx_k_index_set[] = "index_set";
01930 static const char __pyx_k_outer_pow[] = "outer_pow";
01931 static const char __pyx_k_reduce_ex[] = "__reduce_ex__";
01932 static const char __pyx_k_threshold[] = "threshold";
01933 static const char __pyx_k_IndexError[] = "IndexError";
01934 static const char __pyx_k_ValueError[] = "ValueError";
01935 static const char __pyx_k_pyx_vtable[] = "__pyx_vtable__";
01936 static const char __pyx_k_collections[] = "collections";
01937 static const char __pyx_k_e_line_1936[] = "e (line 1936)";
01938 static const char __pyx_k_PyClical_pyx[] = "PyClical.pyx";
01939 static const char __pyx_k_RuntimeError[] = "RuntimeError";
01940 static const char __pyx_k_abs_line_1522[] = "abs (line 1522)";
01941 static const char __pyx_k_cos_line_1651[] = "cos (line 1651)";
01942 static const char __pyx_k_exp_line_1614[] = "exp (line 1614)";
01943 static const char __pyx_k_inv_line_1378[] = "inv (line 1378)";
01944 static const char __pyx_k_log_line_1628[] = "log (line 1628)";
01945 static const char __pyx_k_odd_line_1446[] = "odd (line 1446)";
01946 static const char __pyx_k_pow_line_1543[] = "pow (line 1543)";
01947 static const char __pyx_k_reduce_cython[] = "__reduce_cython__";
01948 static const char __pyx_k_sin_line_1728[] = "sin (line 1728)";
01949 static const char __pyx_k_tan_line_1801[] = "tan (line 1801)";
01950 static const char __pyx_k_using_invalid[] = " using invalid ";
01951 static const char __pyx_k_Cannot_reframe[] = "Cannot reframe";
01952 static const char __pyx_k_NotImplemented[] = "NotImplemented";
01953 static const char __pyx_k_Not_applicable[] = "Not applicable.";
01954 static const char __pyx_k_acos_line_1668[] = "acos (line 1668)";
01955 static const char __pyx_k_agc3_line_1893[] = "agc3 (line 1893)";
01956 static const char __pyx_k_asin_line_1747[] = "asin (line 1747)";
01957 static const char __pyx_k_atan_line_1818[] = "atan (line 1818)";
01958 static const char __pyx_k_cga3_line_1873[] = "cga3 (line 1873)";
01959 static const char __pyx_k_conj_line_1485[] = "conj (line 1485)";
01960 static const char __pyx_k_cosh_line_1689[] = "cosh (line 1689)";
01961 static const char __pyx_k_even_line_1437[] = "even (line 1437)";
01962 static const char __pyx_k_imag_line_1415[] = "imag (line 1415)";
01963 static const char __pyx_k_invalid_string[] = " invalid string ";
01964 static const char __pyx_k_norm_line_1511[] = "norm (line 1511)";
01965 static const char __pyx_k_pure_line_1426[] = "pure (line 1426)";
01966 static const char __pyx_k_quad_line_1500[] = "quad (line 1500)";
01967 static const char __pyx_k_real_line_1404[] = "real (line 1404)";
01968 static const char __pyx_k_scalar_epsilon[] = "scalar_epsilon";
01969 static const char __pyx_k_sinh_line_1768[] = "sinh (line 1768)";
01970 static const char __pyx_k_sqrt_line_1591[] = "sqrt (line 1591)";
01971 static const char __pyx_k_tanh_line_1835[] = "tanh (line 1835)";
01972 static const char __pyx_k_acosh_line_1705[] = "acosh (line 1705)";
01973 static const char __pyx_k_asinh_line_1782[] = "asinh (line 1782)";
01974 static const char __pyx_k_atanh_line_1847[] = "atanh (line 1847)";
01975 static const char __pyx_k_istpq_line_1949[] = "istpq (line 1949)";
01976 static const char __pyx_k_setstate_cython[] = "__setstate_cython__";
01977 static const char __pyx_k_compare_line_492[] = "compare (line 492)";
01978 static const char __pyx_k_index_set__iter[] = "index_set.__iter__";

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01979 static const char __pyx_k_max_pos_line_513[] = "max_pos (line 513)";
01980 static const char __pyx_k_min_neg_line_504[] = "min_neg (line 504)";
01981 static const char __pyx_k_scalar_line_1393[] = "scalar (line 1393)";
01982 static const char __pyx_k_cga3std_line_1882[] = "cga3std (line 1882)";
01983 static const char __pyx_k_max_abs_line_1531[] = "max_abs (line 1531)";
01984 static const char __pyx_k_reverse_line_1470[] = "reverse (line 1470)";
01985 static const char __pyx_k_cline_in_traceback[] = "cline_in_traceback";
01986 static const char __pyx_k_involute_line_1455[] = "involute (line 1455)";
01987 static const char __pyx_k_outer_pow_line_1567[] = "outer_pow (line 1567)";
01988 static const char __pyx_k_clifford_inv_line_926[] = "clifford.inv (line 926)";
01989 static const char __pyx_k_clifford_pow_line_980[] = "clifford.pow (line 980)";
01990 static const char __pyx_k_approx_equal_line_1359[] = "approx_equal (line 1359)";
01991 static const char __pyx_k_clifford_abs_line_1175[] = "clifford.abs (line 1175)";
01992 static const char __pyx_k_clifford_copy_line_556[] = "clifford.copy (line 556)";
01993 static const char __pyx_k_clifford_odd_line_1070[] = "clifford.odd (line 1070)";
01994 static const char __pyx_k_complexifier_line_1576[] = "complexifier (line 1576)";
01995 static const char __pyx_k_index_set_copy_line_65[] = "index_set.copy (line 65)";
01996 static const char __pyx_k_index_set_max_line_351[] = "index_set.max (line 351)";
01997 static const char __pyx_k_index_set_min_line_342[] = "index_set.min (line 342)";
01998 static const char __pyx_k_clifford_conj_line_1138[] = "clifford.conj (line 1138)";
01999 static const char __pyx_k_clifford_even_line_1061[] = "clifford.even (line 1061)";
02000 static const char __pyx_k_clifford_norm_line_1164[] = "clifford.norm (line 1164)";
02001 static const char __pyx_k_clifford_pure_line_1050[] = "clifford.pure (line 1050)";
02002 static const char __pyx_k_clifford_quad_line_1153[] = "clifford.quad (line 1153)";
02003 static const char __pyx_k_error_squared_line_1346[] = "error_squared (line 1346)";
02004 static const char __pyx_k_Unary_print_clifford_l_1[] = "\n          Unary -. \n\n" >>
    print((-clifford("\{1\}")\n          -{1}\n          );
02005 static const char __pyx_k_clifford_or_line_939[] = "clifford.__or__ (line 939)";
02006 static const char __pyx_k_clifford_frame_line_1224[] = "clifford.frame (line 1224)";
02007 static const char __pyx_k_clifford_hidden_doctests[] = "clifford.hidden_doctests";
02008 static const char __pyx_k_clifford_isinf_line_1206[] = "clifford.isinf (line 1206)";
02009 static const char __pyx_k_clifford_isnan_line_1215[] = "clifford.isnan (line 1215)";
02010 static const char __pyx_k_index_set_count_line_315[] = "index_set.count (line 315)";
02011 static const char __pyx_k_clifford_add_line_740[] = "clifford.__add__ (line 740)";
02012 static const char __pyx_k_clifford_and_line_836[] = "clifford.__and__ (line 836)";
02013 static const char __pyx_k_clifford_ior_line_950[] = "clifford.__ior__ (line 950)";
02014 static const char __pyx_k_clifford_mod_line_806[] = "clifford.__mod__ (line 806)";
02015 static const char __pyx_k_clifford_mul_line_780[] = "clifford.__mul__ (line 780)";
02016 static const char __pyx_k_clifford_neg_line_722[] = "clifford.__neg__ (line 722)";
02017 static const char __pyx_k_clifford_pos_line_731[] = "clifford.__pos__ (line 731)";
02018 static const char __pyx_k_clifford_pow_line_961[] = "clifford.__pow__ (line 961)";
02019 static const char __pyx_k_clifford_sub_line_760[] = "clifford.__sub__ (line 760)";
02020 static const char __pyx_k_clifford_xor_line_866[] = "clifford.__xor__ (line 866)";
02021 static const char __pyx_k_clifford_reframe_line_649[] = "clifford.reframe (line 649)";
02022 static const char __pyx_k_clifford_scalar_line_1039[] = "clifford.scalar (line 1039)";
02023 static const char __pyx_k_index_set_or_line_293[] = "index_set.__or__ (line 293)";
02024 static const char __pyx_k_index_set_hidden_doctests[] = "index_set.hidden_doctests";
02025 static const char __pyx_k_random_clifford_line_1864[] = "random_clifford (line 1864)";
02026 static const char __pyx_k_Cannot_take_vector_part_of[] = "Cannot take vector part of ";
02027 static const char __pyx_k_Unary_print_clifford_l_1_2[] = "\n          Unary +. \n\n" >>
    print(+clifford("\{1\}")\n          {1}\n          );
02028 static const char __pyx_k_clifford_iadd_line_751[] = "clifford.__iadd__ (line 751)";
02029 static const char __pyx_k_clifford_iand_line_851[] = "clifford.__iand__ (line 851)";
02030 static const char __pyx_k_clifford_idiv_line_911[] = "clifford.__idiv__ (line 911)";
02031 static const char __pyx_k_clifford_imod_line_821[] = "clifford.__imod__ (line 821)";
02032 static const char __pyx_k_clifford_imul_line_793[] = "clifford.__imul__ (line 793)";
02033 static const char __pyx_k_clifford_isub_line_771[] = "clifford.__isub__ (line 771)";
02034 static const char __pyx_k_clifford_iter_line_638[] = "clifford.__iter__ (line 638)";
02035 static const char __pyx_k_clifford_ior_line_881[] = "clifford.__ior__ (line 881)";
02036 static const char __pyx_k_clifford_str_line_1244[] = "clifford.__str__ (line 1244)";
02037 static const char __pyx_k_clifford_max_abs_line_1184[] = "clifford.max_abs (line 1184)";
02038 static const char __pyx_k_clifford_reverse_line_1123[] = "clifford.reverse (line 1123)";
02039 static const char __pyx_k_index_set_and_line_271[] = "index_set.__and__ (line 271)";
02040 static const char __pyx_k_index_set_ior_line_304[] = "index_set.__ior__ (line 304)";
02041 static const char __pyx_k_index_set_str_line_395[] = "index_set.__str__ (line 395)";
02042 static const char __pyx_k_index_set_xor_line_249[] = "index_set.__xor__ (line 249)";
02043 static const char __pyx_k_clifford_call_line_1020[] = "clifford.__call__ (line 1020)";
02044 static const char __pyx_k_clifford_repr_line_1235[] = "clifford.__repr__ (line 1235)";
02045 static const char __pyx_k_clifford_involute_line_1107[] = "clifford.involute (line 1107)";
02046 static const char __pyx_k_error_squared_tol_line_1337[] = "error_squared_tol (line 1337)";
02047 static const char __pyx_k_index_set_iand_line_282[] = "index_set.__iand__ (line 282)";
02048 static const char __pyx_k_index_set_iter_line_229[] = "index_set.__iter__ (line 229)";
02049 static const char __pyx_k_index_set_ixor_line_260[] = "index_set.__ixor__ (line 260)";
02050 static const char __pyx_k_index_set_repr_line_384[] = "index_set.__repr__ (line 384)";
02051 static const char __pyx_k_clifford_outer_pow_line_1004[] = "clifford.outer_pow (line 1004)";
02052 static const char __pyx_k_clifford_truncated_line_1195[] = "clifford.truncated (line 1195)";
02053 static const char __pyx_k_index_set_count_neg_line_324[] = "index_set.count_neg (line 324)";
02054 static const char __pyx_k_index_set_count_pos_line_333[] = "index_set.count_pos (line 333)";
02055 static const char __pyx_k_clifford_getitem_line_707[] = "clifford.__getitem__ (line 707)";
02056 static const char __pyx_k_clifford_truediv_line_896[] = "clifford.__truediv__ (line 896)";
02057 static const char __pyx_k_index_set_invert_line_240[] = "index_set.__invert__ (line 240)";
02058 static const char __pyx_k_Abbreviation_for_index_set_q_p[] = "\n          Abbreviation for
    index_set((-q,...p)).\n\n" >> print(istpq(2,3))\n          {-3,-2,-1,2}\n          ";
02059 static const char __pyx_k_Conjugation_reverse_o_involute[] = "\n          Conjugation, reverse o
    involute == involute o reverse.\n\n" >> print((clifford("\{1\}")).conj())\n          -{1}\n
    >> print((clifford("\{2\}")) * clifford("\{1\}")).conj())\n          {1,2}\n          >>
    print((clifford("\{1\}")) * clifford("\{2\}")).conj())\n          -{1,2}\n          >>

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print(clifford("\1+\1+\1,2\").conj())\n          1-\{1\}-\{1,2\}\n          ";
02060 static const char __pyx_k_Geometric_product_x_clifford_2[] = "\n          Geometric product.\n\n
>> x = clifford(2); x *= clifford("\{2\}"); print(x)\n          2\{2\}\n          >> x = clifford("\{1\}");
x *= clifford("\{2\}"); print(x)\n          \{1,2\}\n          >> x = clifford("\{1\}"); x *=
clifford("\{1,2\}"); print(x)\n          2\{2\}\n          ";
02061 static const char __pyx_k_Geometric_sum_print_clifford_1[] = "\n          Geometric sum.\n\n          >>
print(clifford(1) + clifford("\{2\}"))\n          1+\{2\}\n          >> print(clifford("\{1\}") +
clifford("\{2\}"))\n          \{1\}+\{2\}\n          ";
02062 static const char __pyx_k_Hyperbolic_sine_of_multivector[] = "\n          Hyperbolic sine of
multivector.\n\n          >> x=clifford("\{1,2\}") * pi/2; print(sinh(x))\n          \{1,2\}\n          >>
x=clifford("\{1,2\}") * pi/6; print(sinh(x))\n          0.5\{1,2\}\n          ";
02063 static const char __pyx_k_Inner_product_print_clifford_1[] = "\n          Inner product.\n\n          >>
print(clifford("\{1\}") & clifford("\{2\}"))\n          0\n          >> print(clifford(2) &
clifford("\{2\}"))\n          0\n          >> print(clifford("\{1\}") & clifford("\{1\}"))\n          1\n
>> print(clifford("\{1\}") & clifford("\{1,2\}"))\n          \{2\}\n          ";
02064 static const char __pyx_k_Inverse_tangent_of_multivector[] = "\n          Inverse tangent of multivector
with optional complexifier.\n\n          >> s=index_set(\{1,2,3\}); x=clifford("\{1\}");
print(tan(atan(x,s))\n          \{1\}\n          >> x=clifford("\{1\}"); print(tan(atan(x))\n          \{1\}\n          ";
02065 static const char __pyx_k_Iterate_over_the_indices_of_an[] = "\n          Iterate over the indices of an
index_set.\n\n          >> for i in index_set(\{-3,4,7\}):print(i, end=",\n")\n          -3,4,7,\n
";
02066 static const char __pyx_k_Maximum_member_index_set_1_1_2[] = "\n          Maximum member.\n\n          >>
index_set(\{-1,1,2\}).max()\n          2\n          ";
02067 static const char __pyx_k_Maximum_positive_index_or_0_if[] = "\n          Maximum positive index, or 0 if
none.\n\n          >> max_pos(index_set(\{1,2\}))\n          2\n          ";
02068 static const char __pyx_k_Minimum_member_index_set_1_1_2[] = "\n          Minimum member.\n\n          >>
index_set(\{-1,1,2\}).min()\n          -1\n          ";
02069 static const char __pyx_k_Minimum_negative_index_or_0_if[] = "\n          Minimum negative index, or 0 if
none.\n\n          >> min_neg(index_set(\{1,2\}))\n          0\n          ";
02070 static const char __pyx_k_Odd_part_of_multivector_sum_of[] = "\n          Odd part of multivector, sum
of odd grade terms.\n\n          >> print(clifford("\1+\1+\1,2\").odd())\n          \{1\}\n          ";
02071 static const char __pyx_k_Outer_product_power_x_clifford[] = "\n          Outer product power.\n\n
>> x=clifford("\2+\1\"); print(x.outer_pow(0))\n          1\n          >> x=clifford("\2+\1\");
print(x.outer_pow(1))\n          2+\{1\}\n          >> x=clifford("\2+\1\"); print(x.outer_pow(2))\n
4+4\{1\}\n          >> print(clifford("\1+\1+\1,2\").outer_pow(3))\n          1+3\{1\}+3\{1,2\}\n\n
";
02072 static const char __pyx_k_Outer_product_print_clifford_1[] = "\n          Outer product.\n\n          >>
print(clifford("\{1\}") ^ clifford("\{2\}"))\n          \{1,2\}\n          >> print(clifford(2) ^
clifford("\{2\}"))\n          2\{2\}\n          >> print(clifford("\{1\}") ^ clifford("\{1\}"))\n
0\n          >> print(clifford("\{1\}") ^ clifford("\{1,2\}"))\n          0\n          ";
02073 static const char __pyx_k_Power_self_to_the_m_x_clifford[] = "\n          Power: self to the m.\n\n
>> x=clifford("\{1\}"); print(x ** 2)\n          1\n          >> x=clifford("\2\"); print(x ** 2)\n
4\n          >> x=clifford("\2+\1\"); print(x ** 0)\n          1\n          >> x=clifford("\2+\1\");
print(x ** 1)\n          2+\{1\}\n          >> x=clifford("\2+\1\"); print(x ** 2)\n          5+4\{1\}\n
>> i=clifford("\{1,2\}"); print(exp(pi/2) * (i ** i))\n          1\n          ";
02074 static const char __pyx_k_Pure_part_print_clifford_1_1_1[] = "\n          Pure part.\n\n          >>
print(clifford("\1+\1+\1,2\").pure())\n          \{1\}+\{1,2\}\n          >>
print(clifford("\{1,2\}").pure())\n          \{1,2\}\n          ";
02075 static const char __pyx_k_Quadratic_form_rev_x_x_0_print[] = "\n          Quadratic form ==
(rev(x)*x)(0).\n\n          >> print(clifford("\1+\1+\1,2\").quad())\n          3.0\n          >>
print(clifford("\1+\{-1\}+\{1,2\}+\{1,2,3\}").quad())\n          2.0\n          ";
02076 static const char __pyx_k_Quadratic_norm_error_tolerance[] = "\n          Quadratic norm error tolerance
relative to a specific multivector.\n\n          >> print(error_squared_tol(clifford("\{1\}")) * 3.0 -
error_squared_tol(clifford("\1\{1\}-2\{2\}+3\{3\}")))\n          0.0\n          ";
02077 static const char __pyx_k_Set_complement_not_print_index[] = "\n          Set complement: not.\n\n
>>
print(~index_set(\{-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}))\n
\{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32\}\n
";
02078 static const char __pyx_k_Set_union_or_print_index_set_1[] = "\n          Set union: or.\n\n          >>
print(index_set(\{1\}) | index_set(\{2\}))\n          \{1,2\}\n          >> print(index_set(\{1,2\}) |
index_set(\{2\}))\n          \{1,2\}\n          ";
02079 static const char __pyx_k_Transform_left_hand_side_using[] = "\n          Transform left hand side,
using right hand side as a transformation.\n\n          >> x=clifford("\{1,2\}") * pi/2;
y=clifford("\{1\}"); print(y|x)\n          -\{1\}\n          >> x=clifford("\{1,2\}") * pi/2;
y=clifford("\{1\}"); print(y|exp(x))\n          -\{1\}\n          ";
02080 static const char __pyx_k_clifford_vector_part_line_1079[] = "clifford.vector_part (line 1079)";
02081 static const char __pyx_k_index_set__getitem__line_191[] = "index_set.__getitem__ (line 191)";
02082 static const char __pyx_k_index_set__setitem__line_179[] = "index_set.__setitem__ (line 179)";
02083 static const char __pyx_k_lexicographic_compare_eg_3_4_5[] = "\n          \lexicographic compare" eg.
\{3,4,5\} is less than \{3,7,8\};\n          -1 if a<b, +1 if a>b, 0 if a==b.\n\n          >>
compare(index_set(\{1,2\}),index_set(\{-1,3\}))\n          -1\n          >>
compare(index_set(\{-1,4\}),index_set(\{-1,3\}))\n          1\n          ";
02084 static const char __pyx_k_Abbreviation_for_clifford_index[] = "\n          Abbreviation for
clifford(index_set(obj)).\n\n          >> print(e(1))\n          \{1\}\n          >> print(e(-1))\n          \{-1\}\n          >>
print(e(0))\n          1\n          ";
02085 static const char __pyx_k_Absolute_value_of_multivector_m[] = "\n          Absolute value of multivector:
multivector 2-norm.\n\n          >> abs(clifford("\1+\{-1\}+\{1,2\}+\{1,2,3\}"))\n          2.0\n          ";
02086 static const char __pyx_k_Absolute_value_square_root_of_n[] = "\n          Absolute value: square root
of norm.\n\n          >> clifford("\1+\{-1\}+\{1,2\}+\{1,2,3\}").abs()\n          2.0\n          ";
02087 static const char __pyx_k_Cardinality_Number_of_indices_i[] = "\n          Cardinality: Number of
indices included in set.\n\n          >> index_set(\{-1,1,2\}).count()\n          3\n          ";
02088 static const char __pyx_k_Check_if_a_multivector_contains[] = "\n          Check if a multivector
contains any infinite values.\n\n          >> clifford().isinfin()\n          False\n          ";
02089 static const char __pyx_k_Contraction_print_clifford_1_c1[] = "\n          Contraction.\n\n          >>
print(clifford("\{1\}") % clifford("\{2\}"))\n          0\n          >> print(clifford(2) %

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        clifford("\{2}\n")\n                2{2}\n                >> print(clifford("\{1}\n") % clifford("\{1}\n"))\n
1\n                >> print(clifford("\{1}\n") % clifford("\{1,2}\n"))\n                {2}\n                ";
02090 static const char __pyx_k_Contraction_x_clifford_1_x_clif[] = "\n                Contraction.\n\n                >> x
= clifford("\{1}\n"); x % = clifford("\{2}\n"); print(x)\n                0\n                >> x = clifford(2); x % =
clifford("\{2}\n"); print(x)\n                2{2}\n                >> x = clifford("\{1}\n"); x % = clifford("\{1}\n");
print(x)\n                1\n                >> x = clifford("\{1}\n"); x % = clifford("\{1,2}\n"); print(x)\n
{2}\n                ";
02091 static const char __pyx_k_Convert_CGA3_null_vector_to_Euc[] = "\n                Convert CGA3 null vector to
Euclidean 3D vector using Doran and Lasenby definition.\n\n                >> x=clifford("\{2{1}+9{2}+{3}\n");
print(agg3(cga3(x))\n                2{1}+9{2}+{3}\n                >> x=clifford("\{2{1}+9{2}+{3}\n");
print(agg3(cga3(x))-x)\n                0\n                ";
02092 static const char __pyx_k_Convert_CGA3_null_vector_to_sta[] = "\n                Convert CGA3 null vector to
standard conformal null vector using Doran and Lasenby definition.\n\n                >>
x=clifford("\{2{1}+9{2}+{3}\n"); print(cga3std(cga3(x))\n                87{-1}+4{1}+18{2}+2{3}+85{4}\n                >>
x=clifford("\{2{1}+9{2}+{3}\n"); print(cga3std(cga3(x))-cga3(x))\n                0\n                ";
02093 static const char __pyx_k_Convert_Euclidean_3D_multivector[] = "\n                Convert Euclidean 3D multivector
to Conformal Geometric Algebra using Doran and Lasenby definition.\n\n                >>
x=clifford("\{2{1}+9{2}+{3}\n"); print(cga3(x))\n                87{-1}+4{1}+18{2}+2{3}+85{4}\n                ";
02094 static const char __pyx_k_Copy_this_clifford_object_x_cli[] = "\n                Copy this clifford
object.\n\n                >> x=clifford("\{1{2}\n"); y=x.copy(); print(y)\n                {2}\n                ";
02095 static const char __pyx_k_Copy_this_index_set_object_s_in[] = "\n                Copy this index_set
object.\n\n                >> s=index_set(1); t=s.copy(); print(t)\n                {1}\n                ";
02096 static const char __pyx_k_Cosine_of_multivector_with_opti[] = "\n                Cosine of multivector with
optional complexifier.\n\n                >> x=clifford("\{1,2}\n"); print(cos(acos(x),"\{1,2,3}\n"))\n                {1,2}\n
>> x=clifford("\{1,2}\n"); print(cos(acos(x)))\n                {1,2}\n                ";
02097 static const char __pyx_k_Even_part_of_multivector_sum_of[] = "\n                Even part of multivector, sum
of even grade terms.\n\n                >> print(clifford("\{1+{1}+{1,2}\n").even())\n                1+{1,2}\n
";
02098 static const char __pyx_k_Exponential_of_multivector_x_cl[] = "\n                Exponential of multivector.\n\n
>> x=clifford("\{1,2}\n") * pi/4; print(exp(x))\n                0.7071+0.7071{1,2}\n                >> x=clifford("\{1,2}\n")
* pi/2; print(exp(x))\n                {1,2}\n                ";
02099 static const char __pyx_k_Geometric_difference_print_clif[] = "\n                Geometric difference.\n\n
>> print(clifford(1) - clifford("\{2}\n"))\n                1-{2}\n                >> print(clifford("\{1}\n") -
clifford("\{2}\n"))\n                {1}-{2}\n                ";
02100 static const char __pyx_k_Geometric_difference_x_clifford[] = "\n                Geometric difference.\n\n
>> x = clifford(1); x -= clifford("\{2}\n"); print(x)\n                1-{2}\n                ";
02101 static const char __pyx_k_Geometric_multiplicative_invers[] = "\n                Geometric multiplicative
inverse.\n\n                >> x = clifford("\{1}\n"); print(x.inv())\n                {1}\n                >> x =
clifford(2); print(x.inv())\n                0.5\n                >> x = clifford("\{1,2}\n"); print(x.inv())\n
-{1,2}\n                ";
02102 static const char __pyx_k_Geometric_product_print_clifor[] = "\n                Geometric product.\n\n
>> print(clifford("\{1}\n") * clifford("\{2}\n"))\n                {1,2}\n                >> print(clifford(2) *
clifford("\{2}\n"))\n                2{2}\n                >> print(clifford("\{1}\n") * clifford("\{1,2}\n"))\n
{2}\n                ";
02103 static const char __pyx_k_Geometric_quotient_print_cliffo[] = "\n                Geometric quotient.\n\n
>> print(clifford("\{1}\n") / clifford("\{2}\n"))\n                {1,2}\n                >> print(clifford(2) /
clifford("\{2}\n"))\n                2{2}\n                >> print(clifford("\{1}\n") / clifford("\{1}\n"))\n
1\n                >> print(clifford("\{1}\n") / clifford("\{1,2}\n"))\n                -{2}\n                ";
02104 static const char __pyx_k_Geometric_quotient_x_clifford_1[] = "\n                Geometric quotient.\n\n
>> x = clifford("\{1}\n"); x /= clifford("\{2}\n"); print(x)\n                {1,2}\n                >> x =
clifford(2); x /= clifford("\{2}\n"); print(x)\n                2{2}\n                >> x = clifford("\{1}\n"); x /=
clifford("\{1}\n"); print(x)\n                1\n                >> x = clifford("\{1}\n"); x /= clifford("\{1,2}\n");
print(x)\n                -{2}\n                ";
02105 static const char __pyx_k_Geometric_sum_x_clifford_1_x_cl[] = "\n                Geometric sum.\n\n                >>
x = clifford(1); x += clifford("\{2}\n"); print(x)\n                1+{2}\n                ";
02106 static const char __pyx_k_Get_the_value_of_an_index_set_o[] = "\n                Get the value of an index_set
object at an index.\n\n                >> index_set({1})[1]\n                True\n                >> index_set({1})[2]\n
False\n                >> index_set({2})[-1]\n                False\n                >> index_set({2})[1]\n                False\n
>> index_set({2})[2]\n                True\n                >> index_set({2})[3]\n                False\n                ";
02107 static const char __pyx_k_Hyperbolic_cosine_of_multivector[] = "\n                Hyperbolic cosine of
multivector.\n\n                >> x=clifford("\{1,2}\n") * pi; print(cosh(x))\n                -1\n                >>
x=clifford("\{1,2,3}\n"); print(cosh(acos(x)))\n                {1,2,3}\n                >> x=clifford("\{1,2}\n");
print(cosh(acos(x)))\n                {1,2}\n                ";
02108 static const char __pyx_k_Hyperbolic_tangent_of_multivect[] = "\n                Hyperbolic tangent of
multivector.\n\n                >> x=clifford("\{1,2}\n") * pi/4; print(tanh(x))\n                {1,2}\n                ";
02109 static const char __pyx_k_Imaginary_part_deprecated_alway[] = "\n                Imaginary part: deprecated
(always 0).\n\n                >> imag(clifford("\{1+{1}+{1,2}\n"))\n                0.0\n                >> imag(clifford("\{1,2}\n"))\n
0.0\n                ";
02110 static const char __pyx_k_Inner_product_x_clifford_1_x_cl[] = "\n                Inner product.\n\n                >>
x = clifford("\{1}\n"); x &= clifford("\{2}\n"); print(x)\n                0\n                >> x = clifford(2); x &=
clifford("\{2}\n"); print(x)\n                0\n                >> x = clifford("\{1}\n"); x &= clifford("\{1}\n");
print(x)\n                1\n                >> x = clifford("\{1}\n"); x &= clifford("\{1,2}\n"); print(x)\n
{2}\n                ";
02111 static const char __pyx_k_Integer_power_of_multivector_ob[] = "\n                Integer power of multivector: obj
to the m.\n\n                >> x=clifford("\{1}\n"); print(pow(x,2))\n                1\n                >> x=clifford("\{2}\n");
print(pow(x,2))\n                4\n                >> x=clifford("\{2+{1}\n"); print(pow(x,0))\n                1\n                >>
x=clifford("\{2+{1}\n"); print(pow(x,1))\n                2+{1}\n                >> x=clifford("\{2+{1}\n"); print(pow(x,2))\n
5+4{1}\n                >> print(pow(clifford("\{1+{1}+{1,2}\n"),3))\n                1+3{1}+3{1,2}\n                >>
i=clifford("\{1,2}\n"); print(exp(pi/2) * pow(i, i))\n                1\n                ";
02112 static const char __pyx_k_Inverse_cosine_of_multivector_w[] = "\n                Inverse cosine of multivector
with optional complexifier.\n\n                >> x=clifford("\{1,2}\n"); print(cos(acos(x),"\{1,2,3}\n"))\n
{1,2}\n                >> x=clifford("\{1,2}\n"); print(cos(acos(x),"\{-1,1,2,3,4}\n"))\n                {1,2}\n                >>
print(acos(0) / pi)\n                0.5\n                >> x=clifford("\{1,2}\n"); print(cos(acos(x)))\n                {1,2}\n                ";
02113 static const char __pyx_k_Inverse_hyperbolic_cosine_of_mu[] = "\n                Inverse hyperbolic cosine of
multivector with optional complexifier.\n\n                >> print(acosh(0,"\{-2,-1,1}\n"))\n                1.571{-2,-1,1}\n
>> x=clifford("\{1,2,3}\n"); print(cosh(acosh(x),"\{-1,1,2,3,4}\n"))\n                {1,2,3}\n                >>
print(acosh(0))\n                1.571{-1}\n                >> x=clifford("\{1,2,3}\n"); print(cosh(acosh(x)))\n                {1,2,3}\n

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    >> x=clifford("\{1,2}\"); print(cosh(acosh(x)))\n    {1,2}\n    ";
02114 static const char __pyx_k_Inverse_hyperbolic_sine_of_mult[] = "\n    Inverse hyperbolic sine of
multivector with optional complexifier.\n\n    >> x=clifford("\{1,2}\"); print(asinh(x, "\{1,2,3}\") *
2/pi)\n    {1,2}\n    >> x=clifford("\{1,2}\"); print(asinh(x) * 2/pi)\n    {1,2}\n    >>
x=clifford("\{1,2}\") / 2; print(asinh(x) * 6/pi)\n    {1,2}\n    ";
02115 static const char __pyx_k_Inverse_hyperbolic_tangent_of_m[] = "\n    Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford("\{1,2}\");
print(tanh(atanh(x,s)))\n    {1,2}\n    >> x=clifford("\{1,2}\"); print(tanh(atanh(x)))\n    {1,2}\n
";
02116 static const char __pyx_k_Inverse_sine_of_multivector_wit[] = "\n    Inverse sine of multivector with
optional complexifier.\n\n    >> s="\{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>
s="\{-1}\"; x=clifford(s); print(asin(sin(x,s), "\{-2,-1,1}\"))\n    {-1}\n    >> print(asin(1) /
pi)\n    0.5\n    >> x=clifford("\{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    ";
02117 static const char __pyx_k_Main_involution_each_i_is_repla[] = "\n    Main involution, each {i} is
replaced by -{i} in each term,\n    eg. clifford("\{1}\") -> -clifford("\{1}\").\n\n    >>
print(clifford("\{1}\").involute())\n    {-1}\n    >> print((clifford("\{2}\") *
clifford("\{1}\")).involute())\n    {-1,2}\n    >> print((clifford("\{1}\") *
clifford("\{2}\")).involute())\n    {1,2}\n    >>
print(clifford("\{1+{1}+{1,2}\").involute())\n    1-{1}+{1,2}\n    ";
02118 static const char __pyx_k_Maximum_absolute_value_of_coord[] = "\n    Maximum absolute value of
coordinates: multivector infinity-norm.\n\n    >>
max_abs(clifford("\{1+{-1}+{1,2}+{1,2,3}\"))\n    1.0\n    >> max_abs(clifford("\{3+2{1}+{1,2}\"))\n
3.0\n    ";
02119 static const char __pyx_k_Maximum_of_absolute_values_of_c[] = "\n    Maximum of absolute values of
components of multivector: multivector infinity norm.\n\n    >>
clifford("\{1+{-1}+{1,2}+{1,2,3}\").max_abs()\n    1.0\n    >>
clifford("\{3+2{1}+{1,2}\").max_abs()\n    3.0\n    ";
02120 static const char __pyx_k_Natural_logarithm_of_multivecto[] = "\n    Natural logarithm of multivector
with optional complexifier.\n\n    >> x=clifford("\{-1}\"); print((log(x, "\{-1}\") * 2/pi))\n
{-1}\n    >> x=clifford("\{1,2}\"); print((log(x, "\{1,2,3}\") * 2/pi))\n    {1,2}\n    >>
x=clifford("\{1,2}\"); print((log(x) * 2/pi))\n    {1,2}\n    >> x=clifford("\{1,2}\");
print((log(x, "\{1,2}\") * 2/pi))\n    Traceback (most recent call last):\n    ...n    RuntimeError:
check_complex(val, i): i is not a valid complexifier for val\n    ";
02121 static const char __pyx_k_Norm_sum_of_squares_of_coordina[] = "\n    Norm == sum of squares of
coordinates.\n\n    >> clifford("\{1+{1}+{1,2}\").norm()\n    3.0\n    >>
clifford("\{1+{-1}+{1,2}+{1,2,3}\").norm()\n    4.0\n    ";
02122 static const char __pyx_k_Not_applicable_for_a_in_cliffor[] = "\n    Not applicable.\n\n    >>
for a in clifford(index_set({-3,4,7})):print(a, end="\,")\n    Traceback (most recent call
last):\n    ...n    TypeError: Not applicable.\n    ";
02123 static const char __pyx_k_Number_of_negative_indices_incl[] = "\n    Number of negative indices
included in set.\n\n    >> index_set({-1,1,2}).count_neg()\n    1\n    ";
02124 static const char __pyx_k_Number_of_positive_indices_incl[] = "\n    Number of positive indices
included in set.\n\n    >> index_set({-1,1,2}).count_pos()\n    2\n    ";
02125 static const char __pyx_k_Outer_product_power_of_multivec[] = "\n    Outer product power of
multivector.\n\n    >> print(outer_pow(clifford("\{1+{1}+{1,2}\"),3))\n    1+3{1}+3{1,2}\n    ";
02126 static const char __pyx_k_Outer_product_x_clifford_1_x_cl[] = "\n    Outer product.\n\n    >>
x = clifford("\{1}\"); x ^= clifford("\{2}\"); print(x)\n    {1,2}\n    >> x = clifford(2); x
^= clifford("\{2}\"); print(x)\n    2{2}\n    >> x = clifford("\{1}\"); x ^=
clifford("\{1}\"); print(x)\n    0\n    >> x = clifford("\{1}\"); x ^= clifford("\{1,2}\");
print(x)\n    0\n    ";
02127 static const char __pyx_k_Pure_grade_vector_part_print_cl[] = "\n    Pure grade-vector part.\n\n
>> print(clifford("\{1}\") (1))\n    {1}\n    >> print(clifford("\{1}\") (0))\n    0\n
>> print(clifford("\{1+{1}+{1,2}\") (0))\n    1\n    >>
print(clifford("\{1+{1}+{1,2}\") (1))\n    {1}\n    >> print(clifford("\{1+{1}+{1,2}\") (2))\n
{1,2}\n    >> print(clifford("\{1+{1}+{1,2}\") (3))\n    0\n    ";
02128 static const char __pyx_k_Pure_part_print_pure_clifford_1[] = "\n    Pure part\n\n    >>
print(pure(clifford("\{1+{1}+{1,2}\"))) \n    {1}+{1,2}\n    >> print(pure(clifford("\{1,2}\"))) \n
{1,2}\n    ";
02129 static const char __pyx_k_Put_self_into_a_larger_frame_co[] = "\n    Put self into a larger frame,
containing the union of self.frame() and index set ixt.\n    This can be used to make
multiplication faster, by multiplying within a common frame.\n\n    >>
clifford("\{2+3{1}\").reframe(index_set({1,2,3}))\n    clifford("\{2+3{1}\")\n    >>
s=index_set({1,2,3}); t=index_set({-3,-2,-1}); x=random_clifford(s); x.reframe(t).frame() == (s|t);\n
True\n    ";
02130 static const char __pyx_k_Random_multivector_within_a_fra[] = "\n    Random multivector within a
frame.\n\n    >> print(random_clifford(index_set({-3,-1,2})).frame())\n    {-3,-1,2}\n    ";
02131 static const char __pyx_k_Real_part_synonym_for_scalar_pa[] = "\n    Real part: synonym for scalar
part.\n\n    >> real(clifford("\{1+{1}+{1,2}\"))\n    1.0\n    >> real(clifford("\{1,2}\"))\n    0.0\n
";
02132 static const char __pyx_k_Relative_or_absolute_error_usin[] = "\n    Relative or absolute error using
the quadratic norm.\n\n    >> err2=scalar_epsilon*scalar_epsilon\n    >>
print(error_squared(clifford("\{1}\"), clifford("\{1}\"), err2))\n    0.0\n    >>
print(error_squared(clifford("\{1{1}-3{2}+4{3}\"), clifford("\{1}\"), err2))\n    25.0\n    ";
02133 static const char __pyx_k_Remove_all_terms_of_self_with_r[] = "\n    Remove all terms of self with
relative size smaller than limit.\n\n    >> clifford("\{1e8+{1}+1e-8{1,2}\").truncated(1.0e-6)\n
clifford("\{100000000}\")\n    >> clifford("\{1e4+{1}+1e-4{1,2}\").truncated(1.0e-6)\n
clifford("\{10000+{1}\")\n    ";
02134 static const char __pyx_k_Reversion_eg_1_2_2_1_print_reve[] = "\n    Reversion, eg. {1}*{2} ->
{2}*{1}\n\n    >> print(reverse(clifford("\{1}\"))) \n    {1}\n    >> print(reverse(clifford("\{2}\")
* clifford("\{1}\"))) \n    {1,2}\n    >> print(reverse(clifford("\{1}\") * clifford("\{2}\"))) \n
{-1,2}\n    >> print(reverse(clifford("\{1+{1}+{1,2}\"))) \n    1+{1}-{1,2}\n    ";
02135 static const char __pyx_k_Reversion_eg_clifford_1_cliffor[] = "\n    Reversion, eg.
clifford("\{1}\") * clifford("\{2}\") -> clifford("\{2}\") * clifford("\{1}\").\n\n    >>
print(clifford("\{1}\").reverse()) \n    {1}\n    >> print((clifford("\{2}\") *
clifford("\{1}\")).reverse()) \n    {1,2}\n    >> print((clifford("\{1}\") *
clifford("\{2}\")).reverse()) \n    {-1,2}\n    >>
print(clifford("\{1+{1}+{1,2}\").reverse()) \n    1+{1}-{1,2}\n    ";

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02136 static const char __pyx_k_Scalar_part_clifford_1_1_2_sc[] = "\n      Scalar part.\n\n      >> clifford(\\"1+{1}+{1,2}\\").scalar()\n      1.0\n      >> clifford(\\"{1,2}\\").scalar()\n      0.0\n      ";
02137 static const char __pyx_k_Scalar_part_scalar_clifford_1_1[] = "\n      Scalar part.\n\n      >> scalar(clifford(\\"1+{1}+{1,2}\\"))\n      1.0\n      >> scalar(clifford(\\"{1,2}\\"))\n      0.0\n      ";
02138 static const char __pyx_k_Set_intersection_and_print_index[] = "\n      Set intersection: and.\n\n      >> print(index_set({1}) & index_set({2}))\n      {2}\n      >> print(index_set({1,2}) & index_set({2}))\n      {2}\n      ";
02139 static const char __pyx_k_Set_intersection_and_x_index_set[] = "\n      Set intersection: and.\n\n      >> x = index_set({1}); x &= index_set({2}); print(x)\n      {2}\n      >> x = index_set({1,2}); x &= index_set({2}); print(x)\n      {2}\n      ";
02140 static const char __pyx_k_Set_the_value_of_an_index_set_object[] = "\n      Set the value of an index_set object at index idx to value val.\n\n      >> s=index_set({1}); s[2] = True; print(s)\n      {1,2}\n      >> s=index_set({1,2}); s[1] = False; print(s)\n      {2}\n      ";
02141 static const char __pyx_k_Set_union_or_x_index_set_1_x_in[] = "\n      Set union: or.\n\n      >> x = index_set({1}); x |= index_set({2}); print(x)\n      {1,2}\n      >> x = index_set({1,2}); x |= index_set({2}); print(x)\n      {1,2}\n      ";
02142 static const char __pyx_k_Sign_of_geometric_product_of_two[] = "\n      Sign of geometric product of two Clifford basis elements.\n\n      >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)\n      1\n      ";
02143 static const char __pyx_k_Sign_of_geometric_square_of_a_C[] = "\n      Sign of geometric square of a Clifford basis element.\n\n      >> s = index_set({1,2}); s.sign_of_square()\n      -1\n      ";
02144 static const char __pyx_k_Sine_of_multivector_with_option[] = "\n      Sine of multivector with optional complexifier.\n\n      >> s=\\"{-1}\\"; x=clifford(s); print(asin(sin(x,s),s))\n      {-1}\n      >> s=\\"{-1}\\"; x=clifford(s); print(asin(sin(x,s),\\"{-2,-1,1}\\"))\n      {-1}\n      >> x=clifford(\\"{1,2,3}\\"); print(asin(sin(x))\n      {1,2,3}\n      ";
02145 static const char __pyx_k_Square_root_of_1_which_commutates[] = "\n      Square root of -1 which commutes with all members of the frame of the given multivector.\n\n      >> print(complexifier(clifford(index_set({1})))\n      {1,2,3}\n      >> print(complexifier(clifford(index_set({-1})))\n      {-1}\n      >> print(complexifier(index_set({1})))\n      {1,2,3}\n      >> print(complexifier(index_set({-1})))\n      {-1}\n      ";
02146 static const char __pyx_k_Square_root_of_multivector_with[] = "\n      Square root of multivector with optional complexifier.\n\n      >> print(sqrt(-1))\n      {-1}\n      >> print(sqrt(clifford(\\"2{-1}\\")))\n      1+{-1}\n      >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)\n      {1,2,3}\n      -1\n      >> j=sqrt(-1,\\"{1,2,3}\\"); print(j); print(j*j)\n      {1,2,3}\n      -1\n      ";
02147 static const char __pyx_k_Subalgebra_generated_by_all_generators[] = "\n      Subalgebra generated by all generators of terms of given multivector.\n\n      >> print(clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\\").frame())\n      {-2,-1,1,2,7}\n      >> s=clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\\").frame(); type(s)\n      <class 'PyClical.index_set'>\n      ";
02148 static const char __pyx_k_Subscripting_map_from_index_set[] = "\n      Subscripting: map from index set to scalar coordinate.\n\n      >> clifford(\\"{1}\\")[index_set(1)]\n      1.0\n      >> clifford(\\"{1}\\")[index_set({1})]\n      0.0\n      >> clifford(\\"{1}\\")[index_set({1,2})]\n      0.0\n      >> clifford(\\"2{1,2}\\")[index_set({1,2})]\n      2.0\n      ";
02149 static const char __pyx_k_Symmetric_set_difference_exclusive_or[] = "\n      Symmetric set difference: exclusive or.\n\n      >> print(index_set({1}) ^ index_set({2}))\n      {1,2}\n      >> print(index_set({1,2}) ^ index_set({2}))\n      {1}\n      ";
02150 static const char __pyx_k_Tangent_of_multivector_with_optional_complexifier[] = "\n      Tangent of multivector with optional complexifier.\n\n      >> x=clifford(\\"{1,2}\\"); print(tan(x,\\"{1,2,3}\\"))\n      0.7616{1,2}\n      >> x=clifford(\\"{1,2}\\"); print(tan(x))\n      0.7616{1,2}\n      ";
02151 static const char __pyx_k_Test_for_approximate_equality_of_multivectors[] = "\n      Test for approximate equality of multivectors.\n\n      >> err2=sqrt(epsilon)*sqrt(epsilon)\n      >> print(approx_equal(clifford(\\"{1}\\"), clifford(\\"1{1}\\"))\n      True\n      >> print(approx_equal(clifford(\\"1{1}-3{2}+4{3}\\"), clifford(\\"{1}\\"))\n      False\n      >> print(approx_equal(clifford(\\"1{1}-3{2}+4{3}+0.001\\"), clifford(\\"1{1}-3{2}+4{3}\\"), err2, err2)\n      False\n      >> print(approx_equal(clifford(\\"1{1}-3{2}+4{3}+1.0e-30\\"), clifford(\\"1{1}-3{2}+4{3}\\"), err2, err2)\n      True\n      ";
02152 static const char __pyx_k_Tests_for_functions_that_Doctest_cannot_see[] = "\n      Tests for functions that Doctest cannot see.\n\n      For index_set.__cinit__: Construct index_set.\n\n      >> print(index_set(1))\n      {1}\n      >> print(index_set({1,2}))\n      {1,2}\n      >> print(index_set(index_set({1,2})))\n      {1,2}\n      >> print(index_set({1,2}))\n      {1,2}\n      >> print(index_set({1,2,1}))\n      {1,2}\n      >> print(index_set(\\"\\"))\n      {}\n      >> print(index_set(\\"{\\"))\n      Traceback (most recent call last):\n      ...n      ValueError: Cannot initialize index_set object from invalid string '{'.\n      >> print(index_set(\\"{1}\\"))\n      Traceback (most recent call last):\n      ...n      ValueError: Cannot initialize index_set object from invalid string '{1'.\n      >> print(index_set(\\"{1,2,100}\\"))\n      Traceback (most recent call last):\n      ...n      ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.\n      >> print(index_set({1,2,100}))\n      Traceback (most recent call last):\n      ...n      IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.\n      >> print(index_set({1,2}))\n      Traceback (most recent call last):\n      ...n      TypeError: Cannot initialize index_set object from <class 'list'>.\n      For index_set.__richcmp__: Compare two objects of class index_set.\n\n      >> index_set(1) == index_set({1})\n      True\n      >> index_set({1}) != index_set({1})\n      False\n      >> index_set({1}) != index_set({2})\n      True\n      >> index_set({1}) == index_set({2})\n      False\n      >> index_set({1}) < index_set({2})\n      True\n      >> index_set({1}) <= index_set({2})\n      True\n      >> index_set({1}) > index_set({2})\n      False\n      >> index_set({1}) >= index_set({2})\n      False\n      >> index_set({1}) >= index_set({2})\n      False\n      >> None == index_set({1,2})\n      False\n      >> None != index_set({1,2})\n      True\n      >> None < index_set({1,2})\n      False\n      >> None <= index_set({1,2})\n      False\n      >> None > index_set({1,2})\n      False\n      >> None >= index_set({1,2})\n      False\n      >> ""index_set({1,2}) == None\n      False\n      >> index_set({1,2}) != None\n      True\n      >> index_set({1,2}) < None\n      False\n      >> index_set({1,2}) <= None\n      False\n      >> index_set({1,2}) > None\n      False\n      >> index_set({1,2}) >= None\n      False\n      ";
02153 static const char __pyx_k_The_informal_string_representation_of_self[] = "\n      The \342\200\234informal\342\200\235 string representation of self.\n\n      >> index_set({1,2}).__str__()\n      '{1,2}'\n      >> str(index_set({1,2}))\n      '{1,2}'\n      ";

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ValueError: Cannot initialize clifford object from invalid string '{1}+'.\n\n For
clifford.__richcmp__: Compare objects of type clifford.\n\n >> clifford("\{1}\") ==
clifford("\{1}\")\n True\n >> clifford("\{1}\") != clifford("\{1.0}\")\n False\n >>
clifford("\{1}\") != clifford("\{1.0}\")\n True\n >> clifford("\{1,2}\") == None\n False\n
>> clifford("\{1,2}\") != None\n True\n >> None == clifford("\{1,2}\")\n False\n >> None
!= clifford("\{1,2}\")\n True\n";
02176     static const char __pyx_k_The_informal_string_representat_2[] = "\n The
\342\200\234informal\342\200\235 string representation of self.\n\n >>
clifford("\{1+3{-1}+2{1,2}+4{-2,7}\").__str__()\n '1+3{-1}+2{1,2}+4{-2,7}'\n";
02177     static const char __pyx_k_The_official_string_representat_2[] = "\n The
\342\200\234official\342\200\235 string representation of self.\n\n >>
clifford("\{1+3{-1}+2{1,2}+4{-2,7}\").__repr__()\n 'clifford("\{1+3{-1}+2{1,2}+4{-2,7}\")'\n
";
02178     static PyObject *__pyx_kp_u_;
02179     static PyObject *__pyx_kp_u_Abbreviation_for_clifford_index;
02180     static PyObject *__pyx_kp_u_Abbreviation_for_index_set_q_p;
02181     static PyObject *__pyx_kp_u_Absolute_value_of_multivector_m;
02182     static PyObject *__pyx_kp_u_Absolute_value_square_root_of_n;
02183     static PyObject *__pyx_kp_u_Cannot_initialize_clifford_objec;
02184     static PyObject *__pyx_kp_u_Cannot_initialize_index_set_obje;
02185     static PyObject *__pyx_kp_u_Cannot_reframe;
02186     static PyObject *__pyx_kp_u_Cannot_take_vector_part_of;
02187     static PyObject *__pyx_kp_u_Cardinality_Number_of_indices_i;
02188     static PyObject *__pyx_kp_u_Check_if_a_multivector_contains;
02189     static PyObject *__pyx_kp_u_Check_if_a_multivector_contains_2;
02190     static PyObject *__pyx_kp_u_Conjugation_reverse_o_involute;
02191     static PyObject *__pyx_kp_u_Conjugation_reverse_o_involute_2;
02192     static PyObject *__pyx_kp_u_Contraction_print_clifford_l_cl;
02193     static PyObject *__pyx_kp_u_Contraction_x_clifford_l_x_clif;
02194     static PyObject *__pyx_kp_u_Convert_CGA3_null_vector_to_Euc;
02195     static PyObject *__pyx_kp_u_Convert_CGA3_null_vector_to_sta;
02196     static PyObject *__pyx_kp_u_Convert_Euclidean_3D_multivecto;
02197     static PyObject *__pyx_kp_u_Copy_this_clifford_object_x_cli;
02198     static PyObject *__pyx_kp_u_Copy_this_index_set_object_s_in;
02199     static PyObject *__pyx_kp_u_Cosine_of_multivector_with_opti;
02200     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of;
02201     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of_2;
02202     static PyObject *__pyx_kp_u_Exponential_of_multivector_x_cl;
02203     static PyObject *__pyx_kp_u_Geometric_difference_print_clif;
02204     static PyObject *__pyx_kp_u_Geometric_difference_x_clifford;
02205     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers;
02206     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers_2;
02207     static PyObject *__pyx_kp_u_Geometric_product_print_cliffor;
02208     static PyObject *__pyx_kp_u_Geometric_product_x_clifford_2;
02209     static PyObject *__pyx_kp_u_Geometric_quotient_print_cliffo;
02210     static PyObject *__pyx_kp_u_Geometric_quotient_x_clifford_l;
02211     static PyObject *__pyx_kp_u_Geometric_sum_print_clifford_l;
02212     static PyObject *__pyx_kp_u_Geometric_sum_x_clifford_l_x_cl;
02213     static PyObject *__pyx_kp_u_Get_the_value_of_an_index_set_o;
02214     static PyObject *__pyx_kp_u_Hyperbolic_cosine_of_multivecto;
02215     static PyObject *__pyx_kp_u_Hyperbolic_sine_of_multivector;
02216     static PyObject *__pyx_kp_u_Hyperbolic_tangent_of_multivect;
02217     static PyObject *__pyx_kp_u_Imaginary_part_deprecated_alway;
02218     static PyObject *__pyx_n_s_IndexError;
02219     static PyObject *__pyx_kp_u_Inner_product_print_clifford_l;
02220     static PyObject *__pyx_kp_u_Inner_product_x_clifford_l_x_cl;
02221     static PyObject *__pyx_kp_u_Integer_power_of_multivector_ob;
02222     static PyObject *__pyx_n_s_Integral;
02223     static PyObject *__pyx_kp_u_Inverse_cosine_of_multivector_w;
02224     static PyObject *__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu;
02225     static PyObject *__pyx_kp_u_Inverse_hyperbolic_sine_of_mult;
02226     static PyObject *__pyx_kp_u_Inverse_hyperbolic_tangent_of_m;
02227     static PyObject *__pyx_kp_u_Inverse_sine_of_multivector_wit;
02228     static PyObject *__pyx_kp_u_Inverse_tangent_of_multivector;
02229     static PyObject *__pyx_kp_u_Iterate_over_the_indices_of_an;
02230     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla;
02231     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla_2;
02232     static PyObject *__pyx_kp_u_Maximum_absolute_value_of_coord;
02233     static PyObject *__pyx_kp_u_Maximum_member_index_set_l_1_2;
02234     static PyObject *__pyx_kp_u_Maximum_of_absolute_values_of_c;
02235     static PyObject *__pyx_kp_u_Maximum_positive_index_or_0_if;
02236     static PyObject *__pyx_kp_u_Minimum_member_index_set_l_1_2;
02237     static PyObject *__pyx_kp_u_Minimum_negative_index_or_0_if;
02238     static PyObject *__pyx_kp_u_Natural_logarithm_of_multivecto;
02239     static PyObject *__pyx_kp_u_Norm_sum_of_squares_of_coordina;
02240     static PyObject *__pyx_n_s_NotImplemented;
02241     static PyObject *__pyx_kp_u_Not_applicable;
02242     static PyObject *__pyx_kp_u_Not_applicable_for_a_in_cliffor;
02243     static PyObject *__pyx_kp_u_Number_of_negative_indices_incl;
02244     static PyObject *__pyx_kp_u_Number_of_positive_indices_incl;
02245     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of;
02246     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of_2;
02247     static PyObject *__pyx_kp_u_Outer_product_power_of_multivec;
02248     static PyObject *__pyx_kp_u_Outer_product_power_x_clifford;
02249     static PyObject *__pyx_kp_u_Outer_product_print_clifford_l;
02250     static PyObject *__pyx_kp_u_Outer_product_x_clifford_l_x_cl;
02251     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford;

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02252     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford_2;
02253     static PyObject *__pyx_kp_u_Pure_grade_vector_part_print_cl;
02254     static PyObject *__pyx_kp_u_Pure_part_print_clifford_1_1_1;
02255     static PyObject *__pyx_kp_u_Pure_part_print_pure_clifford_1;
02256     static PyObject *__pyx_kp_u_Put_self_into_a_larger_frame_co;
02257     static PyObject *__pyx_n_s_PyClical;
02258     static PyObject *__pyx_kp_s_PyClical_pyx;
02259     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print;
02260     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2;
02261     static PyObject *__pyx_kp_u_Quadratic_norm_error_tolerance;
02262     static PyObject *__pyx_kp_u_Random_multivector_within_a_fra;
02263     static PyObject *__pyx_n_s_Real;
02264     static PyObject *__pyx_kp_u_Real_part_synonym_for_scalar_pa;
02265     static PyObject *__pyx_kp_u_Relative_or_absolute_error_usin;
02266     static PyObject *__pyx_kp_u_Remove_all_terms_of_self_with_r;
02267     static PyObject *__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve;
02268     static PyObject *__pyx_kp_u_Reversion_eg_clifford_1_cliffor;
02269     static PyObject *__pyx_n_s_RuntimeError;
02270     static PyObject *__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc;
02271     static PyObject *__pyx_kp_u_Scalar_part_scalar_clifford_1_1;
02272     static PyObject *__pyx_n_s_Sequence;
02273     static PyObject *__pyx_kp_u_Set_complement_not_print_index;
02274     static PyObject *__pyx_kp_u_Set_intersection_and_print_inde;
02275     static PyObject *__pyx_kp_u_Set_intersection_and_x_index_se;
02276     static PyObject *__pyx_kp_u_Set_the_value_of_an_index_set_o;
02277     static PyObject *__pyx_kp_u_Set_union_or_print_index_set_1;
02278     static PyObject *__pyx_kp_u_Set_union_or_x_index_set_1_x_in;
02279     static PyObject *__pyx_kp_u_Sign_of_geometric_product_of_tw;
02280     static PyObject *__pyx_kp_u_Sign_of_geometric_square_of_a_C;
02281     static PyObject *__pyx_kp_u_Sine_of_multivector_with_option;
02282     static PyObject *__pyx_kp_u_Square_root_of_1_which_commutes;
02283     static PyObject *__pyx_kp_u_Square_root_of_multivector_with;
02284     static PyObject *__pyx_kp_u_Subalgebra_generated_by_all_gen;
02285     static PyObject *__pyx_kp_u_Subscripting_map_from_index_set;
02286     static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus;
02287     static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus_2;
02288     static PyObject *__pyx_kp_u_Tangent_of_multivector_with_opt;
02289     static PyObject *__pyx_kp_u_Test_for_approximate_equality_o;
02290     static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes;
02291     static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes_2;
02292     static PyObject *__pyx_kp_u_The_informal_string_representat;
02293     static PyObject *__pyx_kp_u_The_informal_string_representat_2;
02294     static PyObject *__pyx_kp_u_The_official_string_representat;
02295     static PyObject *__pyx_kp_u_The_official_string_representat_2;
02296     static PyObject *__pyx_kp_u_This_comparison_operator_is_not;
02297     static PyObject *__pyx_kp_u_Transform_left_hand_side_using;
02298     static PyObject *__pyx_kp_u_Transform_left_hand_side_using_2;
02299     static PyObject *__pyx_n_s_TypeError;
02300     static PyObject *__pyx_kp_u_UTF_8;
02301     static PyObject *__pyx_kp_u_Unary_print_clifford_1_1;
02302     static PyObject *__pyx_kp_u_Unary_print_clifford_1_1_2;
02303     static PyObject *__pyx_n_s_ValueError;
02304     static PyObject *__pyx_kp_u_Vector_part_of_multivector_as_a;
02305     static PyObject *__pyx_kp_u_2;
02306     static PyObject *__pyx_kp_u_5;
02307     static PyObject *__pyx_kp_u_6;
02308     static PyObject *__pyx_kp_u_7;
02309     static PyObject *__pyx_kp_u_8;
02310     static PyObject *__pyx_kp_u_9;
02311     static PyObject *__pyx_n_s_abc;
02312     static PyObject *__pyx_kp_u_abs_line_1522;
02313     static PyObject *__pyx_n_s_acos;
02314     static PyObject *__pyx_kp_u_acos_line_1668;
02315     static PyObject *__pyx_n_s_acosh;
02316     static PyObject *__pyx_kp_u_acosh_line_1705;
02317     static PyObject *__pyx_kp_u_agc3_line_1893;
02318     static PyObject *__pyx_kp_u_approx_equal_line_1359;
02319     static PyObject *__pyx_n_s_args;
02320     static PyObject *__pyx_kp_u_as_frame;
02321     static PyObject *__pyx_n_s_asin;
02322     static PyObject *__pyx_kp_u_asin_line_1747;
02323     static PyObject *__pyx_n_s_asinh;
02324     static PyObject *__pyx_kp_u_asinh_line_1782;
02325     static PyObject *__pyx_n_s_atan;
02326     static PyObject *__pyx_kp_u_atan_line_1818;
02327     static PyObject *__pyx_n_s_atanh;
02328     static PyObject *__pyx_kp_u_atanh_line_1847;
02329     static PyObject *__pyx_kp_u_cga3_line_1873;
02330     static PyObject *__pyx_kp_u_cga3std_line_1882;
02331     static PyObject *__pyx_n_s_cl;
02332     static PyObject *__pyx_n_s_clifford;
02333     static PyObject *__pyx_kp_u_clifford__add__line_740;
02334     static PyObject *__pyx_kp_u_clifford__and__line_836;
02335     static PyObject *__pyx_kp_u_clifford__call__line_1020;
02336     static PyObject *__pyx_kp_u_clifford__getitem__line_707;
02337     static PyObject *__pyx_kp_u_clifford__iadd__line_751;
02338     static PyObject *__pyx_kp_u_clifford__iand__line_851;

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02339     static PyObject *__pyx_kp_u_clifford_idiv_line_911;
02340     static PyObject *__pyx_kp_u_clifford_imod_line_821;
02341     static PyObject *__pyx_kp_u_clifford_imul_line_793;
02342     static PyObject *__pyx_kp_u_clifford_ior_line_950;
02343     static PyObject *__pyx_kp_u_clifford_isub_line_771;
02344     static PyObject *__pyx_kp_u_clifford_iter_line_638;
02345     static PyObject *__pyx_kp_u_clifford_ixor_line_881;
02346     static PyObject *__pyx_kp_u_clifford_mod_line_806;
02347     static PyObject *__pyx_kp_u_clifford_mul_line_780;
02348     static PyObject *__pyx_kp_u_clifford_neg_line_722;
02349     static PyObject *__pyx_kp_u_clifford_or_line_939;
02350     static PyObject *__pyx_kp_u_clifford_pos_line_731;
02351     static PyObject *__pyx_kp_u_clifford_pow_line_961;
02352     static PyObject *__pyx_kp_u_clifford_repr_line_1235;
02353     static PyObject *__pyx_kp_u_clifford_str_line_1244;
02354     static PyObject *__pyx_kp_u_clifford_sub_line_760;
02355     static PyObject *__pyx_kp_u_clifford_truediv_line_896;
02356     static PyObject *__pyx_kp_u_clifford_xor_line_866;
02357     static PyObject *__pyx_kp_u_clifford_abs_line_1175;
02358     static PyObject *__pyx_kp_u_clifford_conj_line_1138;
02359     static PyObject *__pyx_kp_u_clifford_copy_line_556;
02360     static PyObject *__pyx_kp_u_clifford_even_line_1061;
02361     static PyObject *__pyx_kp_u_clifford_frame_line_1224;
02362     static PyObject *__pyx_n_s_clifford_hidden_doctests;
02363     static PyObject *__pyx_kp_u_clifford_hidden_doctests_line_12;
02364     static PyObject *__pyx_kp_u_clifford_inv_line_926;
02365     static PyObject *__pyx_kp_u_clifford_involute_line_1107;
02366     static PyObject *__pyx_kp_u_clifford_isinf_line_1206;
02367     static PyObject *__pyx_kp_u_clifford_isnan_line_1215;
02368     static PyObject *__pyx_kp_u_clifford_max_abs_line_1184;
02369     static PyObject *__pyx_kp_u_clifford_norm_line_1164;
02370     static PyObject *__pyx_kp_u_clifford_odd_line_1070;
02371     static PyObject *__pyx_kp_u_clifford_outer_pow_line_1004;
02372     static PyObject *__pyx_kp_u_clifford_pow_line_980;
02373     static PyObject *__pyx_kp_u_clifford_pure_line_1050;
02374     static PyObject *__pyx_kp_u_clifford_quad_line_1153;
02375     static PyObject *__pyx_kp_u_clifford_reframe_line_649;
02376     static PyObject *__pyx_kp_u_clifford_reverse_line_1123;
02377     static PyObject *__pyx_kp_u_clifford_scalar_line_1039;
02378     static PyObject *__pyx_kp_u_clifford_truncated_line_1195;
02379     static PyObject *__pyx_kp_u_clifford_vector_part_line_1079;
02380     static PyObject *__pyx_n_s_cline_in_traceback;
02381     static PyObject *__pyx_n_s_close;
02382     static PyObject *__pyx_n_s_collections;
02383     static PyObject *__pyx_kp_u_compare_line_492;
02384     static PyObject *__pyx_kp_u_complexifier_line_1576;
02385     static PyObject *__pyx_n_s_conj;
02386     static PyObject *__pyx_kp_u_conj_line_1485;
02387     static PyObject *__pyx_n_s_copy;
02388     static PyObject *__pyx_n_s_cos;
02389     static PyObject *__pyx_kp_u_cos_line_1651;
02390     static PyObject *__pyx_n_s_cosh;
02391     static PyObject *__pyx_kp_u_cosh_line_1689;
02392     static PyObject *__pyx_n_s_doctest;
02393     static PyObject *__pyx_n_s_e;
02394     static PyObject *__pyx_kp_u_e_line_1936;
02395     static PyObject *__pyx_n_s_encode;
02396     static PyObject *__pyx_kp_u_error_squared_line_1346;
02397     static PyObject *__pyx_kp_u_error_squared_tol_line_1337;
02398     static PyObject *__pyx_n_s_even;
02399     static PyObject *__pyx_kp_u_even_line_1437;
02400     static PyObject *__pyx_n_s_exp;
02401     static PyObject *__pyx_kp_u_exp_line_1614;
02402     static PyObject *__pyx_n_s_fill;
02403     static PyObject *__pyx_n_s_frm;
02404     static PyObject *__pyx_kp_u_from;
02405     static PyObject *__pyx_n_s_getstate;
02406     static PyObject *__pyx_n_s_grade;
02407     static PyObject *__pyx_n_s_i;
02408     static PyObject *__pyx_kp_u_imag_line_1415;
02409     static PyObject *__pyx_n_s_import;
02410     static PyObject *__pyx_n_s_index_set;
02411     static PyObject *__pyx_kp_u_index_set_and_line_271;
02412     static PyObject *__pyx_kp_u_index_set_getitem_line_191;
02413     static PyObject *__pyx_kp_u_index_set_iand_line_282;
02414     static PyObject *__pyx_kp_u_index_set_invert_line_240;
02415     static PyObject *__pyx_kp_u_index_set_ior_line_304;
02416     static PyObject *__pyx_n_s_index_set_iter;
02417     static PyObject *__pyx_kp_u_index_set_iter_line_229;
02418     static PyObject *__pyx_kp_u_index_set_ixor_line_260;
02419     static PyObject *__pyx_kp_u_index_set_or_line_293;
02420     static PyObject *__pyx_kp_u_index_set_repr_line_384;
02421     static PyObject *__pyx_kp_u_index_set_setitem_line_179;
02422     static PyObject *__pyx_kp_u_index_set_str_line_395;
02423     static PyObject *__pyx_kp_u_index_set_xor_line_249;
02424     static PyObject *__pyx_kp_u_index_set_copy_line_65;
02425     static PyObject *__pyx_kp_u_index_set_count_line_315;

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02426     static PyObject *__pyx_kp_u_index_set_count_neg_line_324;
02427     static PyObject *__pyx_kp_u_index_set_count_pos_line_333;
02428     static PyObject *__pyx_n_s_index_set_hidden_doctests;
02429     static PyObject *__pyx_kp_u_index_set_hidden_doctests_line_4;
02430     static PyObject *__pyx_kp_u_index_set_max_line_351;
02431     static PyObject *__pyx_kp_u_index_set_min_line_342;
02432     static PyObject *__pyx_kp_u_index_set_sign_of_mult_line_366;
02433     static PyObject *__pyx_kp_u_index_set_sign_of_square_line_37;
02434     static PyObject *__pyx_n_s_inv;
02435     static PyObject *__pyx_kp_u_inv_line_1378;
02436     static PyObject *__pyx_kp_u_invalid;
02437     static PyObject *__pyx_kp_u_invalid_string;
02438     static PyObject *__pyx_n_s_involute;
02439     static PyObject *__pyx_kp_u_involute_line_1455;
02440     static PyObject *__pyx_n_s_ist;
02441     static PyObject *__pyx_n_s_istpq;
02442     static PyObject *__pyx_kp_u_istpq_line_1949;
02443     static PyObject *__pyx_n_s_iter;
02444     static PyObject *__pyx_n_s_ixt;
02445     static PyObject *__pyx_kp_u_lexicographic_compare_eg_3_4_5;
02446     static PyObject *__pyx_n_s_lhs;
02447     static PyObject *__pyx_n_s_log;
02448     static PyObject *__pyx_kp_u_log_line_1628;
02449     static PyObject *__pyx_n_s_m;
02450     static PyObject *__pyx_n_s_main;
02451     static PyObject *__pyx_n_u_main;
02452     static PyObject *__pyx_n_s_math;
02453     static PyObject *__pyx_n_s_max;
02454     static PyObject *__pyx_kp_u_max_abs_line_1531;
02455     static PyObject *__pyx_kp_u_max_pos_line_513;
02456     static PyObject *__pyx_n_s_min;
02457     static PyObject *__pyx_kp_u_min_neg_line_504;
02458     static PyObject *__pyx_n_s_name;
02459     static PyObject *__pyx_n_s_nbar3;
02460     static PyObject *__pyx_n_s_ninf3;
02461     static PyObject *__pyx_kp_s_no_default__reduce__due_to_non;
02462     static PyObject *__pyx_n_s_norm;
02463     static PyObject *__pyx_kp_u_norm_line_1511;
02464     static PyObject *__pyx_kp_u_norm_sum_of_squares_of_coordina;
02465     static PyObject *__pyx_n_s_numbers;
02466     static PyObject *__pyx_n_s_obj;
02467     static PyObject *__pyx_n_s_odd;
02468     static PyObject *__pyx_kp_u_odd_line_1446;
02469     static PyObject *__pyx_n_s_other;
02470     static PyObject *__pyx_n_s_outer_pow;
02471     static PyObject *__pyx_kp_u_outer_pow_line_1567;
02472     static PyObject *__pyx_n_s_p;
02473     static PyObject *__pyx_n_s_pi;
02474     static PyObject *__pyx_n_s_pow;
02475     static PyObject *__pyx_kp_u_pow_line_1543;
02476     static PyObject *__pyx_n_s_pure;
02477     static PyObject *__pyx_kp_u_pure_line_1426;
02478     static PyObject *__pyx_n_s_pyx_vtable;
02479     static PyObject *__pyx_n_s_q;
02480     static PyObject *__pyx_n_s_quad;
02481     static PyObject *__pyx_kp_u_quad_line_1500;
02482     static PyObject *__pyx_kp_u_random_clifford_line_1864;
02483     static PyObject *__pyx_n_s_range;
02484     static PyObject *__pyx_kp_u_real_line_1404;
02485     static PyObject *__pyx_n_s_reduce;
02486     static PyObject *__pyx_n_s_reduce_cython;
02487     static PyObject *__pyx_n_s_reduce_ex;
02488     static PyObject *__pyx_n_s_reverse;
02489     static PyObject *__pyx_kp_u_reverse_line_1470;
02490     static PyObject *__pyx_n_s_rhs;
02491     static PyObject *__pyx_n_s_scalar;
02492     static PyObject *__pyx_n_s_scalar_epsilon;
02493     static PyObject *__pyx_kp_u_scalar_line_1393;
02494     static PyObject *__pyx_n_s_send;
02495     static PyObject *__pyx_n_s_setstate;
02496     static PyObject *__pyx_n_s_setstate_cython;
02497     static PyObject *__pyx_n_s_sin;
02498     static PyObject *__pyx_kp_u_sin_line_1728;
02499     static PyObject *__pyx_n_s_sinh;
02500     static PyObject *__pyx_kp_u_sinh_line_1768;
02501     static PyObject *__pyx_n_s_sqrt;
02502     static PyObject *__pyx_kp_u_sqrt_line_1591;
02503     static PyObject *__pyx_n_s_tan;
02504     static PyObject *__pyx_kp_u_tan_line_1801;
02505     static PyObject *__pyx_n_s_tanh;
02506     static PyObject *__pyx_kp_u_tanh_line_1835;
02507     static PyObject *__pyx_n_s_tau;
02508     static PyObject *__pyx_n_s_test;
02509     static PyObject *__pyx_n_s_test_2;
02510     static PyObject *__pyx_n_s_testmod;
02511     static PyObject *__pyx_n_s_threshold;
02512     static PyObject *__pyx_n_s_throw;

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02513         static PyObject *__pyx_kp_u_to_frame;
02514         static PyObject *__pyx_n_s_tol;
02515         static PyObject *__pyx_kp_u_using;
02516         static PyObject *__pyx_kp_u_using_invalid;
02517         static PyObject *__pyx_kp_u_utf8;
02518         static PyObject *__pyx_kp_u_value;
02519         static PyObject *__pyx_n_s_version;
02520         static PyObject *__pyx_n_s_xrange;
02521 static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02522 static int __pyx_pf_8PyClical_9index_set_2_cinit__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_other); /* proto */
02523 static void __pyx_pf_8PyClical_9index_set_4_dealloc__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02524 static PyObject *__pyx_pf_8PyClical_9index_set_6_richcmp__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op); /* proto */
02525 static int __pyx_pf_8PyClical_9index_set_8_setitem__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx, PyObject *__pyx_v_val); /* proto */
02526 static PyObject *__pyx_pf_8PyClical_9index_set_10_getitem__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx); /* proto */
02527 static int __pyx_pf_8PyClical_9index_set_12_contains__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx); /* proto */
02528 static PyObject *__pyx_pf_8PyClical_9index_set_14_iter__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02529 static PyObject *__pyx_pf_8PyClical_9index_set_17_invert__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02530 static PyObject *__pyx_pf_8PyClical_9index_set_19_xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /* proto */
02531 static PyObject *__pyx_pf_8PyClical_9index_set_21_ixor__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02532 static PyObject *__pyx_pf_8PyClical_9index_set_23_and__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /* proto */
02533 static PyObject *__pyx_pf_8PyClical_9index_set_25_iand__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02534 static PyObject *__pyx_pf_8PyClical_9index_set_27_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02535 static PyObject *__pyx_pf_8PyClical_9index_set_29_ior__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02536 static PyObject *__pyx_pf_8PyClical_9index_set_31_count__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02537 static PyObject *__pyx_pf_8PyClical_9index_set_33_count_neg__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02538 static PyObject *__pyx_pf_8PyClical_9index_set_35_count_pos__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02539 static PyObject *__pyx_pf_8PyClical_9index_set_37_min__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02540 static PyObject *__pyx_pf_8PyClical_9index_set_39_max__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02541 static PyObject *__pyx_pf_8PyClical_9index_set_41_hash_fn__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02542 static PyObject *__pyx_pf_8PyClical_9index_set_43_sign_of_mult__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02543 static PyObject *__pyx_pf_8PyClical_9index_set_45_sign_of_square__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02544 static PyObject *__pyx_pf_8PyClical_9index_set_47_repr__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02545 static PyObject *__pyx_pf_8PyClical_9index_set_49_str__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self); /* proto */
02546 static PyObject *__pyx_pf_8PyClical_9index_set_51_reduce_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self); /* proto */
02547 static PyObject *__pyx_pf_8PyClical_9index_set_53_setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state); /* proto
*/
02548 static PyObject *__pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject *__pyx_self); /*
proto */
02549 static PyObject *__pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_lhs, PyObject *__pyx_v_rhs); /* proto */
02550 static PyObject *__pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj); /* proto */
02551 static PyObject *__pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj); /* proto */
02552 static PyObject *__pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford *__pyx_v_self);
/* proto */
02553 static int __pyx_pf_8PyClical_8clifford_2_cinit__(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
PyObject *__pyx_v_other, PyObject *__pyx_v_ixt); /* proto */
02554 static void __pyx_pf_8PyClical_8clifford_4_dealloc__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self); /* proto */
02555 static int __pyx_pf_8PyClical_8clifford_6_contains__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v_x); /* proto */
02556 static PyObject *__pyx_pf_8PyClical_8clifford_8_iter__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self); /* proto */
02557 static PyObject *__pyx_pf_8PyClical_8clifford_10_reframe(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_ixt); /* proto */
02558 static PyObject *__pyx_pf_8PyClical_8clifford_12_richcmp__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op); /* proto */
02559 static PyObject *__pyx_pf_8PyClical_8clifford_14_getitem__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_ixt); /* proto */

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02560 static PyObject *__pyx_pf_8PyClical_8clifford_16_neg__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02561 static PyObject *__pyx_pf_8PyClical_8clifford_18_pos__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02562 static PyObject *__pyx_pf_8PyClical_8clifford_20_add__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02563 static PyObject *__pyx_pf_8PyClical_8clifford_22_iadd__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02564 static PyObject *__pyx_pf_8PyClical_8clifford_24_sub__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02565 static PyObject *__pyx_pf_8PyClical_8clifford_26_isub__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02566 static PyObject *__pyx_pf_8PyClical_8clifford_28_mul__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02567 static PyObject *__pyx_pf_8PyClical_8clifford_30_imul__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02568 static PyObject *__pyx_pf_8PyClical_8clifford_32_mod__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02569 static PyObject *__pyx_pf_8PyClical_8clifford_34_imod__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02570 static PyObject *__pyx_pf_8PyClical_8clifford_36_and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02571 static PyObject *__pyx_pf_8PyClical_8clifford_38_iand__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02572 static PyObject *__pyx_pf_8PyClical_8clifford_40_xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02573 static PyObject *__pyx_pf_8PyClical_8clifford_42_ixor__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02574 static PyObject *__pyx_pf_8PyClical_8clifford_44_truediv__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /* proto */
02575 #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
02576 static PyObject *__pyx_pf_8PyClical_8clifford_46_idiv__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02577 #endif
02578 static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02579 static PyObject *__pyx_pf_8PyClical_8clifford_50_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02580 static PyObject *__pyx_pf_8PyClical_8clifford_52_ior__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02581 static PyObject *__pyx_pf_8PyClical_8clifford_54_pow__(PyObject *__pyx_v_self, PyObject *__pyx_v_m,
    CYTHON_UNUSED PyObject *__pyx_v_dummy); /* proto */
02582 static PyObject *__pyx_pf_8PyClical_8clifford_56pow(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
    PyObject *__pyx_v_m); /* proto */
02583 static PyObject *__pyx_pf_8PyClical_8clifford_58outer_pow(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_m); /* proto */
02584 static PyObject *__pyx_pf_8PyClical_8clifford_60_call__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_grade); /* proto */
02585 static PyObject *__pyx_pf_8PyClical_8clifford_62scalar(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02586 static PyObject *__pyx_pf_8PyClical_8clifford_64pure(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02587 static PyObject *__pyx_pf_8PyClical_8clifford_66even(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02588 static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02589 static PyObject *__pyx_pf_8PyClical_8clifford_70vector_part(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_frm); /* proto */
02590 static PyObject *__pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02591 static PyObject *__pyx_pf_8PyClical_8clifford_74reverse(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02592 static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02593 static PyObject *__pyx_pf_8PyClical_8clifford_78quad(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02594 static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02595 static PyObject *__pyx_pf_8PyClical_8clifford_82abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02596 static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02597 static PyObject *__pyx_pf_8PyClical_8clifford_86truncated(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_limit); /* proto */
02598 static PyObject *__pyx_pf_8PyClical_8clifford_88isinf(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02599 static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02600 static PyObject *__pyx_pf_8PyClical_8clifford_92frame(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02601 static PyObject *__pyx_pf_8PyClical_8clifford_94_repr__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02602 static PyObject *__pyx_pf_8PyClical_8clifford_96_str__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02603 static PyObject *__pyx_pf_8PyClical_8clifford_98_reduce_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford *__pyx_v_self); /* proto */
02604 static PyObject *__pyx_pf_8PyClical_8clifford_100_setstate_cython__(CYTHON_UNUSED struct

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    __pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state); /* proto */
02605 static PyObject *__pyx_pf_8PyClical_8clifford_hidden_doctests(CYTHON_UNUSED PyObject *__pyx_self); /*
    proto */
02606 static PyObject *__pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02607 static PyObject *__pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold); /* proto */
02608 static PyObject *__pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol); /* proto */
02609 static PyObject *__pyx_pf_8PyClical_16inv(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02610 static PyObject *__pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02611 static PyObject *__pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02612 static PyObject *__pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02613 static PyObject *__pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02614 static PyObject *__pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02615 static PyObject *__pyx_pf_8PyClical_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02616 static PyObject *__pyx_pf_8PyClical_30involute(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02617 static PyObject *__pyx_pf_8PyClical_32reverse(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02618 static PyObject *__pyx_pf_8PyClical_34conj(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02619 static PyObject *__pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02620 static PyObject *__pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02621 static PyObject *__pyx_pf_8PyClical_40abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02622 static PyObject *__pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02623 static PyObject *__pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_m); /* proto */
02624 static PyObject *__pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj, PyObject *__pyx_v_m); /* proto */
02625 static PyObject *__pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02626 static PyObject *__pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02627 static PyObject *__pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02628 static PyObject *__pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02629 static PyObject *__pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02630 static PyObject *__pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02631 static PyObject *__pyx_pf_8PyClical_60cosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02632 static PyObject *__pyx_pf_8PyClical_62acosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02633 static PyObject *__pyx_pf_8PyClical_64sin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02634 static PyObject *__pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02635 static PyObject *__pyx_pf_8PyClical_68sinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02636 static PyObject *__pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02637 static PyObject *__pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02638 static PyObject *__pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02639 static PyObject *__pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02640 static PyObject *__pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02641 static PyObject *__pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject *__pyx_self, struct
    __pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill); /* proto */
02642 static PyObject *__pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02643 static PyObject *__pyx_pf_8PyClical_84cga3std(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02644 static PyObject *__pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02645 static PyObject *__pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj); /*
    proto */
02646 static PyObject *__pyx_pf_8PyClical_90istpq(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_p,
    PyObject *__pyx_v_q); /* proto */
02647 static PyObject *__pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject *__pyx_self); /* proto */
02648 static PyObject *__pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a, PyObject *k);

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/*proto*/
02649 static PyObject *__pyx_tp_new_8PyClical_clifford(PyTypeObject *t, PyObject *a, PyObject *k); /*proto*/
02650 static PyObject *__pyx_tp_new_8PyClical__pyx_scope_struct____iter__(PyTypeObject *t, PyObject *a,
    PyObject *k); /*proto*/
02651 static PyObject *__pyx_float_0_0;
02652 static PyObject *__pyx_float_1_0;
02653 static PyObject *__pyx_float_4_0;
02654 static PyObject *__pyx_float_8_0;
02655 static PyObject *__pyx_int_0;
02656 static PyObject *__pyx_int_1;
02657 static PyObject *__pyx_int_4;
02658 static PyObject *__pyx_int_neg_1;
02659 static PyObject *__pyx_tuple__3;
02660 static PyObject *__pyx_tuple__4;
02661 static PyObject *__pyx_tuple__10;
02662 static PyObject *__pyx_tuple__11;
02663 static PyObject *__pyx_tuple__12;
02664 static PyObject *__pyx_tuple__15;
02665 static PyObject *__pyx_tuple__16;
02666 static PyObject *__pyx_tuple__18;
02667 static PyObject *__pyx_tuple__20;
02668 static PyObject *__pyx_tuple__21;
02669 static PyObject *__pyx_tuple__22;
02670 static PyObject *__pyx_codeobj__13;
02671 static PyObject *__pyx_codeobj__14;
02672 static PyObject *__pyx_codeobj__17;
02673 static PyObject *__pyx_codeobj__19;
02674 static PyObject *__pyx_codeobj__23;
02675 /* Late includes */
02676
02677 /* "PyClical.pyx":40
02678 * cdef class index_set
02679 *
02680 * cdef inline IndexSet toIndexSet(obj):
02681 *     """
02682 *     Return the C++ IndexSet instance wrapped by index_set(obj).
02683 */
02684
02685 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_toIndexSet(PyObject *__pyx_v_obj) {
02686     IndexSet __pyx_r;
02687     __Pyx_RefNannyDeclarations
02688     PyObject *__pyx_t_1 = NULL;
02689     int __pyx_lineno = 0;
02690     const char *__pyx_filename = NULL;
02691     int __pyx_clineno = 0;
02692     __Pyx_RefNannySetupContext("toIndexSet", 0);
02693
02694     /* "PyClical.pyx":44
02695     *     Return the C++ IndexSet instance wrapped by index_set(obj).
02696     *     """
02697     *     return index_set(obj).instance[0]
02698     *
02699     * cdef class index_set:
02700 */
02701     __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_index_set), __pyx_v_obj);
02702     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 44, __pyx_L1_error)
02703     __Pyx_GOTREF(__pyx_t_1);
02704     __pyx_r = ((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1->instance[0]);
02705     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02706     goto __pyx_L0;
02707
02708     /* "PyClical.pyx":40
02709     * cdef class index_set
02710     *
02711     * cdef inline IndexSet toIndexSet(obj):
02712     *     """
02713     *     Return the C++ IndexSet instance wrapped by index_set(obj).
02714     */
02715     /* function exit code */
02716     __pyx_L1_error:;
02717     __Pyx_XDECREF(__pyx_t_1);
02718     __Pyx_WriteUnraisable("PyClical.toIndexSet", __pyx_clineno, __pyx_lineno, __pyx_filename, 1, 0);
02719     __Pyx_prevent_to_initialize(&__pyx_r);
02720     __pyx_L0:;
02721     __Pyx_RefNannyFinishContext();
02722     return __pyx_r;
02723 }
02724
02725 /* "PyClical.pyx":52
02726 *     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02727 *
02728 *     cdef inline wrap(index_set self, IndexSet other):
02729 *         """
02730 *         Wrap an instance of the C++ class IndexSet.
02731 */
02732

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02733 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
02734 * __pyx_v_self, IndexSet __pyx_v_other) {
02735     PyObject *__pyx_r = NULL;
02736     __Pyx_RefNannyDeclarations
02737     __Pyx_RefNannySetupContext("wrap", 0);
02738     /* "PyClical.pyx":56
02739     *         Wrap an instance of the C++ class IndexSet.
02740     *         """
02741     *         self.instance[0] = other          # ««««««««
02742     *         return self
02743     */
02744     __pyx_v_self->instance[0] = __pyx_v_other;
02745     /* "PyClical.pyx":57
02746     *         """
02747     *         self.instance[0] = other
02748     *         return self          # ««««««««
02749     *
02750     *         cdef inline IndexSet unwrap(index_set self):
02751     */
02752     __Pyx_XDECREF(__pyx_r);
02753     __Pyx_INCREF((PyObject *)__pyx_v_self);
02754     __pyx_r = (PyObject *)__pyx_v_self;
02755     goto __pyx_L0;
02756     /* "PyClical.pyx":52
02757     *         cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02758     *
02759     *         cdef inline wrap(index_set self, IndexSet other):          # ««««««««
02760     *         """
02761     *         Wrap an instance of the C++ class IndexSet.
02762     */
02763     /* function exit code */
02764     __pyx_L0:;
02765     __Pyx_XGIVEREF(__pyx_r);
02766     __Pyx_RefNannyFinishContext();
02767     return __pyx_r;
02768 }
02769 /* "PyClical.pyx":59
02770 *         return self
02771 *
02772 *         cdef inline IndexSet unwrap(index_set self):          # ««««««««
02773 *         """
02774 *         Return the wrapped C++ IndexSet instance.
02775     */
02776 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
02777 * __pyx_v_self) {
02778     IndexSet __pyx_r;
02779     __Pyx_RefNannyDeclarations
02780     __Pyx_RefNannySetupContext("unwrap", 0);
02781     /* "PyClical.pyx":63
02782     *         Return the wrapped C++ IndexSet instance.
02783     *         """
02784     *         return self.instance[0]          # ««««««««
02785     *
02786     *         cpdef copy(index_set self):
02787     */
02788     __pyx_r = (__pyx_v_self->instance[0]);
02789     goto __pyx_L0;
02790     /* "PyClical.pyx":59
02791     *         return self
02792     *
02793     *         cdef inline IndexSet unwrap(index_set self):          # ««««««««
02794     *         """
02795     *         Return the wrapped C++ IndexSet instance.
02796     */
02797     /* function exit code */
02798     __pyx_L0:;
02799     __Pyx_RefNannyFinishContext();
02800     return __pyx_r;
02801 }
02802 /* "PyClical.pyx":65
02803 *         return self.instance[0]
02804 *
02805     *         cpdef copy(index_set self):          # ««««««««
02806     *         """
02807     *         Copy this index_set object.
02808     */

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02818
02819 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused); /*proto*/
02820 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set *__pyx_v_self,
    int __pyx_skip_dispatch) {
02821     PyObject *__pyx_r = NULL;
02822     __Pyx_RefNannyDeclarations
02823     PyObject *__pyx_t_1 = NULL;
02824     PyObject *__pyx_t_2 = NULL;
02825     PyObject *__pyx_t_3 = NULL;
02826     PyObject *__pyx_t_4 = NULL;
02827     int __pyx_lineno = 0;
02828     const char *__pyx_filename = NULL;
02829     int __pyx_clineno = 0;
02830     __Pyx_RefNannySetupContext("copy", 0);
02831     /* Check if called by wrapper */
02832     if (unlikely(__pyx_skip_dispatch)) ;
02833     /* Check if overridden in Python */
02834     else if (unlikely((Py_TYPE((PyObject *)__pyx_v_self)->tp_dictoffset != 0) || (Py_TYPE((PyObject
    *)__pyx_v_self)->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
02835         #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02836             static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version =
    __PYX_DICT_VERSION_INIT;
02837             if (unlikely(!__Pyx_object_dict_version_matches((PyObject *)__pyx_v_self, __pyx_tp_dict_version,
    __pyx_obj_dict_version))) {
02838                 PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version((PyObject *)__pyx_v_self);
02839                 #endif
02840                 __pyx_t_1 = __Pyx_PyObject_GetAttrStr((PyObject *)__pyx_v_self, __pyx_n_s_copy); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 65, __pyx_L1_error)
02841                 __Pyx_GOTREF(__pyx_t_1);
02842                 if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
    (PyCFunction)(void*)__pyx_pw_8PyClical_9index_set_1copy)) {
02843                     __Pyx_XDECREF(__pyx_r);
02844                     __Pyx_INCREF(__pyx_t_1);
02845                     __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;
02846                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
02847                         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
02848                         if (likely(__pyx_t_4)) {
02849                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
02850                             __Pyx_INCREF(__pyx_t_4);
02851                             __Pyx_INCREF(function);
02852                             __Pyx_DECREF_SET(__pyx_t_3, function);
02853                         }
02854                     }
02855                     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
    __Pyx_PyObject_CallNoArg(__pyx_t_3);
02856                     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
02857                     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 65, __pyx_L1_error)
02858                     __Pyx_GOTREF(__pyx_t_2);
02859                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
02860                     __pyx_r = __pyx_t_2;
02861                     __pyx_t_2 = 0;
02862                     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02863                     goto __pyx_L0;
02864                 }
02865                 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02866                     __pyx_tp_dict_version = __Pyx_get_tp_dict_version((PyObject *)__pyx_v_self);
02867                     __pyx_obj_dict_version = __Pyx_get_object_dict_version((PyObject *)__pyx_v_self);
02868                     if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
02869                         __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
02870                     }
02871                 #endif
02872                 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02873                 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02874             }
02875         #endif
02876     }
02877
02878     /* "PyClical.pyx":72
02879     *     {1}
02880     *     """
02881     *     return index_set(self) # ««««««««
02882     *
02883     *     def __cinit__(self, other = 0):
02884     */
02885     __Pyx_XDECREF(__pyx_r);
02886     __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_index_set, (PyObject
    *)__pyx_v_self); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 72, __pyx_L1_error)
02887     __Pyx_GOTREF(__pyx_t_1);
02888     __pyx_r = __pyx_t_1;
02889     __pyx_t_1 = 0;
02890     goto __pyx_L0;
02891
02892     /* "PyClical.pyx":65
02893     *     return self.instance[0]
02894     *
02895     *     cpdef copy(index_set self): # ««««««««

```



```

02896 *          """
02897 *          Copy this index_set object.
02898 */
02899
02900 /* function exit code */
02901 __pyx_L1_error:;
02902 __Pyx_XDECREF(__pyx_t_1);
02903 __Pyx_XDECREF(__pyx_t_2);
02904 __Pyx_XDECREF(__pyx_t_3);
02905 __Pyx_XDECREF(__pyx_t_4);
02906 __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02907 __pyx_r = 0;
02908 __pyx_L0:;
02909 __Pyx_XGIVEREF(__pyx_r);
02910 __Pyx_RefNannyFinishContext();
02911 return __pyx_r;
02912 }
02913
02914 /* Python wrapper */
02915 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
02916 *unused); /*proto*/
02917 static char __pyx_doc_8PyClical_9index_set_copy[] = "\n          Copy this index_set object.\n\n
02918 >> s=index_set(1); t=s.copy(); print(t)\n          {1}\n          ";
02919 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
02920 *unused) {
02921     PyObject *__pyx_r = 0;
02922     __Pyx_RefNannyDeclarations
02923     __Pyx_RefNannySetupContext("copy (wrapper)", 0);
02924     __pyx_r = __pyx_pf_8PyClical_9index_set_copy(((struct __pyx_obj_8PyClical_index_set
02925 *)__pyx_v_self));
02926
02927 /* function exit code */
02928 __Pyx_RefNannyFinishContext();
02929 return __pyx_r;
02930 }
02931
02932 static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
02933 *__pyx_v_self) {
02934     PyObject *__pyx_r = NULL;
02935     __Pyx_RefNannyDeclarations
02936     PyObject *__pyx_t_1 = NULL;
02937     int __pyx_lineno = 0;
02938     const char *__pyx_filename = NULL;
02939     int __pyx_clineno = 0;
02940     __Pyx_RefNannySetupContext("copy", 0);
02941     __Pyx_XDECREF(__pyx_r);
02942     __pyx_t_1 = __pyx_f_8PyClical_9index_set_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
02943     __PYX_ERR(0, 65, __pyx_L1_error)
02944     __Pyx_GOTREF(__pyx_t_1);
02945     __pyx_r = __pyx_t_1;
02946     __pyx_t_1 = 0;
02947     goto __pyx_L0;
02948
02949 /* function exit code */
02950 __pyx_L1_error:;
02951 __Pyx_XDECREF(__pyx_t_1);
02952 __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02953 __pyx_r = NULL;
02954 __pyx_L0:;
02955 __Pyx_XGIVEREF(__pyx_r);
02956 __Pyx_RefNannyFinishContext();
02957 return __pyx_r;
02958 }
02959
02960 /* "PyClical.pyx":74
02961 *          return index_set(self)
02962 *
02963 *          def __cinit__(self, other = 0):          # ««««««««
02964 *          """
02965 *          Construct an object of type index_set.
02966 */
02967
02968 /* Python wrapper */
02969 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
02970 PyObject *__pyx_kwds); /*proto*/
02971 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
02972 PyObject *__pyx_kwds) {
02973     PyObject *__pyx_v_other = 0;
02974     int __pyx_lineno = 0;
02975     const char *__pyx_filename = NULL;
02976     int __pyx_clineno = 0;
02977     int __pyx_r;
02978     __Pyx_RefNannyDeclarations
02979     __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
02980     {
02981         static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_other,0};
02982         PyObject* values[1] = {0};

```

```

02975     values[0] = ((PyObject *) __pyx_int_0);
02976     if (unlikely(__pyx_kwds)) {
02977         Py_ssize_t kw_args;
02978         const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
02979         switch (pos_args) {
02980             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02981                 CYTHON_FALLTHROUGH;
02982             case 0: break;
02983             default: goto __pyx_L5_argtuple_error;
02984         }
02985         kw_args = PyDict_Size(__pyx_kwds);
02986         switch (pos_args) {
02987             case 0:
02988                 if (kw_args > 0) {
02989                     PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
02990                     if (value) { values[0] = value; kw_args--; }
02991                 }
02992             }
02993             if (unlikely(kw_args > 0)) {
02994                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values, pos_args,
02995 " __cinit__" < 0)) __PYX_ERR(0, 74, __pyx_L3_error)
02996             } else {
02997                 switch (PyTuple_GET_SIZE(__pyx_args)) {
02998                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02999                         CYTHON_FALLTHROUGH;
03000                     case 0: break;
03001                     default: goto __pyx_L5_argtuple_error;
03002                 }
03003             }
03004             __pyx_v_other = values[0];
03005         }
03006         goto __pyx_L4_argument_unpacking_done;
03007         __pyx_L5_argtuple_error:;
03008         __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 74,
03009 __pyx_L3_error)
03010         __pyx_L3_error:;
03011         __Pyx_AddTraceback("PyClicl.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03012         __Pyx_RefNannyFinishContext();
03013         return -1;
03014         __pyx_L4_argument_unpacking_done:;
03015         __pyx_r = __pyx_pf_8PyClicl_9index_set_2__cinit__((struct __pyx_obj_8PyClicl_index_set
03016 *)__pyx_v_self), __pyx_v_other);
03017
03018         /* function exit code */
03019         __Pyx_RefNannyFinishContext();
03020         return __pyx_r;
03021     }
03022 }
03023
03024 static int __pyx_pf_8PyClicl_9index_set_2__cinit__(struct __pyx_obj_8PyClicl_index_set
03025 * __pyx_v_self, PyObject * __pyx_v_other) {
03026     PyObject * __pyx_v_error_msg_prefix = NULL;
03027     PyObject * __pyx_v_idx = NULL;
03028     PyObject * __pyx_v_bother = NULL;
03029     int __pyx_r;
03030     __Pyx_RefNannyDeclarations
03031     int __pyx_t_1;
03032     int __pyx_t_2;
03033     IndexSet * __pyx_t_3;
03034     PyObject * __pyx_t_4 = NULL;
03035     PyObject * __pyx_t_5 = NULL;
03036     int __pyx_t_6;
03037     int __pyx_t_7;
03038     PyObject * __pyx_t_8 = NULL;
03039     PyObject * __pyx_t_9 = NULL;
03040     PyObject * __pyx_t_10 = NULL;
03041     Py_ssize_t __pyx_t_11;
03042     PyObject * (* __pyx_t_12) (PyObject *);
03043     PyObject * __pyx_t_13 = NULL;
03044     PyObject * __pyx_t_14 = NULL;
03045     PyObject * __pyx_t_15 = NULL;
03046     PyObject * __pyx_t_16 = NULL;
03047     char * __pyx_t_17;
03048     int __pyx_lineno = 0;
03049     const char * __pyx_filename = NULL;
03050     int __pyx_clineno = 0;
03051     __Pyx_RefNannySetupContext("__cinit__", 0);
03052
03053     /* "PyClicl.pyx":93
03054     {
03055     """
03056     error_msg_prefix = "Cannot initialize index_set object from" # ««««««««
03057     if isinstance(other, index_set):
03058         self.instance = new IndexSet((<index_set>other).unwrap())
03059     */
03060     __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_index_set_obje);
03061     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_index_set_obje;

```



```

03058
03059  /* "PyClical.pyx":94
03060  *      """
03061  *      error_msg_prefix = "Cannot initialize index_set object from"
03062  *      if isinstance(other, index_set):          # ««««««««
03063  *          self.instance = new IndexSet((<index_set>other).unwrap())
03064  *      elif isinstance(other, numbers.Integral):
03065  */
03066  __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_index_set);
03067  __pyx_t_2 = (__pyx_t_1 != 0);
03068  if (__pyx_t_2) {
03069
03070      /* "PyClical.pyx":95
03071      *      error_msg_prefix = "Cannot initialize index_set object from"
03072      *      if isinstance(other, index_set):
03073      *          self.instance = new IndexSet((<index_set>other).unwrap())          # ««««««««
03074      *      elif isinstance(other, numbers.Integral):
03075      *          self.instance = new IndexSet(<int>other)
03076      */
03077      try {
03078          __pyx_t_3 = new IndexSet(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_other)));
03079      } catch(...) {
03080          __Pyx_CppExn2PyErr();
03081          __PYX_ERR(0, 95, __pyx_L1_error)
03082      }
03083      __pyx_v_self->instance = __pyx_t_3;
03084
03085      /* "PyClical.pyx":94
03086      *      """
03087      *      error_msg_prefix = "Cannot initialize index_set object from"
03088      *      if isinstance(other, index_set):          # ««««««««
03089      *          self.instance = new IndexSet((<index_set>other).unwrap())
03090      *      elif isinstance(other, numbers.Integral):
03091      */
03092      goto __pyx_L3;
03093  }
03094
03095      /* "PyClical.pyx":96
03096      *      if isinstance(other, index_set):
03097      *          self.instance = new IndexSet((<index_set>other).unwrap())
03098      *      elif isinstance(other, numbers.Integral):          # ««««««««
03099      *          self.instance = new IndexSet(<int>other)
03100      *      elif isinstance(other, (set, frozenset)):
03101      */
03102      __Pyx_GetModuleGlobalName(__pyx_t_4, __pyx_n_s_numbers); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 96,
__pyx_L1_error)
03103      __Pyx_GOTREF(__pyx_t_4);
03104      __pyx_t_5 = __Pyx_PyObject_GetAttrStr(__pyx_t_4, __pyx_n_s_Integral); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 96, __pyx_L1_error)
03105      __Pyx_GOTREF(__pyx_t_5);
03106      __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03107      __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_5); if (unlikely(__pyx_t_2 == ((int)-1)))
__PYX_ERR(0, 96, __pyx_L1_error)
03108      __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03109      __pyx_t_1 = (__pyx_t_2 != 0);
03110      if (__pyx_t_1) {
03111
03112          /* "PyClical.pyx":97
03113          *          self.instance = new IndexSet((<index_set>other).unwrap())
03114          *      elif isinstance(other, numbers.Integral):
03115          *          self.instance = new IndexSet(<int>other)          # ««««««««
03116          *      elif isinstance(other, (set, frozenset)):
03117          *          try:
03118          */
03119          __pyx_t_6 = __Pyx_PyInt_As_int(__pyx_v_other); if (unlikely((__pyx_t_6 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 97, __pyx_L1_error)
03120          try {
03121              __pyx_t_3 = new IndexSet(((int)__pyx_t_6));
03122          } catch(...) {
03123              __Pyx_CppExn2PyErr();
03124              __PYX_ERR(0, 97, __pyx_L1_error)
03125          }
03126          __pyx_v_self->instance = __pyx_t_3;
03127
03128          /* "PyClical.pyx":96
03129          *      if isinstance(other, index_set):
03130          *          self.instance = new IndexSet((<index_set>other).unwrap())
03131          *      elif isinstance(other, numbers.Integral):          # ««««««««
03132          *          self.instance = new IndexSet(<int>other)
03133          *      elif isinstance(other, (set, frozenset)):
03134          */
03135          goto __pyx_L3;
03136      }
03137
03138      /* "PyClical.pyx":98
03139      *      elif isinstance(other, numbers.Integral):

```

```

03140 *         self.instance = new IndexSet(<int>other)
03141 *         elif isinstance(other, (set, frozenset)):           # ««««««««
03142 *             try:
03143 *                 self.instance = new IndexSet()
03144 */
03145 __pyx_t_2 = PySet_Check(__pyx_v_other);
03146 __pyx_t_7 = (__pyx_t_2 != 0);
03147 if (!__pyx_t_7) {
03148 } else {
03149     __pyx_t_1 = __pyx_t_7;
03150     goto __pyx_L4_bool_binop_done;
03151 }
03152 __pyx_t_7 = PyFrozenSet_Check(__pyx_v_other);
03153 __pyx_t_2 = (__pyx_t_7 != 0);
03154 __pyx_t_1 = __pyx_t_2;
03155 __pyx_L4_bool_binop_done;
03156 __pyx_t_2 = (__pyx_t_1 != 0);
03157 if (__pyx_t_2) {
03158
03159     /* "PyClical.pyx":99
03160 *         self.instance = new IndexSet(<int>other)
03161 *         elif isinstance(other, (set, frozenset)):           # ««««««««
03162 *             try:
03163 *                 self.instance = new IndexSet()
03164 *                 for idx in other:
03165 */
03166     {
03167         __Pyx_PyThreadState_declare
03168         __Pyx_PyThreadState_assign
03169         __Pyx_ExceptionSave(&__pyx_t_8, &__pyx_t_9, &__pyx_t_10);
03170         __Pyx_XGOTREF(__pyx_t_8);
03171         __Pyx_XGOTREF(__pyx_t_9);
03172         __Pyx_XGOTREF(__pyx_t_10);
03173         /*try:*/ {
03174
03175             /* "PyClical.pyx":100
03176 *         elif isinstance(other, (set, frozenset)):           # ««««««««
03177 *             try:
03178 *                 self.instance = new IndexSet()
03179 *                 for idx in other:
03180 *                     self[idx] = True
03181 */
03182         __pyx_t_3 = new IndexSet();
03183         __pyx_v_self->instance = __pyx_t_3;
03184
03185         /* "PyClical.pyx":101
03186 *             try:
03187 *                 self.instance = new IndexSet()
03188 *                 for idx in other:           # ««««««««
03189 *                     self[idx] = True
03190 *             except IndexError:
03191 */
03192         if (likely(PyList_CheckExact(__pyx_v_other)) || PyTuple_CheckExact(__pyx_v_other)) {
03193             __pyx_t_5 = __pyx_v_other; __Pyx_INCREF(__pyx_t_5); __pyx_t_11 = 0;
03194             __pyx_t_12 = NULL;
03195         } else {
03196             __pyx_t_11 = -1; __pyx_t_5 = PyObject_GetIter(__pyx_v_other); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 101, __pyx_L6_error)
__Pyx_GOTREF(__pyx_t_5);
__pyx_t_12 = Py_TYPE(__pyx_t_5)->tp_iternext; if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 101,
__pyx_L6_error)
03199         }
03200         for (;;) {
03201             if (likely(!__pyx_t_12)) {
03202                 if (likely(PyList_CheckExact(__pyx_t_5))) {
03203                     if (__pyx_t_11 >= PyList_GET_SIZE(__pyx_t_5)) break;
03204                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03205                     __pyx_t_4 = PyList_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
__pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03206                     #else
03207                     __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03208                     __Pyx_GOTREF(__pyx_t_4);
03209                     #endif
03210                 } else {
03211                     if (__pyx_t_11 >= PyTuple_GET_SIZE(__pyx_t_5)) break;
03212                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03213                     __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
__pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03214                     #else
03215                     __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03216                     __Pyx_GOTREF(__pyx_t_4);
03217                     #endif
03218                 }
03219             } else {
03220                 __pyx_t_4 = __pyx_t_12(__pyx_t_5);

```

```

03221         if (unlikely(!__pyx_t_4)) {
03222             PyObject* exc_type = PyErr_Occurred();
03223             if (exc_type) {
03224                 if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
PyErr_Clear();
03225             } else __PYX_ERR(0, 101, __pyx_L6_error)
03226         }
03227         break;
03228     }
03229     __Pyx_GOTREF(__pyx_t_4);
03230 }
03231 __Pyx_XDECREF_SET(__pyx_v_idx, __pyx_t_4);
03232 __pyx_t_4 = 0;
03233
03234 /* "PyClical.pyx":102
03235 *         self.instance = new IndexSet()
03236 *         for idx in other:
03237 *             self[idx] = True           # ««««««««
03238 *         except IndexError:
03239 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03240 */
03241     if (unlikely(PyObject_SetItem((PyObject *)__pyx_v_self, __pyx_v_idx, Py_True) < 0))
__PYX_ERR(0, 102, __pyx_L6_error)
03242
03243     /* "PyClical.pyx":101
03244     *         try:
03245     *             self.instance = new IndexSet()
03246     *             for idx in other:           # ««««««««
03247     *                 self[idx] = True
03248     *             except IndexError:
03249     */
03250 }
03251 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03252
03253 /* "PyClical.pyx":99
03254 *         self.instance = new IndexSet(<int>other)
03255 *         elif isinstance(other, (set, frozenset)):
03256 *             try:           # ««««««««
03257 *                 self.instance = new IndexSet()
03258 *                 for idx in other:
03259     */
03260 }
03261 __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
03262 __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
03263 __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
03264 goto __pyx_L11_try_end;
03265 __pyx_L6_error:;
03266 __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
03267 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
03268
03269 /* "PyClical.pyx":103
03270 *         for idx in other:
03271 *             self[idx] = True
03272 *         except IndexError:           # ««««««««
03273 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03274 *         except (RuntimeError, TypeError):
03275     */
03276 __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_IndexError);
03277 if (__pyx_t_6) {
03278     __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03279     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_4, &__pyx_t_13) < 0) __PYX_ERR(0, 103,
__pyx_L8_except_error)
03280     __Pyx_GOTREF(__pyx_t_5);
03281     __Pyx_GOTREF(__pyx_t_4);
03282     __Pyx_GOTREF(__pyx_t_13);
03283
03284     /* "PyClical.pyx":104
03285     *             self[idx] = True
03286     *         except IndexError:
03287     *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")           #
««««««««
03288     *         except (RuntimeError, TypeError):
03289     *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03290     */
03291     __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 104, __pyx_L8_except_error)
03292     __Pyx_GOTREF(__pyx_t_14);
03293     __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 104,
__pyx_L8_except_error)
03294     __Pyx_GOTREF(__pyx_t_15);
03295     __pyx_t_16 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
104, __pyx_L8_except_error)
03296     __Pyx_GOTREF(__pyx_t_16);
03297     __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03298     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03299     __pyx_t_15 = PyNumber_Add(__pyx_t_16, __pyx_kp_u_); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,

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104, __pyx_L8_except_error)
03300     __Pyx_GOTREF(__pyx_t_15);
03301     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03302     __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_IndexError, __pyx_t_15); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 104, __pyx_L8_except_error)
03303     __Pyx_GOTREF(__pyx_t_16);
03304     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03305     __Pyx_Raise(__pyx_t_16, 0, 0, 0);
03306     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03307     __PYX_ERR(0, 104, __pyx_L8_except_error)
03308 }
03309
03310 /* "PyClical.pyx":105
03311 *         except IndexError:
03312 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03313 *         except (RuntimeError, TypeError):
03314 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03315 *         elif isinstance(other, str):
03316 */
03317     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError) ||
__Pyx_PyErr_ExceptionMatches(__pyx_builtin_TypeError);
03318     if (__pyx_t_6) {
03319         __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03320         if (__Pyx_GetException(&__pyx_t_13, &__pyx_t_4, &__pyx_t_5) < 0) __PYX_ERR(0, 105,
__pyx_L8_except_error)
03321         __Pyx_GOTREF(__pyx_t_13);
03322         __Pyx_GOTREF(__pyx_t_4);
03323         __Pyx_GOTREF(__pyx_t_5);
03324
03325         /* "PyClical.pyx":106
03326 *             raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03327 *         except (RuntimeError, TypeError):
03328 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03329 *         elif isinstance(other, str):
03330 *             try:
03331 */
03332         __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03333         __Pyx_GOTREF(__pyx_t_16);
03334         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 106,
__pyx_L8_except_error)
03335         __Pyx_GOTREF(__pyx_t_15);
03336         __pyx_t_14 = PyNumber_Add(__pyx_t_16, __pyx_t_15); if (unlikely(!__pyx_t_14)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03337         __Pyx_GOTREF(__pyx_t_14);
03338         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03339         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03340         __pyx_t_15 = PyNumber_Add(__pyx_t_14, __pyx_kp_u_); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03341         __Pyx_GOTREF(__pyx_t_15);
03342         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03343         __pyx_t_14 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03344         __Pyx_GOTREF(__pyx_t_14);
03345         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03346         __Pyx_Raise(__pyx_t_14, 0, 0, 0);
03347         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03348         __PYX_ERR(0, 106, __pyx_L8_except_error)
03349     }
03350     goto __pyx_L8_except_error;
03351     __pyx_L8_except_error;;
03352
03353 /* "PyClical.pyx":99
03354 *         self.instance = new IndexSet(<int>other)
03355 *         elif isinstance(other, (set, frozenset)):
03356 *             try:
03357 *                 self.instance = new IndexSet()
03358 *                 for idx in other:
03359 */
03360         __Pyx_XGIVEREF(__pyx_t_8);
03361         __Pyx_XGIVEREF(__pyx_t_9);
03362         __Pyx_XGIVEREF(__pyx_t_10);
03363         __Pyx_ExceptionReset(__pyx_t_8, __pyx_t_9, __pyx_t_10);
03364         goto __pyx_L1_error;
03365         __pyx_L11_try_end;;
03366     }
03367
03368 /* "PyClical.pyx":98
03369 *         elif isinstance(other, numbers.Integral):
03370 *             self.instance = new IndexSet(<int>other)
03371 *         elif isinstance(other, (set, frozenset)):
03372 *             try:
03373 *                 self.instance = new IndexSet()
03374 */
03375     goto __pyx_L3;

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03376     }
03377
03378     /* "PyClical.pyx":107
03379     *         except (RuntimeError, TypeError):
03380     *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03381     *             elif isinstance(other, str):
03382     *                 try:
03383     *                     bother = other.encode("UTF-8")
03384     */
03385     __pyx_t_2 = PyUnicode_Check(__pyx_v_other);
03386     __pyx_t_1 = (__pyx_t_2 != 0);
03387     if (likely(__pyx_t_1)) {
03388
03389         /* "PyClical.pyx":108
03390         *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03391         *             elif isinstance(other, str):
03392         *                 try:
03393         *                     bother = other.encode("UTF-8")
03394         *                     self.instance = new IndexSet(<char *>bother)
03395         */
03396         {
03397             __Pyx_PyThreadState_declare
03398             __Pyx_PyThreadState_assign
03399             __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_9, &__pyx_t_8);
03400             __Pyx_XGOTREF(__pyx_t_10);
03401             __Pyx_XGOTREF(__pyx_t_9);
03402             __Pyx_XGOTREF(__pyx_t_8);
03403             /*try:*/ {
03404
03405                 /* "PyClical.pyx":109
03406                 *             elif isinstance(other, str):
03407                 *                 try:
03408                 *                     bother = other.encode("UTF-8")
03409                 *                     self.instance = new IndexSet(<char *>bother)
03410                 *             except RuntimeError:
03411                 */
03412                 __pyx_t_4 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 109, __pyx_L18_error)
03413                 __Pyx_GOTREF(__pyx_t_4);
03414                 __pyx_t_13 = NULL;
03415                 if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_4))) {
03416                     __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_4);
03417                     if (likely(__pyx_t_13)) {
03418                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_4);
03419                         __Pyx_INCREF(__pyx_t_13);
03420                         __Pyx_INCREF(function);
03421                         __Pyx_DECREF_SET(__pyx_t_4, function);
03422                     }
03423                 }
03424                 __pyx_t_5 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_4, __pyx_t_13, __pyx_kp_u_UTF_8) :
__Pyx_PyObject_CallOneArg(__pyx_t_4, __pyx_kp_u_UTF_8);
03425                 __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03426                 if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 109, __pyx_L18_error)
03427                 __Pyx_GOTREF(__pyx_t_5);
03428                 __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03429                 __pyx_v_bother = __pyx_t_5;
03430                 __pyx_t_5 = 0;
03431
03432                 /* "PyClical.pyx":110
03433                 *                 try:
03434                 *                     bother = other.encode("UTF-8")
03435                 *                     self.instance = new IndexSet(<char *>bother)
03436                 *             except RuntimeError:
03437                 *                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03438                 */
03439                 __pyx_t_17 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if (unlikely(!__pyx_t_17) &&
PyErr_Occurred()) __PYX_ERR(0, 110, __pyx_L18_error)
03440                 try {
03441                     __pyx_t_3 = new IndexSet(((char *)__pyx_t_17));
03442                 } catch (...) {
03443                     __Pyx_CppExn2PyErr();
03444                     __PYX_ERR(0, 110, __pyx_L18_error)
03445                 }
03446                 __pyx_v_self->instance = __pyx_t_3;
03447
03448                 /* "PyClical.pyx":108
03449                 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03450                 *             elif isinstance(other, str):
03451                 *                 try:
03452                 *                     bother = other.encode("UTF-8")
03453                 *                     self.instance = new IndexSet(<char *>bother)
03454                 */
03455             }
03456             __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
03457             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
03458             __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
03459             goto __pyx_L23_try_end;

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03460     __pyx_L18_error;;
03461     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03462     __Pyx_XDECREF(__pyx_t_14); __pyx_t_14 = 0;
03463     __Pyx_XDECREF(__pyx_t_15); __pyx_t_15 = 0;
03464     __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
03465     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
03466     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
03467
03468     /* "PyCliclcal.pyx":111
03469     *         bother = other.encode("UTF-8")
03470     *         self.instance = new IndexSet(<char *>bother)
03471     *     except RuntimeError: # ««««««««
03472     *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03473     *     else:
03474     */
03475     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
03476     if (__pyx_t_6) {
03477         __Pyx_AddTraceback("PyCliclcal.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03478         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_4, &__pyx_t_13) < 0) __PYX_ERR(0, 111,
__pyx_L20_except_error)
03479         __Pyx_GOTREF(__pyx_t_5);
03480         __Pyx_GOTREF(__pyx_t_4);
03481         __Pyx_GOTREF(__pyx_t_13);
03482
03483         /* "PyCliclcal.pyx":112
03484         *         self.instance = new IndexSet(<char *>bother)
03485         *     except RuntimeError:
03486         *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03487         *     # ««««««««
03488         *     else:
03489         *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03490         */
03491         __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid_string); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 112, __pyx_L20_except_error)
03492         __Pyx_GOTREF(__pyx_t_14);
03493         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 112,
__pyx_L20_except_error)
03494         __Pyx_GOTREF(__pyx_t_15);
03495         __pyx_t_16 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03496         __Pyx_GOTREF(__pyx_t_16);
03497         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03498         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03499         __pyx_t_15 = PyNumber_Add(__pyx_t_16, __pyx_kp_u); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03500         __Pyx_GOTREF(__pyx_t_15);
03501         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03502         __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 112, __pyx_L20_except_error)
03503         __Pyx_GOTREF(__pyx_t_16);
03504         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03505         __Pyx_Raise(__pyx_t_16, 0, 0, 0);
03506         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03507         __PYX_ERR(0, 112, __pyx_L20_except_error)
03508     }
03509     goto __pyx_L20_except_error;
03510     __pyx_L20_except_error;;
03511
03512     /* "PyCliclcal.pyx":108
03513     *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03514     *     elif isinstance(other, str):
03515     *         try: # ««««««««
03516     *             bother = other.encode("UTF-8")
03517     *             self.instance = new IndexSet(<char *>bother)
03518         */
03519         __Pyx_XGIVEREF(__pyx_t_10);
03520         __Pyx_XGIVEREF(__pyx_t_9);
03521         __Pyx_XGIVEREF(__pyx_t_8);
03522         __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_9, __pyx_t_8);
03523         goto __pyx_L1_error;
03524         __pyx_L23_try_end;;
03525     }
03526
03527     /* "PyCliclcal.pyx":107
03528     *     except (RuntimeError, TypeError):
03529     *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03530     *     elif isinstance(other, str): # ««««««««
03531     *         try:
03532     *             bother = other.encode("UTF-8")
03533         */
03534         goto __pyx_L3;
03535     }
03536
03537     /* "PyCliclcal.pyx":114
03538     *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03539     *     else:

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03539 *             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")           # ««««««««
03540 *
03541 *         def __dealloc__(self):
03542 */
03543 /*else*/ {
03544     __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 114, __pyx_L1_error)
03545     __Pyx_GOTREF(__pyx_t_13);
03546     __pyx_t_4 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), (PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 114, __pyx_L1_error)
03547     __Pyx_GOTREF(__pyx_t_4);
03548     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0,
114, __pyx_L1_error)
03549     __Pyx_GOTREF(__pyx_t_5);
03550     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
03551     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03552     __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u_); if (unlikely(!__pyx_t_4)) __PYX_ERR(0,
114, __pyx_L1_error)
03553     __Pyx_GOTREF(__pyx_t_4);
03554     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03555     __pyx_t_5 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_4); if
(unlikely(!__pyx_t_5)) __PYX_ERR(0, 114, __pyx_L1_error)
03556     __Pyx_GOTREF(__pyx_t_5);
03557     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03558     __Pyx_Raise(__pyx_t_5, 0, 0, 0);
03559     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03560     __PYX_ERR(0, 114, __pyx_L1_error)
03561 }
03562 __pyx_L3:;
03563
03564 /* "PyClical.pyx":74
03565 *         return index_set(self)
03566 *
03567 *     def __cinit__(self, other = 0):           # ««««««««
03568 *         """
03569 *         Construct an object of type index_set.
03570 */
03571
03572 /* function exit code */
03573 __pyx_r = 0;
03574 goto __pyx_L0;
03575 __pyx_L1_error:;
03576 __Pyx_XDECREF(__pyx_t_4);
03577 __Pyx_XDECREF(__pyx_t_5);
03578 __Pyx_XDECREF(__pyx_t_13);
03579 __Pyx_XDECREF(__pyx_t_14);
03580 __Pyx_XDECREF(__pyx_t_15);
03581 __Pyx_XDECREF(__pyx_t_16);
03582 __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03583 __pyx_r = -1;
03584 __pyx_L0:;
03585 __Pyx_XDECREF(__pyx_v_error_msg_prefix);
03586 __Pyx_XDECREF(__pyx_v_idx);
03587 __Pyx_XDECREF(__pyx_v_bother);
03588 __Pyx_RefNannyFinishContext();
03589 return __pyx_r;
03590 }
03591
03592 /* "PyClical.pyx":116
03593 *             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03594 *
03595 *         def __dealloc__(self):           # ««««««««
03596 *             """
03597 *             Clean up by deallocating the instance of C++ class IndexSet.
03598 */
03599
03600 /* Python wrapper */
03601 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self); /*proto*/
03602 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self) {
03603     __Pyx_RefNannyDeclarations
03604     __Pyx_RefNannySetupContext("__dealloc__ (wrapper)", 0);
03605     __pyx_pf_8PyClical_9index_set_4__dealloc__(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_self));
03606
03607 /* function exit code */
03608 __Pyx_RefNannyFinishContext();
03609 }
03610
03611 static void __pyx_pf_8PyClical_9index_set_4__dealloc__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
03612     __Pyx_RefNannyDeclarations
03613     __Pyx_RefNannySetupContext("__dealloc__", 0);
03614
03615 /* "PyClical.pyx":120
03616 *             Clean up by deallocating the instance of C++ class IndexSet.
03617 *             """
03618 *             del self.instance           # ««««««««
03619 *

```

```

03620 *      def __richcmp__(lhs, rhs, int op):
03621 */
03622 delete __pyx_v_self->instance;
03623
03624 /* "PyCliclal.pyx":116
03625 *          raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03626 *
03627 *      def __dealloc__(self):          # ««««««««
03628 *          """
03629 *          Clean up by deallocating the instance of C++ class IndexSet.
03630 */
03631
03632 /* function exit code */
03633 __Pyx_RefNannyFinishContext();
03634 }
03635
03636 /* "PyCliclal.pyx":122
03637 *          del self.instance
03638 *
03639 *      def __richcmp__(lhs, rhs, int op):          # ««««««««
03640 *          """
03641 *          Compare two objects of class index_set.
03642 */
03643
03644 /* Python wrapper */
03645 static PyObject *__pyx_pw_8PyCliclal_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op); /*proto*/
03646 static PyObject *__pyx_pw_8PyCliclal_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op) {
03647     PyObject *__pyx_r = 0;
03648     __Pyx_RefNannyDeclarations
03649     __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);
03650     __pyx_r = __pyx_pf_8PyCliclal_9index_set_6__richcmp__(((struct __pyx_obj_8PyCliclal_index_set
*)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
03651
03652 /* function exit code */
03653 __Pyx_RefNannyFinishContext();
03654 return __pyx_r;
03655 }
03656
03657 static PyObject *__pyx_pf_8PyCliclal_9index_set_6__richcmp__(struct __pyx_obj_8PyCliclal_index_set
*__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
03658     PyObject *__pyx_v_eq = NULL;
03659     PyObject *__pyx_v_lt = NULL;
03660     PyObject *__pyx_r = NULL;
03661     __Pyx_RefNannyDeclarations
03662     int __pyx_t_1;
03663     int __pyx_t_2;
03664     int __pyx_t_3;
03665     PyObject *__pyx_t_4 = NULL;
03666     int __pyx_lineno = 0;
03667     const char *__pyx_filename = NULL;
03668     int __pyx_clineno = 0;
03669     __Pyx_RefNannySetupContext("__richcmp__", 0);
03670
03671 /* "PyCliclal.pyx":143
03672 *          False
03673 *          """
03674 *          if (lhs is None) or (rhs is None):          # ««««««««
03675 *              eq = bool(lhs is rhs)
03676 *              if op == 2: # ==
03677 */
03678     __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
03679     __pyx_t_3 = (__pyx_t_2 != 0);
03680     if (!__pyx_t_3) {
03681     } else {
03682         __pyx_t_1 = __pyx_t_3;
03683         goto __pyx_L4_bool_binop_done;
03684     }
03685     __pyx_t_3 = (__pyx_v_rhs == Py_None);
03686     __pyx_t_2 = (__pyx_t_3 != 0);
03687     __pyx_t_1 = __pyx_t_2;
03688     __pyx_L4_bool_binop_done;
03689     if (__pyx_t_1) {
03690
03691 /* "PyCliclal.pyx":144
03692 *          """
03693 *          if (lhs is None) or (rhs is None):
03694 *              eq = bool(lhs is rhs)          # ««««««««
03695 *              if op == 2: # ==
03696 *                  return eq
03697 */
03698     __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
03699     __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 144,
__pyx_L1_error)
03700     __Pyx_GOTREF(__pyx_t_4);
03701     __pyx_v_eq = __pyx_t_4;

```



```

03702     __pyx_t_4 = 0;
03703
03704     /* "PyClical.pyx":145
03705  *         if (lhs is None) or (rhs is None):
03706  *             eq = bool(lhs is rhs)
03707  *             if op == 2: # ==                # ««««««««
03708  *                 return eq
03709  *             elif op == 3: # !=
03710  */
03711     switch (__pyx_v_op) {
03712     case 2:
03713
03714         /* "PyClical.pyx":146
03715  *             eq = bool(lhs is rhs)
03716  *             if op == 2: # ==                # ««««««««
03717  *                 return eq
03718  *             elif op == 3: # !=
03719  *                 return not eq
03720  */
03721         __Pyx_XDECREF(__pyx_r);
03722         __Pyx_INCREF(__pyx_v_eq);
03723         __pyx_r = __pyx_v_eq;
03724         goto __pyx_L0;
03725
03726         /* "PyClical.pyx":145
03727  *         if (lhs is None) or (rhs is None):
03728  *             eq = bool(lhs is rhs)
03729  *             if op == 2: # ==                # ««««««««
03730  *                 return eq
03731  *             elif op == 3: # !=
03732  */
03733         break;
03734     case 3:
03735
03736         /* "PyClical.pyx":148
03737  *             return eq
03738  *             elif op == 3: # !=
03739  *                 return not eq                # ««««««««
03740  *             else:
03741  *                 if op == 0: # <
03742  */
03743         __Pyx_XDECREF(__pyx_r);
03744         __pyx_t_1 = __PyxPyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 148,
__pyx_L1_error)
03745         __pyx_t_4 = __PyxPyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 148,
__pyx_L1_error)
03746         __Pyx_GOTREF(__pyx_t_4);
03747         __pyx_r = __pyx_t_4;
03748         __pyx_t_4 = 0;
03749         goto __pyx_L0;
03750
03751         /* "PyClical.pyx":147
03752  *             if op == 2: # ==
03753  *                 return eq
03754  *             elif op == 3: # !=                # ««««««««
03755  *                 return not eq
03756  *             else:
03757  */
03758         break;
03759     default:
03760
03761         /* "PyClical.pyx":150
03762  *             return not eq
03763  *             else:
03764  *                 if op == 0: # <                # ««««««««
03765  *                     return False
03766  *                 elif op == 1: # <=
03767  */
03768         switch (__pyx_v_op) {
03769         case 0:
03770
03771             /* "PyClical.pyx":151
03772  *                 else:
03773  *                     if op == 0: # <
03774  *                         return False                # ««««««««
03775  *                     elif op == 1: # <=
03776  *                         return eq
03777  */
03778             __Pyx_XDECREF(__pyx_r);
03779             __Pyx_INCREF(Py_False);
03780             __pyx_r = Py_False;
03781             goto __pyx_L0;
03782
03783             /* "PyClical.pyx":150
03784  *                 return not eq
03785  *                 else:
03786  *                     if op == 0: # <                # ««««««««

```

```

03787 *             return False
03788 *             elif op == 1: # <=
03789 */
03790 break;
03791 case 1:
03792
03793 /* "PyClical.pyx":153
03794 *             return False
03795 *             elif op == 1: # <=
03796 *             return eq # ««««««««
03797 *             elif op == 4: # >
03798 *             return False
03799 */
03800 __Pyx_XDECREF(__pyx_r);
03801 __Pyx_INCREF(__pyx_v_eq);
03802 __pyx_r = __pyx_v_eq;
03803 goto __pyx_L0;
03804
03805 /* "PyClical.pyx":152
03806 *             if op == 0: # <
03807 *             return False
03808 *             elif op == 1: # <= # ««««««««
03809 *             return eq
03810 *             elif op == 4: # >
03811 */
03812 break;
03813 case 4:
03814
03815 /* "PyClical.pyx":155
03816 *             return eq
03817 *             elif op == 4: # >
03818 *             return False # ««««««««
03819 *             elif op == 5: # >=
03820 *             return eq
03821 */
03822 __Pyx_XDECREF(__pyx_r);
03823 __Pyx_INCREF(Py_False);
03824 __pyx_r = Py_False;
03825 goto __pyx_L0;
03826
03827 /* "PyClical.pyx":154
03828 *             elif op == 1: # <=
03829 *             return eq
03830 *             elif op == 4: # > # ««««««««
03831 *             return False
03832 *             elif op == 5: # >=
03833 */
03834 break;
03835 case 5:
03836
03837 /* "PyClical.pyx":157
03838 *             return False
03839 *             elif op == 5: # >=
03840 *             return eq # ««««««««
03841 *             else:
03842 *             return NotImplemented
03843 */
03844 __Pyx_XDECREF(__pyx_r);
03845 __Pyx_INCREF(__pyx_v_eq);
03846 __pyx_r = __pyx_v_eq;
03847 goto __pyx_L0;
03848
03849 /* "PyClical.pyx":156
03850 *             elif op == 4: # >
03851 *             return False
03852 *             elif op == 5: # >= # ««««««««
03853 *             return eq
03854 *             else:
03855 */
03856 break;
03857 default:
03858
03859 /* "PyClical.pyx":159
03860 *             return eq
03861 *             else:
03862 *             return NotImplemented # ««««««««
03863 *             else:
03864 *             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03865 */
03866 __Pyx_XDECREF(__pyx_r);
03867 __Pyx_INCREF(__pyx_builtin_NotImplemented);
03868 __pyx_r = __pyx_builtin_NotImplemented;
03869 goto __pyx_L0;
03870 break;
03871 }
03872 break;
03873 }

```

```

03874
03875  /* "PyClical.pyx":143
03876  *      False
03877  *      ""
03878  *      if (lhs is None) or (rhs is None):          # ««««««««
03879  *          eq = bool(lhs is rhs)
03880  *          if op == 2: # ==
03881  */
03882  }
03883
03884  /* "PyClical.pyx":161
03885  *          return NotImplemented
03886  *      else:
03887  *          eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )          # ««««««««
03888  *          if op == 2: # ==
03889  *              return eq
03890  */
03891  /*else*/ {
03892  __pyx_t_1 = (__pyx_f_8PyClical_toIndexSet(((PyObject *)__pyx_v_lhs)) ==
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03893  __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 161,
__pyx_L1_error)
03894  __Pyx_GOTREF(__pyx_t_4);
03895  __pyx_v_eq = __pyx_t_4;
03896  __pyx_t_4 = 0;
03897
03898  /* "PyClical.pyx":162
03899  *      else:
03900  *          eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03901  *          if op == 2: # ==          # ««««««««
03902  *              return eq
03903  *          elif op == 3: # !=
03904  */
03905  switch (__pyx_v_op) {
03906  case 2:
03907
03908      /* "PyClical.pyx":163
03909  *          eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03910  *          if op == 2: # ==
03911  *              return eq          # ««««««««
03912  *          elif op == 3: # !=
03913  *              return not eq
03914  */
03915  __Pyx_XDECREF(__pyx_r);
03916  __Pyx_INCREF(__pyx_v_eq);
03917  __pyx_r = __pyx_v_eq;
03918  goto __pyx_L0;
03919
03920  /* "PyClical.pyx":162
03921  *      else:
03922  *          eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03923  *          if op == 2: # ==          # ««««««««
03924  *              return eq
03925  *          elif op == 3: # !=
03926  */
03927  break;
03928  case 3:
03929
03930      /* "PyClical.pyx":165
03931  *          return eq
03932  *          elif op == 3: # !=
03933  *              return not eq          # ««««««««
03934  *          else:
03935  *              lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03936  */
03937  __Pyx_XDECREF(__pyx_r);
03938  __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 165,
__pyx_L1_error)
03939  __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 165,
__pyx_L1_error)
03940  __Pyx_GOTREF(__pyx_t_4);
03941  __pyx_r = __pyx_t_4;
03942  __pyx_t_4 = 0;
03943  goto __pyx_L0;
03944
03945  /* "PyClical.pyx":164
03946  *          if op == 2: # ==
03947  *              return eq
03948  *          elif op == 3: # !=          # ««««««««
03949  *              return not eq
03950  *          else:
03951  */
03952  break;
03953  default:
03954
03955  /* "PyClical.pyx":167
03956  *          return not eq

```

```

03957 *           else:
03958 *               lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )           # ««««««««
03959 *               if op == 0: # <
03960 *                   return lt
03961 */
03962 __pyx_t_1 = (__pyx_f_8PyClical_toIndexSet(((PyObject *)__pyx_v_lhs)) <
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03963 __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 167,
__pyx_L1_error)
03964 __Pyx_GOTREF(__pyx_t_4);
03965 __pyx_v_lt = __pyx_t_4;
03966 __pyx_t_4 = 0;
03967
03968 /* "PyClical.pyx":168
03969 *           else:
03970 *               lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03971 *               if op == 0: # <           # ««««««««
03972 *                   return lt
03973 *               elif op == 1: # <=
03974 */
03975 switch (__pyx_v_op) {
03976     case 0:
03977
03978         /* "PyClical.pyx":169
03979 *               lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03980 *               if op == 0: # <
03981 *                   return lt           # ««««««««
03982 *               elif op == 1: # <=
03983 *                   return lt or eq
03984 */
03985 __Pyx_XDECREF(__pyx_r);
03986 __Pyx_INCREF(__pyx_v_lt);
03987 __pyx_r = __pyx_v_lt;
03988 goto __pyx_L0;
03989
03990 /* "PyClical.pyx":168
03991 *           else:
03992 *               lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03993 *               if op == 0: # <           # ««««««««
03994 *                   return lt
03995 *               elif op == 1: # <=
03996 */
03997 break;
03998 case 1:
03999
04000         /* "PyClical.pyx":171
04001 *               return lt
04002 *               elif op == 1: # <=
04003 *                   return lt or eq           # ««««««««
04004 *               elif op == 4: # >
04005 *                   return not (lt or eq)
04006 */
04007 __Pyx_XDECREF(__pyx_r);
04008 __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 171,
__pyx_L1_error)
04009 if (!__pyx_t_1) {
04010     } else {
04011         __Pyx_INCREF(__pyx_v_lt);
04012         __pyx_t_4 = __pyx_v_lt;
04013         goto __pyx_L6_bool_binop_done;
04014     }
04015 __Pyx_INCREF(__pyx_v_eq);
04016 __pyx_t_4 = __pyx_v_eq;
04017 __pyx_L6_bool_binop_done;
04018 __pyx_r = __pyx_t_4;
04019 __pyx_t_4 = 0;
04020 goto __pyx_L0;
04021
04022 /* "PyClical.pyx":170
04023 *               if op == 0: # <
04024 *                   return lt
04025 *               elif op == 1: # <=           # ««««««««
04026 *                   return lt or eq
04027 *               elif op == 4: # >
04028 */
04029 break;
04030 case 4:
04031
04032         /* "PyClical.pyx":173
04033 *               return lt or eq
04034 *               elif op == 4: # >
04035 *                   return not (lt or eq)           # ««««««««
04036 *               elif op == 5: # >=
04037 *                   return not lt
04038 */
04039 __Pyx_XDECREF(__pyx_r);
04040 __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,

```

```

__pyx_L1_error)
04041     if (!__pyx_t_2) {
04042     } else {
04043         __pyx_t_1 = __pyx_t_2;
04044         goto __pyx_L8_bool_binop_done;
04045     }
04046     __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,
__pyx_L1_error)
04047     __pyx_t_1 = __pyx_t_2;
04048     __pyx_L8_bool_binop_done;;
04049     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 173,
__pyx_L1_error)
04050     __Pyx_GOTREF(__pyx_t_4);
04051     __pyx_r = __pyx_t_4;
04052     __pyx_t_4 = 0;
04053     goto __pyx_L0;
04054
04055     /* "PyClical.pyx":172
04056     *         elif op == 1: # <=
04057     *             return lt or eq
04058     *         elif op == 4: # > # ««««««««
04059     *             return not (lt or eq)
04060     *         elif op == 5: # >=
04061     */
04062     break;
04063     case 5:
04064
04065     /* "PyClical.pyx":175
04066     *             return not (lt or eq)
04067     *         elif op == 5: # >=
04068     *             return not lt # ««««««««
04069     *         else:
04070     *             return NotImplemented
04071     */
04072     __Pyx_XDECREF(__pyx_r);
04073     __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 175,
__pyx_L1_error)
04074     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 175,
__pyx_L1_error)
04075     __Pyx_GOTREF(__pyx_t_4);
04076     __pyx_r = __pyx_t_4;
04077     __pyx_t_4 = 0;
04078     goto __pyx_L0;
04079
04080     /* "PyClical.pyx":174
04081     *         elif op == 4: # >
04082     *             return not (lt or eq)
04083     *         elif op == 5: # >= # ««««««««
04084     *             return not lt
04085     *         else:
04086     */
04087     break;
04088     default:
04089
04090     /* "PyClical.pyx":177
04091     *             return not lt
04092     *         else:
04093     *             return NotImplemented # ««««««««
04094     */
04095     def __setitem__(self, idx, val):
04096     */
04097     __Pyx_XDECREF(__pyx_r);
04098     __Pyx_INCREF(__pyx_builtin_NotImplemented);
04099     __pyx_r = __pyx_builtin_NotImplemented;
04100     goto __pyx_L0;
04101     break;
04102     }
04103     break;
04104     }
04105     }
04106
04107     /* "PyClical.pyx":122
04108     *     del self.instance
04109     *
04110     *     def __richcmp__(lhs, rhs, int op): # ««««««««
04111     *         """
04112     *         Compare two objects of class index_set.
04113     */
04114
04115     /* function exit code */
04116     __pyx_L1_error;
04117     __Pyx_XDECREF(__pyx_t_4);
04118     __Pyx_AddTraceback("PyClical.index_set.__richcmp__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04119     __pyx_r = NULL;
04120     __pyx_L0;
04121     __Pyx_XDECREF(__pyx_v_eq);
04122     __Pyx_XDECREF(__pyx_v_lt);

```

```

04123     __Pyx_XGIVEREF(__pyx_r);
04124     __Pyx_RefNannyFinishContext();
04125     return __pyx_r;
04126 }
04127
04128 /* "PyClical.pyx":179
04129 *
04130 *
04131 *     def __setitem__(self, idx, val):
04132 *         """
04133 *         Set the value of an index_set object at index idx to value val.
04134 */
04135
04136 /* Python wrapper */
04137 static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
04138     PyObject *__pyx_v_val); /*proto*/
04139 static char __pyx_doc_8PyClical_9index_set_8__setitem__[] = "\n        Set the value of an index_set
04140 object at index idx to value val.\n\n        >> s=index_set({1}); s[2] = True; print(s)\n
04141 {1,2}\n        >> s=index_set({1,2}); s[1] = False; print(s)\n        {2}\n        ";
04142 #if CYTHON_COMPILING_IN_CPYTHON
04143 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
04144 #endif
04145 static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
04146     PyObject *__pyx_v_val) {
04147     int __pyx_r;
04148     __Pyx_RefNannyDeclarations
04149     __Pyx_RefNannySetupContext("__setitem__ (wrapper)", 0);
04150     __pyx_r = __pyx_pf_8PyClical_9index_set_8__setitem__(((struct __pyx_obj_8PyClical_index_set
04151 *)__pyx_v_self), ((PyObject *)__pyx_v_idx), ((PyObject *)__pyx_v_val));
04152
04153 /* function exit code */
04154 __Pyx_RefNannyFinishContext();
04155 return __pyx_r;
04156 }
04157
04158 static int __pyx_pf_8PyClical_9index_set_8__setitem__(struct __pyx_obj_8PyClical_index_set
04159 * __pyx_v_self, PyObject * __pyx_v_idx, PyObject * __pyx_v_val) {
04160     int __pyx_r;
04161     __Pyx_RefNannyDeclarations
04162     int __pyx_t_1;
04163     int __pyx_t_2;
04164     int __pyx_lineno = 0;
04165     const char * __pyx_filename = NULL;
04166     int __pyx_clineno = 0;
04167     __Pyx_RefNannySetupContext("__setitem__", 0);
04168
04169 /* "PyClical.pyx":188
04170 *
04171 *         {2}
04172 *
04173 *         self.instance.set(idx, val)
04174 *         return
04175 */
04176 __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
04177 PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04178 __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_val); if (unlikely((__pyx_t_2 == (int)-1) &&
04179 PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04180 try {
04181     __pyx_v_self->instance->set(__pyx_t_1, __pyx_t_2);
04182 } catch (...) {
04183     __Pyx_CppExn2PyErr();
04184     __PYX_ERR(0, 188, __pyx_L1_error)
04185 }
04186
04187 /* "PyClical.pyx":189
04188 *
04189 *         """
04190 *         self.instance.set(idx, val)
04191 *         return
04192 *
04193 *         def __getitem__(self, idx):
04194 */
04195 __pyx_r = 0;
04196 goto __pyx_L0;
04197
04198 /* "PyClical.pyx":179
04199 *
04200 *         return NotImplemented
04201 *
04202 *
04203 *     def __setitem__(self, idx, val):
04204 *         """
04205 *         Set the value of an index_set object at index idx to value val.
04206 */
04207
04208 /* function exit code */
04209 __pyx_L1_error:;
04210 __Pyx_AddTraceback("PyClical.index_set.__setitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04211 __pyx_r = -1;
04212 __pyx_L0:;

```

```

04202  __Pyx_RefNannyFinishContext();
04203  return __pyx_r;
04204 }
04205
04206 /* "PyClical.pyx":191
04207 *      return
04208 *
04209 *      def __getitem__(self, idx):          # ««««««««
04210 *          """
04211 *          Get the value of an index_set object at an index.
04212 */
04213
04214 /* Python wrapper */
04215 static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_idx); /*proto*/
04216 static char __pyx_doc_8PyClical_9index_set_10__getitem__[] = "\n      Get the value of an index_set
    object at an index.\n\n      >> index_set({1})[1]\n      True\n      >> index_set({1})[2]\n
    False\n      >> index_set({2})[-1]\n      False\n      >> index_set({2})[1]\n      False\n
    >> index_set({2})[2]\n      True\n      >> index_set({2})[33]\n      False\n      ";
04217 #if CYTHON_COMPILING_IN_CPYTHON
04218 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
04219 #endif
04220 static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_idx) {
04221     PyObject *__pyx_r = 0;
04222     __Pyx_RefNannyDeclarations
04223     __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
04224     __pyx_r = __pyx_pf_8PyClical_9index_set_10__getitem__(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04225
04226     /* function exit code */
04227     __Pyx_RefNannyFinishContext();
04228     return __pyx_r;
04229 }
04230
04231 static PyObject *__pyx_pf_8PyClical_9index_set_10__getitem__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_idx) {
04232     PyObject *__pyx_r = NULL;
04233     __Pyx_RefNannyDeclarations
04234     int __pyx_t_1;
04235     PyObject *__pyx_t_2 = NULL;
04236     int __pyx_lineno = 0;
04237     const char *__pyx_filename = NULL;
04238     int __pyx_clineno = 0;
04239     __Pyx_RefNannySetupContext("__getitem__", 0);
04240
04241     /* "PyClical.pyx":208
04242 *          False
04243 *
04244 *          return self.instance.getitem(idx)          # ««««««««
04245 *
04246 *          def __contains__(self, idx):
04247 */
04248     __Pyx_XDECREF(__pyx_r);
04249     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
    PyErr_Occurred())) __PYX_ERR(0, 208, __pyx_L1_error)
04250     __pyx_t_2 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->operator[](__pyx_t_1)); if
    (unlikely(!__pyx_t_2)) __PYX_ERR(0, 208, __pyx_L1_error)
04251     __Pyx_GOTREF(__pyx_t_2);
04252     __pyx_r = __pyx_t_2;
04253     __pyx_t_2 = 0;
04254     goto __pyx_L0;
04255
04256     /* "PyClical.pyx":191
04257 *      return
04258 *
04259 *      def __getitem__(self, idx):          # ««««««««
04260 *          """
04261 *          Get the value of an index_set object at an index.
04262 */
04263
04264     /* function exit code */
04265     __pyx_L1_error:;
04266     __Pyx_XDECREF(__pyx_t_2);
04267     __Pyx_AddTraceback("PyClical.index_set.__getitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04268     __pyx_r = NULL;
04269     __pyx_L0:;
04270     __Pyx_XGIVEREF(__pyx_r);
04271     __Pyx_RefNannyFinishContext();
04272     return __pyx_r;
04273 }
04274
04275 /* "PyClical.pyx":210
04276 *      return self.instance.getitem(idx)
04277 *
04278 *      def __contains__(self, idx):          # ««««««««
04279 *          """

```

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04280 *          Check that an index_set object contains the index idx: idx in self.
04281 */
04282
04283 /* Python wrapper */
04284 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_idx); /*proto*/
04285 static char __pyx_doc_8PyClical_9index_set_12__contains__[] = "\n          Check that an index_set
    object contains the index idx: idx in self.\n\n          >> 1 in index_set({1})\n          True\n
    >> 2 in index_set({1})\n          False\n          >> -1 in index_set({2})\n          False\n          >> 1
    in index_set({2})\n          False\n          >> 2 in index_set({2})\n          True\n          >> 33 in
    index_set({2})\n          False\n          ";
04286 #if CYTHON_COMPILING_IN_CPYTHON
04287 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_12__contains__;
04288 #endif
04289 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx)
    {
04290     int __pyx_r;
04291     __Pyx_RefNannyDeclarations
04292     __Pyx_RefNannySetupContext("__contains__ (wrapper)", 0);
04293     __pyx_r = __pyx_pf_8PyClical_9index_set_12__contains__(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04294
04295     /* function exit code */
04296     __Pyx_RefNannyFinishContext();
04297     return __pyx_r;
04298 }
04299
04300 static int __pyx_pf_8PyClical_9index_set_12__contains__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_idx) {
04301     int __pyx_r;
04302     __Pyx_RefNannyDeclarations
04303     int __pyx_t_1;
04304     int __pyx_lineno = 0;
04305     const char *__pyx_filename = NULL;
04306     int __pyx_clineno = 0;
04307     __Pyx_RefNannySetupContext("__contains__", 0);
04308
04309     /* "PyClical.pyx":227
    *          False
    *          """
04310     *          return self.instance.getitem(idx)          # ««««««««
04311     *
04312     *      def __iter__(self):
04313     */
04314     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
    PyErr_Occurred())) __PYX_ERR(0, 227, __pyx_L1_error)
04315     __pyx_r = __pyx_v_self->instance->operator[](__pyx_t_1);
04316     goto __pyx_L0;
04317
04318     /* "PyClical.pyx":210
    *          return self.instance.getitem(idx)
04319     *
04320     *      def __contains__(self, idx):          # ««««««««
04321     *          """
04322     *          Check that an index_set object contains the index idx: idx in self.
04323     */
04324     /* function exit code */
04325     __pyx_L1_error;
04326     __Pyx_AddTraceback("PyClical.index_set.__contains__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04327     __pyx_r = -1;
04328     __pyx_L0;
04329     __Pyx_RefNannyFinishContext();
04330     return __pyx_r;
04331 }
04332
04333 static PyObject *__pyx_gb_8PyClical_9index_set_16generator(__pyx_CoroutineObject *__pyx_generator,
    CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject *__pyx_sent_value); /* proto */
04334
04335 /* "PyClical.pyx":229
    *          return self.instance.getitem(idx)
04336     *
04337     *      def __iter__(self):          # ««««««««
04338     *          """
04339     *          Iterate over the indices of an index_set.
04340     */
04341
04342 /* Python wrapper */
04343 static PyObject *__pyx_pw_8PyClical_9index_set_15__iter__(PyObject *__pyx_v_self); /*proto*/
04344 static char __pyx_doc_8PyClical_9index_set_14__iter__[] = "\n          Iterate over the indices of an
    index_set.\n\n          >> for i in index_set({-3,4,7}):print(i, end=',')\n          -3,4,7,\n
    ";
04345 #if CYTHON_COMPILING_IN_CPYTHON
04346 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_14__iter__;
04347 #endif
04348 static PyObject *__pyx_pw_8PyClical_9index_set_15__iter__(PyObject *__pyx_v_self) {
04349     PyObject *__pyx_r = 0;
04350     __Pyx_RefNannyDeclarations

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04355 __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
04356 __pyx_r = __pyx_pf_8PyClical_9index_set_14__iter__((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
04357
04358 /* function exit code */
04359 __Pyx_RefNannyFinishContext();
04360 return __pyx_r;
04361 }
04362
04363 static PyObject *__pyx_pf_8PyClical_9index_set_14__iter__(struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self) {
04364     struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ * __pyx_cur_scope;
04365     PyObject *__pyx_r = NULL;
04366     __Pyx_RefNannyDeclarations
04367     int __pyx_lineno = 0;
04368     const char *__pyx_filename = NULL;
04369     int __pyx_clineno = 0;
04370     __Pyx_RefNannySetupContext("__iter__", 0);
04371     __pyx_cur_scope = (struct __pyx_obj_8PyClical__pyx_scope_struct__iter__
*)__pyx_tp_new_8PyClical__pyx_scope_struct__iter__(__pyx_ptype_8PyClical__pyx_scope_struct__iter__,
__pyx_empty_tuple, NULL);
04372     if (unlikely(!__pyx_cur_scope)) {
04373         __pyx_cur_scope = ((struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ *)Py_None);
04374         __Pyx_INCREF(Py_None);
04375         __PYX_ERR(0, 229, __pyx_L1_error)
04376     } else {
04377         __Pyx_GOTREF(__pyx_cur_scope);
04378     }
04379     __pyx_cur_scope->__pyx_v_self = __pyx_v_self;
04380     __Pyx_INCREF((PyObject *)__pyx_cur_scope->__pyx_v_self);
04381     __Pyx_GIVEREF((PyObject *)__pyx_cur_scope->__pyx_v_self);
04382     {
04383         __pyx_CoroutineObject *gen = __Pyx_Generator_New((__pyx_coroutine_body_t)
__pyx_gb_8PyClical_9index_set_16generator, NULL, (PyObject *) __pyx_cur_scope, __pyx_n_s_iter,
__pyx_n_s_index_set__iter__, __pyx_n_s_PyClical); if (unlikely(!gen)) __PYX_ERR(0, 229,
__pyx_L1_error)
04384         __Pyx_DECREF(__pyx_cur_scope);
04385         __Pyx_RefNannyFinishContext();
04386         return (PyObject *) gen;
04387     }
04388
04389 /* function exit code */
04390 __pyx_L1_error:;
04391 __Pyx_AddTraceback("PyClical.index_set.__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04392 __pyx_r = NULL;
04393 __Pyx_DECREF((PyObject *)__pyx_cur_scope);
04394 __Pyx_XGIVEREF(__pyx_r);
04395 __Pyx_RefNannyFinishContext();
04396 return __pyx_r;
04397 }
04398
04399 static PyObject *__pyx_gb_8PyClical_9index_set_16generator(__pyx_CoroutineObject *__pyx_generator,
CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject *__pyx_sent_value) /* generator body */
04400 {
04401     struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ * __pyx_cur_scope = ((struct
__pyx_obj_8PyClical__pyx_scope_struct__iter__ *)__pyx_generator->closure);
04402     PyObject *__pyx_r = NULL;
04403     PyObject *__pyx_t_1 = NULL;
04404     PyObject *__pyx_t_2 = NULL;
04405     PyObject *__pyx_t_3 = NULL;
04406     PyObject *__pyx_t_4 = NULL;
04407     Py_ssize_t __pyx_t_5;
04408     PyObject *(*__pyx_t_6)(PyObject *);
04409     int __pyx_t_7;
04410     int __pyx_t_8;
04411     int __pyx_lineno = 0;
04412     const char *__pyx_filename = NULL;
04413     int __pyx_clineno = 0;
04414     __Pyx_RefNannyDeclarations
04415     __Pyx_RefNannySetupContext("__iter__", 0);
04416     switch (__pyx_generator->resume_label) {
04417         case 0: goto __pyx_L3_first_run;
04418         case 1: goto __pyx_L7_resume_from_yield;
04419         default: /* CPython raises the right error here */
04420             __Pyx_RefNannyFinishContext();
04421             return NULL;
04422     }
04423     __pyx_L3_first_run;
04424     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 229, __pyx_L1_error)
04425
04426     /* "PyClical.pyx":236
04427     *         -3,4,7,
04428     *         """
04429     *         for idx in range(self.min(), self.max()+1):           # ««««««««
04430     *             if idx in self:
04431     *                 yield idx
04432     */

```

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04433     __pyx_t_2 = __Pyx_PyObject_GetAttrStr(((PyObject *)__pyx_cur_scope->__pyx_v_self), __pyx_n_s_min);
04434     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04435     __Pyx_GOTREF(__pyx_t_2);
04436     __pyx_t_3 = NULL;
04437     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
04438         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
04439         if (likely(__pyx_t_3)) {
04440             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
04441             __Pyx_INCREF(__pyx_t_3);
04442             __Pyx_INCREF(function);
04443             __Pyx_DECREF_SET(__pyx_t_2, function);
04444         }
04445     }
04446     __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_t_3) :
__Pyx_PyObject_CallNoArg(__pyx_t_2);
04447     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
04448     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 236, __pyx_L1_error)
04449     __Pyx_GOTREF(__pyx_t_1);
04450     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04451     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(((PyObject *)__pyx_cur_scope->__pyx_v_self), __pyx_n_s_max);
04452     if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 236, __pyx_L1_error)
04453     __Pyx_GOTREF(__pyx_t_3);
04454     __pyx_t_4 = NULL;
04455     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
04456         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
04457         if (likely(__pyx_t_4)) {
04458             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
04459             __Pyx_INCREF(__pyx_t_4);
04460             __Pyx_INCREF(function);
04461             __Pyx_DECREF_SET(__pyx_t_3, function);
04462         }
04463     }
04464     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
04465     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
04466     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04467     __Pyx_GOTREF(__pyx_t_2);
04468     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04469     __pyx_t_3 = __Pyx_PyInt_AddObjC(__pyx_t_2, __pyx_int_1, 1, 0, 0); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04470     __Pyx_GOTREF(__pyx_t_3);
04471     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04472     __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04473     __Pyx_GOTREF(__pyx_t_2);
04474     __Pyx_GIVEREF(__pyx_t_1);
04475     PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
04476     __Pyx_GIVEREF(__pyx_t_3);
04477     PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_t_3);
04478     __pyx_t_1 = 0;
04479     __pyx_t_3 = 0;
04480     __pyx_t_3 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_2, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04481     __Pyx_GOTREF(__pyx_t_3);
04482     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04483     if (likely(PyList_CheckExact(__pyx_t_3)) || PyTuple_CheckExact(__pyx_t_3)) {
04484         __pyx_t_2 = __pyx_t_3; __Pyx_INCREF(__pyx_t_2); __pyx_t_5 = 0;
04485         __pyx_t_6 = NULL;
04486     } else {
04487         __pyx_t_5 = -1; __pyx_t_2 = PyObject_GetIter(__pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0,
236, __pyx_L1_error)
04488         __Pyx_GOTREF(__pyx_t_2);
04489         __pyx_t_6 = Py_TYPE(__pyx_t_2)->tp_iternext; if (unlikely(!__pyx_t_6)) __PYX_ERR(0, 236,
__pyx_L1_error)
04490     }
04491     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04492     for (;;) {
04493         if (likely(!__pyx_t_6)) {
04494             if (likely(PyList_CheckExact(__pyx_t_2))) {
04495                 if (__pyx_t_5 >= PyList_GET_SIZE(__pyx_t_2)) break;
04496                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04497                 __pyx_t_3 = PyList_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_L1_error)
04498                 #else
04499                 __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04500                 __Pyx_GOTREF(__pyx_t_3);
04501             } #endif
04502         } else {
04503             if (__pyx_t_5 >= PyTuple_GET_SIZE(__pyx_t_2)) break;
04504             #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04505             __pyx_t_3 = PyTuple_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_L1_error)
04506             #else
04507             __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04508             __Pyx_GOTREF(__pyx_t_3);
04509         } #endif

```

```

04508     }
04509 } else {
04510     __pyx_t_3 = __pyx_t_6(__pyx_t_2);
04511     if (unlikely(!__pyx_t_3)) {
04512         PyObject* exc_type = PyErr_Occurred();
04513         if (exc_type) {
04514             if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration))) PyErr_Clear();
04515             else __PYX_ERR(0, 236, __pyx_L1_error)
04516         }
04517         break;
04518     }
04519     __Pyx_GOTREF(__pyx_t_3);
04520 }
04521 __Pyx_XGOTREF(__pyx_cur_scope->__pyx_v_idx);
04522 __Pyx_XDECREF_SET(__pyx_cur_scope->__pyx_v_idx, __pyx_t_3);
04523 __Pyx_GIVEREF(__pyx_t_3);
04524 __pyx_t_3 = 0;
04525
04526 /* "PyClical.pyx":237
04527 *      """
04528 *      for idx in range(self.min(), self.max()+1):
04529 *          if idx in self:          # ««««««««
04530 *              yield idx
04531 *
04532 */
04533 __pyx_t_7 = (__Pyx_PySequence_ContainsTF(__pyx_cur_scope->__pyx_v_idx, ((PyObject
*)__pyx_cur_scope->__pyx_v_self), Py_EQ)); if (unlikely(__pyx_t_7 < 0)) __PYX_ERR(0, 237,
__pyx_L1_error)
04534 __pyx_t_8 = (__pyx_t_7 != 0);
04535 if (__pyx_t_8) {
04536
04537     /* "PyClical.pyx":238
04538     *      for idx in range(self.min(), self.max()+1):
04539     *          if idx in self:
04540     *              yield idx          # ««««««««
04541     *
04542     *      def __invert__(self):
04543     */
04544     __Pyx_INCREF(__pyx_cur_scope->__pyx_v_idx);
04545     __pyx_r = __pyx_cur_scope->__pyx_v_idx;
04546     __Pyx_XGIVEREF(__pyx_t_2);
04547     __pyx_cur_scope->__pyx_t_0 = __pyx_t_2;
04548     __pyx_cur_scope->__pyx_t_1 = __pyx_t_5;
04549     __pyx_cur_scope->__pyx_t_2 = __pyx_t_6;
04550     __Pyx_XGIVEREF(__pyx_r);
04551     __Pyx_RefNannyFinishContext();
04552     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04553     /* return from generator, yielding value */
04554     __pyx_generator->resume_label = 1;
04555     return __pyx_r;
04556     __pyx_L7_resume_from_yield;
04557     __pyx_t_2 = __pyx_cur_scope->__pyx_t_0;
04558     __pyx_cur_scope->__pyx_t_0 = 0;
04559     __Pyx_XGOTREF(__pyx_t_2);
04560     __pyx_t_5 = __pyx_cur_scope->__pyx_t_1;
04561     __pyx_t_6 = __pyx_cur_scope->__pyx_t_2;
04562     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 238, __pyx_L1_error)
04563
04564     /* "PyClical.pyx":237
04565     *      """
04566     *      for idx in range(self.min(), self.max()+1):
04567     *          if idx in self:          # ««««««««
04568     *              yield idx
04569     *
04570     */
04571 }
04572
04573 /* "PyClical.pyx":236
04574 *      -3,4,7,
04575 *      """
04576 *      for idx in range(self.min(), self.max()+1):
04577 *          if idx in self:          # ««««««««
04578 *              yield idx
04579 *
04580     }
04581     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04582     CYTHON_MAYBE_UNUSED_VAR(__pyx_cur_scope);
04583
04584     /* "PyClical.pyx":229
04585     *      return self.instance.getitem(idx)
04586     *
04587     *      def __iter__(self):          # ««««««««
04588     *          """
04589     *          Iterate over the indices of an index_set.
04590     */
04591
04592     /* function exit code */

```

```

04593     PyErr_SetNone(PyExc_StopIteration);
04594     goto __pyx_L0;
04595     __pyx_L1_error:;
04596     __Pyx_XDECREF(__pyx_t_1);
04597     __Pyx_XDECREF(__pyx_t_2);
04598     __Pyx_XDECREF(__pyx_t_3);
04599     __Pyx_XDECREF(__pyx_t_4);
04600     __Pyx_AddTraceback("__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04601     __pyx_L0:;
04602     __Pyx_XDECREF(__pyx_r); __pyx_r = 0;
04603     #if !CYTHON_USE_EXC_INFO_STACK
04604     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04605     #endif
04606     __pyx_generator->resume_label = -1;
04607     __Pyx_Coroutine_clear((PyObject*)__pyx_generator);
04608     __Pyx_RefNannyFinishContext();
04609     return __pyx_r;
04610 }
04611
04612 /* "PyClicl.pyx":240
04613 *         yield idx
04614 *
04615 *     def __invert__(self):          # ««««««««
04616 *         """
04617 *         Set complement: not.
04618 */
04619
04620 /* Python wrapper */
04621 static PyObject *__pyx_pw_8PyClicl_9index_set_18__invert__(PyObject *__pyx_v_self); /*proto*/
04622 static char __pyx_doc_8PyClicl_9index_set_17__invert__[] = "\n        Set complement: not.\n\n\n
>>
print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))\n
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}\n
";
04623 #if CYTHON_COMPILING_IN_CPYTHON
04624 struct wrapperbase __pyx_wrapperbase_8PyClicl_9index_set_17__invert__;
04625 #endif
04626 static PyObject *__pyx_pw_8PyClicl_9index_set_18__invert__(PyObject *__pyx_v_self) {
04627     PyObject *__pyx_r = 0;
04628     __Pyx_RefNannyDeclarations
04629     __Pyx_RefNannySetupContext("__invert__ (wrapper)", 0);
04630     __pyx_r = __pyx_pf_8PyClicl_9index_set_17__invert__(((struct __pyx_obj_8PyClicl_index_set
*)__pyx_v_self));
04631
04632     /* function exit code */
04633     __Pyx_RefNannyFinishContext();
04634     return __pyx_r;
04635 }
04636
04637 static PyObject *__pyx_pf_8PyClicl_9index_set_17__invert__(struct __pyx_obj_8PyClicl_index_set
*__pyx_v_self) {
04638     PyObject *__pyx_r = NULL;
04639     __Pyx_RefNannyDeclarations
04640     PyObject *__pyx_t_1 = NULL;
04641     PyObject *__pyx_t_2 = NULL;
04642     int __pyx_lineno = 0;
04643     const char *__pyx_filename = NULL;
04644     int __pyx_clineno = 0;
04645     __Pyx_RefNannySetupContext("__invert__", 0);
04646
04647     /* "PyClicl.pyx":247
04648 *
04649 *         """
04650 *         return index_set().wrap( self.instance.invert() )          # ««««««««
04651 *
04652 *     def __xor__(lhs, rhs):
04653 */
04654     __Pyx_XDECREF(__pyx_r);
04655     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 247, __pyx_L1_error)
04656     __Pyx_GOTREF(__pyx_t_1);
04657     __pyx_t_2 = __pyx_f_8PyClicl_9index_set_wrap(((struct __pyx_obj_8PyClicl_index_set *)__pyx_t_1),
__pyx_v_self->instance->operator~()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 247, __pyx_L1_error)
04658     __Pyx_GOTREF(__pyx_t_2);
04659     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04660     __pyx_r = __pyx_t_2;
04661     __pyx_t_2 = 0;
04662     goto __pyx_L0;
04663
04664     /* "PyClicl.pyx":240
04665 *         yield idx
04666 *
04667 *     def __invert__(self):          # ««««««««
04668 *         """
04669 *         Set complement: not.

```

```

04670 */
04671
04672 /* function exit code */
04673 __pyx_l1_error++;
04674 __Pyx_XDECREF(__pyx_t_1);
04675 __Pyx_XDECREF(__pyx_t_2);
04676 __Pyx_AddTraceback("PyClical.index_set.__invert__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04677 __pyx_r = NULL;
04678 __pyx_L0;
04679 __Pyx_XGIVEREF(__pyx_r);
04680 __Pyx_RefNannyFinishContext();
04681 return __pyx_r;
04682 }
04683
04684 /* "PyClical.pyx":249
04685 *         return index_set().wrap( self.instance.invert() )
04686 *
04687 *     def __xor__(lhs, rhs):                # ««««««««
04688 *         """
04689 *         Symmetric set difference: exclusive or.
04690 */
04691
04692 /* Python wrapper */
04693 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject
04694 * __pyx_v_rhs); /*proto*/
04695 static char __pyx_doc_8PyClical_9index_set_19__xor__[] = "\n        Symmetric set difference:
04696 exclusive or.\n\n        >> print(index_set({1}) ^ index_set({2}))\n                {1,2}\n        >>
04697 print(index_set({1,2}) ^ index_set({2}))\n                {1}\n        ";
04698 #if CYTHON_COMPILING_IN_CPYTHON
04699 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_19__xor__;
04700 #endif
04701 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
04702 {
04703     PyObject *__pyx_r = 0;
04704     __Pyx_RefNannyDeclarations
04705     __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
04706     __pyx_r = __pyx_pf_8PyClical_9index_set_19__xor__(((PyObject *)__pyx_v_lhs), ((PyObject
04707 *)__pyx_v_rhs));
04708 }
04709
04710 static PyObject *__pyx_pf_8PyClical_9index_set_19__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
04711 {
04712     PyObject *__pyx_r = NULL;
04713     __Pyx_RefNannyDeclarations
04714     PyObject *__pyx_t_1 = NULL;
04715     PyObject *__pyx_t_2 = NULL;
04716     int __pyx_lineno = 0;
04717     const char *__pyx_filename = NULL;
04718     int __pyx_clineno = 0;
04719     __Pyx_RefNannySetupContext("__xor__", 0);
04720
04721 /* "PyClical.pyx":258
04722 *         {1}
04723 *         """
04724 *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )                # ««««««««
04725 *
04726 *     def __ixor__(self, rhs):
04727 */
04728 __Pyx_XDECREF(__pyx_r);
04729 __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
04730 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 258, __pyx_l1_error)
04731 __Pyx_GOTREF(__pyx_t_1);
04732 __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
04733 (__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
04734 (unlikely(!__pyx_t_2)) __PYX_ERR(0, 258, __pyx_l1_error)
04735 __Pyx_GOTREF(__pyx_t_2);
04736 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04737 __pyx_r = __pyx_t_2;
04738 __pyx_t_2 = 0;
04739 goto __pyx_L0;
04740
04741 /* "PyClical.pyx":249
04742 *         return index_set().wrap( self.instance.invert() )
04743 *
04744 *     def __xor__(lhs, rhs):                # ««««««««
04745 *         """
04746 *         Symmetric set difference: exclusive or.
04747 */
04748
04749 /* function exit code */
04750 __pyx_l1_error++;
04751 __Pyx_XDECREF(__pyx_t_1);
04752 __Pyx_XDECREF(__pyx_t_2);

```

```

04748     __Pyx_AddTraceback("PyClical.index_set.__xor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04749     __pyx_r = NULL;
04750     __pyx_L0:;
04751     __Pyx_XGIVEREF(__pyx_r);
04752     __Pyx_RefNannyFinishContext();
04753     return __pyx_r;
04754 }
04755
04756 /* "PyClical.pyx":260
04757 *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04758 *
04759 *     def __ixor__(self, rhs):                # ««««««««
04760 *         """
04761 *         Symmetric set difference: exclusive or.
04762 */
04763
04764 /* Python wrapper */
04765 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_rhs); /*proto*/
04766 static char __pyx_doc_8PyClical_9index_set_21__ixor__[] = "\n        Symmetric set difference:
    exclusive or.\n\n        >> x = index_set({1}); x ^= index_set({2}); print(x)\n        {1,2}\n
    >> x = index_set({1,2}); x ^= index_set({2}); print(x)\n        {1}\n        ";
04767 #if CYTHON_COMPILING_IN_CPYTHON
04768 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_21__ixor__;
04769 #endif
04770 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_rhs) {
04771     PyObject *__pyx_r = 0;
04772     __Pyx_RefNannyDeclarations
04773     __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
04774     __pyx_r = __pyx_pf_8PyClical_9index_set_21__ixor__(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04775
04776     /* function exit code */
04777     __Pyx_RefNannyFinishContext();
04778     return __pyx_r;
04779 }
04780
04781 static PyObject *__pyx_pf_8PyClical_9index_set_21__ixor__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_rhs) {
04782     PyObject *__pyx_r = NULL;
04783     __Pyx_RefNannyDeclarations
04784     PyObject *__pyx_t_1 = NULL;
04785     int __pyx_lineno = 0;
04786     const char *__pyx_filename = NULL;
04787     int __pyx_clineno = 0;
04788     __Pyx_RefNannySetupContext("__ixor__", 0);
04789
04790     /* "PyClical.pyx":269
04791 *         {1}
04792 *         """
04793 *         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )                # ««««««««
04794 *
04795 *     def __and__(lhs, rhs):
04796 */
04797     __Pyx_XDECREF(__pyx_r);
04798     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
    (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 269, __pyx_L1_error)
04799     __Pyx_GOTREF(__pyx_t_1);
04800     __pyx_r = __pyx_t_1;
04801     __pyx_t_1 = 0;
04802     goto __pyx_L0;
04803
04804 /* "PyClical.pyx":260
04805 *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04806 *
04807 *     def __ixor__(self, rhs):                # ««««««««
04808 *         """
04809 *         Symmetric set difference: exclusive or.
04810 */
04811
04812     /* function exit code */
04813     __pyx_L1_error:;
04814     __Pyx_XDECREF(__pyx_t_1);
04815     __Pyx_AddTraceback("PyClical.index_set.__ixor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04816     __pyx_r = NULL;
04817     __pyx_L0:;
04818     __Pyx_XGIVEREF(__pyx_r);
04819     __Pyx_RefNannyFinishContext();
04820     return __pyx_r;
04821 }
04822
04823 /* "PyClical.pyx":271
04824 *         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
04825 *
04826 *     def __and__(lhs, rhs):                # ««««««««

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04827 *          """
04828 *          Set intersection: and.
04829 */
04830
04831 /* Python wrapper */
04832 static PyObject *__pyx_pw_8PyClical_9index_set_24__and__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /*proto*/
04833 static char __pyx_doc_8PyClical_9index_set_23__and__[] = "\n          Set intersection: and.\n\n
    >> print(index_set({1}) & index_set({2}))\n          {}\n          >> print(index_set({1,2}) &
    index_set({2}))\n          {2}\n          ";
04834 #if CYTHON_COMPILING_IN_CPYTHON
04835 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_23__and__;
04836 #endif
04837 static PyObject *__pyx_pf_8PyClical_9index_set_24__and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04838     PyObject *__pyx_r = 0;
04839     __Pyx_RefNannyDeclarations
04840     __Pyx_RefNannySetupContext("__and__ (wrapper)", 0);
04841     __pyx_r = __pyx_pf_8PyClical_9index_set_23__and__((PyObject *)__pyx_v_lhs), ((PyObject
    *)__pyx_v_rhs));
04842
04843     /* function exit code */
04844     __Pyx_RefNannyFinishContext();
04845     return __pyx_r;
04846 }
04847
04848 static PyObject *__pyx_pf_8PyClical_9index_set_23__and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04849     PyObject *__pyx_r = NULL;
04850     __Pyx_RefNannyDeclarations
04851     PyObject *__pyx_t_1 = NULL;
04852     PyObject *__pyx_t_2 = NULL;
04853     int __pyx_lineno = 0;
04854     const char *__pyx_filename = NULL;
04855     int __pyx_clineno = 0;
04856     __Pyx_RefNannySetupContext("__and__", 0);
04857
04858     /* "PyClical.pyx":280
    *          {2}
    *          """
04861     *          return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )          # ««««««««
04862     *
04863     *          def __iand__(self, rhs):
04864     */
04865     __Pyx_XDECREF(__pyx_r);
04866     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 280, __pyx_l1_error)
04867     __Pyx_GOTREF(__pyx_t_1);
04868     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
    (__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) & __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_2)) __PYX_ERR(0, 280, __pyx_l1_error)
04869     __Pyx_GOTREF(__pyx_t_2);
04870     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04871     __pyx_r = __pyx_t_2;
04872     __pyx_t_2 = 0;
04873     goto __pyx_L0;
04874
04875     /* "PyClical.pyx":271
    *          return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
    *
04877     *
04878     *          def __and__(lhs, rhs):          # ««««««««
04879     *          """
04880     *          Set intersection: and.
04881     */
04882
04883     /* function exit code */
04884     __pyx_l1_error:;
04885     __Pyx_XDECREF(__pyx_t_1);
04886     __Pyx_XDECREF(__pyx_t_2);
04887     __Pyx_AddTraceback("PyClical.index_set.__and__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04888     __pyx_r = NULL;
04889     __pyx_L0:;
04890     __Pyx_XGIVEREF(__pyx_r);
04891     __Pyx_RefNannyFinishContext();
04892     return __pyx_r;
04893 }
04894
04895 /* "PyClical.pyx":282
    *          return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04897     *
04898     *          def __iand__(self, rhs):          # ««««««««
04899     *          """
04900     *          Set intersection: and.
04901     */
04902
04903 /* Python wrapper */
04904 static PyObject *__pyx_pw_8PyClical_9index_set_26__iand__(PyObject *__pyx_v_self, PyObject

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    *__pyx_v_rhs); /*proto*/
04905 static char __pyx_doc_8PyClical_9index_set_25_iand__[] = "\n          Set intersection: and.\n\n
    >> x = index_set({1}); x &= index_set({2}); print(x)\n          {} \n          >> x = index_set({1,2}); x
    &= index_set({2}); print(x)\n          {2} \n          ";
04906 #if CYTHON_COMPILING_IN_CPYTHON
04907 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_25_iand__;
04908 #endif
04909 static PyObject *__pyx_pw_8PyClical_9index_set_26_iand__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_rhs) {
04910     PyObject *__pyx_r = 0;
04911     __Pyx_RefNannyDeclarations
04912     __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
04913     __pyx_r = __pyx_pf_8PyClical_9index_set_25_iand__(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04914
04915     /* function exit code */
04916     __Pyx_RefNannyFinishContext();
04917     return __pyx_r;
04918 }
04919
04920 static PyObject *__pyx_pf_8PyClical_9index_set_25_iand__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_rhs) {
04921     PyObject *__pyx_r = NULL;
04922     __Pyx_RefNannyDeclarations
04923     PyObject *__pyx_t_1 = NULL;
04924     int __pyx_lineno = 0;
04925     const char *__pyx_filename = NULL;
04926     int __pyx_clineno = 0;
04927     __Pyx_RefNannySetupContext("__iand__", 0);
04928
04929     /* "PyClical.pyx":291
04930     *         {2}
04931     *         """
04932     *         return self.wrap( self.unwrap() & toIndexSet(rhs) )           # ««««««««
04933     *
04934     *         def __or__(lhs, rhs):
04935     */
04936     __Pyx_XDECREF(__pyx_r);
04937     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
    (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) & __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 291, __pyx_L1_error)
04938     __Pyx_GOTREF(__pyx_t_1);
04939     __pyx_r = __pyx_t_1;
04940     __pyx_t_1 = 0;
04941     goto __pyx_L0;
04942
04943     /* "PyClical.pyx":282
04944     *         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04945     *
04946     *         def __iand__(self, rhs):           # ««««««««
04947     *         """
04948     *         Set intersection: and.
04949     */
04950
04951     /* function exit code */
04952     __pyx_L1_error:;
04953     __Pyx_XDECREF(__pyx_t_1);
04954     __Pyx_AddTraceback("PyClical.index_set.__iand__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04955     __pyx_r = NULL;
04956     __pyx_L0:;
04957     __Pyx_XGIVEREF(__pyx_r);
04958     __Pyx_RefNannyFinishContext();
04959     return __pyx_r;
04960 }
04961
04962 /* "PyClical.pyx":293
04963 *         return self.wrap( self.unwrap() & toIndexSet(rhs) )
04964 *
04965 *         def __or__(lhs, rhs):           # ««««««««
04966 *         """
04967 *         Set union: or.
04968     */
04969
04970 /* Python wrapper */
04971 static PyObject *__pyx_pw_8PyClical_9index_set_28_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /*proto*/
04972 static char __pyx_doc_8PyClical_9index_set_27_or__[] = "\n          Set union: or.\n\n          >>
    print(index_set({1}) | index_set({2}))\n          {1,2} \n          >> print(index_set({1,2}) |
    index_set({2}))\n          {1,2} \n          ";
04973 #if CYTHON_COMPILING_IN_CPYTHON
04974 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_27_or__;
04975 #endif
04976 static PyObject *__pyx_pw_8PyClical_9index_set_28_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04977     PyObject *__pyx_r = 0;
04978     __Pyx_RefNannyDeclarations
04979     __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);

```



```

04980 __pyx_r = __pyx_pf_8PyClical_9index_set_27_or__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
04981
04982 /* function exit code */
04983 __Pyx_RefNannyFinishContext();
04984 return __pyx_r;
04985 }
04986
04987 static PyObject *__pyx_pf_8PyClical_9index_set_27_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
{
04988     PyObject *__pyx_r = NULL;
04989     __Pyx_RefNannyDeclarations
04990     PyObject *__pyx_t_1 = NULL;
04991     PyObject *__pyx_t_2 = NULL;
04992     int __pyx_lineno = 0;
04993     const char *__pyx_filename = NULL;
04994     int __pyx_clineno = 0;
04995     __Pyx_RefNannySetupContext("__or__", 0);
04996
04997     /* "PyClical.pyx":302
04998     *         {1,2}
04999     *         """
05000     *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )           # ««««««««
05001     *
05002     *     def __ior__(self, rhs):
05003     */
05004     __Pyx_XDECREF(__pyx_r);
05005     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 302, __pyx_L1_error)
05006     __Pyx_GOTREF(__pyx_t_1);
05007     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 302, __pyx_L1_error)
05008     __Pyx_GOTREF(__pyx_t_2);
05009     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05010     __pyx_r = __pyx_t_2;
05011     __pyx_t_2 = 0;
05012     goto __pyx_L0;
05013
05014     /* "PyClical.pyx":293
05015     *         return self.wrap( self.unwrap() & toIndexSet(rhs) )
05016     *
05017     *     def __or__(lhs, rhs):           # ««««««««
05018     *         """
05019     *         Set union: or.
05020     */
05021
05022     /* function exit code */
05023     __pyx_L1_error:;
05024     __Pyx_XDECREF(__pyx_t_1);
05025     __Pyx_XDECREF(__pyx_t_2);
05026     __Pyx_AddTraceback("PyClical.index_set.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05027     __pyx_r = NULL;
05028     __pyx_L0:;
05029     __Pyx_XGIVEREF(__pyx_r);
05030     __Pyx_RefNannyFinishContext();
05031     return __pyx_r;
05032 }
05033
05034 /* "PyClical.pyx":304
05035     *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05036     *
05037     *     def __ior__(self, rhs):           # ««««««««
05038     *         """
05039     *         Set union: or.
05040     */
05041
05042 /* Python wrapper */
05043 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
05044 static char __pyx_doc_8PyClical_9index_set_29__ior__[] = "\n        Set union: or.\n\n        >> x =
index_set({1}); x |= index_set({2}); print(x)\n                {1,2}\n        >> x = index_set({1,2}); x |=
index_set({2}); print(x)\n                {1,2}\n        ";
05045 #if CYTHON_COMPILING_IN_CPYTHON
05046 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_29__ior__;
05047 #endif
05048 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
05049     PyObject *__pyx_r = 0;
05050     __Pyx_RefNannyDeclarations
05051     __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
05052     __pyx_r = __pyx_pf_8PyClical_9index_set_29__ior__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05053
05054     /* function exit code */
05055     __Pyx_RefNannyFinishContext();
05056     return __pyx_r;

```

```

05057 }
05058
05059 static PyObject *__pyx_pf_8PyClical_9index_set_29__ior__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs) {
05060     PyObject *__pyx_r = NULL;
05061     __Pyx_RefNannyDeclarations
05062     PyObject *__pyx_t_1 = NULL;
05063     int __pyx_lineno = 0;
05064     const char *__pyx_filename = NULL;
05065     int __pyx_clineno = 0;
05066     __Pyx_RefNannySetupContext("__ior__", 0);
05067
05068     /* "PyClical.pyx":313
05069     *         {1,2}
05070     *         """
05071     *         return self.wrap( self.unwrap() | toIndexSet(rhs) )           # ««««««««
05072     *
05073     *         def count(self):
05074     */
05075     __Pyx_XDECREF(__pyx_r);
05076     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 313, __pyx_L1_error)
05077     __Pyx_GOTREF(__pyx_t_1);
05078     __pyx_r = __pyx_t_1;
05079     __pyx_t_1 = 0;
05080     goto __pyx_L0;
05081
05082     /* "PyClical.pyx":304
05083     *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05084     *
05085     *         def __ior__(self, rhs):           # ««««««««
05086     *         """
05087     *         Set union: or.
05088     */
05089
05090     /* function exit code */
05091     __pyx_L1_error:;
05092     __Pyx_XDECREF(__pyx_t_1);
05093     __Pyx_AddTraceback("PyClical.index_set.__ior__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05094     __pyx_r = NULL;
05095     __pyx_L0:;
05096     __Pyx_XGIVEREF(__pyx_r);
05097     __Pyx_RefNannyFinishContext();
05098     return __pyx_r;
05099 }
05100
05101 /* "PyClical.pyx":315
05102 *         return self.wrap( self.unwrap() | toIndexSet(rhs) )
05103 *
05104 *         def count(self):           # ««««««««
05105 *         """
05106 *         Cardinality: Number of indices included in set.
05107 */
05108
05109 /* Python wrapper */
05110 static PyObject *__pyx_pw_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
05111 static char __pyx_doc_8PyClical_9index_set_31count[] = "\n          Cardinality: Number of indices
included in set.\n\n          >> index_set({-1,1,2}).count()\n          3\n          ";
05112 static PyObject *__pyx_pw_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
05113     PyObject *__pyx_r = 0;
05114     __Pyx_RefNannyDeclarations
05115     __Pyx_RefNannySetupContext("count (wrapper)", 0);
05116     __pyx_r = __pyx_pf_8PyClical_9index_set_31count(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05117
05118     /* function exit code */
05119     __Pyx_RefNannyFinishContext();
05120     return __pyx_r;
05121 }
05122
05123 static PyObject *__pyx_pf_8PyClical_9index_set_31count(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05124     PyObject *__pyx_r = NULL;
05125     __Pyx_RefNannyDeclarations
05126     PyObject *__pyx_t_1 = NULL;
05127     int __pyx_lineno = 0;
05128     const char *__pyx_filename = NULL;
05129     int __pyx_clineno = 0;
05130     __Pyx_RefNannySetupContext("count", 0);
05131
05132     /* "PyClical.pyx":322
05133     *         3
05134     *         """
05135     *         return self.instance.count()           # ««««««««

```

```

05136 *
05137 *     def count_neg(self):
05138 */
05139     __Pyx_XDECREF(__pyx_r);
05140     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count()); if (unlikely(!__pyx_t_1))
05141     __PYX_ERR(0, 322, __pyx_L1_error)
05142     __Pyx_GOTREF(__pyx_t_1);
05143     __pyx_r = __pyx_t_1;
05144     __pyx_t_1 = 0;
05145     goto __pyx_L0;
05146
05147 /* "PyClical.pyx":315
05148 *     return self.wrap( self.unwrap() | toIndexSet(rhs) )
05149 *
05150 *     def count(self):
05151 *         """
05152 *         Cardinality: Number of indices included in set.
05153 */
05154 /* function exit code */
05155 __pyx_L1_error:;
05156 __Pyx_XDECREF(__pyx_t_1);
05157 __Pyx_AddTraceback("PyClical.index_set.count", __pyx_clineno, __pyx_lineno, __pyx_filename);
05158 __pyx_r = NULL;
05159 __pyx_L0:;
05160 __Pyx_XGIVEREF(__pyx_r);
05161 __Pyx_RefNannyFinishContext();
05162 return __pyx_r;
05163 }
05164
05165 /* "PyClical.pyx":324
05166 *     return self.instance.count()
05167 *
05168 *     def count_neg(self):
05169 *         """
05170 *         Number of negative indices included in set.
05171 */
05172
05173 /* Python wrapper */
05174 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
05175 PyObject *unused); /*proto*/
05176 static char __pyx_doc_8PyClical_9index_set_33count_neg[] = "\n        Number of negative indices\n        included in set.\n\n        >> index_set({-1,1,2}).count_neg()\n        1\n        ";
05177 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
05178 PyObject *unused) {
05179     PyObject *__pyx_r = 0;
05180     __Pyx_RefNannyDeclarations
05181     __Pyx_RefNannySetupContext("count_neg (wrapper)", 0);
05182     __pyx_r = __pyx_pf_8PyClical_9index_set_33count_neg(((struct __pyx_obj_8PyClical_index_set
05183 *)__pyx_v_self));
05184
05185 /* function exit code */
05186 __Pyx_RefNannyFinishContext();
05187 return __pyx_r;
05188 }
05189
05190 static PyObject *__pyx_pf_8PyClical_9index_set_33count_neg(struct __pyx_obj_8PyClical_index_set
05191 *__pyx_v_self) {
05192     PyObject *__pyx_r = NULL;
05193     __Pyx_RefNannyDeclarations
05194     PyObject *__pyx_t_1 = NULL;
05195     int __pyx_lineno = 0;
05196     const char *__pyx_filename = NULL;
05197     int __pyx_clineno = 0;
05198     __Pyx_RefNannySetupContext("count_neg", 0);
05199
05200 /* "PyClical.pyx":331
05201 *     1
05202 *     """
05203 *     return self.instance.count_neg()
05204 *         # ««««««««
05205 *
05206 *     def count_pos(self):
05207 */
05208 __Pyx_XDECREF(__pyx_r);
05209 __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_neg()); if (unlikely(!__pyx_t_1))
05210 __PYX_ERR(0, 331, __pyx_L1_error)
05211 __Pyx_GOTREF(__pyx_t_1);
05212 __pyx_r = __pyx_t_1;
05213 __pyx_t_1 = 0;
05214 goto __pyx_L0;
05215
05216 /* "PyClical.pyx":324
05217 *     return self.instance.count()
05218 *
05219 *     def count_neg(self):
05220 *         """
05221 *         Number of negative indices included in set.

```

```

05216 */
05217
05218 /* function exit code */
05219 __pyx_l1_error;;
05220 __Pyx_XDECREF(__pyx_t_1);
05221 __Pyx_AddTraceback("PyClical.index_set.count_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
05222 __pyx_r = NULL;
05223 __pyx_L0;
05224 __Pyx_XGIVEREF(__pyx_r);
05225 __Pyx_RefNannyFinishContext();
05226 return __pyx_r;
05227 }
05228
05229 /* "PyClical.pyx":333
05230 *         return self.instance.count_neg()
05231 *
05232 *         def count_pos(self):           # ««««««««
05233 *             """
05234 *             Number of positive indices included in set.
05235 */
05236
05237 /* Python wrapper */
05238 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
05239 PyObject *unused); /*proto*/
05239 static char __pyx_doc_8PyClical_9index_set_35count_pos[] = "\n        Number of positive indices
05240 included in set.\n\n        >> index_set({-1,1,2}).count_pos()\n                2\n        ";
05240 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
05241 PyObject *unused) {
05242     PyObject *__pyx_r = 0;
05242     __Pyx_RefNannyDeclarations
05243     __Pyx_RefNannySetupContext("count_pos (wrapper)", 0);
05244     __pyx_r = __pyx_pf_8PyClical_9index_set_35count_pos(((struct __pyx_obj_8PyClical_index_set
05245 *)__pyx_v_self));
05246
05247 /* function exit code */
05247 __Pyx_RefNannyFinishContext();
05248 return __pyx_r;
05249 }
05250
05251 static PyObject *__pyx_pf_8PyClical_9index_set_35count_pos(struct __pyx_obj_8PyClical_index_set
05252 *__pyx_v_self) {
05252     PyObject *__pyx_r = NULL;
05253     __Pyx_RefNannyDeclarations
05254     PyObject *__pyx_t_1 = NULL;
05255     int __pyx_lineno = 0;
05256     const char *__pyx_filename = NULL;
05257     int __pyx_clineno = 0;
05258     __Pyx_RefNannySetupContext("count_pos", 0);
05259
05260 /* "PyClical.pyx":340
05261 *         2
05262 *         """
05263 *         return self.instance.count_pos()           # ««««««««
05264 *
05265 *         def min(self):
05266 */
05267     __Pyx_XDECREF(__pyx_r);
05268     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_pos()); if (unlikely(!__pyx_t_1))
05269     __PYX_ERR(0, 340, __pyx_l1_error)
05269     __Pyx_GOTREF(__pyx_t_1);
05270     __pyx_r = __pyx_t_1;
05271     __pyx_t_1 = 0;
05272     goto __pyx_L0;
05273
05274 /* "PyClical.pyx":333
05275 *         return self.instance.count_neg()
05276 *
05277 *         def count_pos(self):           # ««««««««
05278 *             """
05279 *             Number of positive indices included in set.
05280 */
05281
05282 /* function exit code */
05283 __pyx_l1_error;;
05284 __Pyx_XDECREF(__pyx_t_1);
05285 __Pyx_AddTraceback("PyClical.index_set.count_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
05286 __pyx_r = NULL;
05287 __pyx_L0;
05288 __Pyx_XGIVEREF(__pyx_r);
05289 __Pyx_RefNannyFinishContext();
05290 return __pyx_r;
05291 }
05292
05293 /* "PyClical.pyx":342
05294 *         return self.instance.count_pos()
05295 *
05296 *         def min(self):           # ««««««««

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```

05297 *          """
05298 *          Minimum member.
05299 */
05300
05301 /* Python wrapper */
05302 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
05303 static char __pyx_doc_8PyClical_9index_set_37min[] = "\n          Minimum member.\n\n          >>
index_set({-1,1,2}).min()\n          -1\n          ";
05304 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
05305     PyObject *__pyx_r = 0;
05306     __Pyx_RefNannyDeclarations
05307     __Pyx_RefNannySetupContext("min (wrapper)", 0);
05308     __pyx_r = __pyx_pf_8PyClical_9index_set_37min(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05309
05310     /* function exit code */
05311     __Pyx_RefNannyFinishContext();
05312     return __pyx_r;
05313 }
05314
05315 static PyObject *__pyx_pf_8PyClical_9index_set_37min(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05316     PyObject *__pyx_r = NULL;
05317     __Pyx_RefNannyDeclarations
05318     PyObject *__pyx_t_1 = NULL;
05319     int __pyx_lineno = 0;
05320     const char *__pyx_filename = NULL;
05321     int __pyx_clineno = 0;
05322     __Pyx_RefNannySetupContext("min", 0);
05323
05324     /* "PyClical.pyx":349
05325     *          -1
05326     *          """
05327     *          return self.instance.min()
05328     *          # <<<<<<<<
05329     *
05330     *      def max(self):
05331     */
05332     __Pyx_XDECREF(__pyx_r);
05333     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->min()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 349, __pyx_L1_error)
05334     __Pyx_GOTREF(__pyx_t_1);
05335     __pyx_r = __pyx_t_1;
05336     __pyx_t_1 = 0;
05337     goto __pyx_L0;
05338
05339     /* "PyClical.pyx":342
05340     *          return self.instance.count_pos()
05341     *
05342     *      def min(self):
05343     *          # <<<<<<<<
05344     *          """
05345     *          Minimum member.
05346     */
05347     __Pyx_XDECREF(__pyx_r);
05348     __Pyx_GOTREF(__pyx_t_1);
05349     __Pyx_AddTraceback("PyClical.index_set.min", __pyx_clineno, __pyx_lineno, __pyx_filename);
05350     __pyx_r = NULL;
05351     __pyx_L0:;
05352     __Pyx_XGIVEREF(__pyx_r);
05353     __Pyx_RefNannyFinishContext();
05354     return __pyx_r;
05355 }
05356
05357 /* "PyClical.pyx":351
05358 *          return self.instance.min()
05359 *
05360 *      def max(self):
05361 *          # <<<<<<<<
05362 *          """
05363 *          Maximum member.
05364 */
05365 /* Python wrapper */
05366 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
05367 static char __pyx_doc_8PyClical_9index_set_39max[] = "\n          Maximum member.\n\n          >>
index_set({-1,1,2}).max()\n          2\n          ";
05368 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
05369     PyObject *__pyx_r = 0;
05370     __Pyx_RefNannyDeclarations
05371     __Pyx_RefNannySetupContext("max (wrapper)", 0);
05372     __pyx_r = __pyx_pf_8PyClical_9index_set_39max(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05373

```

```

05374  /* function exit code */
05375  __Pyx_RefNannyFinishContext();
05376  return __pyx_r;
05377 }
05378
05379 static PyObject *__pyx_pf_8PyClical_9index_set_39max(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05380  PyObject *__pyx_r = NULL;
05381  __Pyx_RefNannyDeclarations
05382  PyObject *__pyx_t_1 = NULL;
05383  int __pyx_lineno = 0;
05384  const char *__pyx_filename = NULL;
05385  int __pyx_clineno = 0;
05386  __Pyx_RefNannySetupContext("max", 0);
05387
05388  /* "PyClical.pyx":358
05389  *      2
05390  *      """
05391  *      return self.instance.max()          # ««««««««
05392  *
05393  *      def hash_fn(self):
05394  */
05395  __Pyx_XDECREF(__pyx_r);
05396  __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->max()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 358, __pyx_L1_error)
05397  __Pyx_GOTREF(__pyx_t_1);
05398  __pyx_r = __pyx_t_1;
05399  __pyx_t_1 = 0;
05400  goto __pyx_L0;
05401
05402  /* "PyClical.pyx":351
05403  *      return self.instance.min()
05404  *
05405  *      def max(self):          # ««««««««
05406  *      """
05407  *      Maximum member.
05408  */
05409
05410  /* function exit code */
05411  __pyx_L1_error:;
05412  __Pyx_XDECREF(__pyx_t_1);
05413  __Pyx_AddTraceback("PyClical.index_set.max", __pyx_clineno, __pyx_lineno, __pyx_filename);
05414  __pyx_r = NULL;
05415  __pyx_L0:;
05416  __Pyx_XGIVEREF(__pyx_r);
05417  __Pyx_RefNannyFinishContext();
05418  return __pyx_r;
05419 }
05420
05421 /* "PyClical.pyx":360
05422 *      return self.instance.max()
05423 *
05424 *      def hash_fn(self):          # ««««««««
05425 *      """
05426 *      Hash function.
05427 */
05428
05429 /* Python wrapper */
05430 static PyObject *__pyx_pw_8PyClical_9index_set_42hash_fn(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05431 static char __pyx_doc_8PyClical_9index_set_41hash_fn[] = "\n      Hash function.\n      ";
05432 static PyObject *__pyx_pw_8PyClical_9index_set_42hash_fn(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05433  PyObject *__pyx_r = 0;
05434  __Pyx_RefNannyDeclarations
05435  __Pyx_RefNannySetupContext("hash_fn (wrapper)", 0);
05436  __pyx_r = __pyx_pf_8PyClical_9index_set_41hash_fn(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05437
05438  /* function exit code */
05439  __Pyx_RefNannyFinishContext();
05440  return __pyx_r;
05441 }
05442
05443 static PyObject *__pyx_pf_8PyClical_9index_set_41hash_fn(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05444  PyObject *__pyx_r = NULL;
05445  __Pyx_RefNannyDeclarations
05446  PyObject *__pyx_t_1 = NULL;
05447  int __pyx_lineno = 0;
05448  const char *__pyx_filename = NULL;
05449  int __pyx_clineno = 0;
05450  __Pyx_RefNannySetupContext("hash_fn", 0);
05451
05452  /* "PyClical.pyx":364
05453  *      Hash function.
05454  *      """

```

```

05455 *         return self.instance.hash_fn()                # ««««««««
05456 *
05457 *         def sign_of_mult(self, rhs):
05458 */
05459     __Pyx_XDECREF(__pyx_r);
05460     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->hash_fn()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 364, __pyx_L1_error)
05461     __Pyx_GOTREF(__pyx_t_1);
05462     __pyx_r = __pyx_t_1;
05463     __pyx_t_1 = 0;
05464     goto __pyx_L0;
05465
05466 /* "PyClicl.pyx":360
05467 *         return self.instance.max()
05468 *
05469 *         def hash_fn(self):                # ««««««««
05470 *             """
05471 *             Hash function.
05472 */
05473
05474 /* function exit code */
05475 __pyx_L1_error:;
05476 __Pyx_XDECREF(__pyx_t_1);
05477 __Pyx_AddTraceback("PyClicl.index_set.hash_fn", __pyx_clineno, __pyx_lineno, __pyx_filename);
05478 __pyx_r = NULL;
05479 __pyx_L0:;
05480 __Pyx_XGIVEREF(__pyx_r);
05481 __Pyx_RefNannyFinishContext();
05482 return __pyx_r;
05483 }
05484
05485 /* "PyClicl.pyx":366
05486 *         return self.instance.hash_fn()
05487 *
05488 *         def sign_of_mult(self, rhs):                # ««««««««
05489 *             """
05490 *             Sign of geometric product of two Clifford basis elements.
05491 */
05492
05493 /* Python wrapper */
05494 static PyObject *__pyx_pw_8PyClicl_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
05495 static char __pyx_doc_8PyClicl_9index_set_43sign_of_mult[] = "\n          Sign of geometric product of
two Clifford basis elements.\n\n          >> s = index_set({1,2}); t=index_set({-1});
s.sign_of_mult(t)\n          1\n          ";
05496 static PyObject *__pyx_pw_8PyClicl_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
05497     PyObject *__pyx_r = 0;
05498     __Pyx_RefNannyDeclarations
05499     __Pyx_RefNannySetupContext("sign_of_mult (wrapper)", 0);
05500     __pyx_r = __pyx_pf_8PyClicl_9index_set_43sign_of_mult(((struct __pyx_obj_8PyClicl_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05501
05502 /* function exit code */
05503 __Pyx_RefNannyFinishContext();
05504 return __pyx_r;
05505 }
05506
05507 static PyObject *__pyx_pf_8PyClicl_9index_set_43sign_of_mult(struct __pyx_obj_8PyClicl_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs) {
05508     PyObject *__pyx_r = NULL;
05509     __Pyx_RefNannyDeclarations
05510     PyObject *__pyx_t_1 = NULL;
05511     int __pyx_lineno = 0;
05512     const char *__pyx_filename = NULL;
05513     int __pyx_clineno = 0;
05514     __Pyx_RefNannySetupContext("sign_of_mult", 0);
05515
05516 /* "PyClicl.pyx":373
05517 *         1
05518 *         """
05519 *         return self.instance.sign_of_mult(toIndexSet(rhs))                # ««««««««
05520 *
05521 *         def sign_of_square(self):
05522 */
05523     __Pyx_XDECREF(__pyx_r);
05524     __pyx_t_1 =
__Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_mult(__pyx_f_8PyClicl_toIndexSet(__pyx_v_rhs)));
if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 373, __pyx_L1_error)
05525     __Pyx_GOTREF(__pyx_t_1);
05526     __pyx_r = __pyx_t_1;
05527     __pyx_t_1 = 0;
05528     goto __pyx_L0;
05529
05530 /* "PyClicl.pyx":366
05531 *         return self.instance.hash_fn()
05532 *

```

```

05533 *      def sign_of_mult(self, rhs):          # ««««««««
05534 *      """
05535 *          Sign of geometric product of two Clifford basis elements.
05536 */
05537
05538 /* function exit code */
05539 __pyx_l1_error:;
05540 __Pyx_XDECREF(__pyx_t_1);
05541 __Pyx_AddTraceback("PyClicl.index_set.sign_of_mult", __pyx_clineno, __pyx_lineno, __pyx_filename);
05542 __pyx_r = NULL;
05543 __pyx_L0:;
05544 __Pyx_XGIVEREF(__pyx_r);
05545 __Pyx_RefNannyFinishContext();
05546 return __pyx_r;
05547 }
05548
05549 /* "PyClicl.pyx":375
05550 *      return self.instance.sign_of_mult(toIndexSet(rhs))
05551 *
05552 *      def sign_of_square(self):          # ««««««««
05553 *      """
05554 *          Sign of geometric square of a Clifford basis element.
05555 */
05556
05557 /* Python wrapper */
05558 static PyObject* __pyx_pw_8PyClicl_9index_set_46sign_of_square(PyObject* __pyx_v_self, CYTHON_UNUSED
05559 PyObject* __unused); /*proto*/
05559 static char __pyx_doc_8PyClicl_9index_set_45sign_of_square[] = "\n          Sign of geometric square of
05560 a Clifford basis element.\n\n          >> s = index_set({1,2}); s.sign_of_square()\n          -1\n";
05560 static PyObject* __pyx_pw_8PyClicl_9index_set_46sign_of_square(PyObject* __pyx_v_self, CYTHON_UNUSED
05561 PyObject* __unused) {
05561     PyObject* __pyx_r = 0;
05562     __Pyx_RefNannyDeclarations
05563     __Pyx_RefNannySetupContext("sign_of_square (wrapper)", 0);
05564     __pyx_r = __pyx_pf_8PyClicl_9index_set_45sign_of_square((struct __pyx_obj_8PyClicl_index_set
05565 *)__pyx_v_self));
05566
05567 /* function exit code */
05568 __Pyx_RefNannyFinishContext();
05569 return __pyx_r;
05570 }
05571 static PyObject* __pyx_pf_8PyClicl_9index_set_45sign_of_square(struct __pyx_obj_8PyClicl_index_set
05572 *__pyx_v_self) {
05572     PyObject* __pyx_r = NULL;
05573     __Pyx_RefNannyDeclarations
05574     PyObject* __pyx_t_1 = NULL;
05575     int __pyx_lineno = 0;
05576     const char* __pyx_filename = NULL;
05577     int __pyx_clineno = 0;
05578     __Pyx_RefNannySetupContext("sign_of_square", 0);
05579
05580 /* "PyClicl.pyx":382
05581 *      -1
05582 *      """
05583 *      return self.instance.sign_of_square()          # ««««««««
05584 *
05585 *      def __repr__(self):
05586 */
05587     __Pyx_XDECREF(__pyx_r);
05588     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_square()); if
05589 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 382, __pyx_l1_error)
05589     __Pyx_GOTREF(__pyx_t_1);
05590     __pyx_r = __pyx_t_1;
05591     __pyx_t_1 = 0;
05592     goto __pyx_L0;
05593
05594 /* "PyClicl.pyx":375
05595 *      return self.instance.sign_of_mult(toIndexSet(rhs))
05596 *
05597 *      def sign_of_square(self):          # ««««««««
05598 *      """
05599 *          Sign of geometric square of a Clifford basis element.
05600 */
05601
05602 /* function exit code */
05603 __pyx_l1_error:;
05604 __Pyx_XDECREF(__pyx_t_1);
05605 __Pyx_AddTraceback("PyClicl.index_set.sign_of_square", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05606 __pyx_r = NULL;
05607 __pyx_L0:;
05608 __Pyx_XGIVEREF(__pyx_r);
05609 __Pyx_RefNannyFinishContext();
05610 return __pyx_r;
05611 }

```



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```

        index_set({1,2}).__str__()\n                '{1,2}'\n                >>> str(index_set({1,2}))\n                '{1,2}'\n";
05691 #if CYTHON_COMPILING_IN_CPYTHON
05692 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_49__str__;
05693 #endif
05694 static PyObject *__pyx_pw_8PyClical_9index_set_50__str__(PyObject *__pyx_v_self) {
05695     PyObject *__pyx_r = 0;
05696     __Pyx_RefNannyDeclarations
05697     __Pyx_RefNannySetupContext("__str__ (wrapper)", 0);
05698     __pyx_r = __pyx_pf_8PyClical_9index_set_49__str__(((struct __pyx_obj_8PyClical_index_set
05699 *)__pyx_v_self));
05700     /* function exit code */
05701     __Pyx_RefNannyFinishContext();
05702     return __pyx_r;
05703 }
05704
05705 static PyObject *__pyx_pf_8PyClical_9index_set_49__str__(struct __pyx_obj_8PyClical_index_set
05706 *__pyx_v_self) {
05707     PyObject *__pyx_r = NULL;
05708     __Pyx_RefNannyDeclarations
05709     PyObject *__pyx_t_1 = NULL;
05710     int __pyx_lineno = 0;
05711     const char *__pyx_filename = NULL;
05712     int __pyx_clineno = 0;
05713     __Pyx_RefNannySetupContext("__str__", 0);
05714     /* "PyClical.pyx":404
05715     *         '{1,2}'
05716     *         """
05717     *         return index_set_to_str( self.unwrap() ).decode()           # ««««««««
05718     *
05719     * def index_set_hidden_doctests():
05720     */
05721     __Pyx_XDECREF(__pyx_r);
05722     __pyx_t_1 =
05723     __Pyx_decode_cpp_string(index_set_to_str(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self)), 0,
05724     PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 404, __pyx_L1_error)
05725     __Pyx_GOTREF(__pyx_t_1);
05726     __pyx_r = __pyx_t_1;
05727     __pyx_t_1 = 0;
05728     goto __pyx_L0;
05729
05730     /* "PyClical.pyx":395
05731     *         return index_set_to_repr( self.unwrap() ).decode()
05732     *
05733     * def __str__(self):           # ««««««««
05734     *         """
05735     *         The informal string representation of self.
05736     */
05737     __Pyx_XDECREF(__pyx_r);
05738     __Pyx_GOTREF(__pyx_t_1);
05739     __Pyx_AddTraceback("PyClical.index_set.__str__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05740     __pyx_r = NULL;
05741     __pyx_L0:;
05742     __Pyx_XGIVEREF(__pyx_r);
05743     __Pyx_RefNannyFinishContext();
05744     return __pyx_r;
05745 }
05746
05747 /* "(tree fragment)":1
05748 * def __reduce_cython__(self):           # ««««««««
05749 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05750 * def __setstate_cython__(self, __pyx_state):
05751 */
05752
05753 /* Python wrapper */
05754 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05755     CYTHON_UNUSED PyObject *unused); /*proto*/
05756 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05757     CYTHON_UNUSED PyObject *unused) {
05758     PyObject *__pyx_r = 0;
05759     __Pyx_RefNannyDeclarations
05760     __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
05761     __pyx_r = __pyx_pf_8PyClical_9index_set_51__reduce_cython__(((struct __pyx_obj_8PyClical_index_set
05762 *)__pyx_v_self));
05763     /* function exit code */
05764     __Pyx_RefNannyFinishContext();
05765     return __pyx_r;
05766 }
05767
05768 static PyObject *__pyx_pf_8PyClical_9index_set_51__reduce_cython__(CYTHON_UNUSED struct
05769 __pyx_obj_8PyClical_index_set *__pyx_v_self) {
05770     PyObject *__pyx_r = NULL;

```

```

05768 __Pyx_RefNannyDeclarations
05769 PyObject *__pyx_t_1 = NULL;
05770 int __pyx_lineno = 0;
05771 const char *__pyx_filename = NULL;
05772 int __pyx_clineno = 0;
05773 __Pyx_RefNannySetupContext("__reduce_cython__", 0);
05774
05775 /* "(tree fragment)":2
05776 * def __reduce_cython__(self):
05777 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
05778 * def __setstate_cython__(self, __pyx_state):
05779 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05780 */
05781 __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__3, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
05782 __Pyx_GOTREF(__pyx_t_1);
05783 __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05784 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05785 __PYX_ERR(1, 2, __pyx_L1_error)
05786
05787 /* "(tree fragment)":1
05788 * def __reduce_cython__(self): # ««««««««
05789 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05790 * def __setstate_cython__(self, __pyx_state):
05791 */
05792
05793 /* function exit code */
05794 __pyx_L1_error:;
05795 __Pyx_XDECREF(__pyx_t_1);
05796 __Pyx_AddTraceback("PyClical.index_set.__reduce_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05797 __pyx_r = NULL;
05798 __Pyx_XGIVEREF(__pyx_r);
05799 __Pyx_RefNannyFinishContext();
05800 return __pyx_r;
05801 }
05802
05803 /* "(tree fragment)":3
05804 * def __reduce_cython__(self):
05805 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05806 * def __setstate_cython__(self, __pyx_state): # ««««««««
05807 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05808 */
05809
05810 /* Python wrapper */
05811 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state); /*proto*/
05812 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state) {
05813     PyObject *__pyx_r = 0;
05814     __Pyx_RefNannyDeclarations
05815     __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
05816     __pyx_r = __pyx_pf_8PyClical_9index_set_53__setstate_cython__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
05817
05818 /* function exit code */
05819 __Pyx_RefNannyFinishContext();
05820 return __pyx_r;
05821 }
05822
05823 static PyObject *__pyx_pf_8PyClical_9index_set_53__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
05824     PyObject *__pyx_r = NULL;
05825     __Pyx_RefNannyDeclarations
05826     PyObject *__pyx_t_1 = NULL;
05827     int __pyx_lineno = 0;
05828     const char *__pyx_filename = NULL;
05829     int __pyx_clineno = 0;
05830     __Pyx_RefNannySetupContext("__setstate_cython__", 0);
05831
05832 /* "(tree fragment)":4
05833 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05834 * def __setstate_cython__(self, __pyx_state):
05835 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
05836 */
05837 __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__4, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
05838 __Pyx_GOTREF(__pyx_t_1);
05839 __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05840 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05841 __PYX_ERR(1, 4, __pyx_L1_error)
05842
05843 /* "(tree fragment)":3
05844 * def __reduce_cython__(self):
05845 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05846 * def __setstate_cython__(self, __pyx_state): # ««««««««
05847 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")

```

```

05848 */
05849
05850 /* function exit code */
05851 __pyx_ll_error;;
05852 __Pyx_XDECREF(__pyx_t_1);
05853 __Pyx_AddTraceback("PyClical.index_set.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05854 __pyx_r = NULL;
05855 __Pyx_XGIVEREF(__pyx_r);
05856 __Pyx_RefNannyFinishContext();
05857 return __pyx_r;
05858 }
05859
05860 /* "PyClical.pyx":406
05861 *         return index_set_to_str( self.unwrap() ).decode()
05862 *
05863 * def index_set_hidden_doctests(): # ««««««««
05864 *     """
05865 *     Tests for functions that Doctest cannot see.
05866 */
05867
05868 /* Python wrapper */
05869 static PyObject *__pyx_pw_8PyClical_lindex_set_hidden_doctests(PyObject *__pyx_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05870 static char __pyx_doc_8PyClical_index_set_hidden_doctests[] = "\n    Tests for functions that Doctest
cannot see.\n\n    For index_set.__cinit__: Construct index_set.\n\n    >> print(index_set({1}))\n
{1}\n    >> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set(index_set({1,2})))\n
{1,2}\n    >> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set({1,2,1}))\n    {1,2}\n    >>
print(index_set({1,2,1}))\n    {1,2}\n    >> print(index_set("\\"))\n    {}\n    >>
print(index_set("\\{\\")\n    Traceback (most recent call last):\n    ...n    ValueError: Cannot
initialize index_set object from invalid string '\\.\n    >> print(index_set("\\{1\\")\n    Traceback
(most recent call last):\n    ...n    ValueError: Cannot initialize index_set object from invalid
string '\\.\n    >> print(index_set("\\{1,2,100}\\")\n    Traceback (most recent call last):\n
...n    ValueError: Cannot initialize index_set object from invalid string '\\{1,2,100}\\.\n    >>
print(index_set({1,2,100}))\n    Traceback (most recent call last):\n    ...n    IndexError: Cannot
initialize index_set object from invalid {1, 2, 100}.\n    >> print(index_set([1,2]))\n    Traceback
(most recent call last):\n    ...n    TypeError: Cannot initialize index_set object from <class
'list'>.\n\n    For index_set.__richcmp__: Compare two objects of class index_set.\n\n    >>
index_set({1}) == index_set({1})\n    True\n    >> index_set({1}) != index_set({1})\n    False\n    >>
index_set({1}) != index_set({2})\n    True\n    >> index_set({1}) == index_set({2})\n    False\n
>> index_set({1}) < index_set({2})\n    True\n    >> index_set({1}) <= index_set({2})\n    True\n
>> index_set({1}) > index_set({2})\n    False\n    >> index_set({1}) >= index_set({2})\n    False\n
>> None == index_set({1,2})\n    False\n    >> None != index_set({1,2})\n    True\n    >> None <
index_set({1,2})\n    False\n    >> None <= index_set({1,2})\n    False\n    >> None >
index_set({1,2})\n    False\n    >> None >= index_set({1,2})\n    False\n    >> "index_set({1,2}) ==
None\n    False\n    >> index_set({1,2}) != None\n    True\n    >> index_set({1,2}) < None\n
False\n    >> index_set({1,2}) <= None\n    False\n    >> index_set({1,2}) > None\n    False\n    >>
index_set({1,2}) >= None\n    False\n    ";
05871 static PyMethodDef __pyx_mdef_8PyClical_lindex_set_hidden_doctests =
{"index_set_hidden_doctests", (PyCFunction)__pyx_pw_8PyClical_lindex_set_hidden_doctests,
METH_NOARGS, __pyx_doc_8PyClical_index_set_hidden_doctests};
05872 static PyObject *__pyx_pw_8PyClical_lindex_set_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused) {
    PyObject *__pyx_r = 0;
    __Pyx_RefNannyDeclarations
    __Pyx_RefNannySetupContext("index_set_hidden_doctests (wrapper)", 0);
    __pyx_r = __pyx_pf_8PyClical_index_set_hidden_doctests(__pyx_self);
05877
    /* function exit code */
05879 __Pyx_RefNannyFinishContext();
05880 return __pyx_r;
05881 }
05882
05883 static PyObject *__pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject
*__pyx_self) {
05884     PyObject *__pyx_r = NULL;
05885     __Pyx_RefNannyDeclarations
05886     __Pyx_RefNannySetupContext("index_set_hidden_doctests", 0);
05887
05888     /* "PyClical.pyx":490
05889 *         False
05890 *         """
05891 *         return # ««««««««
05892 *
05893 * cpdef inline compare(lhs,rhs):
05894 */
05895     __Pyx_XDECREF(__pyx_r);
05896     __pyx_r = Py_None; __Pyx_INCREF(Py_None);
05897     goto __pyx_L0;
05898
05899     /* "PyClical.pyx":406
05900 *         return index_set_to_str( self.unwrap() ).decode()
05901 *
05902 * def index_set_hidden_doctests(): # ««««««««
05903 *     """
05904 *     Tests for functions that Doctest cannot see.
05905 */

```

```

05906
05907     /* function exit code */
05908     __pyx_L0;;
05909     __Pyx_XGIVEREF(__pyx_r);
05910     __Pyx_RefNannyFinishContext();
05911     return __pyx_r;
05912 }
05913
05914     /* "PyClical.pyx":492
05915 *     return
05916 *
05917 * cpdef inline compare(lhs,rhs):          # ««««««««
05918 *     """
05919 *     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05920 */
05921
05922     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
05923     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_compare(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch) {
05924         PyObject *__pyx_r = NULL;
05925         __Pyx_RefNannyDeclarations
05926         PyObject *__pyx_t_1 = NULL;
05927         int __pyx_lineno = 0;
05928         const char *__pyx_filename = NULL;
05929         int __pyx_clineno = 0;
05930         __Pyx_RefNannySetupContext("compare", 0);
05931
05932     /* "PyClical.pyx":502
05933 *     1
05934 *     """
05935 *     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )          # ««««««««
05936 *
05937 * cpdef inline min_neg(obj):
05938 */
05939         __Pyx_XDECREF(__pyx_r);
05940         __pyx_t_1 = __Pyx_PyInt_From_int(compare(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs),
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 502,
__pyx_L1_error)
05941         __Pyx_GOTREF(__pyx_t_1);
05942         __pyx_r = __pyx_t_1;
05943         __pyx_t_1 = 0;
05944         goto __pyx_L0;
05945
05946     /* "PyClical.pyx":492
05947 *     return
05948 *
05949 * cpdef inline compare(lhs,rhs):          # ««««««««
05950 *     """
05951 *     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05952 */
05953
05954     /* function exit code */
05955     __pyx_L1_error;;
05956     __Pyx_XDECREF(__pyx_t_1);
05957     __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
05958     __pyx_r = 0;
05959     __pyx_L0;;
05960     __Pyx_XGIVEREF(__pyx_r);
05961     __Pyx_RefNannyFinishContext();
05962     return __pyx_r;
05963 }
05964
05965     /* Python wrapper */
05966     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
05967     static char __pyx_doc_8PyClical_2compare[] = "\n    \"lexicographic compare\" eg. {3,4,5} is
less than {3,7,8};\n    -1 if a<b, +1 if a>b, 0 if a==b.\n\n    >>
compare(index_set({1,2}),index_set({-1,3}))\n    -1\n    >>
compare(index_set({-1,4}),index_set({-1,3}))\n    1\n    ";
05968     static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
05969         PyObject *__pyx_v_lhs = 0;
05970         PyObject *__pyx_v_rhs = 0;
05971         int __pyx_lineno = 0;
05972         const char *__pyx_filename = NULL;
05973         int __pyx_clineno = 0;
05974         PyObject *__pyx_r = 0;
05975         __Pyx_RefNannyDeclarations
05976         __Pyx_RefNannySetupContext("compare (wrapper)", 0);
05977         {
05978             static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_lhs,&__pyx_n_s_rhs,0};
05979             PyObject* values[2] = {0,0};
05980             if (unlikely(__pyx_kwds)) {
05981                 Py_ssize_t kw_args;
05982                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
05983                 switch (pos_args) {

```

```

05984         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
05985         CYTHON_FALLTHROUGH;
05986         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
05987         CYTHON_FALLTHROUGH;
05988         case 0: break;
05989         default: goto __pyx_L5_argtuple_error;
05990     }
05991     kw_args = PyDict_Size(__pyx_kwds);
05992     switch (pos_args) {
05993     case 0:
05994         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs) != 0))
kw_args--;
05995         else goto __pyx_L5_argtuple_error;
05996         CYTHON_FALLTHROUGH;
05997         case 1:
05998         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs) != 0))
kw_args--;
05999         else {
06000             __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, 1); __PYX_ERR(0, 492, __pyx_L3_error)
06001         }
06002     }
06003     if (unlikely(kw_args > 0)) {
06004         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
pos_args, "compare") < 0)) __PYX_ERR(0, 492, __pyx_L3_error)
06005     }
06006     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
06007         goto __pyx_L5_argtuple_error;
06008     } else {
06009         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06010         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06011     }
06012     __pyx_v_lhs = values[0];
06013     __pyx_v_rhs = values[1];
06014 }
06015 goto __pyx_L4_argument_unpacking_done;
06016 __pyx_L5_argtuple_error:;
06017 __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
492, __pyx_L3_error)
06018 __pyx_L3_error:;
06019 __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06020 __Pyx_RefNannyFinishContext();
06021 return NULL;
06022 __pyx_L4_argument_unpacking_done:;
06023 __pyx_r = __pyx_pf_8PyClical_2compare(__pyx_self, __pyx_v_lhs, __pyx_v_rhs);
06024
06025 /* function exit code */
06026 __Pyx_RefNannyFinishContext();
06027 return __pyx_r;
06028 }
06029
06030 static PyObject *__pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_lhs, PyObject *__pyx_v_rhs) {
06031     PyObject *__pyx_r = NULL;
06032     __Pyx_RefNannyDeclarations
06033     PyObject *__pyx_t_1 = NULL;
06034     int __pyx_lineno = 0;
06035     const char *__pyx_filename = NULL;
06036     int __pyx_clineno = 0;
06037     __Pyx_RefNannySetupContext("compare", 0);
06038     __Pyx_XDECREF(__pyx_r);
06039     __pyx_t_1 = __pyx_pf_8PyClical_compare(__pyx_v_lhs, __pyx_v_rhs, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 492, __pyx_L1_error)
06040     __Pyx_GOTREF(__pyx_t_1);
06041     __pyx_r = __pyx_t_1;
06042     __pyx_t_1 = 0;
06043     goto __pyx_L0;
06044
06045 /* function exit code */
06046 __pyx_L1_error:;
06047 __Pyx_XDECREF(__pyx_t_1);
06048 __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06049 __pyx_r = NULL;
06050 __pyx_L0:;
06051 __Pyx_XGIVEREF(__pyx_r);
06052 __Pyx_RefNannyFinishContext();
06053 return __pyx_r;
06054 }
06055
06056 /* "PyClical.pyx":504
06057 * return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06058 *
06059 * cpdef inline min_neg(obj): # ««««««««
06060 *     """
06061 *     Minimum negative index, or 0 if none.
06062 */
06063
06064 static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);

```

```

/*proto*/
06065 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_min_neg(PyObject *__pyx_v_obj, CYTHON_UNUSED
int __pyx_skip_dispatch) {
06066     PyObject *__pyx_r = NULL;
06067     __Pyx_RefNannyDeclarations
06068     PyObject *__pyx_t_1 = NULL;
06069     int __pyx_lineno = 0;
06070     const char *__pyx_filename = NULL;
06071     int __pyx_clineno = 0;
06072     __Pyx_RefNannySetupContext("min_neg", 0);
06073
06074     /* "PyClical.pyx":511
06075     *      0
06076     *      """
06077     *      return glucat.min_neg( toIndexSet(obj) )          # ««««««««
06078     *
06079     * cpdef inline max_pos(obj):
06080     */
06081     __Pyx_XDECREF(__pyx_r);
06082     __pyx_t_1 = __Pyx_PyInt_From_int(min_neg(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 511, __pyx_L1_error)
06083     __Pyx_GOTREF(__pyx_t_1);
06084     __pyx_r = __pyx_t_1;
06085     __pyx_t_1 = 0;
06086     goto __pyx_L0;
06087
06088     /* "PyClical.pyx":504
06089     *      return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06090     *
06091     * cpdef inline min_neg(obj):          # ««««««««
06092     *      """
06093     *      Minimum negative index, or 0 if none.
06094     */
06095
06096     /* function exit code */
06097     __pyx_L1_error++;
06098     __Pyx_XDECREF(__pyx_t_1);
06099     __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
06100     __pyx_r = 0;
06101     __pyx_L0:;
06102     __Pyx_XGIVEREF(__pyx_r);
06103     __Pyx_RefNannyFinishContext();
06104     return __pyx_r;
06105 }
06106
06107 /* Python wrapper */
06108 static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);
/*proto*/
06109 static char __pyx_doc_8PyClical_4min_neg[] = "\n      Minimum negative index, or 0 if none.\n\n
>> min_neg(index_set({1,2}))\n      0\n      ";
06110 static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06111     PyObject *__pyx_r = 0;
06112     __Pyx_RefNannyDeclarations
06113     __Pyx_RefNannySetupContext("min_neg (wrapper)", 0);
06114     __pyx_r = __pyx_pf_8PyClical_4min_neg(__pyx_self, ((PyObject *)__pyx_v_obj));
06115
06116     /* function exit code */
06117     __Pyx_RefNannyFinishContext();
06118     return __pyx_r;
06119 }
06120
06121 static PyObject *__pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
06122     PyObject *__pyx_r = NULL;
06123     __Pyx_RefNannyDeclarations
06124     PyObject *__pyx_t_1 = NULL;
06125     int __pyx_lineno = 0;
06126     const char *__pyx_filename = NULL;
06127     int __pyx_clineno = 0;
06128     __Pyx_RefNannySetupContext("min_neg", 0);
06129     __Pyx_XDECREF(__pyx_r);
06130     __pyx_t_1 = __pyx_f_8PyClical_min_neg(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 504, __pyx_L1_error)
06131     __Pyx_GOTREF(__pyx_t_1);
06132     __pyx_r = __pyx_t_1;
06133     __pyx_t_1 = 0;
06134     goto __pyx_L0;
06135
06136     /* function exit code */
06137     __pyx_L1_error++;
06138     __Pyx_XDECREF(__pyx_t_1);
06139     __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
06140     __pyx_r = NULL;
06141     __pyx_L0:;
06142     __Pyx_XGIVEREF(__pyx_r);
06143     __Pyx_RefNannyFinishContext();
06144     return __pyx_r;

```

```

06145     }
06146
06147     /* "PyClical.pyx":513
06148 *     return glucat.min_neg( toIndexSet(obj) )
06149 *
06150 * cpdef inline max_pos(obj):                # ««««««««
06151 *     """
06152 *     Maximum positive index, or 0 if none.
06153 */
06154
06155     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06156 /*proto*/
06157     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_pos(PyObject *__pyx_v_obj, CYTHON_UNUSED
int __pyx_skip_dispatch) {
06158         PyObject *__pyx_r = NULL;
06159         __Pyx_RefNannyDeclarations
06160         PyObject *__pyx_t_1 = NULL;
06161         int __pyx_lineno = 0;
06162         const char *__pyx_filename = NULL;
06163         int __pyx_clineno = 0;
06164         __Pyx_RefNannySetupContext("max_pos", 0);
06165
06166     /* "PyClical.pyx":520
06167 *     2
06168 *     """
06169 *     return glucat.max_pos( toIndexSet(obj) )                # ««««««««
06170 * cdef inline vector[scalar_t] list_to_vector(lst):
06171 */
06172         __Pyx_XDECREF(__pyx_r);
06173         __pyx_t_1 = __Pyx_PyInt_From_int(max_pos(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 520, __pyx_L1_error)
06174         __Pyx_GOTREF(__pyx_t_1);
06175         __pyx_r = __pyx_t_1;
06176         __pyx_t_1 = 0;
06177         goto __pyx_L0;
06178
06179     /* "PyClical.pyx":513
06180 *     return glucat.min_neg( toIndexSet(obj) )
06181 *
06182 * cpdef inline max_pos(obj):                # ««««««««
06183 *     """
06184 *     Maximum positive index, or 0 if none.
06185 */
06186
06187     /* function exit code */
06188     __pyx_L1_error:;
06189     __Pyx_XDECREF(__pyx_t_1);
06190     __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06191     __pyx_r = 0;
06192     __pyx_L0:;
06193     __Pyx_XGIVEREF(__pyx_r);
06194     __Pyx_RefNannyFinishContext();
06195     return __pyx_r;
06196 }
06197
06198 /* Python wrapper */
06199     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06200 /*proto*/
06201     static char __pyx_doc_8PyClical_6max_pos[] = "\n    Maximum positive index, or 0 if none.\n\n
>> max_pos(index_set({1,2}))\n    2\n    ";
06202     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06203         PyObject *__pyx_r = 0;
06204         __Pyx_RefNannyDeclarations
06205         __Pyx_RefNannySetupContext("max_pos (wrapper)", 0);
06206         __pyx_r = __pyx_pf_8PyClical_6max_pos(__pyx_self, ((PyObject *)__pyx_v_obj));
06207
06208     /* function exit code */
06209     __Pyx_RefNannyFinishContext();
06210     return __pyx_r;
06211 }
06212
06213     static PyObject *__pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
06214         PyObject *__pyx_r = NULL;
06215         __Pyx_RefNannyDeclarations
06216         PyObject *__pyx_t_1 = NULL;
06217         int __pyx_lineno = 0;
06218         const char *__pyx_filename = NULL;
06219         int __pyx_clineno = 0;
06220         __Pyx_RefNannySetupContext("max_pos", 0);
06221         __Pyx_XDECREF(__pyx_r);
06222         __pyx_t_1 = __pyx_f_8PyClical_max_pos(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 513, __pyx_L1_error)
06223         __Pyx_GOTREF(__pyx_t_1);
06224         __pyx_r = __pyx_t_1;
06225         __pyx_t_1 = 0;

```



```

06225         goto __pyx_L0;
06226
06227         /* function exit code */
06228         __pyx_L1_error++;
06229         __Pyx_XDECREF(__pyx_t_1);
06230         __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06231         __pyx_r = NULL;
06232         __pyx_L0++;
06233         __Pyx_XGIVEREF(__pyx_r);
06234         __Pyx_RefNannyFinishContext();
06235         return __pyx_r;
06236     }
06237
06238     /* "PyClical.pyx":522
06239  *     return glucat.max_pos( toIndexSet(obj) )
06240  *
06241  * cdef inline vector[scalar_t] list_to_vector(lst):
06242  *     """
06243  *     Create a C++ std::vector[scalar_t] from an iterable Python object.
06244  * */
06245
06246     static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClical_list_to_vector(PyObject
*__pyx_v_lst) {
06247         std::vector<scalar_t> __pyx_v_v;
06248         PyObject *__pyx_v_s = NULL;
06249         std::vector<scalar_t> __pyx_r;
06250         __Pyx_RefNannyDeclarations
06251         PyObject *__pyx_t_1 = NULL;
06252         Py_ssize_t __pyx_t_2;
06253         PyObject *(*__pyx_t_3)(PyObject *);
06254         PyObject *__pyx_t_4 = NULL;
06255         scalar_t __pyx_t_5;
06256         int __pyx_lineno = 0;
06257         const char *__pyx_filename = NULL;
06258         int __pyx_clineno = 0;
06259         __Pyx_RefNannySetupContext("list_to_vector", 0);
06260
06261         /* "PyClical.pyx":527
06262  *     """
06263  *     cdef vector[scalar_t] v
06264  *     for s in lst:
06265  *         v.push_back(<scalar_t>s)
06266  *     return v
06267  * */
06268         if (likely(PyList_CheckExact(__pyx_v_lst)) || PyTuple_CheckExact(__pyx_v_lst)) {
06269             __pyx_t_1 = __pyx_v_lst; __Pyx_INCREF(__pyx_t_1); __pyx_t_2 = 0;
06270             __pyx_t_3 = NULL;
06271         } else {
06272             __pyx_t_2 = -1; __pyx_t_1 = PyObject_GetIter(__pyx_v_lst); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 527, __pyx_L1_error)
06273             __Pyx_GOTREF(__pyx_t_1);
06274             __pyx_t_3 = Py_TYPE(__pyx_t_1)->tp_iternext; if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 527,
__pyx_L1_error)
06275         }
06276         for (;;) {
06277             if (likely(!__pyx_t_3)) {
06278                 if (likely(PyList_CheckExact(__pyx_t_1))) {
06279                     if (__pyx_t_2 >= PyList_GET_SIZE(__pyx_t_1)) break;
06280                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06281                     __pyx_t_4 = PyList_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06282                     __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06283                     #else
06284                     __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06285                     __Pyx_GOTREF(__pyx_t_4);
06286                     #endif
06287                 } else {
06288                     if (__pyx_t_2 >= PyTuple_GET_SIZE(__pyx_t_1)) break;
06289                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06290                     __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06291                     __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06292                     #else
06293                     __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06294                     __Pyx_GOTREF(__pyx_t_4);
06295                     #endif
06296                 }
06297             } else {
06298                 __pyx_t_4 = __pyx_t_3(__pyx_t_1);
06299                 if (unlikely(!__pyx_t_4)) {
06300                     PyObject* exc_type = PyErr_Occurred();
06301                     if (exc_type) {
06302                         if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
PyErr_Clear();
06303                         else __PYX_ERR(0, 527, __pyx_L1_error)
06304                     }
06305                     break;

```

```

06304         }
06305         __Pyx_GOTREF(__pyx_t_4);
06306     }
06307     __Pyx_XDECREF_SET(__pyx_v_s, __pyx_t_4);
06308     __pyx_t_4 = 0;
06309
06310     /* "PyClical.pyx":528
06311     * cdef vector[scalar_t] v
06312     * for s in lst:
06313     *     v.push_back(<scalar_t>s)          # ««««««««
06314     * return v
06315     */
06316
06317     __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_s); if (unlikely((__pyx_t_5 == ((scalar_t)-1))
06318     && PyErr_Occurred())) __PYX_ERR(0, 528, __pyx_L1_error)
06319     try {
06320         __pyx_v_v.push_back(((scalar_t)__pyx_t_5));
06321     } catch (...) {
06322         __Pyx_CppExn2PyErr();
06323         __PYX_ERR(0, 528, __pyx_L1_error)
06324     }
06325
06326     /* "PyClical.pyx":527
06327     * """
06328     * cdef vector[scalar_t] v
06329     * for s in lst:          # ««««««««
06330     *     v.push_back(<scalar_t>s)
06331     * return v
06332     */
06333     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06334
06335     /* "PyClical.pyx":529
06336     * for s in lst:
06337     *     v.push_back(<scalar_t>s)
06338     * return v          # ««««««««
06339     *
06340     * # Forward reference.
06341     */
06342     __pyx_r = __pyx_v_v;
06343     goto __pyx_L0;
06344
06345     /* "PyClical.pyx":522
06346     * return glucat.max_pos( toIndexSet(obj) )
06347     *
06348     * cdef inline vector[scalar_t] list_to_vector(lst):          # ««««««««
06349     * """
06350     * Create a C++ std::vector[scalar_t] from an iterable Python object.
06351     */
06352
06353     /* function exit code */
06354     __pyx_L1_error:;
06355     __Pyx_XDECREF(__pyx_t_1);
06356     __Pyx_XDECREF(__pyx_t_4);
06357     __Pyx_WriteUnraisable("PyClical.list_to_vector", __pyx_clineno, __pyx_lineno,
__pyx_filename, 1, 0);
06358     __Pyx_pretend_to_initialize(&__pyx_r);
06359     __pyx_L0:;
06360     __Pyx_XDECREF(__pyx_v_s);
06361     __Pyx_RefNannyFinishContext();
06362     return __pyx_r;
06363 }
06364
06365     /* "PyClical.pyx":534
06366     * cdef class clifford
06367     *
06368     * cdef inline Clifford toClifford(obj):          # ««««««««
06369     *     return clifford(obj).instance[0]
06370     *
06371     */
06372
06373     static CYTHON_INLINE Clifford __pyx_f_8PyClical_toClifford(PyObject *__pyx_v_obj) {
06374         Clifford __pyx_r;
06375         __Pyx_RefNannyDeclarations
06376         PyObject *__pyx_t_1 = NULL;
06377         int __pyx_lineno = 0;
06378         const char *__pyx_filename = NULL;
06379         int __pyx_clineno = 0;
06380         __Pyx_RefNannySetupContext("toClifford", 0);
06381
06382     /* "PyClical.pyx":535
06383     *
06384     * cdef inline Clifford toClifford(obj):
06385     *     return clifford(obj).instance[0]          # ««««««««
06386     *
06387     * cdef class clifford:
06388     */

```

```

06389     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 535, __pyx_L1_error)
06390     __Pyx_GOTREF(__pyx_t_1);
06391     __pyx_r = (((struct __pyx_obj_8PyClical_clifford *) __pyx_t_1)->instance[0]);
06392     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06393     goto __pyx_L0;
06394
06395     /* "PyClical.pyx":534
06396     * cdef class clifford
06397     *
06398     * cdef inline Clifford toClifford(obj):          # ««««««««
06399     *     return clifford(obj).instance[0]
06400     *
06401     */
06402
06403     /* function exit code */
06404     __pyx_L1_error:;
06405     __Pyx_XDECREF(__pyx_t_1);
06406     __Pyx_WriteUnraisable("PyClical.toClifford", __pyx_clineno, __pyx_lineno, __pyx_filename, 1,
0);
06407     __Pyx_pretend_to_initialize(&__pyx_r);
06408     __pyx_L0:;
06409     __Pyx_RefNannyFinishContext();
06410     return __pyx_r;
06411 }
06412
06413     /* "PyClical.pyx":543
06414     * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06415     *
06416     * cdef inline wrap(clifford self, Clifford other):          # ««««««««
06417     *     """
06418     *     Wrap an instance of the C++ class Clifford.
06419     */
06420
06421     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, Clifford __pyx_v_other) {
06422     PyObject *__pyx_r = NULL;
06423     __Pyx_RefNannyDeclarations
06424     __Pyx_RefNannySetupContext("wrap", 0);
06425
06426     /* "PyClical.pyx":547
06427     * Wrap an instance of the C++ class Clifford.
06428     *
06429     * self.instance[0] = other          # ««««««««
06430     * return self
06431     *
06432     */
06433     (__pyx_v_self->instance[0]) = __pyx_v_other;
06434
06435     /* "PyClical.pyx":548
06436     *
06437     * self.instance[0] = other
06438     * return self          # ««««««««
06439     *
06440     * cdef inline Clifford unwrap(clifford self):
06441     */
06442     __Pyx_XDECREF(__pyx_r);
06443     __Pyx_INCREF(((PyObject *) __pyx_v_self));
06444     __pyx_r = ((PyObject *) __pyx_v_self);
06445     goto __pyx_L0;
06446
06447     /* "PyClical.pyx":543
06448     * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06449     *
06450     * cdef inline wrap(clifford self, Clifford other):          # ««««««««
06451     *     """
06452     *     Wrap an instance of the C++ class Clifford.
06453     */
06454
06455     /* function exit code */
06456     __pyx_L0:;
06457     __Pyx_XGIVEREF(__pyx_r);
06458     __Pyx_RefNannyFinishContext();
06459     return __pyx_r;
06460 }
06461
06462     /* "PyClical.pyx":550
06463     * return self
06464     *
06465     * cdef inline Clifford unwrap(clifford self):          # ««««««««
06466     *     """
06467     *     Return the wrapped C++ Clifford instance.
06468     */
06469
06470     static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
06471     Clifford __pyx_r;

```

```

06472     __Pyx_RefNannyDeclarations
06473     __Pyx_RefNannySetupContext("unwrap", 0);
06474
06475     /* "PyClical.pyx":554
06476     *     Return the wrapped C++ Clifford instance.
06477     *     """
06478     *     return self.instance[0] # ««««««««
06479     *
06480     * cpdef copy(clifford self):
06481     */
06482     __pyx_r = (__pyx_v_self->instance[0]);
06483     goto __pyx_L0;
06484
06485     /* "PyClical.pyx":550
06486     *     return self
06487     *
06488     * cdef inline Clifford unwrap(clifford self): # ««««««««
06489     *     """
06490     *     Return the wrapped C++ Clifford instance.
06491     */
06492
06493     /* function exit code */
06494     __pyx_L0:;
06495     __Pyx_RefNannyFinishContext();
06496     return __pyx_r;
06497 }
06498
06499     /* "PyClical.pyx":556
06500     *     return self.instance[0]
06501     *
06502     * cpdef copy(clifford self): # ««««««««
06503     *     """
06504     *     Copy this clifford object.
06505     */
06506
06507     static PyObject *__pyx_pw_8PyClical_8clifford_lcopy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06508     static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, int __pyx_skip_dispatch) {
06509         PyObject *__pyx_r = NULL;
06510         __Pyx_RefNannyDeclarations
06511         PyObject *__pyx_t_1 = NULL;
06512         PyObject *__pyx_t_2 = NULL;
06513         PyObject *__pyx_t_3 = NULL;
06514         PyObject *__pyx_t_4 = NULL;
06515         int __pyx_lineno = 0;
06516         const char *__pyx_filename = NULL;
06517         int __pyx_clineno = 0;
06518         __Pyx_RefNannySetupContext("copy", 0);
06519         /* Check if called by wrapper */
06520         if (unlikely(__pyx_skip_dispatch)) ;
06521         /* Check if overridden in Python */
06522         else if (unlikely((Py_TYPE(((PyObject *)__pyx_v_self))->tp_dictoffset != 0) ||
(Py_TYPE(((PyObject *)__pyx_v_self))->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
06523             #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06524             static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version
= __PYX_DICT_VERSION_INIT;
06525             if (unlikely(!__Pyx_object_dict_version_matches(((PyObject *)__pyx_v_self),
__pyx_tp_dict_version, __pyx_obj_dict_version))) {
06526                 PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version(((PyObject
*)__pyx_v_self));
06527                 #endif
06528                 __pyx_t_1 = __Pyx_PyObject_GetAttrStr(((PyObject *)__pyx_v_self), __pyx_n_s_copy); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 556, __pyx_L1_error)
06529                 __Pyx_GOTREF(__pyx_t_1);
06530                 if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
(PyCFunction)(void*)__pyx_pw_8PyClical_8clifford_lcopy)) {
06531                     __Pyx_XDECREF(__pyx_r);
06532                     __Pyx_INCREF(__pyx_t_1);
06533                     __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;
06534                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
06535                         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
06536                         if (likely(__pyx_t_4)) {
06537                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
06538                             __Pyx_INCREF(__pyx_t_4);
06539                             __Pyx_INCREF(function);
06540                             __Pyx_DECREF_SET(__pyx_t_3, function);
06541                         }
06542                     }
06543                     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
06544                     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
06545                     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 556, __pyx_L1_error)
06546                     __Pyx_GOTREF(__pyx_t_2);
06547                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
06548                     __pyx_r = __pyx_t_2;
06549                     __pyx_t_2 = 0;

```

```

06550         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06551         goto __pyx_L0;
06552     }
06553     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06554     __pyx_tp_dict_version = __Pyx_get_tp_dict_version(((PyObject *)__pyx_v_self));
06555     __pyx_obj_dict_version = __Pyx_get_object_dict_version(((PyObject *)__pyx_v_self));
06556     if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
06557         __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
06558     }
06559     #endif
06560     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06561     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06562     }
06563     #endif
06564 }
06565
06566 /* "PyClical.pyx":563
06567 * {2}
06568 * """
06569 *     return clifford(self)                # ««««««««
06570 *
06571 * def __cinit__(self, other = 0, ixt = None):
06572 */
06573     __Pyx_XDECREF(__pyx_r);
06574     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
06575 ((PyObject *)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 563, __pyx_L1_error)
06576     __Pyx_GOTREF(__pyx_t_1);
06577     __pyx_r = __pyx_t_1;
06578     __pyx_t_1 = 0;
06579     goto __pyx_L0;
06580
06581 /* "PyClical.pyx":556
06582 *     return self.instance[0]
06583 *
06584 * cpdef copy(clifford self):                # ««««««««
06585 *     """
06586 *     Copy this clifford object.
06587 */
06588
06589 /* function exit code */
06590 __pyx_L1_error:;
06591 __Pyx_XDECREF(__pyx_t_1);
06592 __Pyx_XDECREF(__pyx_t_2);
06593 __Pyx_XDECREF(__pyx_t_3);
06594 __Pyx_XDECREF(__pyx_t_4);
06595 __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06596 __pyx_r = 0;
06597 __pyx_L0:;
06598 __Pyx_XGIVEREF(__pyx_r);
06599 __Pyx_RefNannyFinishContext();
06600 return __pyx_r;
06601 }
06602
06603 /* Python wrapper */
06604 static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06605 static char __pyx_doc_8PyClical_8clifford_copy[] = "\n        Copy this clifford object.\n\n"
">> x=clifford(\"1{2}\"); y=x.copy(); print(y)\n        {2}\n        ";
06606 static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
06607     PyObject *__pyx_r = 0;
06608     __Pyx_RefNannyDeclarations
06609     __Pyx_RefNannySetupContext("copy (wrapper)", 0);
06610     __pyx_r = __pyx_pf_8PyClical_8clifford_copy(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
06611
06612 /* function exit code */
06613 __Pyx_RefNannyFinishContext();
06614 return __pyx_r;
06615 }
06616
06617 static PyObject *__pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
06618     PyObject *__pyx_r = NULL;
06619     __Pyx_RefNannyDeclarations
06620     PyObject *__pyx_t_1 = NULL;
06621     int __pyx_lineno = 0;
06622     const char *__pyx_filename = NULL;
06623     int __pyx_clineno = 0;
06624     __Pyx_RefNannySetupContext("copy", 0);
06625     __Pyx_XDECREF(__pyx_r);
06626     __pyx_t_1 = __pyx_f_8PyClical_8clifford_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 556, __pyx_L1_error)
06627     __Pyx_GOTREF(__pyx_t_1);
06628     __pyx_r = __pyx_t_1;
06629     __pyx_t_1 = 0;
06630     goto __pyx_L0;

```

```

06630
06631     /* function exit code */
06632     __pyx_L1_error++;
06633     __Pyx_XDECREF(__pyx_t_1);
06634     __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06635     __pyx_r = NULL;
06636     __pyx_L0++;
06637     __Pyx_XGIVEREF(__pyx_r);
06638     __Pyx_RefNannyFinishContext();
06639     return __pyx_r;
06640 }
06641
06642     /* "PyClical.pyx":565
06643     *     return clifford(self)
06644     *
06645     * def __cinit__(self, other = 0, ixt = None):          # ««««««««
06646     *     """
06647     *     Construct an object of type clifford.
06648     */
06649
06650     /* Python wrapper */
06651     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject
06652 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
06653     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject
06654 *__pyx_args, PyObject *__pyx_kwds) {
06655     PyObject *__pyx_v_other = 0;
06656     PyObject *__pyx_v_ixt = 0;
06657     int __pyx_lineno = 0;
06658     const char *__pyx_filename = NULL;
06659     int __pyx_clineno = 0;
06660     int __pyx_r;
06661     __Pyx_RefNannyDeclarations
06662     __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
06663     {
06664         static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_other,&__pyx_n_s_ixt,0};
06665         PyObject* values[2] = {0,0};
06666         values[0] = ((PyObject *)__pyx_int_0);
06667         values[1] = ((PyObject *)__Py_None);
06668         if (unlikely(__pyx_kwds)) {
06669             Py_ssize_t kw_args;
06670             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
06671             switch (pos_args) {
06672                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06673                     CYTHON_FALLTHROUGH;
06674                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06675                     CYTHON_FALLTHROUGH;
06676                 case 0: break;
06677                 default: goto __pyx_L5_argtuple_error;
06678             }
06679             kw_args = PyDict_Size(__pyx_kwds);
06680             switch (pos_args) {
06681                 case 0:
06682                     if (kw_args > 0) {
06683                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
06684                         if (value) { values[0] = value; kw_args--; }
06685                     }
06686                     CYTHON_FALLTHROUGH;
06687                 case 1:
06688                     if (kw_args > 0) {
06689                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt);
06690                         if (value) { values[1] = value; kw_args--; }
06691                     }
06692             }
06693             if (unlikely(kw_args > 0)) {
06694                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
06695 pos_args, "__cinit__") < 0)) __PYX_ERR(0, 565, __pyx_L3_error)
06696             }
06697             else {
06698                 switch (PyTuple_GET_SIZE(__pyx_args)) {
06699                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06700                         CYTHON_FALLTHROUGH;
06701                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06702                         CYTHON_FALLTHROUGH;
06703                     case 0: break;
06704                     default: goto __pyx_L5_argtuple_error;
06705                 }
06706             }
06707             __pyx_v_other = values[0];
06708             __pyx_v_ixt = values[1];
06709         }
06710         goto __pyx_L4_argument_unpacking_done;
06711         __pyx_L5_argtuple_error:;
06712         __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
06713 565, __pyx_L3_error)
06714         __pyx_L3_error:;
06715         __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
06716 __pyx_filename);

```

```

06712     __Pyx_RefNannyFinishContext();
06713     return -1;
06714     __pyx_L4_argument_unpacking_done;;
06715     __pyx_r = __pyx_pf_8PyClical_8clifford_2__cinit__((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), __pyx_v_other, __pyx_v_ixt);

06716
06717     /* function exit code */
06718     __Pyx_RefNannyFinishContext();
06719     return __pyx_r;
06720 }
06721
06722 static int __pyx_pf_8PyClical_8clifford_2__cinit__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_other, PyObject *__pyx_v_ixt) {
06723     PyObject *__pyx_v_error_msg_prefix = NULL;
06724     PyObject *__pyx_v_bother = NULL;
06725     PyObject *__pyx_v_err = NULL;
06726     int __pyx_r;
06727     __Pyx_RefNannyDeclarations
06728     int __pyx_t_1;
06729     int __pyx_t_2;
06730     PyObject *__pyx_t_3 = NULL;
06731     PyObject *__pyx_t_4 = NULL;
06732     PyObject *__pyx_t_5 = NULL;
06733     Clifford *__pyx_t_6;
06734     PyObject *__pyx_t_7 = NULL;
06735     PyObject *__pyx_t_8 = NULL;
06736     scalar_t __pyx_t_9;
06737     PyObject *__pyx_t_10 = NULL;
06738     PyObject *__pyx_t_11 = NULL;
06739     PyObject *__pyx_t_12 = NULL;
06740     PyObject *__pyx_t_13 = NULL;
06741     char *__pyx_t_14;
06742     int __pyx_t_15;
06743     PyObject *__pyx_t_16 = NULL;
06744     PyObject *__pyx_t_17 = NULL;
06745     PyObject *__pyx_t_18 = NULL;
06746     int __pyx_t_19;
06747     char const *__pyx_t_20;
06748     PyObject *__pyx_t_21 = NULL;
06749     PyObject *__pyx_t_22 = NULL;
06750     PyObject *__pyx_t_23 = NULL;
06751     int __pyx_lineno = 0;
06752     const char *__pyx_filename = NULL;
06753     int __pyx_clineno = 0;
06754     __Pyx_RefNannySetupContext("__cinit__", 0);
06755
06756     /* "PyClical.pyx":588
06757     * 2{1}+3{2}
06758     * """
06759     * error_msg_prefix = "Cannot initialize clifford object from" # ««««««««
06760     * if ixt is None:
06761     *     try:
06762     */
06763     __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_clifford_objec);
06764     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_clifford_objec;
06765
06766     /* "PyClical.pyx":589
06767     * """
06768     * error_msg_prefix = "Cannot initialize clifford object from"
06769     * if ixt is None: # ««««««««
06770     *     try:
06771     *         if isinstance(other, clifford):
06772     */
06773     __pyx_t_1 = (__pyx_v_ixt == Py_None);
06774     __pyx_t_2 = (__pyx_t_1 != 0);
06775     if (__pyx_t_2) {
06776
06777         /* "PyClical.pyx":590
06778         * error_msg_prefix = "Cannot initialize clifford object from"
06779         * if ixt is None: # ««««««««
06780         *     try:
06781         *         if isinstance(other, clifford):
06782         *             self.instance = new Clifford(<clifford>other).unwrap()
06783         */
06784         {
06785             __Pyx_PyThreadState_declare
06786             __Pyx_PyThreadState_assign
06787             __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
06788             __Pyx_XGOTREF(__pyx_t_3);
06789             __Pyx_XGOTREF(__pyx_t_4);
06790             __Pyx_XGOTREF(__pyx_t_5);
06791             /*try:*/ {
06792
06793                 /* "PyClical.pyx":591
06794                 * if ixt is None:
06795                 *     try:
06796                 *         if isinstance(other, clifford): # ««««««««

```

```

06797 *         self.instance = new Clifford((<clifford>other).unwrap())
06798 *         elif isinstance(other, index_set):
06799 */
06800     __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_clifford);
06801     __pyx_t_1 = (__pyx_t_2 != 0);
06802     if (__pyx_t_1) {
06803
06804         /* "PyClical.pyx":592
06805     try:
06806     *         if isinstance(other, clifford):
06807     *             self.instance = new Clifford((<clifford>other).unwrap())           # ««««««««
06808     *         elif isinstance(other, index_set):
06809     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06810 */
06811         try {
06812             __pyx_t_6 = new Clifford(__pyx_f_8PyClical_8clifford_unwrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_other)));
06813         } catch(...) {
06814             __Pyx_CppExn2PyErr();
06815             __PYX_ERR(0, 592, __pyx_L4_error)
06816         }
06817         __pyx_v_self->instance = __pyx_t_6;
06818
06819         /* "PyClical.pyx":591
06820     *         if ixt is None:
06821     *         try:
06822     *             if isinstance(other, clifford):           # ««««««««
06823     *                 self.instance = new Clifford((<clifford>other).unwrap())
06824     *             elif isinstance(other, index_set):
06825 */
06826         goto __pyx_L10;
06827     }
06828
06829     /* "PyClical.pyx":593
06830     *         if isinstance(other, clifford):
06831     *             self.instance = new Clifford((<clifford>other).unwrap())
06832     *         elif isinstance(other, index_set):           # ««««««««
06833     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06834     *         elif isinstance(other, numbers.Real):
06835 */
06836     __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_index_set);
06837     __pyx_t_2 = (__pyx_t_1 != 0);
06838     if (__pyx_t_2) {
06839
06840         /* "PyClical.pyx":594
06841     *             self.instance = new Clifford((<clifford>other).unwrap())
06842     *         elif isinstance(other, index_set):
06843     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06844     *         # ««««««««
06845     *         elif isinstance(other, numbers.Real):
06846     *             self.instance = new Clifford(<scalar_t>other)
06847 */
06848         try {
06849             __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_other)), ((scalar_t)1.0));
06849         } catch(...) {
06850             __Pyx_CppExn2PyErr();
06851             __PYX_ERR(0, 594, __pyx_L4_error)
06852         }
06853         __pyx_v_self->instance = __pyx_t_6;
06854
06855         /* "PyClical.pyx":593
06856     *         if isinstance(other, clifford):
06857     *             self.instance = new Clifford((<clifford>other).unwrap())
06858     *         elif isinstance(other, index_set):           # ««««««««
06859     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06860     *         elif isinstance(other, numbers.Real):
06861 */
06862         goto __pyx_L10;
06863     }
06864
06865     /* "PyClical.pyx":595
06866     *         elif isinstance(other, index_set):
06867     *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06868     *         elif isinstance(other, numbers.Real):           # ««««««««
06869     *             self.instance = new Clifford(<scalar_t>other)
06870     *         elif isinstance(other, str):
06871 */
06872     __PYX_ERR(0, 595, __pyx_L4_error); if (unlikely(!__pyx_t_7))
06873     __Pyx_GOTREF(__pyx_t_7);
06874     __pyx_t_8 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_Real); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 595, __pyx_L4_error)
06875     __Pyx_GOTREF(__pyx_t_8);
06876     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06877     __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_8); if (unlikely(__pyx_t_2 ==
((int)-1))) __PYX_ERR(0, 595, __pyx_L4_error)

```



```

06878         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
06879         __pyx_t_1 = (__pyx_t_2 != 0);
06880         if (__pyx_t_1) {
06881
06882             /* "PyClical.pyx":596
06883             *         self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06884             *         elif isinstance(other, numbers.Real):
06885             *             self.instance = new Clifford(<scalar_t>other) # ««««««««
06886             *         elif isinstance(other, str):
06887             *             try:
06888             */
06889             __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 596, __pyx_L4_error)
06890             try {
06891                 __pyx_t_6 = new Clifford(((scalar_t)__pyx_t_9));
06892             } catch (...) {
06893                 __Pyx_CppExn2PyErr();
06894                 __PYX_ERR(0, 596, __pyx_L4_error)
06895             }
06896             __pyx_v_self->instance = __pyx_t_6;
06897
06898             /* "PyClical.pyx":595
06899             *         elif isinstance(other, index_set):
06900             *             self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06901             *         elif isinstance(other, numbers.Real): # ««««««««
06902             *             self.instance = new Clifford(<scalar_t>other)
06903             *         elif isinstance(other, str):
06904             */
06905             goto __pyx_L10;
06906         }
06907
06908         /* "PyClical.pyx":597
06909         *         elif isinstance(other, numbers.Real):
06910         *             self.instance = new Clifford(<scalar_t>other)
06911         *         elif isinstance(other, str): # ««««««««
06912         *             try:
06913         *                 bother = other.encode("UTF-8")
06914         */
06915         __pyx_t_1 = PyUnicode_Check(__pyx_v_other);
06916         __pyx_t_2 = (__pyx_t_1 != 0);
06917         if (likely(__pyx_t_2)) {
06918
06919             /* "PyClical.pyx":598
06920             *         self.instance = new Clifford(<scalar_t>other)
06921             *         elif isinstance(other, str):
06922             *             try: # ««««««««
06923             *                 bother = other.encode("UTF-8")
06924             *                 self.instance = new Clifford(<char *>bother)
06925             */
06926             {
06927                 __Pyx_PyThreadState_declare
06928                 __Pyx_PyThreadState_assign
06929                 __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_11, &__pyx_t_12);
06930                 __Pyx_XGOTREF(__pyx_t_10);
06931                 __Pyx_XGOTREF(__pyx_t_11);
06932                 __Pyx_XGOTREF(__pyx_t_12);
06933                 /*try:*/ {
06934
06935                     /* "PyClical.pyx":599
06936                     *         elif isinstance(other, str):
06937                     *             try:
06938                     *                 bother = other.encode("UTF-8") # ««««««««
06939                     *                 self.instance = new Clifford(<char *>bother)
06940                     *                 except RuntimeError:
06941                     */
06942                     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 599, __pyx_L11_error)
06943                     __Pyx_GOTREF(__pyx_t_7);
06944                     __pyx_t_13 = NULL;
06945                     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_7))) {
06946                         __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_7);
06947                         if (likely(__pyx_t_13)) {
06948                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_7);
06949                             __Pyx_INCREF(__pyx_t_13);
06950                             __Pyx_INCREF(function);
06951                             __Pyx_DECREF_SET(__pyx_t_7, function);
06952                         }
06953                     }
06954                     __pyx_t_8 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_7, __pyx_t_13,
__pyx_kp_u_UTF_8) : __Pyx_PyObject_CallOneArg(__pyx_t_7, __pyx_kp_u_UTF_8);
06955                     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06956                     if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 599, __pyx_L11_error)
06957                     __Pyx_GOTREF(__pyx_t_8);
06958                     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06959                     __pyx_v_bother = __pyx_t_8;
06960                     __pyx_t_8 = 0;
06961

```

```

06962         /* "PyClical.pyx":600
06963         *         try:
06964         *             bother = other.encode("UTF-8")
06965         *             self.instance = new Clifford(<char *>bother) # ««««««««
06966         *         except RuntimeError:
06967         *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
06968         */
06969         __pyx_t_14 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if
(unlikely(!__pyx_t_14) && PyErr_Occurred()) __PYX_ERR(0, 600, __pyx_L11_error)
06970         try {
06971             __pyx_t_6 = new Clifford(((char *)__pyx_t_14));
06972         } catch (...) {
06973             __Pyx_CppExn2PyErr();
06974             __PYX_ERR(0, 600, __pyx_L11_error)
06975         }
06976         __pyx_v_self->instance = __pyx_t_6;
06977
06978         /* "PyClical.pyx":598
06979         *         self.instance = new Clifford(<scalar_t>other)
06980         *     elif isinstance(other, str):
06981         *         try: # ««««««««
06982         *             bother = other.encode("UTF-8")
06983         *             self.instance = new Clifford(<char *>bother)
06984         */
06985         }
06986         __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
06987         __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;
06988         __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
06989         goto __pyx_L16_try_end;
06990         __pyx_L11_error:;
06991         __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06992         __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
06993         __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
06994
06995         /* "PyClical.pyx":601
06996         *         bother = other.encode("UTF-8")
06997         *         self.instance = new Clifford(<char *>bother)
06998         *         except RuntimeError: # ««««««««
06999         *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07000         *     else:
07001         */
07002         __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07003         if (__pyx_t_15) {
07004             __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07005             if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0,
601, __pyx_L13_except_error)
07006             __Pyx_GOTREF(__pyx_t_8);
07007             __Pyx_GOTREF(__pyx_t_7);
07008             __Pyx_GOTREF(__pyx_t_13);
07009
07010             /* "PyClical.pyx":602
07011             *         self.instance = new Clifford(<char *>bother)
07012             *         except RuntimeError:
07013             *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07014             *         # ««««««««
07015             *         else:
07016             *             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07017             */
07018             __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix,
__pyx_kp_u_invalid_string); if (unlikely(!__pyx_t_16)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07019             __Pyx_GOTREF(__pyx_t_16);
07020             __pyx_t_17 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07021             __Pyx_GOTREF(__pyx_t_17);
07022             __pyx_t_18 = PyNumber_Add(__pyx_t_16, __pyx_t_17); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07023             __Pyx_GOTREF(__pyx_t_18);
07024             __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07025             __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07026             __pyx_t_17 = PyNumber_Add(__pyx_t_18, __pyx_kp_u_); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07027             __Pyx_GOTREF(__pyx_t_17);
07028             __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07029             __pyx_t_18 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_17); if
(unlikely(!__pyx_t_18)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07030             __Pyx_GOTREF(__pyx_t_18);
07031             __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07032             __Pyx_Raise(__pyx_t_18, 0, 0, 0);
07033             __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07034             __PYX_ERR(0, 602, __pyx_L13_except_error)
07035         }
07036         goto __pyx_L13_except_error;
07037         __pyx_L13_except_error:;
07038
07039         /* "PyClical.pyx":598
07040         *         self.instance = new Clifford(<scalar_t>other)

```

```

07040 *             elif isinstance(other, str):
07041 *                 try:
07042 *                     # ««««««««
07043 *                     bother = other.encode("UTF-8")
07044 *                     self.instance = new Clifford(<char *>bother)
07045 */
07046     __Pyx_XGIVEREF(__pyx_t_10);
07047     __Pyx_XGIVEREF(__pyx_t_11);
07048     __Pyx_XGIVEREF(__pyx_t_12);
07049     __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_11, __pyx_t_12);
07050     goto __pyx_L4_error;
07051     __pyx_L16_try_end;
07052 }
07053
07054 /* "PyClical.pyx":597
07055 *     elif isinstance(other, numbers.Real):
07056 *         self.instance = new Clifford(<scalar_t>other)
07057 *     elif isinstance(other, str):
07058 *         # ««««««««
07059 *         try:
07060 *             bother = other.encode("UTF-8")
07061 */
07062     goto __pyx_L10;
07063 }
07064
07065 /* "PyClical.pyx":604
07066 *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07067 *     else:
07068 *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07069 *                                     #
07070 * ««««««««
07071 * except RuntimeError as err:
07072 *     raise ValueError(error_msg_prefix + " " + str(type(other))
07073 */
07074
07075 /*else*/ {
07076     __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
07077 (unlikely(!__pyx_t_13)) __PYX_ERR(0, 604, __pyx_L4_error)
07078     __Pyx_GOTREF(__pyx_t_13);
07079     __pyx_t_7 = __PyxPyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
07080 *)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07081     __Pyx_GOTREF(__pyx_t_7);
07082     __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_7); if (unlikely(!__pyx_t_8))
07083 __PYX_ERR(0, 604, __pyx_L4_error)
07084     __Pyx_GOTREF(__pyx_t_8);
07085     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07086     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07087     __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_); if
07088 (unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07089     __Pyx_GOTREF(__pyx_t_7);
07090     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07091     __pyx_t_8 = __PyxPyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_7); if
07092 (unlikely(!__pyx_t_8)) __PYX_ERR(0, 604, __pyx_L4_error)
07093     __Pyx_GOTREF(__pyx_t_8);
07094     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07095     __Pyx_Raise(__pyx_t_8, 0, 0, 0);
07096     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07097     __PYX_ERR(0, 604, __pyx_L4_error)
07098 }
07099     __pyx_L10;
07100
07101 /* "PyClical.pyx":590
07102 * error_msg_prefix = "Cannot initialize clifford object from"
07103 * if ixt is None:
07104 *     try:
07105 *         # ««««««««
07106 *         if isinstance(other, clifford):
07107 *             self.instance = new Clifford(<clifford>other).unwrap()
07108 */
07109 }
07110 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07111 __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07112 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07113 goto __pyx_L9_try_end;
07114 __pyx_L4_error;
07115 __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07116 __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
07117 __Pyx_XDECREF(__pyx_t_17); __pyx_t_17 = 0;
07118 __Pyx_XDECREF(__pyx_t_18); __pyx_t_18 = 0;
07119 __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
07120 __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
07121
07122 /* "PyClical.pyx":605
07123 *     else:
07124 *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07125 * except RuntimeError as err:
07126 *                                     # ««««««««
07127 *         raise ValueError(error_msg_prefix + " " + str(type(other))
07128 *                             + " value " + repr(other) + ":",
07129 */
07130
07131 __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07132 if (__pyx_t_15) {
07133     __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,

```

```

__pyx_filename);
07121     if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0, 605,
__pyx_L6_except_error)
07122     __Pyx_GOTREF(__pyx_t_8);
07123     __Pyx_GOTREF(__pyx_t_7);
07124     __Pyx_GOTREF(__pyx_t_13);
07125     __Pyx_INCREF(__pyx_t_7);
07126     __pyx_v_err = __pyx_t_7;
07127     /*try:*/ {
07128
07129         /* "PyClical.pyx":606
07130         *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07131         *     except RuntimeError as err:
07132         *         raise ValueError(error_msg_prefix + " " + str(type(other))          # ««««««««
07133         *             + " value " + repr(other) + ":"
07134         *             + "\n\t" + str(err))
07135         */
07136         __pyx_t_18 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_18)) __PYX_ERR(0, 606, __pyx_L24_error)
07137         __Pyx_GOTREF(__pyx_t_18);
07138         __pyx_t_17 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), (PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_17)) __PYX_ERR(0, 606, __pyx_L24_error)
07139         __Pyx_GOTREF(__pyx_t_17);
07140         __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_t_18, __pyx_t_17); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 606, __pyx_L24_error)
07141         __Pyx_GOTREF(__pyx_t_16);
07142         __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07143         __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07144
07145         /* "PyClical.pyx":607
07146         *     except RuntimeError as err:
07147         *         raise ValueError(error_msg_prefix + " " + str(type(other))
07148         *             + " value " + repr(other) + ":"
07149         *             + "\n\t" + str(err))
07150         *     elif isinstance(ixt, index_set):
07151         */
07152         __pyx_t_17 = __Pyx_PyUnicode_Concat(__pyx_t_16, __pyx_kp_u_value); if
(unlikely(!__pyx_t_17)) __PYX_ERR(0, 607, __pyx_L24_error)
07153         __Pyx_GOTREF(__pyx_t_17);
07154         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07155         __pyx_t_16 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
607, __pyx_L24_error)
07156         __Pyx_GOTREF(__pyx_t_16);
07157         __pyx_t_18 = PyNumber_Add(__pyx_t_17, __pyx_t_16); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 607, __pyx_L24_error)
07158         __Pyx_GOTREF(__pyx_t_18);
07159         __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07160         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07161         __pyx_t_16 = PyNumber_Add(__pyx_t_18, __pyx_kp_u_5); if (unlikely(!__pyx_t_16))
__PYX_ERR(0, 607, __pyx_L24_error)
07162         __Pyx_GOTREF(__pyx_t_16);
07163         __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07164
07165         /* "PyClical.pyx":608
07166         *         raise ValueError(error_msg_prefix + " " + str(type(other))
07167         *             + " value " + repr(other) + ":"
07168         *             + "\n\t" + str(err))
07169         *     elif isinstance(ixt, index_set):
07170         *         if isinstance(other, numbers.Real):
07171         */
07172         __pyx_t_18 = PyNumber_Add(__pyx_t_16, __pyx_kp_u_6); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 608, __pyx_L24_error)
07173         __Pyx_GOTREF(__pyx_t_18);
07174         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07175         __pyx_t_16 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type),
__pyx_v_err); if (unlikely(!__pyx_t_16)) __PYX_ERR(0, 608, __pyx_L24_error)
07176         __Pyx_GOTREF(__pyx_t_16);
07177         __pyx_t_17 = PyNumber_Add(__pyx_t_18, __pyx_t_16); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 608, __pyx_L24_error)
07178         __Pyx_GOTREF(__pyx_t_17);
07179         __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07180         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07181
07182         /* "PyClical.pyx":606
07183         *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07184         *     except RuntimeError as err:
07185         *         raise ValueError(error_msg_prefix + " " + str(type(other))          # ««««««««
07186         *             + " value " + repr(other) + ":"
07187         *             + "\n\t" + str(err))
07188         */
07189         __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_17); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 606, __pyx_L24_error)
07190         __Pyx_GOTREF(__pyx_t_16);
07191         __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07192         __Pyx_Raise(__pyx_t_16, 0, 0, 0);
07193         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;

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07194         __PYX_ERR(0, 606, __pyx_L24_error)
07195     }
07196
07197     /* "PyClical.pyx":605
07198     else:
07199         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07200     except RuntimeError as err:
07201         raise ValueError(error_msg_prefix + " " + str(type(other))
07202         + " value " + repr(other) + ":")
07203 */
07204     /*finally:*/ {
07205         __pyx_L24_error;;
07206         /*exception exit:*/{
07207             __Pyx_PyThreadState_declare
07208             __Pyx_PyThreadState_assign
07209             __pyx_t_12 = 0; __pyx_t_11 = 0; __pyx_t_10 = 0; __pyx_t_21 = 0; __pyx_t_22 = 0;
07210             __pyx_t_23 = 0;
07211             __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
07212             __Pyx_XDECREF(__pyx_t_17); __pyx_t_17 = 0;
07213             __Pyx_XDECREF(__pyx_t_18); __pyx_t_18 = 0;
07214             if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_21, &__pyx_t_22,
07215             &__pyx_t_23);
07216             if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_12,
07217             &__pyx_t_11, &__pyx_t_10) < 0)) __Pyx_ErrFetch(&__pyx_t_12, &__pyx_t_11, &__pyx_t_10);
07218             __Pyx_XGOTREF(__pyx_t_12);
07219             __Pyx_XGOTREF(__pyx_t_11);
07220             __Pyx_XGOTREF(__pyx_t_10);
07221             __Pyx_XGOTREF(__pyx_t_21);
07222             __Pyx_XGOTREF(__pyx_t_22);
07223             __Pyx_XGOTREF(__pyx_t_23);
07224             __pyx_t_15 = __pyx_lineno; __pyx_t_19 = __pyx_clineno; __pyx_t_20 =
07225             __pyx_filename;
07226             {
07227                 __Pyx_DECREF(__pyx_v_err);
07228                 __pyx_v_err = NULL;
07229             }
07230             if (PY_MAJOR_VERSION >= 3) {
07231                 __Pyx_XGIVEREF(__pyx_t_21);
07232                 __Pyx_XGIVEREF(__pyx_t_22);
07233                 __Pyx_XGIVEREF(__pyx_t_23);
07234                 __Pyx_ExceptionReset(__pyx_t_21, __pyx_t_22, __pyx_t_23);
07235             }
07236             __Pyx_XGIVEREF(__pyx_t_12);
07237             __Pyx_XGIVEREF(__pyx_t_11);
07238             __Pyx_XGIVEREF(__pyx_t_10);
07239             __Pyx_ErrRestore(__pyx_t_12, __pyx_t_11, __pyx_t_10);
07240             __pyx_t_12 = 0; __pyx_t_11 = 0; __pyx_t_10 = 0; __pyx_t_21 = 0; __pyx_t_22 = 0;
07241             __pyx_t_23 = 0;
07242             __pyx_lineno = __pyx_t_15; __pyx_clineno = __pyx_t_19; __pyx_filename =
07243             __pyx_t_20;
07244             goto __pyx_L6_except_error;
07245         }
07246     }
07247     goto __pyx_L6_except_error;
07248     __pyx_L6_except_error;;
07249
07250     /* "PyClical.pyx":590
07251     error_msg_prefix = "Cannot initialize clifford object from"
07252     if ixt is None:
07253         try:
07254             # ««««««««
07255             if isinstance(other, clifford):
07256                 self.instance = new Clifford(<<clifford>other).unwrap()
07257 */
07258         __Pyx_XGIVEREF(__pyx_t_3);
07259         __Pyx_XGIVEREF(__pyx_t_4);
07260         __Pyx_XGIVEREF(__pyx_t_5);
07261         __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07262         goto __pyx_L1_error;
07263         __pyx_L9_try_end;;
07264     }
07265
07266     /* "PyClical.pyx":589
07267     """
07268     error_msg_prefix = "Cannot initialize clifford object from"
07269     if ixt is None:
07270         # ««««««««
07271         try:
07272             if isinstance(other, clifford):
07273 */
07274         goto __pyx_L3;
07275     }
07276
07277     /* "PyClical.pyx":609
07278     + " value " + repr(other) + ":"
07279     + "\n\t" + str(err))
07280     # ««««««««
07281     elif isinstance(ixt, index_set):
07282     if isinstance(other, numbers.Real):

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07275 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07276 */
07277 __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClical_index_set);
07278 __pyx_t_1 = (__pyx_t_2 != 0);
07279 if (likely(__pyx_t_1)) {
07280
07281     /* "PyClical.pyx":610
07282 *                                     + "\n\t" + str(err))
07283 *     elif isinstance(ixt, index_set):
07284 *         if isinstance(other, numbers.Real):             # ««««««««
07285 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07286 *         elif isinstance(other, collections.abc.Sequence):
07287 */
07288     __Pyx_GetModuleGlobalName(__pyx_t_13, __pyx_n_s_numbers); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 610, __pyx_L1_error)
07289     __Pyx_GOTREF(__pyx_t_13);
07290     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Real); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 610, __pyx_L1_error)
07291     __Pyx_GOTREF(__pyx_t_7);
07292     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07293     __pyx_t_1 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_1 ==
((int)-1))) __PYX_ERR(0, 610, __pyx_L1_error)
07294     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07295     __pyx_t_2 = (__pyx_t_1 != 0);
07296     if (__pyx_t_2) {
07297
07298     /* "PyClical.pyx":611
07299 *     elif isinstance(ixt, index_set):
07300 *         if isinstance(other, numbers.Real):
07301 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07302 * # ««««««««
07303 *     elif isinstance(other, collections.abc.Sequence):
07304 *         self.instance = new Clifford(list_to_vector(other), <index_set>ixt).unwrap())
07305 */
07306     __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 611, __pyx_L1_error)
07307     try {
07308         __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_ixt)), ((scalar_t)__pyx_t_9));
07309     } catch (...) {
07310         __Pyx_CppExn2PyErr();
07311         __PYX_ERR(0, 611, __pyx_L1_error)
07312     }
07313     __pyx_v_self->instance = __pyx_t_6;
07314
07315     /* "PyClical.pyx":610
07316 *                                     + "\n\t" + str(err))
07317 *     elif isinstance(ixt, index_set):
07318 *         if isinstance(other, numbers.Real):             # ««««««««
07319 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07320 *         elif isinstance(other, collections.abc.Sequence):
07321 */
07322     goto __pyx_L30;
07323
07324     /* "PyClical.pyx":612
07325 *         if isinstance(other, numbers.Real):
07326 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07327 *         elif isinstance(other, collections.abc.Sequence):             # ««««««««
07328 *             self.instance = new Clifford(list_to_vector(other), <index_set>ixt).unwrap())
07329 *         else:
07330 */
07331     __Pyx_GetModuleGlobalName(__pyx_t_7, __pyx_n_s_collections); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 612, __pyx_L1_error)
07332     __Pyx_GOTREF(__pyx_t_7);
07333     __pyx_t_13 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_abc); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 612, __pyx_L1_error)
07334     __Pyx_GOTREF(__pyx_t_13);
07335     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07336     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Sequence); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 612, __pyx_L1_error)
07337     __Pyx_GOTREF(__pyx_t_7);
07338     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07339     __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_2 ==
((int)-1))) __PYX_ERR(0, 612, __pyx_L1_error)
07340     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07341     __pyx_t_1 = (__pyx_t_2 != 0);
07342     if (likely(__pyx_t_1)) {
07343
07344     /* "PyClical.pyx":613
07345 *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07346 *         elif isinstance(other, collections.abc.Sequence):
07347 *             self.instance = new Clifford(list_to_vector(other), <index_set>ixt).unwrap())
07348 * # ««««««««
07349 *         else:
07350 *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07351 */

```

```

07351         try {
07352             __pyx_t_6 = new Clifford(__pyx_f_8PyClical_list_to_vector(__pyx_v_other),
__pyx_f_8PyClical_9index_set_unwrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_ixt)));
07353         } catch (...) {
07354             __Pyx_CppExn2PyErr();
07355             __PYX_ERR(0, 613, __pyx_L1_error)
07356         }
07357         __pyx_v_self->instance = __pyx_t_6;
07358
07359         /* "PyClical.pyx":612
07360         *         if isinstance(other, numbers.Real):
07361         *             self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07362         *         elif isinstance(other, collections.abc.Sequence):
07363         *             self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07364         *         else:
07365         */
07366         goto __pyx_L30;
07367     }
07368
07369     /* "PyClical.pyx":615
07370     *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07371     *         else:
07372     *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07373     *                                     + ", " + repr(ixt) + ").")
07374     *         else:
07375     */
07376     /*else*/ {
07377         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
07378         __Pyx_GOTREF(__pyx_t_7);
07379         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), (PyObject
*)Py_TYPE(__pyx_v_other)); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 615, __pyx_L1_error)
07380         __Pyx_GOTREF(__pyx_t_13);
07381         __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_13); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 615, __pyx_L1_error)
07382         __Pyx_GOTREF(__pyx_t_8);
07383         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07384         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07385
07386         /* "PyClical.pyx":616
07387         *         else:
07388         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07389         *                                     + ", " + repr(ixt) + ").")
07390         *         else:
07391         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07392         */
07393         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_8); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 616, __pyx_L1_error)
07394         __Pyx_GOTREF(__pyx_t_13);
07395         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07396         __pyx_t_8 = PyObject_Repr(__pyx_v_ixt); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 616,
__pyx_L1_error)
07397         __Pyx_GOTREF(__pyx_t_8);
07398         __pyx_t_7 = PyNumber_Add(__pyx_t_13, __pyx_t_8); if (unlikely(!__pyx_t_7)) __PYX_ERR(0,
616, __pyx_L1_error)
07399         __Pyx_GOTREF(__pyx_t_7);
07400         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07401         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07402         __pyx_t_8 = PyNumber_Add(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 616, __pyx_L1_error)
07403         __Pyx_GOTREF(__pyx_t_8);
07404         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07405
07406         /* "PyClical.pyx":615
07407         *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07408         *         else:
07409         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07410         *                                     + ", " + repr(ixt) + ").")
07411         *         else:
07412         */
07413         __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_8); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
07414         __Pyx_GOTREF(__pyx_t_7);
07415         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07416         __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07417         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07418         __PYX_ERR(0, 615, __pyx_L1_error)
07419     }
07420     __pyx_L30:;
07421
07422     /* "PyClical.pyx":609
07423     *                                     + " value " + repr(other) + ":"
07424     *                                     + "\n\t" + str(err))
07425     *         elif isinstance(ixt, index_set):
07426     *             if isinstance(other, numbers.Real):
07427     *                 self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07428     */

```

```

07429         goto __pyx_L3;
07430     }
07431
07432     /* "PyClical.pyx":618
07433     *                                     + ", " + repr(ixt) + ").")
07434     *     else:
07435     *         raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07436     *         + ", " + str(type(ixt)) + ").")
07437     *
07438     */
07439     /*else*/ {
07440         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07441         __Pyx_GOTREF(__pyx_t_7);
07442         __pyx_t_8 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 618, __pyx_L1_error)
07443         __Pyx_GOTREF(__pyx_t_8);
07444         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_8); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 618, __pyx_L1_error)
07445         __Pyx_GOTREF(__pyx_t_13);
07446         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07447         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07448
07449         /* "PyClical.pyx":619
07450     *     else:
07451     *         raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07452     *         + ", " + str(type(ixt)) + ").")
07453     *
07454     *     def __dealloc__(self):
07455     */
07456         __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_8); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 619, __pyx_L1_error)
07457         __Pyx_GOTREF(__pyx_t_8);
07458         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07459         __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_ixt))); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 619, __pyx_L1_error)
07460         __Pyx_GOTREF(__pyx_t_13);
07461         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_t_13); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 619, __pyx_L1_error)
07462         __Pyx_GOTREF(__pyx_t_7);
07463         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07464         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07465         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 619, __pyx_L1_error)
07466         __Pyx_GOTREF(__pyx_t_13);
07467         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07468
07469         /* "PyClical.pyx":618
07470     *                                     + ", " + repr(ixt) + ").")
07471     *     else:
07472     *         raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07473     *         + ", " + str(type(ixt)) + ").")
07474     *
07475     */
07476         __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_13); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07477         __Pyx_GOTREF(__pyx_t_7);
07478         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07479         __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07480         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07481         __PYX_ERR(0, 618, __pyx_L1_error)
07482     }
07483     __pyx_L3:;
07484
07485     /* "PyClical.pyx":565
07486     *     return clifford(self)
07487     *
07488     *     def __cinit__(self, other = 0, ixt = None):          # ««««««««
07489     *         """
07490     *         Construct an object of type clifford.
07491     */
07492
07493     /* function exit code */
07494     __pyx_r = 0;
07495     goto __pyx_L0;
07496     __pyx_L1_error:;
07497     __Pyx_XDECREF(__pyx_t_7);
07498     __Pyx_XDECREF(__pyx_t_8);
07499     __Pyx_XDECREF(__pyx_t_13);
07500     __Pyx_XDECREF(__pyx_t_16);
07501     __Pyx_XDECREF(__pyx_t_17);
07502     __Pyx_XDECREF(__pyx_t_18);
07503     __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07504     __pyx_r = -1;
07505     __pyx_L0:;
07506     __Pyx_XDECREF(__pyx_v_error_msg_prefix);

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07586         int __pyx_lineno = 0;
07587         const char *__pyx_filename = NULL;
07588         int __pyx_clineno = 0;
07589         __Pyx_RefNannySetupContext("__contains__", 0);
07590
07591         /* "PyClicl.pyx":636
07592  *         TypeError: Not applicable.
07593  *         """
07594  *         raise TypeError("Not applicable.")          # ««««««««
07595  *
07596  *     def __iter__(self):
07597  */
07598         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 636, __pyx_L1_error)
07599         __Pyx_GOTREF(__pyx_t_1);
07600         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
07601         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
07602         __PYX_ERR(0, 636, __pyx_L1_error)
07603
07604         /* "PyClicl.pyx":627
07605  *         del self.instance
07606  *
07607  *     def __contains__(self, x):          # ««««««««
07608  *         """
07609  *         Not applicable.
07610  */
07611
07612         /* function exit code */
07613         __pyx_L1_error:;
07614         __Pyx_XDECREF(__pyx_t_1);
07615         __Pyx_AddTraceback("PyClicl.clifford.__contains__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07616         __pyx_r = -1;
07617         __Pyx_RefNannyFinishContext();
07618         return __pyx_r;
07619     }
07620
07621     /* "PyClicl.pyx":638
07622  *         raise TypeError("Not applicable.")
07623  *
07624  *     def __iter__(self):          # ««««««««
07625  *         """
07626  *         Not applicable.
07627  */
07628
07629     /* Python wrapper */
07630     static PyObject *__pyx_pw_8PyClicl_8clifford_9__iter__(PyObject *__pyx_v_self); /*proto*/
07631     static char __pyx_doc_8PyClicl_8clifford_8__iter__[] = "\n        Not applicable.\n\n
>> for a in clifford(index_set({-3,4,7})):print(a, end=\",\")\n        Traceback (most recent call
last):\n        ..\n        TypeError: Not applicable.\n        ";
07632     #if CYTHON_COMPILING_IN_CPYTHON
07633     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_8__iter__;
07634     #endif
07635     static PyObject *__pyx_pw_8PyClicl_8clifford_9__iter__(PyObject *__pyx_v_self) {
07636         PyObject *__pyx_r = 0;
07637         __Pyx_RefNannyDeclarations
07638         __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
07639         __pyx_r = __pyx_pf_8PyClicl_8clifford_8__iter__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
07640
07641         /* function exit code */
07642         __Pyx_RefNannyFinishContext();
07643         return __pyx_r;
07644     }
07645
07646     static PyObject *__pyx_pf_8PyClicl_8clifford_8__iter__(CYTHON_UNUSED struct
__pyx_obj_8PyClicl_clifford *__pyx_v_self) {
07647         PyObject *__pyx_r = NULL;
07648         __Pyx_RefNannyDeclarations
07649         PyObject *__pyx_t_1 = NULL;
07650         int __pyx_lineno = 0;
07651         const char *__pyx_filename = NULL;
07652         int __pyx_clineno = 0;
07653         __Pyx_RefNannySetupContext("__iter__", 0);
07654
07655         /* "PyClicl.pyx":647
07656  *         TypeError: Not applicable.
07657  *         """
07658  *         raise TypeError("Not applicable.")          # ««««««««
07659  *
07660  *     def reframe(self, ixt):
07661  */
07662         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 647, __pyx_L1_error)
07663         __Pyx_GOTREF(__pyx_t_1);
07664         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
07665         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;

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07666         __PYX_ERR(0, 647, __pyx_L1_error)
07667
07668         /* "PyClical.pyx":638
07669  *         raise TypeError("Not applicable.")
07670  *
07671  *     def __iter__(self):                # ««««««««
07672  *         """
07673  *         Not applicable.
07674  */
07675
07676         /* function exit code */
07677         __pyx_L1_error++;
07678         __Pyx_XDECREF(__pyx_t_1);
07679         __Pyx_AddTraceback("PyClical.clifford.__iter__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07680         __pyx_r = NULL;
07681         __Pyx_XGIVEREF(__pyx_r);
07682         __Pyx_RefNannyFinishContext();
07683         return __pyx_r;
07684     }
07685
07686     /* "PyClical.pyx":649
07687  *         raise TypeError("Not applicable.")
07688  *
07689  *     def reframe(self, ixt):            # ««««««««
07690  *         """
07691  *         Put self into a larger frame, containing the union of self.frame() and index set ixt.
07692  */
07693
07694         /* Python wrapper */
07695         static PyObject *__pyx_pw_8PyClical_8clifford_11reframe(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt); /*proto*/
07696         static char __pyx_doc_8PyClical_8clifford_10reframe[] = "\n        Put self into a larger
frame, containing the union of self.frame() and index set ixt.\n        This can be used to make
multiplication faster, by multiplying within a common frame.\n\n        >>
clifford(\"2+3{1}\").reframe(index_set({1,2,3}))\n        clifford(\"2+3{1}\")\n        >>
s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);\n
True\n        ";
07697         static PyObject *__pyx_pw_8PyClical_8clifford_11reframe(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt) {
07698             PyObject *__pyx_r = 0;
07699             __Pyx_RefNannyDeclarations
07700             __Pyx_RefNannySetupContext("reframe (wrapper)", 0);
07701             __pyx_r = __pyx_pf_8PyClical_8clifford_10reframe(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
07702
07703             /* function exit code */
07704             __Pyx_RefNannyFinishContext();
07705             return __pyx_r;
07706         }
07707
07708         static PyObject *__pyx_pf_8PyClical_8clifford_10reframe(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_ixt) {
07709             PyObject *__pyx_v_error_msg_prefix = NULL;
07710             struct __pyx_obj_8PyClical_clifford *__pyx_v_result = NULL;
07711             PyObject *__pyx_v_err = NULL;
07712             PyObject *__pyx_r = NULL;
07713             __Pyx_RefNannyDeclarations
07714             int __pyx_t_1;
07715             int __pyx_t_2;
07716             PyObject *__pyx_t_3 = NULL;
07717             PyObject *__pyx_t_4 = NULL;
07718             PyObject *__pyx_t_5 = NULL;
07719             PyObject *__pyx_t_6 = NULL;
07720             Clifford *__pyx_t_7;
07721             int __pyx_t_8;
07722             PyObject *__pyx_t_9 = NULL;
07723             PyObject *__pyx_t_10 = NULL;
07724             PyObject *__pyx_t_11 = NULL;
07725             PyObject *__pyx_t_12 = NULL;
07726             PyObject *__pyx_t_13 = NULL;
07727             int __pyx_t_14;
07728             char const *__pyx_t_15;
07729             PyObject *__pyx_t_16 = NULL;
07730             PyObject *__pyx_t_17 = NULL;
07731             PyObject *__pyx_t_18 = NULL;
07732             PyObject *__pyx_t_19 = NULL;
07733             PyObject *__pyx_t_20 = NULL;
07734             PyObject *__pyx_t_21 = NULL;
07735             int __pyx_lineno = 0;
07736             const char *__pyx_filename = NULL;
07737             int __pyx_clineno = 0;
07738             __Pyx_RefNannySetupContext("reframe", 0);
07739
07740             /* "PyClical.pyx":659
07741  *         True
07742  *         """

```

```

07743 *         error_msg_prefix = "Cannot reframe"                # ««««««««
07744 *         if isinstance(ixt, index_set):
07745 *             try:
07746 */
07747         __Pyx_INCREF(__pyx_kp_u_Cannot_reframe);
07748         __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_reframe;
07749
07750         /* "PyClicl.pyx":660
07751         """
07752         error_msg_prefix = "Cannot reframe"
07753         if isinstance(ixt, index_set):                # ««««««««
07754             try:
07755                 result = clifford()
07756 */
07757         __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClicl_index_set);
07758         __pyx_t_2 = (__pyx_t_1 != 0);
07759         if (likely(__pyx_t_2)) {
07760
07761             /* "PyClicl.pyx":661
07762             error_msg_prefix = "Cannot reframe"
07763             if isinstance(ixt, index_set):
07764                 try:                # ««««««««
07765                     result = clifford()
07766                     result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07767 */
07768             {
07769                 __Pyx_PyThreadState_declare
07770                 __Pyx_PyThreadState_assign
07771                 __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
07772                 __Pyx_XGOTREF(__pyx_t_3);
07773                 __Pyx_XGOTREF(__pyx_t_4);
07774                 __Pyx_XGOTREF(__pyx_t_5);
07775                 /*try:*/ {
07776
07777                     /* "PyClicl.pyx":662
07778                     if isinstance(ixt, index_set):
07779                         try:
07780                             result = clifford()                # ««««««««
07781                             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07782                             except RuntimeError as err:
07783 */
07784                         __pyx_t_6 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 662, __pyx_L4_error)
07785                         __Pyx_GOTREF(__pyx_t_6);
07786                         __pyx_v_result = ((struct __pyx_obj_8PyClicl_clifford *) __pyx_t_6);
07787                         __pyx_t_6 = 0;
07788
07789                         /* "PyClicl.pyx":663
07790                         try:
07791                             result = clifford()
07792                             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07793                         # ««««««««
07794                         except RuntimeError as err:
07795                             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07796 */
07797                         try {
07798                             __pyx_t_7 = new Clifford(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self),
__pyx_f_8PyClicl_9index_set_unwrap(((struct __pyx_obj_8PyClicl_index_set *) __pyx_v_ixt)));
07799                         } catch (...) {
07800                             __Pyx_CppExn2PyErr();
07801                             __PYX_ERR(0, 663, __pyx_L4_error)
07802                         }
07803                         __pyx_v_result->instance = __pyx_t_7;
07804
07805                         /* "PyClicl.pyx":664
07806                         error_msg_prefix = "Cannot reframe"
07807                         if isinstance(ixt, index_set):
07808                             try:                # ««««««««
07809                                 result = clifford()
07810                                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07811 */
07812                             }
07813                             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07814                             __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07815                             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07816                             goto __pyx_L9_try_end;
07817                             __pyx_L4_error:;
07818                             __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
07819
07820                             /* "PyClicl.pyx":664
07821                             result = clifford()
07822                             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07823                             except RuntimeError as err:                # ««««««««
07824                                 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07825 */
07826                                 + str(ixt) + ":",

```

```

07827         if (__pyx_t_8) {
07828             __Pyx_AddTraceback("PyClical.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07829         if (__Pyx_GetException(&__pyx_t_6, &__pyx_t_9, &__pyx_t_10) < 0) __PYX_ERR(0, 664,
__pyx_L6_except_error)
07830             __Pyx_GOTREF(__pyx_t_6);
07831             __Pyx_GOTREF(__pyx_t_9);
07832             __Pyx_GOTREF(__pyx_t_10);
07833             __Pyx_INCREF(__pyx_t_9);
07834             __pyx_v_err = __pyx_t_9;
07835             /*try:*/ {
07836
07837                 /* "PyClical.pyx":665
07838                 *     result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07839                 *     except RuntimeError as err:
07840                 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07841                 * # ««««««««
07842                 *                                     + str(ixt) + ":"
07843                 *                                     + "\n\t" + str(err))
07844                 */
07845                 __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_from); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 665, __pyx_L15_error)
07846                 __Pyx_GOTREF(__pyx_t_11);
07847                 __pyx_t_12 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)__pyx_v_self)); if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07848                 __Pyx_GOTREF(__pyx_t_12);
07849                 __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)
07850                 __Pyx_GOTREF(__pyx_t_13);
07851                 __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07852                 __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07853                 __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_to_frame); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07854                 __Pyx_GOTREF(__pyx_t_12);
07855                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07856
07857                 /* "PyClical.pyx":666
07858                 *     except RuntimeError as err:
07859                 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07860                 *                                     + str(ixt) + ":"
07861                 *                                     + "\n\t" + str(err))
07862                 * # ««««««««
07863                 *
07864                 *     else:
07865                 *         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_ixt); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07866                 __Pyx_GOTREF(__pyx_t_13);
07867                 __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_12, __pyx_t_13); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 666, __pyx_L15_error)
07868                 __Pyx_GOTREF(__pyx_t_11);
07869                 __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07870                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07871                 __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_kp_u_5); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07872                 __Pyx_GOTREF(__pyx_t_13);
07873                 __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07874
07875                 /* "PyClical.pyx":667
07876                 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07877                 *                                     + str(ixt) + ":"
07878                 *                                     + "\n\t" + str(err))
07879                 * #
07880                 *
07881                 *     else:
07882                 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
07883                 */
07884                 __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_6); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 667, __pyx_L15_error)
07885                 __Pyx_GOTREF(__pyx_t_11);
07886                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07887                 __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_err); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 667, __pyx_L15_error)
07888                 __Pyx_GOTREF(__pyx_t_13);
07889                 __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_13); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 667, __pyx_L15_error)
07890                 __Pyx_GOTREF(__pyx_t_12);
07891                 __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07892                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07893
07894                 /* "PyClical.pyx":665
07895                 *     result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07896                 *     except RuntimeError as err:
07897                 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07898                 * # ««««««««
07899                 *                                     + str(ixt) + ":"
07900                 *                                     + "\n\t" + str(err))
07901                 */
07902                 __pyx_t_13 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)

```

```

07898         __Pyx_GOTREF(__pyx_t_13);
07899         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07900         __Pyx_Raise(__pyx_t_13, 0, 0, 0);
07901         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07902         __PYX_ERR(0, 665, __pyx_L15_error)
07903     }
07904
07905     /* "PyClicical.pyx":664
07906     *
07907     *     result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07908     * except RuntimeError as err:
07909     *     raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07910     *                     + str(ixt) + ":")
07911 */
07912     /*finally:*/ {
07913         __pyx_L15_error;;
07914         /*exception exit:*/{
07915             __Pyx_PyThreadState_declare
07916             __Pyx_PyThreadState_assign
07917             __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
07918             __pyx_t_21 = 0;
07919             __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;
07920             __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
07921             __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07922             if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_19, &__pyx_t_20,
07923                 &__pyx_t_21);
07924             if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_16,
07925                 &__pyx_t_17, &__pyx_t_18) < 0)) __Pyx_ErrFetch(&__pyx_t_16, &__pyx_t_17, &__pyx_t_18);
07926             __Pyx_XGOTREF(__pyx_t_16);
07927             __Pyx_XGOTREF(__pyx_t_17);
07928             __Pyx_XGOTREF(__pyx_t_18);
07929             __Pyx_XGOTREF(__pyx_t_19);
07930             __Pyx_XGOTREF(__pyx_t_20);
07931             __Pyx_XGOTREF(__pyx_t_21);
07932             __pyx_t_8 = __pyx_lineno; __pyx_t_14 = __pyx_clineno; __pyx_t_15 = __pyx_filename;
07933             {
07934                 __Pyx_DECREF(__pyx_v_err);
07935                 __pyx_v_err = NULL;
07936             }
07937             if (PY_MAJOR_VERSION >= 3) {
07938                 __Pyx_XGIVEREF(__pyx_t_19);
07939                 __Pyx_XGIVEREF(__pyx_t_20);
07940                 __Pyx_XGIVEREF(__pyx_t_21);
07941                 __Pyx_ExceptionReset(__pyx_t_19, __pyx_t_20, __pyx_t_21);
07942             }
07943             __Pyx_XGIVEREF(__pyx_t_16);
07944             __Pyx_XGIVEREF(__pyx_t_17);
07945             __Pyx_XGIVEREF(__pyx_t_18);
07946             __Pyx_ErrRestore(__pyx_t_16, __pyx_t_17, __pyx_t_18);
07947             __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
07948             __pyx_t_21 = 0;
07949             __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_14; __pyx_filename = __pyx_t_15;
07950             goto __pyx_L6_except_error;
07951         }
07952     }
07953     goto __pyx_L6_except_error;
07954     __pyx_L6_except_error;;
07955
07956     /* "PyClicical.pyx":661
07957     * error_msg_prefix = "Cannot reframe"
07958     * if isinstance(ixt, index_set):
07959     *     try:
07960     *         # ««««««««
07961     *         result = clifford()
07962     *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07963 */
07964     __Pyx_XGIVEREF(__pyx_t_3);
07965     __Pyx_XGIVEREF(__pyx_t_4);
07966     __Pyx_XGIVEREF(__pyx_t_5);
07967     __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07968     goto __pyx_L1_error;
07969     __pyx_L9_try_end;;
07970 }
07971
07972 /* "PyClicical.pyx":660
07973 * """
07974 * error_msg_prefix = "Cannot reframe"
07975 * if isinstance(ixt, index_set):
07976 *     # ««««««««
07977 *     try:
07978 *         result = clifford()
07979 */
07980     goto __pyx_L3;
07981 }
07982
07983 /* "PyClicical.pyx":669
07984 *
07985 *         + "\n\t" + str(err))
07986 *
07987     else:

```

```

07981 *          raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")          #
07982 *          return result
07983 *
07984 */
07985 /*else*/ {
07986     __pyx_t_10 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_using); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 669, __pyx_L1_error)
07987     __Pyx_GOTREF(__pyx_t_10);
07988     __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), (PyObject
*)Py_TYPE(__pyx_v_ixt)); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 669, __pyx_L1_error)
07989     __Pyx_GOTREF(__pyx_t_9);
07990     __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_10, __pyx_t_9); if (unlikely(!__pyx_t_6))
__PYX_ERR(0, 669, __pyx_L1_error)
07991     __Pyx_GOTREF(__pyx_t_6);
07992     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
07993     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
07994     __pyx_t_9 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_9); if (unlikely(!__pyx_t_9))
__PYX_ERR(0, 669, __pyx_L1_error)
07995     __Pyx_GOTREF(__pyx_t_9);
07996     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
07997     __pyx_t_6 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_9); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 669, __pyx_L1_error)
07998     __Pyx_GOTREF(__pyx_t_6);
07999     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
08000     __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08001     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08002     __PYX_ERR(0, 669, __pyx_L1_error)
08003 }
08004 __pyx_L3:;
08005
08006 /* "PyClical.pyx":670
08007 *     else:
08008 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
08009 *         return result          # ««««««««
08010 *
08011 * def __richcmp__(lhs, rhs, int op):
08012 */
08013     __Pyx_XDECREF(__pyx_r);
08014     __Pyx_INCREF((PyObject *)__pyx_v_result);
08015     __pyx_r = ((PyObject *)__pyx_v_result);
08016     goto __pyx_L0;
08017
08018 /* "PyClical.pyx":649
08019 *     raise TypeError("Not applicable.")
08020 *
08021 * def reframe(self, ixt):          # ««««««««
08022 *     """
08023 *     Put self into a larger frame, containing the union of self.frame() and index set ixt.
08024 */
08025
08026 /* function exit code */
08027 __pyx_L1_error:;
08028 __Pyx_XDECREF(__pyx_t_6);
08029 __Pyx_XDECREF(__pyx_t_9);
08030 __Pyx_XDECREF(__pyx_t_10);
08031 __Pyx_XDECREF(__pyx_t_11);
08032 __Pyx_XDECREF(__pyx_t_12);
08033 __Pyx_XDECREF(__pyx_t_13);
08034 __Pyx_AddTraceback("PyClical.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08035 __pyx_r = NULL;
08036 __pyx_L0:;
08037 __Pyx_XDECREF(__pyx_v_error_msg_prefix);
08038 __Pyx_XDECREF((PyObject *)__pyx_v_result);
08039 __Pyx_XDECREF(__pyx_v_err);
08040 __Pyx_XGIVEREF(__pyx_r);
08041 __Pyx_RefNannyFinishContext();
08042 return __pyx_r;
08043 }
08044
08045 /* "PyClical.pyx":672
08046 *     return result
08047 *
08048 * def __richcmp__(lhs, rhs, int op):          # ««««««««
08049 *     """
08050 *     Compare objects of type clifford.
08051 */
08052
08053 /* Python wrapper */
08054 static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op); /*proto*/
08055 static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op) {
08056     PyObject *__pyx_r = 0;
08057     __Pyx_RefNannyDeclarations
08058     __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);

```

```

08059     __pyx_r = __pyx_pf_8PyClical_8clifford_12__richcmp__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
08060
08061     /* function exit code */
08062     __Pyx_RefNannyFinishContext();
08063     return __pyx_r;
08064 }
08065
08066 static PyObject *__pyx_pf_8PyClical_8clifford_12__richcmp__(struct
__pyx_obj_8PyClical_clifford *__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
08067     PyObject *__pyx_r = NULL;
08068     __Pyx_RefNannyDeclarations
08069     int __pyx_t_1;
08070     int __pyx_t_2;
08071     int __pyx_t_3;
08072     PyObject *__pyx_t_4 = NULL;
08073     PyObject *__pyx_t_5 = NULL;
08074     PyObject *__pyx_t_6 = NULL;
08075     int __pyx_lineno = 0;
08076     const char *__pyx_filename = NULL;
08077     int __pyx_clineno = 0;
08078     __Pyx_RefNannySetupContext("__richcmp__", 0);
08079
08080     /* "PyClical.pyx":691
08081     *
08082     * """
08083     *     if op == 2: # ==
08084     *         if (lhs is None) or (rhs is None):
08085     *             return bool(lhs is rhs)
08086     */
08087     __pyx_t_1 = ((__pyx_v_op == 2) != 0);
08088     if (__pyx_t_1) {
08089
08090         /* "PyClical.pyx":692
08091         * """
08092         *     if op == 2: # ==
08093         *         if (lhs is None) or (rhs is None):
08094         *             return bool(lhs is rhs)
08095         *         else:
08096         */
08097         __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
08098         __pyx_t_3 = (__pyx_t_2 != 0);
08099         if (!__pyx_t_3) {
08100         } else {
08101             __pyx_t_1 = __pyx_t_3;
08102             goto __pyx_L5_bool_binop_done;
08103         }
08104         __pyx_t_3 = (__pyx_v_rhs == Py_None);
08105         __pyx_t_2 = (__pyx_t_3 != 0);
08106         __pyx_t_1 = __pyx_t_2;
08107         __pyx_L5_bool_binop_done;
08108         if (__pyx_t_1) {
08109
08110             /* "PyClical.pyx":693
08111             *     if op == 2: # ==
08112             *         if (lhs is None) or (rhs is None):
08113             *             return bool(lhs is rhs)
08114             *         else:
08115             *             return bool( toClifford(lhs) == toClifford(rhs) )
08116             */
08117             __Pyx_XDECREF(__pyx_r);
08118             __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
08119             __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 693, __pyx_L1_error)
08120             __Pyx_GOTREF(__pyx_t_4);
08121             __pyx_r = __pyx_t_4;
08122             __pyx_t_4 = 0;
08123             goto __pyx_L0;
08124
08125             /* "PyClical.pyx":692
08126             * """
08127             *     if op == 2: # ==
08128             *         if (lhs is None) or (rhs is None):
08129             *             return bool(lhs is rhs)
08130             *         else:
08131             */
08132         }
08133
08134         /* "PyClical.pyx":695
08135         *     return bool(lhs is rhs)
08136         *     else:
08137         *         return bool( toClifford(lhs) == toClifford(rhs) )
08138         *     elif op == 3: # !=
08139         *         if (lhs is None) or (rhs is None):
08140         */
08141         /*else*/ {
08142             __Pyx_XDECREF(__pyx_r);

```



```

08143         __pyx_t_1 = (__pyx_f_8PyClical_toClifford(((PyObject *)__pyx_v_lhs)) ==
__pyx_f_8PyClical_toClifford(__pyx_v_rhs));
08144         __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 695, __pyx_L1_error)
08145         __Pyx_GOTREF(__pyx_t_4);
08146         __pyx_r = __pyx_t_4;
08147         __pyx_t_4 = 0;
08148         goto __pyx_L0;
08149     }
08150
08151     /* "PyClical.pyx":691
08152     * True
08153     * """
08154     * if op == 2: # == # ««««««««
08155     *     if (lhs is None) or (rhs is None):
08156     *         return bool(lhs is rhs)
08157     */
08158 }
08159
08160 /* "PyClical.pyx":696
08161 * else:
08162 *     return bool( toClifford(lhs) == toClifford(rhs) )
08163 * elif op == 3: # != # ««««««««
08164 *     if (lhs is None) or (rhs is None):
08165 *         return not bool(lhs is rhs)
08166 */
__pyx_t_1 = ((__pyx_v_op == 3) != 0);
08167 if (__pyx_t_1) {
08168     /* "PyClical.pyx":697
08169     *     return bool( toClifford(lhs) == toClifford(rhs) )
08170     * elif op == 3: # !=
08171     *     if (lhs is None) or (rhs is None): # ««««««««
08172     *         return not bool(lhs is rhs)
08173     * else:
08174     */
__pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
08175 __pyx_t_3 = (__pyx_t_2 != 0);
08176 if (!__pyx_t_3) {
08177     } else {
08178         __pyx_t_1 = __pyx_t_3;
08179         goto __pyx_L8_bool_binop_done;
08180     }
08181     __pyx_t_3 = (__pyx_v_rhs == Py_None);
08182     __pyx_t_2 = (__pyx_t_3 != 0);
08183     __pyx_t_1 = __pyx_t_2;
08184     __pyx_L8_bool_binop_done;
08185     if (__pyx_t_1) {
08186         /* "PyClical.pyx":698
08187         * elif op == 3: # !=
08188         *     if (lhs is None) or (rhs is None): # ««««««««
08189         *         return not bool(lhs is rhs)
08190         * else:
08191         *         return bool( toClifford(lhs) != toClifford(rhs) )
08192         */
__Pyx_XDECREF(__pyx_r);
08193 __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
08194 __pyx_t_4 = __Pyx_PyBool_FromLong(!(!(!__pyx_t_1) != 0)); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 698, __pyx_L1_error)
08195 __Pyx_GOTREF(__pyx_t_4);
08196 __pyx_r = __pyx_t_4;
08197 __pyx_t_4 = 0;
08198         goto __pyx_L0;
08199
08200     /* "PyClical.pyx":697
08201     *     return bool( toClifford(lhs) == toClifford(rhs) )
08202     * elif op == 3: # !=
08203     *     if (lhs is None) or (rhs is None): # ««««««««
08204     *         return not bool(lhs is rhs)
08205     * else:
08206     */
08207 }
08208
08209 /* "PyClical.pyx":700
08210 *     return not bool(lhs is rhs)
08211 * else:
08212 *     return bool( toClifford(lhs) != toClifford(rhs) ) # ««««««««
08213 * elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08214 *     raise TypeError("This comparison operator is not implemented for "
08215 */
/*else*/ {
08216     __Pyx_XDECREF(__pyx_r);
08217     __pyx_t_1 = (__pyx_f_8PyClical_toClifford(((PyObject *)__pyx_v_lhs)) !=
__pyx_f_8PyClical_toClifford(__pyx_v_rhs));
08218     __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 700, __pyx_L1_error)

```

```

08225         __Pyx_GOTREF(__pyx_t_4);
08226         __pyx_r = __pyx_t_4;
08227         __pyx_t_4 = 0;
08228         goto __pyx_L0;
08229     }
08230
08231     /* "PyClical.pyx":696
08232     *         else:
08233     *             return bool( toClifford(lhs) == toClifford(rhs) )
08234     *         elif op == 3: # != # ««««««««
08235     *             if (lhs is None) or (rhs is None):
08236     *                 return not bool(lhs is rhs)
08237     */
08238 }
08239
08240 /* "PyClical.pyx":701
08241 *         else:
08242 *             return bool( toClifford(lhs) != toClifford(rhs) )
08243 *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford): # ««««««««
08244 *             raise TypeError("This comparison operator is not implemented for "
08245 *                 + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08246 */
08247 __pyx_t_2 = __Pyx_TypeCheck((PyObject *)__pyx_v_lhs, __pyx_ptype_8PyClical_clifford);
08248 __pyx_t_3 = (__pyx_t_2 != 0);
08249 if (!__pyx_t_3) {
08250 } else {
08251     __pyx_t_1 = __pyx_t_3;
08252     goto __pyx_L10_bool_binop_done;
08253 }
08254 __pyx_t_3 = __Pyx_TypeCheck(__pyx_v_rhs, __pyx_ptype_8PyClical_clifford);
08255 __pyx_t_2 = (__pyx_t_3 != 0);
08256 __pyx_t_1 = __pyx_t_2;
08257 __pyx_L10_bool_binop_done;
08258 if (unlikely(__pyx_t_1)) {
08259
08260     /* "PyClical.pyx":703
08261     *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08262     *             raise TypeError("This comparison operator is not implemented for "
08263     *                 + str(type(lhs)) + ", " + str(type(rhs)) + ".") # ««««««««
08264     *         else:
08265     *             return NotImplemented
08266     */
08267     __pyx_t_4 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
08268 *)Py_TYPE((PyObject *)__pyx_v_lhs))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 703, __pyx_L1_error)
08269     __Pyx_GOTREF(__pyx_t_4);
08270     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_kp_u_This_comparison_operator_is_not, __pyx_t_4);
08271     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08272     __Pyx_GOTREF(__pyx_t_5);
08273     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08274     __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u_8); if (unlikely(!__pyx_t_4))
08275     __PYX_ERR(0, 703, __pyx_L1_error)
08276     __Pyx_GOTREF(__pyx_t_4);
08277     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08278     __pyx_t_5 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
08279 *)Py_TYPE(__pyx_v_rhs))); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08280     __Pyx_GOTREF(__pyx_t_5);
08281     __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_4, __pyx_t_5); if (unlikely(!__pyx_t_6))
08282     __PYX_ERR(0, 703, __pyx_L1_error)
08283     __Pyx_GOTREF(__pyx_t_6);
08284     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08285     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08286     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_); if (unlikely(!__pyx_t_5))
08287     __PYX_ERR(0, 703, __pyx_L1_error)
08288     __Pyx_GOTREF(__pyx_t_5);
08289     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08290
08291     /* "PyClical.pyx":702
08292     *             return bool( toClifford(lhs) != toClifford(rhs) )
08293     *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08294     *             raise TypeError("This comparison operator is not implemented for " #
08295     *                 + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08296     *         else:
08297     */
08298     __pyx_t_6 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_5); if
08299     (unlikely(!__pyx_t_6)) __PYX_ERR(0, 702, __pyx_L1_error)
08300     __Pyx_GOTREF(__pyx_t_6);
08301     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08302     __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08303     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08304     __PYX_ERR(0, 702, __pyx_L1_error)
08305
08306     /* "PyClical.pyx":701
08307     *         else:
08308     *             return bool( toClifford(lhs) != toClifford(rhs) )
08309     *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford): # ««««««««
08310     *             raise TypeError("This comparison operator is not implemented for "

```

```

08304 *                                + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08305 */
08306     }
08307
08308     /* "PyClical.pyx":705
08309 *                                + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08310 *                                else:
08311 *                                    return NotImplemented                # ««««««««
08312 *
08313 *     def __getitem__(self, ixt):
08314 */
08315     /*else*/ {
08316         __Pyx_XDECREF(__pyx_r);
08317         __Pyx_INCREF(__pyx_builtin_NotImplemented);
08318         __pyx_r = __pyx_builtin_NotImplemented;
08319         goto __pyx_L0;
08320     }
08321
08322     /* "PyClical.pyx":672
08323 *     return result
08324 *
08325 *     def __richcmp__(lhs, rhs, int op):                # ««««««««
08326 *         """
08327 *         Compare objects of type clifford.
08328 */
08329
08330     /* function exit code */
08331     __pyx_L1_error:;
08332     __Pyx_XDECREF(__pyx_t_4);
08333     __Pyx_XDECREF(__pyx_t_5);
08334     __Pyx_XDECREF(__pyx_t_6);
08335     __Pyx_AddTraceback("PyClical.clifford.__richcmp__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08336     __pyx_r = NULL;
08337     __pyx_L0:;
08338     __Pyx_XGIVEREF(__pyx_r);
08339     __Pyx_RefNannyFinishContext();
08340     return __pyx_r;
08341 }
08342
08343     /* "PyClical.pyx":707
08344 *     return NotImplemented
08345 *
08346 *     def __getitem__(self, ixt):                # ««««««««
08347 *         """
08348 *         Subscripting: map from index set to scalar coordinate.
08349 */
08350
08351     /* Python wrapper */
08352     static PyObject *__pyx_pw_8PyClical_8clifford_15_getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt); /*proto*/
08353     static char __pyx_doc_8PyClical_8clifford_14_getitem__[] = "\n        Subscripting: map from
index set to scalar coordinate.\n\n        >> clifford(\"{1}\") [index_set(1)]\n        1.0\n
>> clifford(\"{1}\") [index_set({1})]\n        1.0\n        >> clifford(\"{1}\") [index_set({1,2})]\n
0.0\n        >> clifford(\"2{1,2}\") [index_set({1,2})]\n        2.0\n        ";
08354     #if CYTHON_COMPILING_IN_CPYTHON
08355     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_14_getitem__;
08356     #endif
08357     static PyObject *__pyx_pw_8PyClical_8clifford_15_getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt) {
08358         PyObject *__pyx_r = 0;
08359         __Pyx_RefNannyDeclarations
08360         __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
08361         __pyx_r = __pyx_pf_8PyClical_8clifford_14_getitem__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
08362
08363     /* function exit code */
08364     __Pyx_RefNannyFinishContext();
08365     return __pyx_r;
08366 }
08367
08368     static PyObject *__pyx_pf_8PyClical_8clifford_14_getitem__(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, PyObject *__pyx_v_ixt) {
08369         PyObject *__pyx_r = NULL;
08370         __Pyx_RefNannyDeclarations
08371         PyObject *__pyx_t_1 = NULL;
08372         int __pyx_lineno = 0;
08373         const char *__pyx_filename = NULL;
08374         int __pyx_clineno = 0;
08375         __Pyx_RefNannySetupContext("__getitem__", 0);
08376
08377     /* "PyClical.pyx":720
08378 *     2.0
08379 *     """
08380 *     return self.instance.getitem(toIndexSet(ixt))                # ««««««««
08381 *
08382 *     def __neg__(self):

```

```

08383 */
08384     __Pyx_XDECREF(__pyx_r);
08385     __pyx_t_1 =
PyFloat_FromDouble(__pyx_v_self->instance->operator[](__pyx_f_8PyClical_toIndexSet(__pyx_v_ixt))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 720, __pyx_L1_error)
08386     __Pyx_GOTREF(__pyx_t_1);
08387     __pyx_r = __pyx_t_1;
08388     __pyx_t_1 = 0;
08389     goto __pyx_L0;
08390
08391     /* "PyClical.pyx":707
08392     *         return NotImplemented
08393     *
08394     *     def __getitem__(self, ixt):                # ««««««««
08395     *         """
08396     *         Subscripting: map from index set to scalar coordinate.
08397     */
08398
08399     /* function exit code */
08400     __pyx_L1_error:;
08401     __Pyx_XDECREF(__pyx_t_1);
08402     __Pyx_AddTraceback("PyClical.clifford.__getitem__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08403     __pyx_r = NULL;
08404     __pyx_L0:;
08405     __Pyx_XGIVEREF(__pyx_r);
08406     __Pyx_RefNannyFinishContext();
08407     return __pyx_r;
08408 }
08409
08410     /* "PyClical.pyx":722
08411     *         return self.instance.getitem(toIndexSet(ixt))
08412     *
08413     *     def __neg__(self):                # ««««««««
08414     *         """
08415     *         Unary -.
08416     */
08417
08418     /* Python wrapper */
08419     static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self); /*proto*/
08420     static char __pyx_doc_8PyClical_8clifford_16__neg__[] = "\n        Unary -. \n\n        »>
print(-clifford(\{1\})\n\n        -{1}\n\n        ");
08421     #if CYTHON_COMPILING_IN_CPYTHON
08422     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_16__neg__;
08423     #endif
08424     static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self) {
08425         PyObject *__pyx_r = 0;
08426         __Pyx_RefNannyDeclarations
08427         __Pyx_RefNannySetupContext("__neg__ (wrapper)", 0);
08428         __pyx_r = __pyx_pf_8PyClical_8clifford_16__neg__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08429
08430     /* function exit code */
08431     __Pyx_RefNannyFinishContext();
08432     return __pyx_r;
08433 }
08434
08435     static PyObject *__pyx_pf_8PyClical_8clifford_16__neg__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08436         PyObject *__pyx_r = NULL;
08437         __Pyx_RefNannyDeclarations
08438         PyObject *__pyx_t_1 = NULL;
08439         PyObject *__pyx_t_2 = NULL;
08440         int __pyx_lineno = 0;
08441         const char *__pyx_filename = NULL;
08442         int __pyx_clineno = 0;
08443         __Pyx_RefNannySetupContext("__neg__", 0);
08444
08445     /* "PyClical.pyx":729
08446     *         -{1}
08447     *         """
08448     *         return clifford().wrap( self.instance.neg() )                # ««««««««
08449     *
08450     *     def __pos__(self):
08451     */
08452     __Pyx_XDECREF(__pyx_r);
08453     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 729, __pyx_L1_error)
08454     __Pyx_GOTREF(__pyx_t_1);
08455     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->operator-()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 729,
__pyx_L1_error)
08456     __Pyx_GOTREF(__pyx_t_2);
08457     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08458     __pyx_r = __pyx_t_2;
08459     __pyx_t_2 = 0;
08460     goto __pyx_L0;

```

```

08461
08462     /* "PyClical.pyx":722
08463     *         return self.instance.getitem(toIndexSet(ixt))
08464     *
08465     *     def __neg__(self):
08466     *         """
08467     *         Unary -.
08468     */
08469
08470     /* function exit code */
08471     __pyx_L1_error:;
08472     __Pyx_XDECREF(__pyx_t_1);
08473     __Pyx_XDECREF(__pyx_t_2);
08474     __Pyx_AddTraceback("PyClical.clifford.__neg__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08475     __pyx_r = NULL;
08476     __pyx_L0:;
08477     __Pyx_XGIVEREF(__pyx_r);
08478     __Pyx_RefNannyFinishContext();
08479     return __pyx_r;
08480 }
08481
08482     /* "PyClical.pyx":731
08483     *         return clifford().wrap( self.instance.neg() )
08484     *
08485     *     def __pos__(self):
08486     *         """
08487     *         Unary +.
08488     */
08489
08490     /* Python wrapper */
08491     static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self); /*proto*/
08492     static char __pyx_doc_8PyClical_8clifford_18__pos__[] = "\n        Unary +.\n\n    >>
print(+clifford(\"{1}\"))\n        {1}\n        ";
08493     #if CYTHON_COMPILING_IN_CPYTHON
08494     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_18__pos__;
08495     #endif
08496     static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self) {
08497         PyObject *__pyx_r = 0;
08498         __Pyx_RefNannyDeclarations
08499         __Pyx_RefNannySetupContext("__pos__ (wrapper)", 0);
08500         __pyx_r = __pyx_pf_8PyClical_8clifford_18__pos__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08501
08502     /* function exit code */
08503     __Pyx_RefNannyFinishContext();
08504     return __pyx_r;
08505 }
08506
08507     static PyObject *__pyx_pf_8PyClical_8clifford_18__pos__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08508         PyObject *__pyx_r = NULL;
08509         __Pyx_RefNannyDeclarations
08510         PyObject *__pyx_t_1 = NULL;
08511         int __pyx_lineno = 0;
08512         const char *__pyx_filename = NULL;
08513         int __pyx_clineno = 0;
08514         __Pyx_RefNannySetupContext("__pos__", 0);
08515
08516     /* "PyClical.pyx":738
08517     *         {1}
08518     *         """
08519     *         return clifford(self)
08520     *         # <<<<<<<<<
08521     *     def __add__(lhs, rhs):
08522     */
08523     __Pyx_XDECREF(__pyx_r);
08524     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
((PyObject *)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 738, __pyx_L1_error)
__Pyx_GOTREF(__pyx_t_1);
08525     __pyx_r = __pyx_t_1;
08526     __pyx_t_1 = 0;
08527     goto __pyx_L0;
08528
08529     /* "PyClical.pyx":731
08530     *         return clifford().wrap( self.instance.neg() )
08531     *
08532     *
08533     *     def __pos__(self):
08534     *         """
08535     *         Unary +.
08536     */
08537
08538     /* function exit code */
08539     __pyx_L1_error:;
08540     __Pyx_XDECREF(__pyx_t_1);
08541     __Pyx_AddTraceback("PyClical.clifford.__pos__", __pyx_clineno, __pyx_lineno,
__pyx_filename);

```

```

08542         __pyx_r = NULL;
08543         __pyx_L0:;
08544         __Pyx_XGIVEREF(__pyx_r);
08545         __Pyx_RefNannyFinishContext();
08546         return __pyx_r;
08547     }
08548
08549     /* "PyClicl.pyx":740
08550     *         return clifford(self)
08551     *
08552     *     def __add__(lhs, rhs):
08553     *         """
08554     *         Geometric sum.
08555     */
08556
08557     /* Python wrapper */
08558     static PyObject *__pyx_pw_8PyClicl_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
08559 *__pyx_v_rhs); /*proto*/
08560     static char __pyx_doc_8PyClicl_8clifford_20__add__[] = "\n        Geometric sum.\n\n
>> print(clifford(1) + clifford(\"{2}\"))\n        1+{2}\n        >> print(clifford(\"{1}\" )
clifford(\"{2}\")\n        {1}+{2}\n        ";
08561     #if CYTHON_COMPILING_IN_CPYTHON
08562     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_20__add__;
08563     #endif
08564     static PyObject *__pyx_pw_8PyClicl_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
08565 *__pyx_v_rhs) {
08566         PyObject *__pyx_r = 0;
08567         __Pyx_RefNannyDeclarations
08568         __Pyx_RefNannySetupContext("__add__ (wrapper)", 0);
08569         __pyx_r = __pyx_pf_8PyClicl_8clifford_20__add__((PyObject *)__pyx_v_lhs), ((PyObject
08570 *)__pyx_v_rhs));
08571
08572         /* function exit code */
08573         __Pyx_RefNannyFinishContext();
08574         return __pyx_r;
08575     }
08576
08577     static PyObject *__pyx_pf_8PyClicl_8clifford_20__add__(PyObject *__pyx_v_lhs, PyObject
08578 *__pyx_v_rhs) {
08579         PyObject *__pyx_r = NULL;
08580         __Pyx_RefNannyDeclarations
08581         PyObject *__pyx_t_1 = NULL;
08582         PyObject *__pyx_t_2 = NULL;
08583         int __pyx_lineno = 0;
08584         const char *__pyx_filename = NULL;
08585         int __pyx_clineno = 0;
08586         __Pyx_RefNannySetupContext("__add__", 0);
08587
08588     /* "PyClicl.pyx":749
08589     *         {1}+{2}
08590     *         """
08591     *         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08592     *         # ««««««««
08593     *
08594     *     def __iadd__(self, rhs):
08595     */
08596         __Pyx_XDECREF(__pyx_r);
08597         __pyx_t_1 = __Pyx_PyObject_CallNoArg((PyObject *)__pyx_ptype_8PyClicl_clifford); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 749, __pyx_L1_error)
08598         __Pyx_GOTREF(__pyx_t_1);
08599         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
08600 *)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) +
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 749,
__pyx_L1_error)
08601         __Pyx_GOTREF(__pyx_t_2);
08602         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08603         __pyx_r = __pyx_t_2;
08604         __pyx_t_2 = 0;
08605         goto __pyx_L0;
08606
08607     /* "PyClicl.pyx":740
08608     *         return clifford(self)
08609     *
08610     *     def __add__(lhs, rhs):
08611     *         """
08612     *         Geometric sum.
08613     */
08614
08615     /* function exit code */
08616     __pyx_L1_error:;
08617     __Pyx_XDECREF(__pyx_t_1);
08618     __Pyx_XDECREF(__pyx_t_2);
08619     __Pyx_AddTraceback("PyClicl.clifford.__add__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08620     __pyx_r = NULL;
08621     __pyx_L0:;
08622     __Pyx_XGIVEREF(__pyx_r);
08623     __Pyx_RefNannyFinishContext();

```

```

08618         return __pyx_r;
08619     }
08620
08621     /* "PyClical.pyx":751
08622     *         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08623     *
08624     *     def __iadd__(self, rhs):
08625     *         """
08626     *         Geometric sum.
08627     */
08628
08629     /* Python wrapper */
08630     static PyObject *__pyx_pw_8PyClical_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
08631 *__pyx_v_rhs); /*proto*/
08632     static char __pyx_doc_8PyClical_8clifford_22__iadd__[] = "\n        Geometric sum.\n\n
08633 >> x = clifford(1); x += clifford(\"{2}\"); print(x)\n        1+{2}\n        ";
08634     #if CYTHON_COMPILING_IN_CPYTHON
08635     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_22__iadd__;
08636     #endif
08637     static PyObject *__pyx_pw_8PyClical_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
08638 *__pyx_v_rhs) {
08639     PyObject *__pyx_r = 0;
08640     __Pyx_RefNannyDeclarations
08641     __Pyx_RefNannySetupContext("__iadd__ (wrapper)", 0);
08642     __pyx_r = __pyx_pf_8PyClical_8clifford_22__iadd__(((struct __pyx_obj_8PyClical_clifford
08643 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08644
08645     /* function exit code */
08646     __Pyx_RefNannyFinishContext();
08647     return __pyx_r;
08648 }
08649
08650 static PyObject *__pyx_pf_8PyClical_8clifford_22__iadd__(struct __pyx_obj_8PyClical_clifford
08651 *__pyx_v_self, PyObject *__pyx_v_rhs) {
08652     PyObject *__pyx_r = NULL;
08653     __Pyx_RefNannyDeclarations
08654     PyObject *__pyx_t_1 = NULL;
08655     int __pyx_lineno = 0;
08656     const char *__pyx_filename = NULL;
08657     int __pyx_clineno = 0;
08658     __Pyx_RefNannySetupContext("__iadd__", 0);
08659
08660     /* "PyClical.pyx":758
08661     *         1+{2}
08662     *         """
08663     *         return self.wrap( self.unwrap() + toClifford(rhs) )
08664     *         # ««««««««
08665     *
08666     *     def __sub__(lhs, rhs):
08667     *         """
08668     *         Geometric sum.
08669     */
08670
08671     __Pyx_XDECREF(__pyx_r);
08672     __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
08673 (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) + __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
08674 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 758, __pyx_L1_error)
08675     __Pyx_GOTREF(__pyx_t_1);
08676     __pyx_r = __pyx_t_1;
08677     __pyx_t_1 = 0;
08678     goto __pyx_L0;
08679
08680     /* "PyClical.pyx":751
08681     *         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08682     *
08683     *     def __iadd__(self, rhs):
08684     *         """
08685     *         Geometric sum.
08686     */
08687
08688     /* function exit code */
08689     __pyx_L1_error:;
08690     __Pyx_XDECREF(__pyx_t_1);
08691     __Pyx_AddTraceback("PyClical.clifford.__iadd__", __pyx_clineno, __pyx_lineno,
08692 __pyx_filename);
08693     __pyx_r = NULL;
08694     __pyx_L0:;
08695     __Pyx_XGIVEREF(__pyx_r);
08696     __Pyx_RefNannyFinishContext();
08697     return __pyx_r;
08698 }
08699
08700     /* "PyClical.pyx":760
08701     *         return self.wrap( self.unwrap() + toClifford(rhs) )
08702     *
08703     *     def __sub__(lhs, rhs):
08704     *         """
08705     *         Geometric difference.
08706     */
08707
08708     /* Python wrapper */

```

```

08697     static PyObject * __pyx_pw_8PyClical_8clifford_25__sub__(PyObject * __pyx_v_lhs, PyObject
* __pyx_v_rhs); /*proto*/
08698     static char __pyx_doc_8PyClical_8clifford_24__sub__[] = "\n          Geometric difference.\n\n
>> print(clifford(1) - clifford(\"{2}\"))\n          1-{2}\n          >> print(clifford(\"{1}\") -
clifford(\"{2}\"))\n          {1}-{2}\n          ";
08699     #if CYTHON_COMPILING_IN_CPYTHON
08700     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_24__sub__;
08701     #endif
08702     static PyObject * __pyx_pw_8PyClical_8clifford_25__sub__(PyObject * __pyx_v_lhs, PyObject
* __pyx_v_rhs) {
08703         PyObject * __pyx_r = 0;
08704         __Pyx_RefNannyDeclarations
08705         __Pyx_RefNannySetupContext("__sub__ (wrapper)", 0);
08706         __pyx_r = __pyx_pf_8PyClical_8clifford_24__sub__(((PyObject *) __pyx_v_lhs), ((PyObject
*) __pyx_v_rhs));
08707
08708         /* function exit code */
08709         __Pyx_RefNannyFinishContext();
08710         return __pyx_r;
08711     }
08712
08713     static PyObject * __pyx_pf_8PyClical_8clifford_24__sub__(PyObject * __pyx_v_lhs, PyObject
* __pyx_v_rhs) {
08714         PyObject * __pyx_r = NULL;
08715         __Pyx_RefNannyDeclarations
08716         PyObject * __pyx_t_1 = NULL;
08717         PyObject * __pyx_t_2 = NULL;
08718         int __pyx_lineno = 0;
08719         const char * __pyx_filename = NULL;
08720         int __pyx_clineno = 0;
08721         __Pyx_RefNannySetupContext("__sub__", 0);
08722
08723         /* "PyClical.pyx":769
08724         *         {1}-{2}
08725         *         """
08726         *         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08727         *         # <<<<<<<<<
08728         *
08729         *         def __isub__(self, rhs):
08730         */
08731         __Pyx_XDECREF(__pyx_r);
08732         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 769, __pyx_L1_error)
08733         __Pyx_GOTREF(__pyx_t_1);
08734         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) -
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 769,
__pyx_L1_error)
08735         __Pyx_GOTREF(__pyx_t_2);
08736         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08737         __pyx_r = __pyx_t_2;
08738         __pyx_t_2 = 0;
08739         goto __pyx_L0;
08740
08741         /* "PyClical.pyx":760
08742         *         return self.wrap( self.unwrap() + toClifford(rhs) )
08743         *
08744         *         def __sub__(lhs, rhs):
08745         *         # <<<<<<<<<
08746         *         """
08747         *         Geometric difference.
08748         *         """
08749         */
08750         /* function exit code */
08751         __pyx_L1_error:;
08752         __Pyx_XDECREF(__pyx_t_1);
08753         __Pyx_XDECREF(__pyx_t_2);
08754         __Pyx_AddTraceback("PyClical.clifford.__sub__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08755         __pyx_r = NULL;
08756         __pyx_L0:;
08757         __Pyx_XGIVEREF(__pyx_r);
08758         __Pyx_RefNannyFinishContext();
08759         return __pyx_r;
08760
08761         /* "PyClical.pyx":771
08762         *         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08763         *
08764         *         def __isub__(self, rhs):
08765         *         # <<<<<<<<<
08766         *         """
08767         *         Geometric difference.
08768         *         """
08769         */
08770         /* Python wrapper */
08771         static PyObject * __pyx_pw_8PyClical_8clifford_27__isub__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs); /*proto*/
08772         static char __pyx_doc_8PyClical_8clifford_26__isub__[] = "\n          Geometric difference.\n\n
>> x = clifford(1); x -= clifford(\"{2}\"); print(x)\n          1-{2}\n          ";

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08846
08847     /* function exit code */
08848     __Pyx_RefNannyFinishContext();
08849     return __pyx_r;
08850 }
08851
08852 static PyObject *__pyx_pf_8PyClical_8clifford_28__mul__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08853     PyObject *__pyx_r = NULL;
08854     __Pyx_RefNannyDeclarations
08855     PyObject *__pyx_t_1 = NULL;
08856     PyObject *__pyx_t_2 = NULL;
08857     int __pyx_lineno = 0;
08858     const char *__pyx_filename = NULL;
08859     int __pyx_clineno = 0;
08860     __Pyx_RefNannySetupContext("__mul__", 0);
08861
08862     /* "PyClical.pyx":791
08863     *     {2}
08864     *     """
08865     *     return clifford().wrap( toClifford(lhs) * toClifford(rhs) )           # ««««««««
08866     *
08867     *     def __imul__(self, rhs):
08868     */
08869     __Pyx_XDECREF(__pyx_r);
08870     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 791, __pyx_L1_error)
08871     __Pyx_GOTREF(__pyx_t_1);
08872     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) *
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 791,
__pyx_L1_error)
08873     __Pyx_GOTREF(__pyx_t_2);
08874     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08875     __pyx_r = __pyx_t_2;
08876     __pyx_t_2 = 0;
08877     goto __pyx_L0;
08878
08879     /* "PyClical.pyx":780
08880     *     return self.wrap( self.unwrap() - toClifford(rhs) )
08881     *
08882     *     def __mul__(lhs, rhs):           # ««««««««
08883     *     """
08884     *     Geometric product.
08885     */
08886
08887     /* function exit code */
08888     __pyx_L1_error:;
08889     __Pyx_XDECREF(__pyx_t_1);
08890     __Pyx_XDECREF(__pyx_t_2);
08891     __Pyx_AddTraceback("PyClical.clifford.__mul__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08892     __pyx_r = NULL;
08893     __pyx_L0:;
08894     __Pyx_XGIVEREF(__pyx_r);
08895     __Pyx_RefNannyFinishContext();
08896     return __pyx_r;
08897 }
08898
08899     /* "PyClical.pyx":793
08900     *     return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08901     *
08902     *     def __imul__(self, rhs):           # ««««««««
08903     *     """
08904     *     Geometric product.
08905     */
08906
08907     /* Python wrapper */
08908     static PyObject *__pyx_pw_8PyClical_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08909     static char __pyx_doc_8PyClical_8clifford_30__imul__[] = "\n          Geometric product.\n\n
>> x = clifford(2); x *= clifford('{2}'); print(x)\n          2{2}\n          >> x =
clifford('{1}'); x *= clifford('{2}'); print(x)\n          {1,2}\n          >> x = clifford('{1}');
x *= clifford('{1,2}'); print(x)\n          {2}\n          ";
08910     #if CYTHON_COMPILING_IN_CPYTHON
08911     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_30__imul__;
08912     #endif
08913     static PyObject *__pyx_pw_8PyClical_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08914         PyObject *__pyx_r = 0;
08915         __Pyx_RefNannyDeclarations
08916         __Pyx_RefNannySetupContext("__imul__ (wrapper)", 0);
08917         __pyx_r = __pyx_pf_8PyClical_8clifford_30__imul__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08918
08919     /* function exit code */
08920     __Pyx_RefNannyFinishContext();

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08921         return __pyx_r;
08922     }
08923
08924     static PyObject *__pyx_pf_8PyClicl_8clifford_30__imul__(struct __pyx_obj_8PyClicl_clifford
08925 *__pyx_v_self, PyObject *__pyx_v_rhs) {
08926         PyObject *__pyx_r = NULL;
08927         __Pyx_RefNannyDeclarations
08928         PyObject *__pyx_t_1 = NULL;
08929         int __pyx_lineno = 0;
08930         const char *__pyx_filename = NULL;
08931         int __pyx_clineno = 0;
08932         __Pyx_RefNannySetupContext("__imul__", 0);
08933
08934         /* "PyClicl.pyx":804
08935          *
08936          *
08937          *
08938          * def __mod__(lhs, rhs):
08939          */
08940         __Pyx_XDECREF(__pyx_r);
08941         __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
08942 (__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) * __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
08943 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 804, __pyx_l1_error)
08944         __Pyx_GOTREF(__pyx_t_1);
08945         __pyx_r = __pyx_t_1;
08946         __pyx_t_1 = 0;
08947         goto __pyx_L0;
08948
08949         /* "PyClicl.pyx":793
08950          *
08951          *
08952          * def __imul__(self, rhs):
08953          */
08954         __Pyx_XDECREF(__pyx_r);
08955         __Pyx_GOTREF(__pyx_t_1);
08956         __Pyx_ADDTRACEBACK("PyClicl.clifford.__imul__", __pyx_clineno, __pyx_lineno,
08957 __pyx_filename);
08958         __pyx_r = NULL;
08959         __pyx_L0:;
08960         __Pyx_XGIVEREF(__pyx_r);
08961         __Pyx_RefNannyFinishContext();
08962         return __pyx_r;
08963     }
08964
08965     /* "PyClicl.pyx":806
08966          *
08967          *
08968          *
08969          * def __mod__(lhs, rhs):
08970          *
08971          *
08972          *
08973          */
08974         /* Python wrapper */
08975         static PyObject *__pyx_pw_8PyClicl_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
08976 *__pyx_v_rhs); /*proto*/
08977         static char __pyx_doc_8PyClicl_8clifford_32__mod__[] = "\n          Contraction.\n\n          >>
08978 print(clifford(\\"{1}\\") % clifford(\\"{2}\\"))\n          0\n          >> print(clifford(2) %
08979 clifford(\\"{2}\\"))\n          2\n          >> print(clifford(\\"{1}\\") % clifford(\\"{1}\\"))\n
08980 1\n          >> print(clifford(\\"{1}\\") % clifford(\\"{1,2}\\"))\n          {2}\n          ";
08981         #if CYTHON_COMPILING_IN_CPYTHON
08982         struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_32__mod__;
08983         #endif
08984         static PyObject *__pyx_pw_8PyClicl_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
08985 *__pyx_v_rhs) {
08986         PyObject *__pyx_r = 0;
08987         __Pyx_RefNannyDeclarations
08988         __Pyx_RefNannySetupContext("__mod__ (wrapper)", 0);
08989         __pyx_r = __pyx_pf_8PyClicl_8clifford_32__mod__((PyObject *)__pyx_v_lhs), ((PyObject
08990 *)__pyx_v_rhs));
08991
08992         /* function exit code */
08993         __Pyx_RefNannyFinishContext();
08994         return __pyx_r;
08995     }
08996
08997     static PyObject *__pyx_pf_8PyClicl_8clifford_32__mod__(PyObject *__pyx_v_lhs, PyObject
08998 *__pyx_v_rhs) {
08999         PyObject *__pyx_r = NULL;
09000         __Pyx_RefNannyDeclarations
09001         PyObject *__pyx_t_1 = NULL;
09002         PyObject *__pyx_t_2 = NULL;
09003         int __pyx_lineno = 0;

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```

08997         const char *__pyx_filename = NULL;
08998         int __pyx_clineno = 0;
08999         __Pyx_RefNannySetupContext("__mod__", 0);
09000
09001         /* "PyClicl.pyx":819
09002         {
09003         """
09004         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )           # ««««««««
09005         *
09006         def __imod__(self, rhs):
09007         */
09008         __Pyx_XDECREF(__pyx_r);
09009         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 819, __pyx_L1_error)
09010         __Pyx_GOTREF(__pyx_t_1);
09011         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) %
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 819,
__pyx_L1_error)
09012         __Pyx_GOTREF(__pyx_t_2);
09013         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09014         __pyx_r = __pyx_t_2;
09015         __pyx_t_2 = 0;
09016         goto __pyx_L0;
09017
09018         /* "PyClicl.pyx":806
09019         return self.wrap( self.unwrap() * toClifford(rhs) )
09020         *
09021         def __mod__(lhs, rhs):           # ««««««««
09022         """
09023         Contraction.
09024         */
09025
09026         /* function exit code */
09027         __pyx_L1_error:;
09028         __Pyx_XDECREF(__pyx_t_1);
09029         __Pyx_XDECREF(__pyx_t_2);
09030         __Pyx_AddTraceback("PyClicl.clifford.__mod__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09031         __pyx_r = NULL;
09032         __pyx_L0:;
09033         __Pyx_XGIVEREF(__pyx_r);
09034         __Pyx_RefNannyFinishContext();
09035         return __pyx_r;
09036     }
09037
09038     /* "PyClicl.pyx":821
09039     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
09040     *
09041     def __imod__(self, rhs):           # ««««««««
09042     """
09043     Contraction.
09044     */
09045
09046     /* Python wrapper */
09047     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09048     static char __pyx_doc_8PyClicl_8clifford_34__imod__[] = "\n          Contraction.\n\n          >> x = clifford(2); x %="
x = clifford("\{1}\"); x %= clifford("\{2}\"); print(x)\n          0\n          >> x = clifford(2); x %="
clifford("\{2}\"); print(x)\n          2{2}\n          >> x = clifford("\{1}\"); x %= clifford("\{1}\");
print(x)\n          1\n          >> x = clifford("\{1}\"); x %= clifford("\{1,2}\"); print(x)\n
{2}\n          ";
09049     #if CYTHON_COMPILING_IN_CPYTHON
09050     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_34__imod__;
09051     #endif
09052     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09053         PyObject *__pyx_r = 0;
09054         __Pyx_RefNannyDeclarations
09055         __Pyx_RefNannySetupContext("__imod__ (wrapper)", 0);
09056         __pyx_r = __pyx_pf_8PyClicl_8clifford_34__imod__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09057
09058         /* function exit code */
09059         __Pyx_RefNannyFinishContext();
09060         return __pyx_r;
09061     }
09062
09063     static PyObject *__pyx_pf_8PyClicl_8clifford_34__imod__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09064         PyObject *__pyx_r = NULL;
09065         __Pyx_RefNannyDeclarations
09066         PyObject *__pyx_t_1 = NULL;
09067         int __pyx_lineno = 0;
09068         const char *__pyx_filename = NULL;
09069         int __pyx_clineno = 0;
09070         __Pyx_RefNannySetupContext("__imod__", 0);

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09148     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 849, __pyx_L1_error)
09149     __Pyx_GOTREF(__pyx_t_1);
09150     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) &
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 849,
__pyx_L1_error)
09151     __Pyx_GOTREF(__pyx_t_2);
09152     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09153     __pyx_r = __pyx_t_2;
09154     __pyx_t_2 = 0;
09155     goto __pyx_L0;
09156
09157     /* "PyClical.pyx":836
09158     *         return self.wrap( self.unwrap() % toClifford(rhs) )
09159     *
09160     *     def __and__(lhs, rhs):
09161     *         """
09162     *         Inner product.
09163     */
09164
09165     /* function exit code */
09166     __pyx_L1_error;
09167     __Pyx_XDECREF(__pyx_t_1);
09168     __Pyx_XDECREF(__pyx_t_2);
09169     __Pyx_AddTraceback("PyClical.clifford.__and__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09170     __pyx_r = NULL;
09171     __pyx_L0;
09172     __Pyx_XGIVEREF(__pyx_r);
09173     __Pyx_RefNannyFinishContext();
09174     return __pyx_r;
09175 }
09176
09177     /* "PyClical.pyx":851
09178     *         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09179     *
09180     *     def __iand__(self, rhs):
09181     *         """
09182     *         Inner product.
09183     */
09184
09185     /* Python wrapper */
09186     static PyObject *__pyx_pw_8PyClical_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09187     static char __pyx_doc_8PyClical_8clifford_38__iand__[] = "\n        Inner product.\n\n
>> x = clifford('{1}'); x &= clifford('{2}'); print(x)\n        0\n        >> x = clifford(2); x
&= clifford('{2}'); print(x)\n        0\n        >> x = clifford('{1}'); x &= clifford('{1}');
print(x)\n        1\n        >> x = clifford('{1}'); x &= clifford('{1,2}'); print(x)\n
{2}\n        ";
09188     #if CYTHON_COMPILING_IN_CPYTHON
09189     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_38__iand__;
09190     #endif
09191     static PyObject *__pyx_pw_8PyClical_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09192         PyObject *__pyx_r = 0;
09193         __Pyx_RefNannyDeclarations
09194         __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
09195         __pyx_r = __pyx_pf_8PyClical_8clifford_38__iand__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09196
09197         /* function exit code */
09198         __Pyx_RefNannyFinishContext();
09199         return __pyx_r;
09200     }
09201
09202     static PyObject *__pyx_pf_8PyClical_8clifford_38__iand__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09203         PyObject *__pyx_r = NULL;
09204         __Pyx_RefNannyDeclarations
09205         PyObject *__pyx_t_1 = NULL;
09206         int __pyx_lineno = 0;
09207         const char *__pyx_filename = NULL;
09208         int __pyx_clineno = 0;
09209         __Pyx_RefNannySetupContext("__iand__", 0);
09210
09211         /* "PyClical.pyx":864
09212     *     {2}
09213     *     """
09214     *         return self.wrap( self.unwrap() & toClifford(rhs) )
09215     *
09216     *     def __xor__(lhs, rhs):
09217     */
09218         __Pyx_XDECREF(__pyx_r);
09219         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) & __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 864, __pyx_L1_error)

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09220     __Pyx_GOTREF(__pyx_t_1);
09221     __pyx_r = __pyx_t_1;
09222     __pyx_t_1 = 0;
09223     goto __pyx_L0;
09224
09225     /* "PyClical.pyx":851
09226  *     return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09227  *
09228  *     def __iand__(self, rhs):           # ««««««««
09229  *         """
09230  *         Inner product.
09231  */
09232
09233     /* function exit code */
09234     __pyx_L1_error:;
09235     __Pyx_XDECREF(__pyx_t_1);
09236     __Pyx_AddTraceback("PyClical.clifford.__iand__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09237     __pyx_r = NULL;
09238     __pyx_L0:;
09239     __Pyx_XGIVEREF(__pyx_r);
09240     __Pyx_RefNannyFinishContext();
09241     return __pyx_r;
09242 }
09243
09244     /* "PyClical.pyx":866
09245  *     return self.wrap( self.unwrap() & toClifford(rhs) )
09246  *
09247  *     def __xor__(lhs, rhs):           # ««««««««
09248  *         """
09249  *         Outer product.
09250  */
09251
09252     /* Python wrapper */
09253     static PyObject *__pyx_pw_8PyClical_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09254     static char __pyx_doc_8PyClical_8clifford_40__xor__[] = "\n        Outer product.\n\n
>> print(clifford(\"{1}\") ^ clifford(\"{2}\"))\n        {1,2}\n        >> print(clifford(2) ^
clifford(\"{2}\"))\n        2{2}\n        >> print(clifford(\"{1}\") ^ clifford(\"{1}\"))\n
0\n        >> print(clifford(\"{1}\") ^ clifford(\"{1,2}\"))\n        0\n        ";
09255     #if CYTHON_COMPILING_IN_CPYTHON
09256     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_40__xor__;
09257     #endif
09258     static PyObject *__pyx_pw_8PyClical_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09259         PyObject *__pyx_r = 0;
09260         __Pyx_RefNannyDeclarations
09261         __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
09262         __pyx_r = __pyx_pf_8PyClical_8clifford_40__xor__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09263
09264     /* function exit code */
09265     __Pyx_RefNannyFinishContext();
09266     return __pyx_r;
09267 }
09268
09269     static PyObject *__pyx_pf_8PyClical_8clifford_40__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09270         PyObject *__pyx_r = NULL;
09271         __Pyx_RefNannyDeclarations
09272         PyObject *__pyx_t_1 = NULL;
09273         PyObject *__pyx_t_2 = NULL;
09274         int __pyx_lineno = 0;
09275         const char *__pyx_filename = NULL;
09276         int __pyx_clineno = 0;
09277         __Pyx_RefNannySetupContext("__xor__", 0);
09278
09279     /* "PyClical.pyx":879
09280  *     0
09281  *     """
09282  *     return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )           # ««««««««
09283  *
09284  *     def __ixor__(self, rhs):
09285  */
09286     __Pyx_XDECREF(__pyx_r);
09287     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 879, __pyx_L1_error)
09288     __Pyx_GOTREF(__pyx_t_1);
09289     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) ^
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 879,
__pyx_L1_error)
09290     __Pyx_GOTREF(__pyx_t_2);
09291     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09292     __pyx_r = __pyx_t_2;
09293     __pyx_t_2 = 0;
09294     goto __pyx_L0;

```

```

09295
09296         /* "PyClicl.pyx":866
09297         *         return self.wrap( self.unwrap() & toClifford(rhs) )
09298         *
09299         *         def __xor__(lhs, rhs):           # ««««««««
09300         *         """
09301         *         Outer product.
09302         */
09303
09304         /* function exit code */
09305         __pyx_L1_error:;
09306         __Pyx_XDECREF(__pyx_t_1);
09307         __Pyx_XDECREF(__pyx_t_2);
09308         __Pyx_AddTraceback("PyClicl.clifford.__xor__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09309         __pyx_r = NULL;
09310         __pyx_L0:;
09311         __Pyx_XGIVEREF(__pyx_r);
09312         __Pyx_RefNannyFinishContext();
09313         return __pyx_r;
09314     }
09315
09316     /* "PyClicl.pyx":881
09317     *         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09318     *
09319     *         def __ixor__(self, rhs):           # ««««««««
09320     *         """
09321     *         Outer product.
09322     */
09323
09324     /* Python wrapper */
09325     static PyObject* __pyx_pw_8PyClicl_8clifford_43__ixor__(PyObject* __pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09326     static char __pyx_doc_8PyClicl_8clifford_42__ixor__[] = "\n        Outer product.\n\n
>> x = clifford('{1}\n'); x ^= clifford('{2}\n'); print(x)\n        {1,2}\n        >> x =
clifford(2); x ^= clifford('{2}\n'); print(x)\n                2{2}\n        >> x = clifford('{1}\n'); x ^=
clifford('{1}\n'); print(x)\n                0\n        >> x = clifford('{1}\n'); x ^= clifford('{1,2}\n');
print(x)\n                0\n        ";
09327     #if CYTHON_COMPILING_IN_CPYTHON
09328     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_42__ixor__;
09329     #endif
09330     static PyObject* __pyx_pw_8PyClicl_8clifford_43__ixor__(PyObject* __pyx_v_self, PyObject
*__pyx_v_rhs) {
09331         PyObject* __pyx_r = 0;
09332         __Pyx_RefNannyDeclarations
09333         __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
09334         __pyx_r = __pyx_pf_8PyClicl_8clifford_42__ixor__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject*)__pyx_v_rhs));
09335
09336         /* function exit code */
09337         __Pyx_RefNannyFinishContext();
09338         return __pyx_r;
09339     }
09340
09341     static PyObject* __pyx_pf_8PyClicl_8clifford_42__ixor__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject* __pyx_v_rhs) {
09342         PyObject* __pyx_r = NULL;
09343         __Pyx_RefNannyDeclarations
09344         PyObject* __pyx_t_1 = NULL;
09345         int __pyx_lineno = 0;
09346         const char* __pyx_filename = NULL;
09347         int __pyx_clineno = 0;
09348         __Pyx_RefNannySetupContext("__ixor__", 0);
09349
09350         /* "PyClicl.pyx":894
09351         *         0
09352         *         """
09353         *         return self.wrap( self.unwrap() ^ toClifford(rhs) )           # ««««««««
09354         *
09355         *         def __truediv__(lhs, rhs):
09356         */
09357         __Pyx_XDECREF(__pyx_r);
09358         __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) ^ __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 894, __pyx_L1_error)
09359         __Pyx_GOTREF(__pyx_t_1);
09360         __pyx_r = __pyx_t_1;
09361         __pyx_t_1 = 0;
09362         goto __pyx_L0;
09363
09364         /* "PyClicl.pyx":881
09365         *         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09366         *
09367         *         def __ixor__(self, rhs):           # ««««««««
09368         *         """
09369         *         Outer product.
09370         */

```



```

09371
09372     /* function exit code */
09373     __pyx_L1_error++;
09374     __Pyx_XDECREF(__pyx_t_1);
09375     __Pyx_AddTraceback("PyClical.clifford.__ixor__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09376     __pyx_r = NULL;
09377     __pyx_L0;
09378     __Pyx_XGIVEREF(__pyx_r);
09379     __Pyx_RefNannyFinishContext();
09380     return __pyx_r;
09381 }
09382
09383     /* "PyClical.pyx":896
09384     *     return self.wrap( self.unwrap() ^ toClifford(rhs) )
09385     *
09386     * def __truediv__(lhs, rhs):
09387     *     """
09388     *     Geometric quotient.
09389     */
09390
09391     /* Python wrapper */
09392     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09393     static char __pyx_doc_8PyClical_8clifford_44__truediv__[] = "\n          Geometric quotient.\n\n
>> print(clifford(\"{1}\") / clifford(\"{2}\"))\n          {1,2}\n          >> print(clifford(2) /
clifford(\"{2}\"))\n          2{2}\n          >> print(clifford(\"{1}\") / clifford(\"{1}\"))\n
1\n          >> print(clifford(\"{1}\") / clifford(\"{1,2}\"))\n          -{2}\n          ";
09394     #if CYTHON_COMPILING_IN_CPYTHON
09395     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_44__truediv__;
09396     #endif
09397     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09398     PyObject *__pyx_r = 0;
09399     __Pyx_RefNannyDeclarations
09400     __Pyx_RefNannySetupContext("__truediv__ (wrapper)", 0);
09401     __pyx_r = __pyx_pf_8PyClical_8clifford_44__truediv__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09402
09403     /* function exit code */
09404     __Pyx_RefNannyFinishContext();
09405     return __pyx_r;
09406 }
09407
09408     static PyObject *__pyx_pf_8PyClical_8clifford_44__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09409     PyObject *__pyx_r = NULL;
09410     __Pyx_RefNannyDeclarations
09411     PyObject *__pyx_t_1 = NULL;
09412     PyObject *__pyx_t_2 = NULL;
09413     int __pyx_lineno = 0;
09414     const char *__pyx_filename = NULL;
09415     int __pyx_clineno = 0;
09416     __Pyx_RefNannySetupContext("__truediv__", 0);
09417
09418     /* "PyClical.pyx":909
09419     *     -{2}
09420     *
09421     *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09422     *
09423     * def __idiv__(self, rhs):
09424     */
09425     __Pyx_XDECREF(__pyx_r);
09426     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 909, __pyx_L1_error)
09427     __Pyx_GOTREF(__pyx_t_1);
09428     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) /
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 909,
__pyx_L1_error)
09429     __Pyx_GOTREF(__pyx_t_2);
09430     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09431     __pyx_r = __pyx_t_2;
09432     __pyx_t_2 = 0;
09433     goto __pyx_L0;
09434
09435     /* "PyClical.pyx":896
09436     *     return self.wrap( self.unwrap() ^ toClifford(rhs) )
09437     *
09438     * def __truediv__(lhs, rhs):
09439     *     """
09440     *     Geometric quotient.
09441     */
09442
09443     /* function exit code */
09444     __pyx_L1_error++;
09445     __Pyx_XDECREF(__pyx_t_1);

```

```

09446     __Pyx_XDECREF(__pyx_t_2);
09447     __Pyx_AddTraceback("PyClical.clifford.__truediv__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09448     __pyx_r = NULL;
09449     __pyx_L0;
09450     __Pyx_XGIVEREF(__pyx_r);
09451     __Pyx_RefNannyFinishContext();
09452     return __pyx_r;
09453 }
09454
09455     /* "PyClical.pyx":911
09456     *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09457     *
09458     * def __idiv__(self, rhs):                                # ««««««««
09459     *     """
09460     *     Geometric quotient.
09461     */
09462
09463     /* Python wrapper */
09464     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09465     static PyObject * __pyx_pw_8PyClical_8clifford_47__idiv__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs); /*proto*/
09466     static char __pyx_doc_8PyClical_8clifford_46__idiv__[] = "\n        Geometric quotient.\n\n
>> x = clifford(\"{1}\"); x /= clifford(\"{2}\"); print(x)\n        {1,2}\n        >> x =
clifford(2); x /= clifford(\"{2}\"); print(x)\n        2{2}\n        >> x = clifford(\"{1}\"); x /=
clifford(\"{1}\"); print(x)\n        1\n        >> x = clifford(\"{1}\"); x /= clifford(\"{1,2}\");
print(x)\n        -{2}\n        ";
09467     #if CYTHON_COMPILING_IN_CPYTHON
09468     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_46__idiv__;
09469     #endif
09470     static PyObject * __pyx_pw_8PyClical_8clifford_47__idiv__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs) {
09471         PyObject * __pyx_r = 0;
09472         __Pyx_RefNannyDeclarations
09473         __Pyx_RefNannySetupContext("__idiv__ (wrapper)", 0);
09474         __pyx_r = __pyx_pf_8PyClical_8clifford_46__idiv__((struct __pyx_obj_8PyClical_clifford
*) __pyx_v_self), ((PyObject *) __pyx_v_rhs));
09475
09476         /* function exit code */
09477         __Pyx_RefNannyFinishContext();
09478         return __pyx_r;
09479     }
09480     #endif
09481     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09482     static PyObject * __pyx_pf_8PyClical_8clifford_46__idiv__(struct __pyx_obj_8PyClical_clifford
* __pyx_v_self, PyObject * __pyx_v_rhs) {
09483         PyObject * __pyx_r = NULL;
09484         __Pyx_RefNannyDeclarations
09485         PyObject * __pyx_t_1 = NULL;
09486         int __pyx_lineno = 0;
09487         const char * __pyx_filename = NULL;
09488         int __pyx_clineno = 0;
09489         __Pyx_RefNannySetupContext("__idiv__", 0);
09490
09491         /* "PyClical.pyx":924
09492         *     -{2}
09493         *     """
09494         *     return self.wrap( self.unwrap() / toClifford(rhs) )                # ««««««««
09495         *
09496         * def inv(self):
09497         */
09498         __Pyx_XDECREF(__pyx_r);
09499         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) / __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 924, __pyx_L1_error)
09500         __Pyx_GOTREF(__pyx_t_1);
09501         __pyx_r = __pyx_t_1;
09502         __pyx_t_1 = 0;
09503         goto __pyx_L0;
09504
09505         /* "PyClical.pyx":911
09506         *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09507         *
09508         * def __idiv__(self, rhs):                                # ««««««««
09509         *     """
09510         *     Geometric quotient.
09511         */
09512
09513         /* function exit code */
09514         __pyx_L1_error;
09515         __Pyx_XDECREF(__pyx_t_1);
09516         __Pyx_AddTraceback("PyClical.clifford.__idiv__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09517         __pyx_r = NULL;
09518         __pyx_L0;
09519         __Pyx_XGIVEREF(__pyx_r);
09520         __Pyx_RefNannyFinishContext();
09521

```

```

09522         return __pyx_r;
09523     }
09524 #endif
09525 /* "PyClical.pyx":926
09526 *     return self.wrap( self.unwrap() / toClifford(rhs) )
09527 *
09528 *
09529 *     def inv(self):          # ««««««««
09530 *         """
09531 *         Geometric multiplicative inverse.
09532 */
09533
09534 /* Python wrapper */
09535 static PyObject *__pyx_pw_8PyClical_8clifford_49inv(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
09536 static char __pyx_doc_8PyClical_8clifford_48inv[] = "\n          Geometric multiplicative
inverse.\n\n          >> x = clifford(\"{1}\"); print(x.inv())\n          {1}\n          >> x =
clifford(2); print(x.inv())\n          0.5\n          >> x = clifford(\"{1,2}\"); print(x.inv())\n
-{1,2}\n          ";
09537 static PyObject *__pyx_pw_8PyClical_8clifford_49inv(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
09538     PyObject *__pyx_r = 0;
09539     __Pyx_RefNannyDeclarations
09540     __Pyx_RefNannySetupContext("inv (wrapper)", 0);
09541     __pyx_r = __pyx_pf_8PyClical_8clifford_48inv(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
09542
09543     /* function exit code */
09544     __Pyx_RefNannyFinishContext();
09545     return __pyx_r;
09546 }
09547
09548 static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
09549     PyObject *__pyx_r = NULL;
09550     __Pyx_RefNannyDeclarations
09551     PyObject *__pyx_t_1 = NULL;
09552     PyObject *__pyx_t_2 = NULL;
09553     int __pyx_lineno = 0;
09554     const char *__pyx_filename = NULL;
09555     int __pyx_clineno = 0;
09556     __Pyx_RefNannySetupContext("inv", 0);
09557
09558     /* "PyClical.pyx":937
09559 *     -{1,2}
09560 *     """
09561 *     return clifford().wrap( self.instance.inv() )          # ««««««««
09562 *
09563 *     def __or__(lhs, rhs):
09564 */
09565     __Pyx_XDECREF(__pyx_r);
09566     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 937, __pyx_l1_error)
09567     __Pyx_GOTREF(__pyx_t_1);
09568     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1, __pyx_v_self->instance->inv()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 937,
__pyx_l1_error)
09569     __Pyx_GOTREF(__pyx_t_2);
09570     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09571     __pyx_r = __pyx_t_2;
09572     __pyx_t_2 = 0;
09573     goto __pyx_l0;
09574
09575     /* "PyClical.pyx":926
09576 *     return self.wrap( self.unwrap() / toClifford(rhs) )
09577 *
09578 *
09579 *     def inv(self):          # ««««««««
09580 *         """
09581 *         Geometric multiplicative inverse.
09582 */
09583
09584     /* function exit code */
09585     __pyx_l1_error;
09586     __Pyx_XDECREF(__pyx_t_1);
09587     __Pyx_XDECREF(__pyx_t_2);
09588     __Pyx_AddTraceback("PyClical.clifford.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
09589     __pyx_r = NULL;
09590     __pyx_l0;
09591     __Pyx_XGIVEREF(__pyx_r);
09592     __Pyx_RefNannyFinishContext();
09593     return __pyx_r;
09594 }
09595
09596 /* "PyClical.pyx":939
09597 *     return clifford().wrap( self.instance.inv() )
09598 *
09599 *
09600 *     def __or__(lhs, rhs):          # ««««««««
09601 *         """

```

```

09600 *          Transform left hand side, using right hand side as a transformation.
09601 */
09602
09603     /* Python wrapper */
09604     static PyObject *__pyx_pw_8PyClical_8clifford_51_or__(PyObject *__pyx_v_lhs, PyObject
09605 *__pyx_v_rhs); /*proto*/
09606     static char __pyx_doc_8PyClical_8clifford_50_or__[] = "\n          Transform left hand side,
using right hand side as a transformation.\n\n          >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|x)\n          -{1}\n          >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|exp(x))\n          -{1}\n          ";
09606     #if CYTHON_COMPILING_IN_CPYTHON
09607     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_50_or__;
09608     #endif
09609     static PyObject *__pyx_pw_8PyClical_8clifford_51_or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09610         PyObject *__pyx_r = 0;
09611         __Pyx_RefNannyDeclarations
09612         __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);
09613         __pyx_r = __pyx_pf_8PyClical_8clifford_50_or__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09614
09615         /* function exit code */
09616         __Pyx_RefNannyFinishContext();
09617         return __pyx_r;
09618     }
09619
09620     static PyObject *__pyx_pf_8PyClical_8clifford_50_or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09621         PyObject *__pyx_r = NULL;
09622         __Pyx_RefNannyDeclarations
09623         PyObject *__pyx_t_1 = NULL;
09624         PyObject *__pyx_t_2 = NULL;
09625         int __pyx_lineno = 0;
09626         const char *__pyx_filename = NULL;
09627         int __pyx_clineno = 0;
09628         __Pyx_RefNannySetupContext("__or__", 0);
09629
09630         /* "PyClical.pyx":948
09631 *          -{1}
09632 *          """
09633 *          return clifford().wrap( toClifford(lhs) | toClifford(rhs) )          # ««««««««
09634 *
09635 *          def __ior__(self, rhs):
09636 */
09637         __Pyx_XDECREF(__pyx_r);
09638         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 948, __pyx_L1_error)
09639         __Pyx_GOTREF(__pyx_t_1);
09640         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) |
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 948,
__pyx_L1_error)
09641         __Pyx_GOTREF(__pyx_t_2);
09642         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09643         __pyx_r = __pyx_t_2;
09644         __pyx_t_2 = 0;
09645         goto __pyx_L0;
09646
09647         /* "PyClical.pyx":939
09648 *          return clifford().wrap( self.instance.inv() )
09649 *
09650 *          def __or__(lhs, rhs):          # ««««««««
09651 *          """
09652 *          Transform left hand side, using right hand side as a transformation.
09653 */
09654
09655         /* function exit code */
09656         __pyx_L1_error:;
09657         __Pyx_XDECREF(__pyx_t_1);
09658         __Pyx_XDECREF(__pyx_t_2);
09659         __Pyx_AddTraceback("PyClical.clifford.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
09660         __pyx_r = NULL;
09661         __pyx_L0:;
09662         __Pyx_XGIVEREF(__pyx_r);
09663         __Pyx_RefNannyFinishContext();
09664         return __pyx_r;
09665     }
09666
09667     /* "PyClical.pyx":950
09668 *          return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09669 *
09670 *          def __ior__(self, rhs):          # ««««««««
09671 *          """
09672 *          Transform left hand side, using right hand side as a transformation.
09673 */
09674
09675     /* Python wrapper */

```

```

09676     static PyObject * __pyx_pw_8PyClical_8clifford_53__ior__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs); /*proto*/
09677     static char __pyx_doc_8PyClical_8clifford_52__ior__[] = "\n          Transform left hand side,
using right hand side as a transformation.\n\n          >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); y|=x; print(y)\n          -{1}\n          >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); y|=exp(x); print(y)\n          -{1}\n          ";
09678     #if CYTHON_COMPILING_IN_CPYTHON
09679     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_52__ior__;
09680     #endif
09681     static PyObject * __pyx_pw_8PyClical_8clifford_53__ior__(PyObject * __pyx_v_self, PyObject
* __pyx_v_rhs) {
09682         PyObject * __pyx_r = 0;
09683         __Pyx_RefNannyDeclarations
09684         __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
09685         __pyx_r = __pyx_pf_8PyClical_8clifford_52__ior__(((struct __pyx_obj_8PyClical_clifford
*) __pyx_v_self), ((PyObject *) __pyx_v_rhs));
09686
09687         /* function exit code */
09688         __Pyx_RefNannyFinishContext();
09689         return __pyx_r;
09690     }
09691
09692     static PyObject * __pyx_pf_8PyClical_8clifford_52__ior__(struct __pyx_obj_8PyClical_clifford
* __pyx_v_self, PyObject * __pyx_v_rhs) {
09693         PyObject * __pyx_r = NULL;
09694         __Pyx_RefNannyDeclarations
09695         PyObject * __pyx_t_1 = NULL;
09696         int __pyx_lineno = 0;
09697         const char * __pyx_filename = NULL;
09698         int __pyx_clineno = 0;
09699         __Pyx_RefNannySetupContext("__ior__", 0);
09700
09701         /* "PyClical.pyx":959
09702         *     -{1}
09703         *     """
09704         *     return self.wrap( self.unwrap() | toClifford(rhs) )
09705         *
09706         *     def __pow__(self, m, dummy):
09707         */
09708         __Pyx_XDECREF(__pyx_r);
09709         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) | __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 959, __pyx_L1_error)
09710         __Pyx_GOTREF(__pyx_t_1);
09711         __pyx_r = __pyx_t_1;
09712         __pyx_t_1 = 0;
09713         goto __pyx_L0;
09714
09715         /* "PyClical.pyx":950
09716         *     return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09717         *
09718         *     def __ior__(self, rhs):
09719         *         """
09720         *         Transform left hand side, using right hand side as a transformation.
09721         */
09722         /* function exit code */
09723         __pyx_L1_error:;
09724         __Pyx_XDECREF(__pyx_t_1);
09725         __Pyx_AddTraceback("PyClical.clifford.__ior__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09726         __pyx_r = NULL;
09727         __pyx_L0:;
09728         __Pyx_XGIVEREF(__pyx_r);
09729         __Pyx_RefNannyFinishContext();
09730         return __pyx_r;
09731     }
09732
09733
09734     /* "PyClical.pyx":961
09735     *     return self.wrap( self.unwrap() | toClifford(rhs) )
09736     *
09737     *     def __pow__(self, m, dummy):
09738     *         """
09739     *         Power: self to the m.
09740     */
09741
09742     /* Python wrapper */
09743     static PyObject * __pyx_pw_8PyClical_8clifford_55__pow__(PyObject * __pyx_v_self, PyObject
* __pyx_v_m, PyObject * __pyx_v_dummy); /*proto*/
09744     static char __pyx_doc_8PyClical_8clifford_54__pow__[] = "\n          Power: self to the m.\n\n
>> x=clifford(\"{1}\"); print(x ** 2)\n          1\n          >> x=clifford(\"2\"); print(x ** 2)\n
4\n          >> x=clifford(\"2+{1}\"); print(x ** 0)\n          1\n          >>
x=clifford(\"2+{1}\"); print(x ** 1)\n          2+{1}\n          >> x=clifford(\"2+{1}\"); print(x **
2)\n          5+4{1}\n          >> i=clifford(\"{1,2}\"); print(exp(pi/2) * (i ** i))\n          1\n
";
09745     #if CYTHON_COMPILING_IN_CPYTHON
09746     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_54__pow__;

```

```

09747         #endif
09748         static PyObject *__pyx_pw_8PyClical_8clifford_55_pow__(PyObject *__pyx_v_self, PyObject
*__pyx_v_m, PyObject *__pyx_v_dummy) {
09749             PyObject *__pyx_r = 0;
09750             __Pyx_RefNannyDeclarations
09751             __Pyx_RefNannySetupContext("__pow__ (wrapper)", 0);
09752             __pyx_r = __pyx_pf_8PyClical_8clifford_54_pow__((PyObject *)__pyx_v_self), ((PyObject
*)__pyx_v_m), ((PyObject *)__pyx_v_dummy));
09753
09754             /* function exit code */
09755             __Pyx_RefNannyFinishContext();
09756             return __pyx_r;
09757         }
09758
09759         static PyObject *__pyx_pf_8PyClical_8clifford_54_pow__(PyObject *__pyx_v_self, PyObject
*__pyx_v_m, CYTHON_UNUSED PyObject *__pyx_v_dummy) {
09760             PyObject *__pyx_r = NULL;
09761             __Pyx_RefNannyDeclarations
09762             PyObject *__pyx_t_1 = NULL;
09763             int __pyx_lineno = 0;
09764             const char *__pyx_filename = NULL;
09765             int __pyx_clineno = 0;
09766             __Pyx_RefNannySetupContext("__pow__", 0);
09767
09768             /* "PyClical.pyx":978
09769             *
09770             *     """
09771             *         return pow(self, m)                # ««««««««
09772             *
09773             *     def pow(self, m):
09774             */
09775             __Pyx_XDECREF(__pyx_r);
09776             __pyx_t_1 = __pyx_pf_8PyClical_pow(__pyx_v_self, __pyx_v_m, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 978, __pyx_L1_error)
09777             __Pyx_GOTREF(__pyx_t_1);
09778             __pyx_r = __pyx_t_1;
09779             __pyx_t_1 = 0;
09780             goto __pyx_L0;
09781
09782             /* "PyClical.pyx":961
09783             *         return self.wrap( self.unwrap() | toClifford(rhs) )
09784             *
09785             *     def __pow__(self, m, dummy):
09786             *         """
09787             *         Power: self to the m.
09788             */
09789
09790             /* function exit code */
09791             __pyx_L1_error:;
09792             __Pyx_XDECREF(__pyx_t_1);
09793             __Pyx_AddTraceback("PyClical.clifford.__pow__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09794             __pyx_r = NULL;
09795             __pyx_L0:;
09796             __Pyx_XGIVEREF(__pyx_r);
09797             __Pyx_RefNannyFinishContext();
09798             return __pyx_r;
09799         }
09800
09801             /* "PyClical.pyx":980
09802             *         return pow(self, m)
09803             *
09804             *     def pow(self, m):
09805             *         """
09806             *         Power: self to the m.
09807             */
09808
09809             /* Python wrapper */
09810             static PyObject *__pyx_pw_8PyClical_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m); /*proto*/
09811             static char __pyx_doc_8PyClical_8clifford_56pow[] = "\n                Power: self to the m.\n\n
>> x=clifford(\"{1}\"); print(x.pow(2))\n                1\n                >> x=clifford(\"2\"); print(x.pow(2))\n
4\n                >> x=clifford(\"2+{1}\"); print(x.pow(0))\n                1\n                >>
x=clifford(\"2+{1}\"); print(x.pow(1))\n                2+{1}\n                >> x=clifford(\"2+{1}\");
print(x.pow(2))\n                5+4{1}\n                >> print(clifford(\"1+{1}+{1,2}\").pow(3))\n
1+3{1}+3{1,2}\n                >> i=clifford(\"{1,2}\"); print(exp(pi/2) * i.pow(i))\n                1\n
";
09812             static PyObject *__pyx_pw_8PyClical_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m) {
09813                 PyObject *__pyx_r = 0;
09814                 __Pyx_RefNannyDeclarations
09815                 __Pyx_RefNannySetupContext("pow (wrapper)", 0);
09816                 __pyx_r = __pyx_pf_8PyClical_8clifford_56pow(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_m));
09817
09818                 /* function exit code */
09819                 __Pyx_RefNannyFinishContext();
09820                 return __pyx_r;

```

```

09821     }
09822
09823     static PyObject * __pyx_pf_8PyClical_8clifford_56pow(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject * __pyx_v_m) {
09824         PyObject * __pyx_r = NULL;
09825         __Pyx_RefNannyDeclarations
09826         PyObject * __pyx_t_1 = NULL;
09827         PyObject * __pyx_t_2 = NULL;
09828         int __pyx_t_3;
09829         int __pyx_t_4;
09830         int __pyx_t_5;
09831         int __pyx_lineno = 0;
09832         const char * __pyx_filename = NULL;
09833         int __pyx_clineno = 0;
09834         __Pyx_RefNannySetupContext("pow", 0);
09835
09836         /* "PyClical.pyx":999
09837         *
09838         *
09839         * if isinstance(m, numbers.Integral): # ««««««««
09840         *     return clifford().wrap( self.instance.pow(m) )
09841         * else:
09842         */
09843         __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_numbers); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 999, __pyx_L1_error)
09844         __Pyx_GOTREF(__pyx_t_1);
09845         __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_t_1, __pyx_n_s_Integral); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 999, __pyx_L1_error)
09846         __Pyx_GOTREF(__pyx_t_2);
09847         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09848         __pyx_t_3 = PyObject_IsInstance(__pyx_v_m, __pyx_t_2); if (unlikely(__pyx_t_3 == ((int)-1)))
__PYX_ERR(0, 999, __pyx_L1_error)
09849         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09850         __pyx_t_4 = (__pyx_t_3 != 0);
09851         if (__pyx_t_4) {
09852
09853             /* "PyClical.pyx":1000
09854             *
09855             * if isinstance(m, numbers.Integral):
09856             *     return clifford().wrap( self.instance.pow(m) ) # ««««««««
09857             * else:
09858             *     return exp(m * log(self))
09859             */
09860             __Pyx_XDECREF(__pyx_r);
09861             __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1000, __pyx_L1_error)
09862             __Pyx_GOTREF(__pyx_t_2);
09863             __pyx_t_5 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_5 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1000, __pyx_L1_error)
09864             __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_2), __pyx_v_self->instance->pow(__pyx_t_5)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1000,
__pyx_L1_error)
09865             __Pyx_GOTREF(__pyx_t_1);
09866             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09867             __pyx_r = __pyx_t_1;
09868             __pyx_t_1 = 0;
09869             goto __pyx_L0;
09870
09871             /* "PyClical.pyx":999
09872             *
09873             *
09874             * if isinstance(m, numbers.Integral): # ««««««««
09875             *     return clifford().wrap( self.instance.pow(m) )
09876             * else:
09877             */
09878         }
09879
09880         /* "PyClical.pyx":1002
09881         *     return clifford().wrap( self.instance.pow(m) )
09882         * else:
09883         *     return exp(m * log(self)) # ««««««««
09884         *
09885         * def outer_pow(self, m):
09886         */
09887         /*else*/ {
09888             __Pyx_XDECREF(__pyx_r);
09889             __pyx_t_1 = __pyx_f_8PyClical_log(((PyObject *) __pyx_v_self), 0, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1002, __pyx_L1_error)
09890             __Pyx_GOTREF(__pyx_t_1);
09891             __pyx_t_2 = PyNumber_Multiply(__pyx_v_m, __pyx_t_1); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1002, __pyx_L1_error)
09892             __Pyx_GOTREF(__pyx_t_2);
09893             __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09894             __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_t_2, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1002, __pyx_L1_error)
09895             __Pyx_GOTREF(__pyx_t_1);
09896             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;

```

```

09897         __pyx_r = __pyx_t_1;
09898         __pyx_t_1 = 0;
09899         goto __pyx_L0;
09900     }
09901
09902     /* "PyClical.pyx":980
09903     *         return pow(self, m)
09904     *
09905     *     def pow(self, m):                # ««««««««
09906     *         """
09907     *         Power: self to the m.
09908     */
09909
09910     /* function exit code */
09911     __pyx_L1_error:;
09912     __Pyx_XDECREF(__pyx_t_1);
09913     __Pyx_XDECREF(__pyx_t_2);
09914     __Pyx_AddTraceback("PyClical.clifford.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
09915     __pyx_r = NULL;
09916     __pyx_L0:;
09917     __Pyx_XGIVEREF(__pyx_r);
09918     __Pyx_RefNannyFinishContext();
09919     return __pyx_r;
09920 }
09921
09922     /* "PyClical.pyx":1004
09923     *         return exp(m * log(self))
09924     *
09925     *     def outer_pow(self, m):          # ««««««««
09926     *         """
09927     *         Outer product power.
09928     */
09929
09930     /* Python wrapper */
09931     static PyObject* __pyx_pw_8PyClical_8clifford_59outer_pow(PyObject* __pyx_v_self, PyObject
09932 *__pyx_v_m); /*proto*/
09933     static char __pyx_doc_8PyClical_8clifford_58outer_pow[] = "\n        Outer product power.\n\n
    >> x=clifford(\"2+{1}\"); print(x.outer_pow(0))\n        1\n    >> x=clifford(\"2+{1}\");
    print(x.outer_pow(1))\n        2+{1}\n    >> x=clifford(\"2+{1}\"); print(x.outer_pow(2))\n
    4+4{1}\n    >> print(clifford(\"1+{1}+{1,2}\").outer_pow(3))\n        1+3{1}+3{1,2}\n\n
    ";
09934     static PyObject* __pyx_pw_8PyClical_8clifford_59outer_pow(PyObject* __pyx_v_self, PyObject
09935 *__pyx_v_m) {
09936     PyObject* __pyx_r = 0;
09937     __Pyx_RefNannyDeclarations
09938     __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
09939     __pyx_r = __pyx_pf_8PyClical_8clifford_58outer_pow(((struct __pyx_obj_8PyClical_clifford
09940 *)__pyx_v_self), ((PyObject *)__pyx_v_m));
09941
09942     /* function exit code */
09943     __Pyx_RefNannyFinishContext();
09944     return __pyx_r;
09945 }
09946
09947     static PyObject* __pyx_pf_8PyClical_8clifford_58outer_pow(struct __pyx_obj_8PyClical_clifford
09948 *__pyx_v_self, PyObject* __pyx_v_m) {
09949     PyObject* __pyx_r = NULL;
09950     __Pyx_RefNannyDeclarations
09951     PyObject* __pyx_t_1 = NULL;
09952     int __pyx_t_2;
09953     PyObject* __pyx_t_3 = NULL;
09954     int __pyx_lineno = 0;
09955     const char* __pyx_filename = NULL;
09956     int __pyx_clineno = 0;
09957     __Pyx_RefNannySetupContext("outer_pow", 0);
09958
09959     /* "PyClical.pyx":1018
09960     *
09961     *         return clifford().wrap( self.instance.outer_pow(m) )                # ««««««««
09962     *
09963     *     def __call__(self, grade):
09964     */
09965     __Pyx_XDECREF(__pyx_r);
09966     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1018, __pyx_L1_error)
09967     __Pyx_GOTREF(__pyx_t_1);
09968     __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_2 == (int)-1) &&
    PyErr_Occurred())) __PYX_ERR(0, 1018, __pyx_L1_error)
09969     __pyx_t_3 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
    *)__pyx_t_1), __pyx_v_self->instance->outer_pow(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
    1018, __pyx_L1_error)
09970     __Pyx_GOTREF(__pyx_t_3);
09971     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09972     __pyx_r = __pyx_t_3;
09973     __pyx_t_3 = 0;
09974     goto __pyx_L0;

```



```

09972
09973         /* "PyClicl.pyx":1004
09974         *         return exp(m * log(self))
09975         *
09976         *         def outer_pow(self, m):             # ««««««««
09977         *         """
09978         *         Outer product power.
09979         */
09980
09981         /* function exit code */
09982         __pyx_L1_error:;
09983         __Pyx_XDECREF(__pyx_t_1);
09984         __Pyx_XDECREF(__pyx_t_3);
09985         __Pyx_AddTraceback("PyClicl.clifford.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09986         __pyx_r = NULL;
09987         __pyx_L0:;
09988         __Pyx_XGIVEREF(__pyx_r);
09989         __Pyx_RefNannyFinishContext();
09990         return __pyx_r;
09991     }
09992
09993         /* "PyClicl.pyx":1020
09994         *         return clifford().wrap( self.instance.outer_pow(m) )
09995         *
09996         *         def __call__(self, grade):             # ««««««««
09997         *         """
09998         *         Pure grade-vector part.
09999         */
10000
10001         /* Python wrapper */
10002         static PyObject *__pyx_pw_8PyClicl_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
10003         static char __pyx_doc_8PyClicl_8clifford_60__call__[] = "\n         Pure grade-vector
part.\n\n         >> print(clifford(\"{1}\") (1))\n         {1}\n         >>
print(clifford(\"{1}\") (0))\n         0\n         >> print(clifford(\"1+{1}+{1,2}\") (0))\n         1\n
         >> print(clifford(\"1+{1}+{1,2}\") (1))\n         {1}\n         >>
print(clifford(\"1+{1}+{1,2}\") (2))\n         {1,2}\n         >> print(clifford(\"1+{1}+{1,2}\") (3))\n
         0\n         ";
10004         #if CYTHON_COMPILING_IN_CPYTHON
10005         struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_60__call__;
10006         #endif
10007         static PyObject *__pyx_pw_8PyClicl_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
10008             PyObject *__pyx_v_grade = 0;
10009             int __pyx_lineno = 0;
10010             const char *__pyx_filename = NULL;
10011             int __pyx_clineno = 0;
10012             PyObject *__pyx_r = 0;
10013             __Pyx_RefNannyDeclarations
10014             __Pyx_RefNannySetupContext("__call__ (wrapper)", 0);
10015             {
10016                 static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_grade,0};
10017                 PyObject* values[1] = {0};
10018                 if (unlikely(__pyx_kwds)) {
10019                     Py_ssize_t kw_args;
10020                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10021                     switch (pos_args) {
10022                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10023                         CYTHON_FALLTHROUGH;
10024                         case 0: break;
10025                         default: goto __pyx_L5_argtuple_error;
10026                     }
10027                     kw_args = PyDict_Size(__pyx_kwds);
10028                     switch (pos_args) {
10029                         case 0:
10030                             if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_grade)) != 0))
10031                                 kw_args--;
10032                             else goto __pyx_L5_argtuple_error;
10033                     }
10034                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
pos_args, "__call__") < 0)) __PYX_ERR(0, 1020, __pyx_L3_error)
10035                 }
10036                 else if (PyTuple_GET_SIZE(__pyx_args) != 1) {
10037                     goto __pyx_L5_argtuple_error;
10038                 } else {
10039                     values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10040                 }
10041                 __pyx_v_grade = values[0];
10042             }
10043             goto __pyx_L4_argument_unpacking_done;
10044             __pyx_L5_argtuple_error:;
10045             __Pyx_RaiseArgtupleInvalid("__call__", 1, 1, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
1020, __pyx_L3_error)
10046             __pyx_L3_error:;
10047             __Pyx_AddTraceback("PyClicl.clifford.__call__", __pyx_clineno, __pyx_lineno,

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    __pyx_filename);
10048     __Pyx_RefNannyFinishContext();
10049     return NULL;
10050     __pyx_L4_argument_unpacking_done;;
10051     __pyx_r = __pyx_pf_8PyClical_8clifford_60__call__((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), __pyx_v_grade);

10052
10053     /* function exit code */
10054     __Pyx_RefNannyFinishContext();
10055     return __pyx_r;
10056 }
10057
10058 static PyObject *__pyx_pf_8PyClical_8clifford_60__call__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_grade) {
10059     PyObject *__pyx_r = NULL;
10060     __Pyx_RefNannyDeclarations
10061     PyObject *__pyx_t_1 = NULL;
10062     int __pyx_t_2;
10063     PyObject *__pyx_t_3 = NULL;
10064     int __pyx_lineno = 0;
10065     const char *__pyx_filename = NULL;
10066     int __pyx_clineno = 0;
10067     __Pyx_RefNannySetupContext("__call__", 0);
10068
10069     /* "PyClical.pyx":1037
10070     *
10071     *
10072     *     return clifford().wrap( self.instance.call(grade) )           # ««««««««
10073     *
10074     *     def scalar(self):
10075     */
10076     __Pyx_XDECREF(__pyx_r);
10077     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1037, __pyx_L1_error)
10078     __Pyx_GOTREF(__pyx_t_1);
10079     __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_grade); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1037, __pyx_L1_error)
10080     __pyx_t_3 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->operator)(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1037, __pyx_L1_error)
10081     __Pyx_GOTREF(__pyx_t_3);
10082     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10083     __pyx_r = __pyx_t_3;
10084     __pyx_t_3 = 0;
10085     goto __pyx_L0;
10086
10087     /* "PyClical.pyx":1020
10088     *     return clifford().wrap( self.instance.outer_pow(m) )
10089     *
10090     *     def __call__(self, grade):           # ««««««««
10091     *         """
10092     *         Pure grade-vector part.
10093     */
10094
10095     /* function exit code */
10096     __pyx_L1_error:;
10097     __Pyx_XDECREF(__pyx_t_1);
10098     __Pyx_XDECREF(__pyx_t_3);
10099     __Pyx_AddTraceback("PyClical.clifford.__call__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10100     __pyx_r = NULL;
10101     __pyx_L0:;
10102     __Pyx_XGIVEREF(__pyx_r);
10103     __Pyx_RefNannyFinishContext();
10104     return __pyx_r;
10105 }
10106
10107     /* "PyClical.pyx":1039
10108     *     return clifford().wrap( self.instance.call(grade) )
10109     *
10110     *     def scalar(self):           # ««««««««
10111     *         """
10112     *         Scalar part.
10113     */
10114
10115     /* Python wrapper */
10116     static PyObject *__pyx_pw_8PyClical_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10117     static char __pyx_doc_8PyClical_8clifford_62scalar[] = "\n          Scalar part.\n\n          >>
clifford(\"1+{1,2}\").scalar()\n          1.0\n          >> clifford(\"{1,2}\").scalar()\n          0.0\n          ";
10118     static PyObject *__pyx_pw_8PyClical_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10119         PyObject *__pyx_r = 0;
10120         __Pyx_RefNannyDeclarations
10121         __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
10122         __pyx_r = __pyx_pf_8PyClical_8clifford_62scalar(((struct __pyx_obj_8PyClical_clifford

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        *)__pyx_v_self));
10123
10124     /* function exit code */
10125     __Pyx_RefNannyFinishContext();
10126     return __pyx_r;
10127 }
10128
10129 static PyObject *__pyx_pf_8PyClical_8clifford_62scalar(struct __pyx_obj_8PyClical_clifford
 *)__pyx_v_self) {
10130     PyObject *__pyx_r = NULL;
10131     __Pyx_RefNannyDeclarations
10132     PyObject *__pyx_t_1 = NULL;
10133     int __pyx_lineno = 0;
10134     const char *__pyx_filename = NULL;
10135     int __pyx_clineno = 0;
10136     __Pyx_RefNannySetupContext("scalar", 0);
10137
10138     /* "PyClical.pyx":1048
10139     *     0.0
10140     *     """
10141     *     return self.instance.scalar()          # ««««««««
10142     *
10143     * def pure(self):
10144     */
10145     __Pyx_XDECREF(__pyx_r);
10146     __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->scalar()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1048, __pyx_L1_error)
10147     __Pyx_GOTREF(__pyx_t_1);
10148     __pyx_r = __pyx_t_1;
10149     __pyx_t_1 = 0;
10150     goto __pyx_L0;
10151
10152     /* "PyClical.pyx":1039
10153     *     return clifford().wrap( self.instance.call(grade) )
10154     *
10155     * def scalar(self):          # ««««««««
10156     *     """
10157     *     Scalar part.
10158     */
10159
10160     /* function exit code */
10161     __pyx_L1_error:;
10162     __Pyx_XDECREF(__pyx_t_1);
10163     __Pyx_AddTraceback("PyClical.clifford.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
10164     __pyx_r = NULL;
10165     __pyx_L0:;
10166     __Pyx_XGIVEREF(__pyx_r);
10167     __Pyx_RefNannyFinishContext();
10168     return __pyx_r;
10169 }
10170
10171     /* "PyClical.pyx":1050
10172     *     return self.instance.scalar()
10173     *
10174     * def pure(self):          # ««««««««
10175     *     """
10176     *     Pure part.
10177     */
10178
10179     /* Python wrapper */
10180     static PyObject *__pyx_pw_8PyClical_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10181     static char __pyx_doc_8PyClical_8clifford_64pure[] = "\n        Pure part.\n\n        >>
print(clifford(\n"+{1}+{1,2}\n").pure())\n        {1}+{1,2}\n        >>
print(clifford(\n"+{1,2}\n").pure())\n        {1,2}\n        ";
10182     static PyObject *__pyx_pw_8PyClical_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10183         PyObject *__pyx_r = 0;
10184         __Pyx_RefNannyDeclarations
10185         __Pyx_RefNannySetupContext("pure (wrapper)", 0);
10186         __pyx_r = __pyx_pf_8PyClical_8clifford_64pure(((struct __pyx_obj_8PyClical_clifford
 *)__pyx_v_self));
10187
10188     /* function exit code */
10189     __Pyx_RefNannyFinishContext();
10190     return __pyx_r;
10191 }
10192
10193 static PyObject *__pyx_pf_8PyClical_8clifford_64pure(struct __pyx_obj_8PyClical_clifford
 *)__pyx_v_self) {
10194     PyObject *__pyx_r = NULL;
10195     __Pyx_RefNannyDeclarations
10196     PyObject *__pyx_t_1 = NULL;
10197     PyObject *__pyx_t_2 = NULL;
10198     int __pyx_lineno = 0;
10199     const char *__pyx_filename = NULL;
10200     int __pyx_clineno = 0;

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10201         __Pyx_RefNannySetupContext("pure", 0);
10202
10203         /* "PyClicl.pyx":1059
10204         *         {1,2}
10205         *         """
10206         *         return clifford().wrap( self.instance.pure() )           # ««««««««
10207         *
10208         *         def even(self):
10209         */
10210         __Pyx_XDECREF(__pyx_r);
10211         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1059, __pyx_L1_error)
10212         __Pyx_GOTREF(__pyx_t_1);
10213         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), __pyx_v_self->instance->pure()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1059,
__pyx_L1_error)
10214         __Pyx_GOTREF(__pyx_t_2);
10215         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10216         __pyx_r = __pyx_t_2;
10217         __pyx_t_2 = 0;
10218         goto __pyx_L0;
10219
10220         /* "PyClicl.pyx":1050
10221         *         return self.instance.scalar()
10222         *
10223         *         def pure(self):           # ««««««««
10224         *         """
10225         *         Pure part.
10226         */
10227
10228         /* function exit code */
10229         __pyx_L1_error:;
10230         __Pyx_XDECREF(__pyx_t_1);
10231         __Pyx_XDECREF(__pyx_t_2);
10232         __Pyx_AddTraceback("PyClicl.clifford.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
10233         __pyx_r = NULL;
10234         __pyx_L0:;
10235         __Pyx_XGIVEREF(__pyx_r);
10236         __Pyx_RefNannyFinishContext();
10237         return __pyx_r;
10238     }
10239
10240     /* "PyClicl.pyx":1061
10241     *         return clifford().wrap( self.instance.pure() )
10242     *
10243     *         def even(self):           # ««««««««
10244     *         """
10245     *         Even part of multivector, sum of even grade terms.
10246     */
10247
10248     /* Python wrapper */
10249     static PyObject *__pyx_pw_8PyClicl_8clifford_67even(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10250     static char __pyx_doc_8PyClicl_8clifford_66even[] = "\n        Even part of multivector, sum
of even grade terms.\n\n        >> print(clifford(\"1+{1}+{1,2}\").even())\n        1+{1,2}\n
";
10251     static PyObject *__pyx_pw_8PyClicl_8clifford_67even(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10252         PyObject *__pyx_r = 0;
10253         __Pyx_RefNannyDeclarations
10254         __Pyx_RefNannySetupContext("even (wrapper)", 0);
10255         __pyx_r = __pyx_pf_8PyClicl_8clifford_66even(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
10256
10257         /* function exit code */
10258         __Pyx_RefNannyFinishContext();
10259         return __pyx_r;
10260     }
10261
10262     static PyObject *__pyx_pf_8PyClicl_8clifford_66even(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self) {
10263         PyObject *__pyx_r = NULL;
10264         __Pyx_RefNannyDeclarations
10265         PyObject *__pyx_t_1 = NULL;
10266         PyObject *__pyx_t_2 = NULL;
10267         int __pyx_lineno = 0;
10268         const char *__pyx_filename = NULL;
10269         int __pyx_clineno = 0;
10270         __Pyx_RefNannySetupContext("even", 0);
10271
10272         /* "PyClicl.pyx":1068
10273         *         1+{1,2}
10274         *         """
10275         *         return clifford().wrap( self.instance.even() )           # ««««««««
10276         *
10277         *         def odd(self):
10278         */

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10279     __Pyx_XDECREF(__pyx_r);
10280     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1068, __pyx_L1_error)
10281     __Pyx_GOTREF(__pyx_t_1);
10282     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->even()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1068,
__pyx_L1_error)
10283     __Pyx_GOTREF(__pyx_t_2);
10284     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10285     __pyx_r = __pyx_t_2;
10286     __pyx_t_2 = 0;
10287     goto __pyx_L0;
10288
10289     /* "PyClical.pyx":1061
10290     *     return clifford().wrap( self.instance.pure() )
10291     *
10292     *     def even(self):
10293     *         """
10294     *         Even part of multivector, sum of even grade terms.
10295     */
10296
10297     /* function exit code */
10298     __pyx_L1_error:;
10299     __Pyx_XDECREF(__pyx_t_1);
10300     __Pyx_XDECREF(__pyx_t_2);
10301     __Pyx_AddTraceback("PyClical.clifford.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
10302     __pyx_r = NULL;
10303     __pyx_L0:;
10304     __Pyx_XGIVEREF(__pyx_r);
10305     __Pyx_RefNannyFinishContext();
10306     return __pyx_r;
10307 }
10308
10309     /* "PyClical.pyx":1070
10310     *     return clifford().wrap( self.instance.even() )
10311     *
10312     *     def odd(self):
10313     *         """
10314     *         Odd part of multivector, sum of odd grade terms.
10315     */
10316
10317     /* Python wrapper */
10318     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10319     static char __pyx_doc_8PyClical_8clifford_68odd[] = "\n        Odd part of multivector, sum of
odd grade terms.\n\n        >> print(clifford(\"1+{1}+{1,2}\").odd())\n        {1}\n        ";
10320     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10321         PyObject *__pyx_r = 0;
10322         __Pyx_RefNannyDeclarations
10323         __Pyx_RefNannySetupContext("odd (wrapper)", 0);
10324         __pyx_r = __pyx_pf_8PyClical_8clifford_68odd(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10325
10326         /* function exit code */
10327         __Pyx_RefNannyFinishContext();
10328         return __pyx_r;
10329     }
10330
10331     static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10332         PyObject *__pyx_r = NULL;
10333         __Pyx_RefNannyDeclarations
10334         PyObject *__pyx_t_1 = NULL;
10335         PyObject *__pyx_t_2 = NULL;
10336         int __pyx_lineno = 0;
10337         const char *__pyx_filename = NULL;
10338         int __pyx_clineno = 0;
10339         __Pyx_RefNannySetupContext("odd", 0);
10340
10341         /* "PyClical.pyx":1077
10342         *     {1}
10343         *     """
10344         *     return clifford().wrap( self.instance.odd() )
10345         *
10346         *     def vector_part(self, frm = None):
10347         */
10348         __Pyx_XDECREF(__pyx_r);
10349         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1077, __pyx_L1_error)
10350         __Pyx_GOTREF(__pyx_t_1);
10351         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->odd()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1077,
__pyx_L1_error)
10352         __Pyx_GOTREF(__pyx_t_2);
10353         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10354         __pyx_r = __pyx_t_2;

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10355     __pyx_t_2 = 0;
10356     goto __pyx_L0;
10357
10358     /* "PyCliclcal.pyx":1070
10359  *         return clifford().wrap( self.instance.even() )
10360  *
10361  *     def odd(self):
10362  *         """
10363  *         Odd part of multivector, sum of odd grade terms.
10364  */
10365
10366     /* function exit code */
10367     __pyx_L1_error:;
10368     __Pyx_XDECREF(__pyx_t_1);
10369     __Pyx_XDECREF(__pyx_t_2);
10370     __Pyx_AddTraceback("PyCliclcal.clifford.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
10371     __pyx_r = NULL;
10372     __pyx_L0:;
10373     __Pyx_XGIVEREF(__pyx_r);
10374     __Pyx_RefNannyFinishContext();
10375     return __pyx_r;
10376 }
10377
10378     /* "PyCliclcal.pyx":1079
10379  *         return clifford().wrap( self.instance.odd() )
10380  *
10381  *     def vector_part(self, frm = None):
10382  *         """
10383  *         Vector part of multivector, as a Python list, with respect to frm.
10384  */
10385
10386     /* Python wrapper */
10387     static PyObject *__pyx_pw_8PyCliclcal_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
10388 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
10389     static char __pyx_doc_8PyCliclcal_8clifford_70vector_part[] = "\n          Vector part of
multivector, as a Python list, with respect to frm.\n\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part())\n          [2.0, 3.0]\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part(index_set((-1,1,2))))\n          [0.0, 2.0, 3.0]\n
";
10390     static PyObject *__pyx_pw_8PyCliclcal_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
10391         PyObject *__pyx_v_frm = 0;
10392         int __pyx_lineno = 0;
10393         const char *__pyx_filename = NULL;
10394         int __pyx_clineno = 0;
10395         PyObject *__pyx_r = 0;
10396         __Pyx_RefNannyDeclarations
10397         __Pyx_RefNannySetupContext("vector_part (wrapper)", 0);
10398         {
10399             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_frm,0};
10400             PyObject* values[1] = {0};
10401             values[0] = ((PyObject *)Py_None);
10402             if (unlikely(__pyx_kwds)) {
10403                 Py_ssize_t kw_args;
10404                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10405                 switch (pos_args) {
10406                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
10407                     case 0: break;
10408                     default: goto __pyx_L5_argtuple_error;
10409                 }
10410                 kw_args = PyDict_Size(__pyx_kwds);
10411                 switch (pos_args) {
10412                     case 0:
10413                         if (kw_args > 0) {
10414                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_frm);
10415                             if (value) { values[0] = value; kw_args--; }
10416                         }
10417                     if (unlikely(kw_args > 0)) {
10418                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
10419 pos_args, "vector_part") < 0)) __PYX_ERR(0, 1079, __pyx_L3_error)
10420                     }
10421                     else {
10422                         switch (PyTuple_GET_SIZE(__pyx_args)) {
10423                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
10424                             case 0: break;
10425                             default: goto __pyx_L5_argtuple_error;
10426                         }
10427                     }
10428                 }
10429                 __pyx_v_frm = values[0];
10430             }
10431             goto __pyx_L4_argument_unpacking_done;
10432             __pyx_L5_argtuple_error:;
10433             __Pyx_RaiseArgtupleInvalid("vector_part", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1079, __pyx_L3_error)

```

```

10434     __pyx_L3_error;;
10435     __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10436     __Pyx_RefNannyFinishContext();
10437     return NULL;
10438     __pyx_L4_argument_unpacking_done;;
10439     __pyx_r = __pyx_pf_8PyClical_8clifford_70vector_part(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), __pyx_v_frm);
10440
10441     /* function exit code */
10442     __Pyx_RefNannyFinishContext();
10443     return __pyx_r;
10444 }
10445
10446 static PyObject *__pyx_pf_8PyClical_8clifford_70vector_part(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, PyObject *__pyx_v_frm) {
10447     PyObject *__pyx_v_error_msg_prefix = NULL;
10448     std::vector<scalar_t> __pyx_v_vec;
10449     int __pyx_v_n;
10450     int __pyx_v_i;
10451     PyObject *__pyx_v_lst = NULL;
10452     PyObject *__pyx_v_err = NULL;
10453     PyObject *__pyx_r = NULL;
10454     __Pyx_RefNannyDeclarations
10455     PyObject *__pyx_t_1 = NULL;
10456     PyObject *__pyx_t_2 = NULL;
10457     PyObject *__pyx_t_3 = NULL;
10458     int __pyx_t_4;
10459     int __pyx_t_5;
10460     std::vector<scalar_t> __pyx_t_6;
10461     PyObject *__pyx_t_7 = NULL;
10462     int __pyx_t_8;
10463     int __pyx_t_9;
10464     int __pyx_t_10;
10465     PyObject *__pyx_t_11 = NULL;
10466     PyObject *__pyx_t_12 = NULL;
10467     PyObject *__pyx_t_13 = NULL;
10468     PyObject *__pyx_t_14 = NULL;
10469     PyObject *__pyx_t_15 = NULL;
10470     char const *__pyx_t_16;
10471     PyObject *__pyx_t_17 = NULL;
10472     PyObject *__pyx_t_18 = NULL;
10473     PyObject *__pyx_t_19 = NULL;
10474     PyObject *__pyx_t_20 = NULL;
10475     PyObject *__pyx_t_21 = NULL;
10476     PyObject *__pyx_t_22 = NULL;
10477     int __pyx_lineno = 0;
10478     const char *__pyx_filename = NULL;
10479     int __pyx_clineno = 0;
10480     __Pyx_RefNannySetupContext("vector_part", 0);
10481
10482     /* "PyClical.pyx":1088
10483     * [0.0, 2.0, 3.0]
10484     * """
10485     * error_msg_prefix = "Cannot take vector part of " # ««««««««
10486     * cdef vector[scalar_t] vec
10487     * cdef int n
10488     */
10489     __Pyx_INCREF(__pyx_kp_u_Cannot_take_vector_part_of);
10490     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_take_vector_part_of;
10491
10492     /* "PyClical.pyx":1092
10493     * cdef int n
10494     * cdef int i
10495     * try: # ««««««««
10496     *     if frm is None:
10497     *         vec = self.instance.vector_part()
10498     */
10499     {
10500         __Pyx_PyThreadState_declare
10501         __Pyx_PyThreadState_assign
10502         __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
10503         __Pyx_XGOTREF(__pyx_t_1);
10504         __Pyx_XGOTREF(__pyx_t_2);
10505         __Pyx_XGOTREF(__pyx_t_3);
10506         /*try:*/ {
10507
10508             /* "PyClical.pyx":1093
10509             * cdef int i
10510             * try:
10511             *     if frm is None: # ««««««««
10512             *         vec = self.instance.vector_part()
10513             *     else:
10514             */
10515             __pyx_t_4 = (__pyx_v_frm == Py_None);
10516             __pyx_t_5 = (__pyx_t_4 != 0);
10517             if (__pyx_t_5) {

```

```

10518
10519
10520 *      try:
10521 *          if frm is None:
10522 *              vec = self.instance.vector_part() # ««««««««
10523 *          else:
10524 *              vec = self.instance.vector_part((<index_set>frm).unwrap())
10525 */
10526
10527     __pyx_t_6 = __pyx_v_self->instance->vector_part();
10528     __pyx_v_vec = __pyx_t_6;
10529
10530     /* "PyClical.pyx":1093
10531 *      cdef int i
10532 *      try:
10533 *          if frm is None: # ««««««««
10534 *              vec = self.instance.vector_part()
10535 *          else:
10536 */
10537     goto __pyx_L9;
10538 }
10539
10540 /* "PyClical.pyx":1096
10541 *      vec = self.instance.vector_part()
10542 *      else:
10543 *          vec = self.instance.vector_part((<index_set>frm).unwrap()) # ««««««««
10544 *          n = vec.size()
10545 *          lst = [0.0]*n
10546 */
10547
10548     /*else*/ {
10549         try {
10550             __pyx_t_6 =
10551             __pyx_v_self->instance->vector_part(__pyx_f_8PyClical_9index_set_unwrap(((struct
10552             __pyx_obj_8PyClical_index_set *)__pyx_v_frm)));
10553         } catch (...) {
10554             __Pyx_CppExn2PyErr();
10555             __PYX_ERR(0, 1096, __pyx_L3_error)
10556         }
10557         __pyx_v_vec = __pyx_t_6;
10558     }
10559     __pyx_L9;;
10560
10561     /* "PyClical.pyx":1097
10562 *      else:
10563 *          vec = self.instance.vector_part((<index_set>frm).unwrap())
10564 *          n = vec.size() # ««««««««
10565 *          lst = [0.0]*n
10566 *          for i in xrange(n):
10567 */
10568     __pyx_v_n = __pyx_v_vec.size();
10569
10570     /* "PyClical.pyx":1098
10571 *      vec = self.instance.vector_part((<index_set>frm).unwrap())
10572 *      n = vec.size()
10573 *      lst = [0.0]*n # ««««««««
10574 *      for i in xrange(n):
10575 *          lst[i] = vec[i]
10576 */
10577     __pyx_t_7 = PyList_New(1 * ((__pyx_v_n<0) ? 0: __pyx_v_n)); if (unlikely(!__pyx_t_7))
10578     __PYX_ERR(0, 1098, __pyx_L3_error)
10579     __Pyx_GOTREF(__pyx_t_7);
10580     { Py_ssize_t __pyx_temp;
10581         for (__pyx_temp=0; __pyx_temp < __pyx_v_n; __pyx_temp++) {
10582             __Pyx_INCREF(__pyx_float_0_0);
10583             __Pyx_GIVEREF(__pyx_float_0_0);
10584             PyList_SET_ITEM(__pyx_t_7, __pyx_temp, __pyx_float_0_0);
10585         }
10586     }
10587     __pyx_v_lst = ((PyObject*)__pyx_t_7);
10588     __pyx_t_7 = 0;
10589
10590     /* "PyClical.pyx":1099
10591 *      n = vec.size()
10592 *      lst = [0.0]*n
10593 *      for i in xrange(n): # ««««««««
10594 *          lst[i] = vec[i]
10595 *      return lst
10596 */
10597     __pyx_t_8 = __pyx_v_n;
10598     __pyx_t_9 = __pyx_t_8;
10599     for (__pyx_t_10 = 0; __pyx_t_10 < __pyx_t_9; __pyx_t_10+=1) {
10600         __pyx_v_i = __pyx_t_10;
10601
10602         /* "PyClical.pyx":1100
10603 *      lst = [0.0]*n
10604 *      for i in xrange(n):
10605 *          lst[i] = vec[i] # ««««««««
10606 *      return lst

```



```

10602 *         except RuntimeError as err:
10603 */
10604     __pyx_t_7 = PyFloat_FromDouble((__pyx_v_vec[__pyx_v_i])); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 1100, __pyx_L3_error)
10605     __Pyx_GOTREF(__pyx_t_7);
10606     if (unlikely(__Pyx_SetItemInt(__pyx_v_lst, __pyx_v_i, __pyx_t_7, int, 1,
__Pyx_PyInt_From_int, 1, 1, 1) < 0)) __PYX_ERR(0, 1100, __pyx_L3_error)
10607     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
10608 }
10609
10610 /* "PyClical.pyx":1101
10611 *     for i in xrange(n):
10612 *         lst[i] = vec[i]
10613 *     return lst
10614 *     except RuntimeError as err:
10615 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10616 */
10617     __Pyx_XDECREF(__pyx_r);
10618     __Pyx_INCREF(__pyx_v_lst);
10619     __pyx_r = __pyx_v_lst;
10620     goto __pyx_L7_try_return;
10621
10622 /* "PyClical.pyx":1092
10623 * cdef int n
10624 * cdef int i
10625 * try:
10626 *     if frm is None:
10627 *         vec = self.instance.vector_part()
10628 */
10629 }
10630 __pyx_L3_error:;
10631 __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
10632
10633 /* "PyClical.pyx":1102
10634 *     lst[i] = vec[i]
10635 *     return lst
10636 *     except RuntimeError as err:
10637 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10638 *             + repr(frm) + " as frame:\n\t"
10639 */
10640     __pyx_t_8 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
10641     if (__pyx_t_8) {
10642         __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10643         if (__Pyx_GetException(&__pyx_t_7, &__pyx_t_11, &__pyx_t_12) < 0) __PYX_ERR(0, 1102,
__pyx_L5_except_error)
10644         __Pyx_GOTREF(__pyx_t_7);
10645         __Pyx_GOTREF(__pyx_t_11);
10646         __Pyx_GOTREF(__pyx_t_12);
10647         __Pyx_INCREF(__pyx_t_11);
10648         __pyx_v_err = __pyx_t_11;
10649         /*try:*/ {
10650
10651         /* "PyClical.pyx":1103
10652 *         return lst
10653 *     except RuntimeError as err:
10654 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10655 *             + repr(frm) + " as frame:\n\t"
10656 *             + str(err))
10657 */
10658         __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)__pyx_v_self)); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10659         __Pyx_GOTREF(__pyx_t_13);
10660         __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_t_13); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 1103, __pyx_L17_error)
10661         __Pyx_GOTREF(__pyx_t_14);
10662         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10663         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_14, __pyx_kp_u_using_invalid); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10664         __Pyx_GOTREF(__pyx_t_13);
10665         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10666
10667         /* "PyClical.pyx":1104
10668 *     except RuntimeError as err:
10669 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10670 *             + repr(frm) + " as frame:\n\t"
10671 *             + str(err))
10672 */
10673         __pyx_t_14 = PyObject_Repr(__pyx_v_frm); if (unlikely(!__pyx_t_14)) __PYX_ERR(0, 1104,
__pyx_L17_error)
10674         __Pyx_GOTREF(__pyx_t_14);
10675         __pyx_t_15 = PyNumber_Add(__pyx_t_13, __pyx_t_14); if (unlikely(!__pyx_t_15))
__PYX_ERR(0, 1104, __pyx_L17_error)
10676         __Pyx_GOTREF(__pyx_t_15);
10677         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10678         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;

```

```

10680     __pyx_t_14 = PyNumber_Add(__pyx_t_15, __pyx_kp_u_as_frame); if (unlikely(!__pyx_t_14))
10681     __PYX_ERR(0, 1104, __pyx_L17_error)
10682     __Pyx_GOTREF(__pyx_t_14);
10683     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10684     /* "PyClical.pyx":1105
10685     *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10686     *                         + repr(frm) + " as frame:\n\t"
10687     *                         + str(err))
10688     *                         # ««««««««
10689     *     def involute(self):
10690     */
10691     __pyx_t_15 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), __pyx_v_err);
10692     if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 1105, __pyx_L17_error)
10693     __Pyx_GOTREF(__pyx_t_15);
10694     __pyx_t_13 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_13))
10695     __PYX_ERR(0, 1105, __pyx_L17_error)
10696     __Pyx_GOTREF(__pyx_t_13);
10697     __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10698     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10699     /* "PyClical.pyx":1103
10700     *     return lst
10701     * except RuntimeError as err:
10702     *     raise ValueError(error_msg_prefix + str(self) + " using invalid "
10703     *                     + repr(frm) + " as frame:\n\t"
10704     *                     + str(err))
10705     */
10706     __pyx_t_15 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_13); if
10707     (unlikely(!__pyx_t_15)) __PYX_ERR(0, 1103, __pyx_L17_error)
10708     __Pyx_GOTREF(__pyx_t_15);
10709     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10710     __Pyx_Raise(__pyx_t_15, 0, 0, 0);
10711     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10712     __PYX_ERR(0, 1103, __pyx_L17_error)
10713     }
10714     /* "PyClical.pyx":1102
10715     *     lst[i] = vec[i]
10716     *     return lst
10717     * except RuntimeError as err:
10718     *     raise ValueError(error_msg_prefix + str(self) + " using invalid "
10719     *                     + repr(frm) + " as frame:\n\t"
10720     *                     # ««««««««
10721     */
10722     /*finally:*/ {
10723     __pyx_L17_error;;
10724     /*exception exit:*/{
10725     __Pyx_PyThreadState_declare
10726     __Pyx_PyThreadState_assign
10727     __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
10728     __pyx_t_22 = 0;
10729     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
10730     __Pyx_XDECREF(__pyx_t_14); __pyx_t_14 = 0;
10731     __Pyx_XDECREF(__pyx_t_15); __pyx_t_15 = 0;
10732     if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_20, &__pyx_t_21,
10733     &__pyx_t_22);
10734     if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_17, &__pyx_t_18,
10735     &__pyx_t_19) < 0)) __Pyx_ErrFetch(&__pyx_t_17, &__pyx_t_18, &__pyx_t_19);
10736     __Pyx_XGOTREF(__pyx_t_17);
10737     __Pyx_XGOTREF(__pyx_t_18);
10738     __Pyx_XGOTREF(__pyx_t_19);
10739     __Pyx_XGOTREF(__pyx_t_20);
10740     __Pyx_XGOTREF(__pyx_t_21);
10741     __Pyx_XGOTREF(__pyx_t_22);
10742     __pyx_t_8 = __pyx_lineno; __pyx_t_9 = __pyx_clineno; __pyx_t_16 = __pyx_filename;
10743     {
10744     __Pyx_DECREF(__pyx_v_err);
10745     __pyx_v_err = NULL;
10746     }
10747     if (PY_MAJOR_VERSION >= 3) {
10748     __Pyx_XGIVEREF(__pyx_t_20);
10749     __Pyx_XGIVEREF(__pyx_t_21);
10750     __Pyx_XGIVEREF(__pyx_t_22);
10751     __Pyx_ExceptionReset(__pyx_t_20, __pyx_t_21, __pyx_t_22);
10752     }
10753     __Pyx_XGIVEREF(__pyx_t_17);
10754     __Pyx_XGIVEREF(__pyx_t_18);
10755     __Pyx_XGIVEREF(__pyx_t_19);
10756     __Pyx_ErrRestore(__pyx_t_17, __pyx_t_18, __pyx_t_19);
10757     __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
10758     __pyx_t_22 = 0;
10759     __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_9; __pyx_filename = __pyx_t_16;
10760     goto __pyx_L5_except_error;
10761     }
10762     }
10763     }
10764     goto __pyx_L5_except_error;

```

```

10759         __pyx_L5_except_error;;
10760
10761         /* "PyClical.pyx":1092
10762  *         cdef int n
10763  *         cdef int i
10764  *         try:
10765  *             # ««««««««
10766  *             if frm is None:
10767  *                 vec = self.instance.vector_part()
10768  */
10769         __Pyx_XGIVEREF(__pyx_t_1);
10770         __Pyx_XGIVEREF(__pyx_t_2);
10771         __Pyx_XGIVEREF(__pyx_t_3);
10772         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10773         goto __pyx_L1_error;
10774         __pyx_L7_try_return;;
10775         __Pyx_XGIVEREF(__pyx_t_1);
10776         __Pyx_XGIVEREF(__pyx_t_2);
10777         __Pyx_XGIVEREF(__pyx_t_3);
10778         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10779         goto __pyx_L0;
10780     }
10781
10782     /* "PyClical.pyx":1079
10783  *         return clifford().wrap( self.instance.odd() )
10784  *
10785  *         def vector_part(self, frm = None):
10786  *             # ««««««««
10787  *             """
10788  *             Vector part of multivector, as a Python list, with respect to frm.
10789  */
10790
10791     /* function exit code */
10792     __pyx_L1_error;;
10793     __Pyx_XDECREF(__pyx_t_7);
10794     __Pyx_XDECREF(__pyx_t_11);
10795     __Pyx_XDECREF(__pyx_t_12);
10796     __Pyx_XDECREF(__pyx_t_13);
10797     __Pyx_XDECREF(__pyx_t_14);
10798     __Pyx_XDECREF(__pyx_t_15);
10799     __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10800     __pyx_r = NULL;
10801     __pyx_L0;;
10802     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
10803     __Pyx_XDECREF(__pyx_v_lst);
10804     __Pyx_XDECREF(__pyx_v_err);
10805     __Pyx_XGIVEREF(__pyx_r);
10806     __Pyx_RefNannyFinishContext();
10807     return __pyx_r;
10808 }
10809
10810     /* "PyClical.pyx":1107
10811  *
10812  *         + str(err))
10813  *
10814  *         def involute(self):
10815  *             # ««««««««
10816  *             """
10817  *             Main involution, each {i} is replaced by -{i} in each term,
10818  */
10819
10820     /* Python wrapper */
10821     static PyObject * __pyx_pw_8PyClical_8clifford_73involute(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10822     static char __pyx_doc_8PyClical_8clifford_72involute[] = "\n            Main involution, each {i}
is replaced by -{i} in each term,\n            eg. clifford(\"{1}\") -> -clifford(\"{1}\").\n\n            >>
print(clifford(\"{1}\").involute())\n            -{1}\n            >> print((clifford(\"{2}\") *
clifford(\"{1}\")).involute())\n            -{1,2}\n            >> print((clifford(\"{1}\") *
clifford(\"{2}\")).involute())\n            {1,2}\n            >>
print(clifford(\"1+{1}+{1,2}\").involute())\n            1-{1}+{1,2}\n            ";
10823     static PyObject * __pyx_pw_8PyClical_8clifford_73involute(PyObject * __pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10824         PyObject * __pyx_r = 0;
10825         __Pyx_RefNannyDeclarations
10826         __Pyx_RefNannySetupContext("involute (wrapper)", 0);
10827         __pyx_r = __pyx_pf_8PyClical_8clifford_72involute((struct __pyx_obj_8PyClical_clifford
*) __pyx_v_self));
10828     }
10829
10830     /* function exit code */
10831     __Pyx_RefNannyFinishContext();
10832     return __pyx_r;
10833 }
10834
10835     static PyObject * __pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
* __pyx_v_self) {
10836         PyObject * __pyx_r = NULL;
10837         __Pyx_RefNannyDeclarations
10838         PyObject * __pyx_t_1 = NULL;
10839         PyObject * __pyx_t_2 = NULL;
10840         int __pyx_lineno = 0;

```

```

10836     const char *__pyx_filename = NULL;
10837     int __pyx_clineno = 0;
10838     __Pyx_RefNannySetupContext("involute", 0);
10839
10840     /* "PyCliclal.pyx":1121
10841     *     1-{}+{1,2}
10842     *     """
10843     *     return clifford().wrap( self.instance.involute() )           # ««««««««
10844     *
10845     *     def reverse(self):
10846     */
10847         __Pyx_XDECREF(__pyx_r);
10848         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyCliclal_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1121, __pyx_L1_error)
10849         __Pyx_GOTREF(__pyx_t_1);
10850         __pyx_t_2 = __pyx_f_8PyCliclal_8clifford_wrap(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_t_1), __pyx_v_self->instance->involute()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1121,
__pyx_L1_error)
10851         __Pyx_GOTREF(__pyx_t_2);
10852         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10853         __pyx_r = __pyx_t_2;
10854         __pyx_t_2 = 0;
10855         goto __pyx_L0;
10856
10857     /* "PyCliclal.pyx":1107
10858     *
10859     *
10860     *     def involute(self):           # ««««««««
10861     *     """
10862     *     Main involution, each {i} is replaced by -{i} in each term,
10863     */
10864
10865     /* function exit code */
10866     __pyx_L1_error:;
10867     __Pyx_XDECREF(__pyx_t_1);
10868     __Pyx_XDECREF(__pyx_t_2);
10869     __Pyx_AddTraceback("PyCliclal.clifford.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10870     __pyx_r = NULL;
10871     __pyx_L0:;
10872     __Pyx_XGIVEREF(__pyx_r);
10873     __Pyx_RefNannyFinishContext();
10874     return __pyx_r;
10875 }
10876
10877     /* "PyCliclal.pyx":1123
10878     *     return clifford().wrap( self.instance.involute() )
10879     *
10880     *     def reverse(self):           # ««««««««
10881     *     """
10882     *     Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10883     */
10884
10885     /* Python wrapper */
10886     static PyObject *__pyx_pw_8PyCliclal_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10887     static char __pyx_doc_8PyCliclal_8clifford_74reverse[] = "\n        Reversion, eg.
clifford(\"{1}\")*clifford(\"{2}\") -> clifford(\"{2}\")*clifford(\"{1}\").\n\n        »>
print(clifford(\"{1}\").reverse())\n        {1}\n        »> print((clifford(\"{2}\") *
clifford(\"{1}\")).reverse())\n        {1,2}\n        »> print((clifford(\"{1}\") *
clifford(\"{2}\")).reverse())\n        -{1,2}\n        »>
print(clifford(\"1+{1}+{1,2}\").reverse())\n        1+{1}-{1,2}\n        ";
10888     static PyObject *__pyx_pw_8PyCliclal_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10889         PyObject *__pyx_r = 0;
10890         __Pyx_RefNannyDeclarations
10891         __Pyx_RefNannySetupContext("reverse (wrapper)", 0);
10892         __pyx_r = __pyx_pf_8PyCliclal_8clifford_74reverse(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_v_self));
10893
10894     /* function exit code */
10895     __Pyx_RefNannyFinishContext();
10896     return __pyx_r;
10897 }
10898
10899     static PyObject *__pyx_pf_8PyCliclal_8clifford_74reverse(struct __pyx_obj_8PyCliclal_clifford
*__pyx_v_self) {
10900         PyObject *__pyx_r = NULL;
10901         __Pyx_RefNannyDeclarations
10902         PyObject *__pyx_t_1 = NULL;
10903         PyObject *__pyx_t_2 = NULL;
10904         int __pyx_lineno = 0;
10905         const char *__pyx_filename = NULL;
10906         int __pyx_clineno = 0;
10907         __Pyx_RefNannySetupContext("reverse", 0);
10908
10909     /* "PyCliclal.pyx":1136

```

```

10910 *          1+{1}-{1,2}
10911 *          """
10912 *          return clifford().wrap( self.instance.reverse() )          # ««««««««
10913 *
10914 *      def conj(self):
10915 */
10916         __Pyx_XDECREF(__pyx_r);
10917         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1136, __pyx_L1_error)
10918         __Pyx_GOTREF(__pyx_t_1);
10919         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1, __pyx_v_self->instance->reverse())); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1136,
__pyx_L1_error)
10920         __Pyx_GOTREF(__pyx_t_2);
10921         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10922         __pyx_r = __pyx_t_2;
10923         __pyx_t_2 = 0;
10924         goto __pyx_L0;
10925
10926         /* "PyClical.pyx":1123
10927 *          return clifford().wrap( self.instance.involute() )
10928 *
10929 *      def reverse(self):          # ««««««««
10930 *          """
10931 *          Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10932 */
10933
10934         /* function exit code */
10935         __pyx_L1_error:;
10936         __Pyx_XDECREF(__pyx_t_1);
10937         __Pyx_XDECREF(__pyx_t_2);
10938         __Pyx_AddTraceback("PyClical.clifford.reverse", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10939         __pyx_r = NULL;
10940         __pyx_L0:;
10941         __Pyx_XGIVEREF(__pyx_r);
10942         __Pyx_RefNannyFinishContext();
10943         return __pyx_r;
10944     }
10945
10946     /* "PyClical.pyx":1138
10947 *          return clifford().wrap( self.instance.reverse() )
10948 *
10949 *      def conj(self):          # ««««««««
10950 *          """
10951 *          Conjugation, reverse o involute == involute o reverse.
10952 */
10953
10954     /* Python wrapper */
10955     static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10956     static char __pyx_doc_8PyClical_8clifford_76conj[] = "\n          Conjugation, reverse o
involute == involute o reverse.\n\n          >> print((clifford(\"{1}\")).conj())\n          -{1}\n
>> print((clifford(\"{2}\") * clifford(\"{1}\")).conj())\n          {1,2}\n          >>
print((clifford(\"{1}\") * clifford(\"{2}\")).conj())\n          -(1,2)\n          >>
print(clifford(\"1+{1}+{1,2}\").conj())\n          1-{1}-{1,2}\n          ";
10957     static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10958         PyObject *__pyx_r = 0;
10959         __Pyx_RefNannyDeclarations
10960         __Pyx_RefNannySetupContext("conj (wrapper)", 0);
10961         __pyx_r = __pyx_pf_8PyClical_8clifford_76conj(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10962
10963         /* function exit code */
10964         __Pyx_RefNannyFinishContext();
10965         return __pyx_r;
10966     }
10967
10968     static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10969         PyObject *__pyx_r = NULL;
10970         __Pyx_RefNannyDeclarations
10971         PyObject *__pyx_t_1 = NULL;
10972         PyObject *__pyx_t_2 = NULL;
10973         int __pyx_lineno = 0;
10974         const char *__pyx_filename = NULL;
10975         int __pyx_clineno = 0;
10976         __Pyx_RefNannySetupContext("conj", 0);
10977
10978         /* "PyClical.pyx":1151
10979 *          1-{1}-{1,2}
10980 *          """
10981 *          return clifford().wrap( self.instance.conj() )          # ««««««««
10982 *
10983 *      def quad(self):
10984 */

```

```

10985     __Pyx_XDECREF(__pyx_r);
10986     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyCliclal_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1151, __pyx_L1_error)
10987     __Pyx_GOTREF(__pyx_t_1);
10988     __pyx_t_2 = __pyx_f_8PyCliclal_8clifford_wrap(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_t_1), __pyx_v_self->instance->conj()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1151,
__pyx_L1_error)
10989     __Pyx_GOTREF(__pyx_t_2);
10990     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10991     __pyx_r = __pyx_t_2;
10992     __pyx_t_2 = 0;
10993     goto __pyx_L0;
10994
10995     /* "PyCliclal.pyx":1138
10996     *     return clifford().wrap( self.instance.reverse() )
10997     *
10998     *     def conj(self):
10999     *         """
11000     *         Conjugation, reverse o involute == involute o reverse.
11001     */
11002
11003     /* function exit code */
11004     __pyx_L1_error:;
11005     __Pyx_XDECREF(__pyx_t_1);
11006     __Pyx_XDECREF(__pyx_t_2);
11007     __Pyx_AddTraceback("PyCliclal.clifford.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
11008     __pyx_r = NULL;
11009     __pyx_L0:;
11010     __Pyx_XGIVEREF(__pyx_r);
11011     __Pyx_RefNannyFinishContext();
11012     return __pyx_r;
11013 }
11014
11015     /* "PyCliclal.pyx":1153
11016     *     return clifford().wrap( self.instance.conj() )
11017     *
11018     *     def quad(self):
11019     *         """
11020     *         Quadratic form == (rev(x)*x)(0).
11021     */
11022
11023     /* Python wrapper */
11024     static PyObject *__pyx_pw_8PyCliclal_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11025     static char __pyx_doc_8PyCliclal_8clifford_78quad[] = "\n          Quadratic form ==
(rev(x)*x)(0).\n\n          >> print(clifford(\"1+{1}+{1,2}\").quad())\n          3.0\n          >>
print(clifford(\"1+{-1}+{1,2}+{1,2,3}\").quad())\n          2.0\n          ";
11026     static PyObject *__pyx_pw_8PyCliclal_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11027         PyObject *__pyx_r = 0;
11028         __Pyx_RefNannyDeclarations
11029         __Pyx_RefNannySetupContext("quad (wrapper)", 0);
11030         __pyx_r = __pyx_pf_8PyCliclal_8clifford_78quad(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_v_self));
11031
11032     /* function exit code */
11033     __Pyx_RefNannyFinishContext();
11034     return __pyx_r;
11035 }
11036
11037     static PyObject *__pyx_pf_8PyCliclal_8clifford_78quad(struct __pyx_obj_8PyCliclal_clifford
*__pyx_v_self) {
11038         PyObject *__pyx_r = NULL;
11039         __Pyx_RefNannyDeclarations
11040         PyObject *__pyx_t_1 = NULL;
11041         int __pyx_lineno = 0;
11042         const char *__pyx_filename = NULL;
11043         int __pyx_clineno = 0;
11044         __Pyx_RefNannySetupContext("quad", 0);
11045
11046     /* "PyCliclal.pyx":1162
11047     *     2.0
11048     *     """
11049     *     return self.instance.quad()
11050     *
11051     *     def norm(self):
11052     */
11053     __Pyx_XDECREF(__pyx_r);
11054     __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->quad()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1162, __pyx_L1_error)
11055     __Pyx_GOTREF(__pyx_t_1);
11056     __pyx_r = __pyx_t_1;
11057     __pyx_t_1 = 0;
11058     goto __pyx_L0;
11059
11060     /* "PyCliclal.pyx":1153
11061     *     return clifford().wrap( self.instance.conj() )

```

```

11062 *
11063 *     def quad(self):                # ««««««««
11064 *         """
11065 *             Quadratic form == (rev(x)*x)(0).
11066 */
11067
11068     /* function exit code */
11069     __pyx_L1_error:;
11070     __Pyx_XDECREF(__pyx_t_1);
11071     __Pyx_AddTraceback("PyClical.clifford.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
11072     __pyx_r = NULL;
11073     __pyx_L0:;
11074     __Pyx_XGIVEREF(__pyx_r);
11075     __Pyx_RefNannyFinishContext();
11076     return __pyx_r;
11077 }
11078
11079 /* "PyClical.pyx":1164
11080 *     return self.instance.quad()
11081 *
11082 *     def norm(self):                # ««««««««
11083 *         """
11084 *             Norm == sum of squares of coordinates.
11085 */
11086
11087     /* Python wrapper */
11088     static PyObject *__pyx_pw_8PyClical_8clifford_8lnorm(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11089     static char __pyx_doc_8PyClical_8clifford_80norm[] = "\n            Norm == sum of squares of
coordinates.\n\n            >> clifford(\"1+{1}+{1,2}\").norm()\n            3.0\n            >>
clifford(\"1+{-1}+{1,2}+{1,2,3}\").norm()\n            4.0\n            ";
11090     static PyObject *__pyx_pw_8PyClical_8clifford_8lnorm(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11091         PyObject *__pyx_r = 0;
11092         __Pyx_RefNannyDeclarations
11093         __Pyx_RefNannySetupContext("norm (wrapper)", 0);
11094         __pyx_r = __pyx_pf_8PyClical_8clifford_80norm(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11095
11096     /* function exit code */
11097     __Pyx_RefNannyFinishContext();
11098     return __pyx_r;
11099 }
11100
11101     static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11102         PyObject *__pyx_r = NULL;
11103         __Pyx_RefNannyDeclarations
11104         PyObject *__pyx_t_1 = NULL;
11105         int __pyx_lineno = 0;
11106         const char *__pyx_filename = NULL;
11107         int __pyx_clineno = 0;
11108         __Pyx_RefNannySetupContext("norm", 0);
11109
11110     /* "PyClical.pyx":1173
11111 *         4.0
11112 *         """
11113 *         return self.instance.norm()                # ««««««««
11114 *
11115 *     def abs(self):
11116 */
11117         __Pyx_XDECREF(__pyx_r);
11118         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->norm()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1173, __pyx_L1_error)
11119         __Pyx_GOTREF(__pyx_t_1);
11120         __pyx_r = __pyx_t_1;
11121         __pyx_t_1 = 0;
11122         goto __pyx_L0;
11123
11124     /* "PyClical.pyx":1164
11125 *     return self.instance.quad()
11126 *
11127 *     def norm(self):                # ««««««««
11128 *         """
11129 *             Norm == sum of squares of coordinates.
11130 */
11131
11132     /* function exit code */
11133     __pyx_L1_error:;
11134     __Pyx_XDECREF(__pyx_t_1);
11135     __Pyx_AddTraceback("PyClical.clifford.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
11136     __pyx_r = NULL;
11137     __pyx_L0:;
11138     __Pyx_XGIVEREF(__pyx_r);
11139     __Pyx_RefNannyFinishContext();
11140     return __pyx_r;
11141 }

```





```

PyObject *unused) {
11219     PyObject *__pyx_r = 0;
11220     __Pyx_RefNannyDeclarations
11221     __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
11222     __pyx_r = __pyx_pf_8PyClical_8clifford_84max_abs(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11223
11224     /* function exit code */
11225     __Pyx_RefNannyFinishContext();
11226     return __pyx_r;
11227 }
11228
11229 static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11230     PyObject *__pyx_r = NULL;
11231     __Pyx_RefNannyDeclarations
11232     PyObject *__pyx_t_1 = NULL;
11233     int __pyx_lineno = 0;
11234     const char *__pyx_filename = NULL;
11235     int __pyx_clineno = 0;
11236     __Pyx_RefNannySetupContext("max_abs", 0);
11237
11238     /* "PyClical.pyx":1193
11239     *
11240     *
11241     *     return self.instance.max_abs()
11242     *
11243     *     def truncated(self, limit):
11244     */
11245     __Pyx_XDECREF(__pyx_r);
11246     __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->max_abs()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1193, __pyx_L1_error)
11247     __Pyx_GOTREF(__pyx_t_1);
11248     __pyx_r = __pyx_t_1;
11249     __pyx_t_1 = 0;
11250     goto __pyx_L0;
11251
11252     /* "PyClical.pyx":1184
11253     *
11254     *
11255     *     return glucat.abs( self.unwrap() )
11256     *
11257     *
11258     *     def max_abs(self):
11259     *
11260     *         Maximum of absolute values of components of multivector: multivector infinity norm.
11261     */
11262     /* function exit code */
11263     __pyx_L1_error:;
11264     __Pyx_XDECREF(__pyx_t_1);
11265     __Pyx_AddTraceback("PyClical.clifford.max_abs", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11266     __pyx_r = NULL;
11267     __pyx_L0:;
11268     __Pyx_XGIVEREF(__pyx_r);
11269     __Pyx_RefNannyFinishContext();
11270     return __pyx_r;
11271 }
11272
11273 /* "PyClical.pyx":1195
11274 *
11275 *
11276 *     return self.instance.max_abs()
11277 *
11278 *
11279 *     def truncated(self, limit):
11280 *
11281 *         Remove all terms of self with relative size smaller than limit.
11282 *
11283 *
11284 *     Python wrapper */
11285 static PyObject *__pyx_pw_8PyClical_8clifford_87truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit); /*proto*/
11286 static char __pyx_doc_8PyClical_8clifford_86truncated[] = "\n
Remove all terms of self
with relative size smaller than limit.\n\n
>>
clifford(\\"1e8+{1}+1e-8{1,2}\\").truncated(1.0e-6)\n
clifford(\\"100000000\\")\n
>>
clifford(\\"1e4+{1}+1e-4{1,2}\\").truncated(1.0e-6)\n
clifford(\\"10000+{1}\\")\n
";
11287 static PyObject *__pyx_pw_8PyClical_8clifford_87truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit) {
11288     PyObject *__pyx_r = 0;
11289     __Pyx_RefNannyDeclarations
11290     __Pyx_RefNannySetupContext("truncated (wrapper)", 0);
11291     __pyx_r = __pyx_pf_8PyClical_8clifford_86truncated(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_limit));
11292
11293     /* function exit code */
11294     __Pyx_RefNannyFinishContext();
11295     return __pyx_r;
11296 }
11297
11298 static PyObject *__pyx_pf_8PyClical_8clifford_86truncated(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_limit) {

```

```

11294     PyObject *__pyx_r = NULL;
11295     __Pyx_RefNannyDeclarations
11296     PyObject *__pyx_t_1 = NULL;
11297     scalar_t __pyx_t_2;
11298     PyObject *__pyx_t_3 = NULL;
11299     int __pyx_lineno = 0;
11300     const char *__pyx_filename = NULL;
11301     int __pyx_clineno = 0;
11302     __Pyx_RefNannySetupContext("truncated", 0);
11303
11304     /* "PyClicl.pyx":1204
11305     *
11306     *
11307     *     return clifford().wrap( self.instance.truncated(limit) )
11308     *
11309     *     def isinf(self):
11310     */
11311     __Pyx_XDECREF(__pyx_r);
11312     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
11313 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1204, __pyx_L1_error)
11314     __Pyx_GOTREF(__pyx_t_1);
11315     __pyx_t_2 = __pyx_PyFloat_AsDouble(__pyx_v_limit); if (unlikely((__pyx_t_2 ==
11316 ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1204, __pyx_L1_error)
11317     __pyx_t_3 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
11318 *)__pyx_t_1), __pyx_v_self->instance->truncated(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
11319 1204, __pyx_L1_error)
11320     __Pyx_GOTREF(__pyx_t_3);
11321     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11322     __pyx_r = __pyx_t_3;
11323     __pyx_t_3 = 0;
11324     goto __pyx_L0;
11325
11326     /* "PyClicl.pyx":1195
11327     *
11328     *     return self.instance.max_abs()
11329     *
11330     *     def truncated(self, limit):
11331     *
11332     *         Remove all terms of self with relative size smaller than limit.
11333     */
11334
11335     /* function exit code */
11336     __pyx_L1_error:;
11337     __Pyx_XDECREF(__pyx_t_1);
11338     __Pyx_XDECREF(__pyx_t_3);
11339     __Pyx_AddTraceback("PyClicl.clifford.truncated", __pyx_clineno, __pyx_lineno,
11340 __pyx_filename);
11341     __pyx_r = NULL;
11342     __pyx_L0:;
11343     __Pyx_XGIVEREF(__pyx_r);
11344     __Pyx_RefNannyFinishContext();
11345     return __pyx_r;
11346 }
11347
11348     /* "PyClicl.pyx":1206
11349     *
11350     *     return clifford().wrap( self.instance.truncated(limit) )
11351     *
11352     *     def isinf(self):
11353     *
11354     *         Check if a multivector contains any infinite values.
11355     */
11356
11357     /* Python wrapper */
11358     static PyObject *__pyx_pw_8PyClicl_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
11359 PyObject *unused); /*proto*/
11360     static char __pyx_doc_8PyClicl_8clifford_88isinf[] = "\n        Check if a multivector
11361 contains any infinite values.\n\n        >> clifford().isinf()\n        False\n        ";
11362     static PyObject *__pyx_pw_8PyClicl_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
11363 PyObject *unused) {
11364     PyObject *__pyx_r = 0;
11365     __Pyx_RefNannyDeclarations
11366     __Pyx_RefNannySetupContext("isinf (wrapper)", 0);
11367     __pyx_r = __pyx_pf_8PyClicl_8clifford_88isinf(((struct __pyx_obj_8PyClicl_clifford
11368 *)__pyx_v_self));
11369
11370     /* function exit code */
11371     __Pyx_RefNannyFinishContext();
11372     return __pyx_r;
11373 }
11374
11375     static PyObject *__pyx_pf_8PyClicl_8clifford_88isinf(struct __pyx_obj_8PyClicl_clifford
11376 *__pyx_v_self) {
11377     PyObject *__pyx_r = NULL;
11378     __Pyx_RefNannyDeclarations
11379     PyObject *__pyx_t_1 = NULL;
11380     int __pyx_lineno = 0;
11381     const char *__pyx_filename = NULL;
11382     int __pyx_clineno = 0;

```

```

11371     __Pyx_RefNannySetupContext("isinf", 0);
11372
11373     /* "PyClical.pyx":1213
11374     *     False
11375     *     """
11376     *     return self.instance.isnan()          # ««««««««
11377     *
11378     *     def isnan(self):
11379     */
11380     __Pyx_XDECREF(__pyx_r);
11381     __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
11382 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1213, __pyx_L1_error)
11383     __Pyx_GOTREF(__pyx_t_1);
11384     __pyx_r = __pyx_t_1;
11385     __pyx_t_1 = 0;
11386     goto __pyx_L0;
11387
11388     /* "PyClical.pyx":1206
11389     *     return clifford().wrap( self.instance.truncated(limit) )
11390     *
11391     *     def isinf(self):          # ««««««««
11392     *     """
11393     *     Check if a multivector contains any infinite values.
11394     */
11395     /* function exit code */
11396     __pyx_L1_error++;
11397     __Pyx_XDECREF(__pyx_t_1);
11398     __Pyx_AddTraceback("PyClical.clifford.isinf", __pyx_clineno, __pyx_lineno, __pyx_filename);
11399     __pyx_r = NULL;
11400     __pyx_L0++;
11401     __Pyx_XGIVEREF(__pyx_r);
11402     __Pyx_RefNannyFinishContext();
11403     return __pyx_r;
11404 }
11405
11406     /* "PyClical.pyx":1215
11407     *     return self.instance.isnan()
11408     *
11409     *     def isnan(self):          # ««««««««
11410     *     """
11411     *     Check if a multivector contains any IEEE NaN values.
11412     */
11413
11414     /* Python wrapper */
11415     static PyObject *__pyx_pw_8PyClical_8clifford_91isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11416     static char __pyx_doc_8PyClical_8clifford_90isnan[] = "\n        Check if a multivector
contains any IEEE NaN values.\n\n        >> clifford().isnan()\n        False\n        ";
11417     static PyObject *__pyx_pw_8PyClical_8clifford_91isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11418     PyObject *__pyx_r = 0;
11419     __Pyx_RefNannyDeclarations
11420     __Pyx_RefNannySetupContext("isnan (wrapper)", 0);
11421     __pyx_r = __pyx_pf_8PyClical_8clifford_90isnan(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11422
11423     /* function exit code */
11424     __Pyx_RefNannyFinishContext();
11425     return __pyx_r;
11426 }
11427
11428     static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11429     PyObject *__pyx_r = NULL;
11430     __Pyx_RefNannyDeclarations
11431     PyObject *__pyx_t_1 = NULL;
11432     int __pyx_lineno = 0;
11433     const char *__pyx_filename = NULL;
11434     int __pyx_clineno = 0;
11435     __Pyx_RefNannySetupContext("isnan", 0);
11436
11437     /* "PyClical.pyx":1222
11438     *     False
11439     *     """
11440     *     return self.instance.isnan()          # ««««««««
11441     *
11442     *     def frame(self):
11443     */
11444     __Pyx_XDECREF(__pyx_r);
11445     __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
11446 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1222, __pyx_L1_error)
11447     __Pyx_GOTREF(__pyx_t_1);
11448     __pyx_r = __pyx_t_1;
11449     __pyx_t_1 = 0;
11450     goto __pyx_L0;

```



```

11527         /* function exit code */
11528         __pyx_L1_error++;
11529         __Pyx_XDECREF(__pyx_t_1);
11530         __Pyx_XDECREF(__pyx_t_2);
11531         __Pyx_AddTraceback("PyClical.clifford.frame", __pyx_clineno, __pyx_lineno, __pyx_filename);
11532         __pyx_r = NULL;
11533         __pyx_L0;
11534         __Pyx_XGIVEREF(__pyx_r);
11535         __Pyx_RefNannyFinishContext();
11536         return __pyx_r;
11537     }
11538
11539     /* "PyClical.pyx":1235
11540     *     return index_set().wrap( self.instance.frame() )
11541     *
11542     * def __repr__(self):
11543     *     """
11544     *     The official string representation of self.
11545     */
11546
11547     /* Python wrapper */
11548     static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self); /*proto*/
11549     static char __pyx_doc_8PyClical_8clifford_94__repr__[] = "\n        The\n        \342\200\234official\342\200\235 string representation of self.\n        \n        clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\") .__repr__()\n        'clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\")'\n        ";
11550     #if CYTHON_COMPILING_IN_CPYTHON
11551     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_94__repr__;
11552     #endif
11553     static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self) {
11554         PyObject *__pyx_r = 0;
11555         __Pyx_RefNannyDeclarations
11556         __Pyx_RefNannySetupContext("__repr__ (wrapper)", 0);
11557         __pyx_r = __pyx_pf_8PyClical_8clifford_94__repr__(((struct __pyx_obj_8PyClical_clifford
11558 *)__pyx_v_self));
11559         /* function exit code */
11560         __Pyx_RefNannyFinishContext();
11561         return __pyx_r;
11562     }
11563
11564     static PyObject *__pyx_pf_8PyClical_8clifford_94__repr__(struct __pyx_obj_8PyClical_clifford
11565 *__pyx_v_self) {
11566         PyObject *__pyx_r = NULL;
11567         __Pyx_RefNannyDeclarations
11568         PyObject *__pyx_t_1 = NULL;
11569         int __pyx_lineno = 0;
11570         const char *__pyx_filename = NULL;
11571         int __pyx_clineno = 0;
11572         __Pyx_RefNannySetupContext("__repr__", 0);
11573
11574         /* "PyClical.pyx":1242
11575         *     'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
11576         *     """
11577         *     return clifford_to_repr( self.unwrap() ).decode()
11578         *
11579         * def __str__(self):
11580         *
11581         *     __Pyx_XDECREF(__pyx_r);
11582         *     __pyx_t_1 =
11583         *     __Pyx_decode_cpp_string(clifford_to_repr(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self)), 0,
11584         *     PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1242, __pyx_L1_error)
11585         *     __Pyx_GOTREF(__pyx_t_1);
11586         *     __pyx_r = __pyx_t_1;
11587         *     __pyx_t_1 = 0;
11588         *     goto __pyx_L0;
11589         *
11590         *     /* "PyClical.pyx":1235
11591         *     *     return index_set().wrap( self.instance.frame() )
11592         *     *
11593         *     def __repr__(self):
11594         *     *     """
11595         *     *     The official string representation of self.
11596         *     */
11597         *
11598         *     /* function exit code */
11599         *     __pyx_L1_error++;
11600         *     __Pyx_XDECREF(__pyx_t_1);
11601         *     __Pyx_AddTraceback("PyClical.clifford.__repr__", __pyx_clineno, __pyx_lineno,
11602         *     __pyx_filename);
11603         *     __pyx_r = NULL;
11604         *     __pyx_L0;
11605         *     __Pyx_XGIVEREF(__pyx_r);
11606         *     __Pyx_RefNannyFinishContext();
11607         *     return __pyx_r;
11608         *
11609         * }
11610     }

```



```

11684     __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
11685     __pyx_r = __pyx_pf_8PyClical_8clifford_98__reduce_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self));
11686
11687     /* function exit code */
11688     __Pyx_RefNannyFinishContext();
11689     return __pyx_r;
11690 }
11691
11692 static PyObject *__pyx_pf_8PyClical_8clifford_98__reduce_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
11693     PyObject *__pyx_r = NULL;
11694     __Pyx_RefNannyDeclarations
11695     PyObject *__pyx_t_1 = NULL;
11696     int __pyx_lineno = 0;
11697     const char *__pyx_filename = NULL;
11698     int __pyx_clineno = 0;
11699     __Pyx_RefNannySetupContext("__reduce_cython__", 0);
11700
11701     /* "(tree fragment)":2
11702 * def __reduce_cython__(self):
11703 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
11704 * def __setstate_cython__(self, __pyx_state):
11705 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11706 */
11707     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__11, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
11708     __Pyx_GOTREF(__pyx_t_1);
11709     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11710     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11711     __PYX_ERR(1, 2, __pyx_L1_error)
11712
11713     /* "(tree fragment)":1
11714 * def __reduce_cython__(self): # ««««««««
11715 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11716 * def __setstate_cython__(self, __pyx_state):
11717 */
11718
11719     /* function exit code */
11720     __pyx_L1_error:;
11721     __Pyx_XDECREF(__pyx_t_1);
11722     __Pyx_AddTraceback("PyClical.clifford.__reduce_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11723     __pyx_r = NULL;
11724     __Pyx_XGIVEREF(__pyx_r);
11725     __Pyx_RefNannyFinishContext();
11726     return __pyx_r;
11727 }
11728
11729 /* "(tree fragment)":3
11730 * def __reduce_cython__(self):
11731 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11732 * def __setstate_cython__(self, __pyx_state): # ««««««««
11733 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11734 */
11735
11736     /* Python wrapper */
11737     static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state); /*proto*/
11738     static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state) {
11739         PyObject *__pyx_r = 0;
11740         __Pyx_RefNannyDeclarations
11741         __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
11742         __pyx_r = __pyx_pf_8PyClical_8clifford_100__setstate_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
11743
11744         /* function exit code */
11745         __Pyx_RefNannyFinishContext();
11746         return __pyx_r;
11747     }
11748
11749     static PyObject *__pyx_pf_8PyClical_8clifford_100__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
11750         PyObject *__pyx_r = NULL;
11751         __Pyx_RefNannyDeclarations
11752         PyObject *__pyx_t_1 = NULL;
11753         int __pyx_lineno = 0;
11754         const char *__pyx_filename = NULL;
11755         int __pyx_clineno = 0;
11756         __Pyx_RefNannySetupContext("__setstate_cython__", 0);
11757
11758         /* "(tree fragment)":4
11759 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11760 * def __setstate_cython__(self, __pyx_state):
11761 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
11762 */

```

```

11763     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__12, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
11764     __Pyx_GOTREF(__pyx_t_1);
11765     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11766     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11767     __PYX_ERR(1, 4, __pyx_L1_error)
11768
11769     /* "(tree fragment)":3
11770 * def __reduce_cython__(self):
11771 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11772 * def __setstate_cython__(self, __pyx_state): # <<<<<<<<
11773 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11774 */
11775
11776     /* function exit code */
11777     __pyx_L1_error:;
11778     __Pyx_XDECREF(__pyx_t_1);
11779     __Pyx_AddTraceback("PyClical.clifford.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11780     __pyx_r = NULL;
11781     __Pyx_XGIVEREF(__pyx_r);
11782     __Pyx_RefNannyFinishContext();
11783     return __pyx_r;
11784 }
11785
11786     /* "PyClical.pyx":1253
11787 *     return clifford_to_str( self.unwrap() ).decode()
11788 *
11789 * def clifford_hidden_doctests(): # <<<<<<<<
11790 *     """
11791 *     Tests for functions that Doctest cannot see.
11792 */
11793
11794     /* Python wrapper */
11795     static PyObject *__pyx_pw_8PyClical_9clifford_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused); /*proto*/
11796     static char __pyx_doc_8PyClical_8clifford_hidden_doctests[] = "\n    Tests for functions that
Doctest cannot see.\n\n    For clifford.__cinit__: Construct an object of type clifford.\n\n    >>
print(clifford(2))\n    2\n    >> print(clifford(2.0))\n    2\n    >> print(clifford(1.0e-1))\n
0.1\n    >> print(clifford(\"2\"))\n    2\n    >> print(clifford(\"2{1,2,3}\"))\n    2{1,2,3}\n    >>
print(clifford(clifford(\"2{1,2,3}\")))\n    2{1,2,3}\n    >> print(clifford(\"-{1}\"))\n    -{1}\n
>> print(clifford(2,index_set({1,2})))\n    2{1,2}\n    >> print(clifford([2,3],index_set({1,2})))\n
2{1}+3{2}\n    >> print(clifford([1,2]))\n    Traceback (most recent call last):\n    ...
TypeError: Cannot initialize clifford object from <class 'list'>.\n    >> print(clifford(None))\n
Traceback (most recent call last):\n    ...
TypeError: Cannot initialize clifford object from
<class 'NoneType'>.\n    >> print(clifford(None,[1,2]))\n    Traceback (most recent call last):\n
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).\n
>> print(clifford([1,2],[1,2]))\n    Traceback (most recent call last):\n    ...
TypeError:
Cannot initialize clifford object from (<class 'list'>, <class 'list'>).\n    >>
print(clifford(\"\")\n    Traceback (most recent call last):\n    ...
ValueError: Cannot
initialize clifford object from invalid string \".\n    >> print(clifford(\"{\")\n    Traceback (most
recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string
'{'.\n    >> print(clifford(\"{1}\")\n    Traceback (most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1'.\n    >>
print(clifford(\"{+}\")\n    Traceback (most recent call last):\n    ...
ValueError: Cannot
initialize clifford object from invalid string '+'.n    >> print(clifford(\"{-}\")\n    Traceback
(most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid
string '-'.n    >> print(clifford(\"{1}+\")\n    Traceback (most recent call last):\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.\n\n    For
clifford.__richcmp__: Compare objects of type clifford.\n\n    >> clifford(\"{1}\") ==
clifford(\"{1}\")\n    True\n    >> clifford(\"{1}\") != clifford(\"1.0{1}\")\n    False\n    >>
clifford(\"{1}\") != clifford(\"1.0\")\n    True\n    >> clifford(\"{1,2}\") == None\n    False\n
>> clifford(\"{1,2}\") != None\n    True\n    >> None == clifford(\"{1,2}\")\n    False\n    >> None
!= clifford(\"{1,2}\")\n    True\n    ";
11797     static PyMethodDef __pyx_mdef_8PyClical_9clifford_hidden_doctests =
{"clifford_hidden_doctests", (PyCFunction)__pyx_pw_8PyClical_9clifford_hidden_doctests, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_hidden_doctests};
11798     static PyObject *__pyx_pw_8PyClical_9clifford_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused) {
11799         PyObject *__pyx_r = 0;
11800         __Pyx_RefNannyDeclarations
11801         __Pyx_RefNannySetupContext("clifford_hidden_doctests (wrapper)", 0);
11802         __pyx_r = __pyx_pf_8PyClical_8clifford_hidden_doctests(__pyx_self);
11803
11804         /* function exit code */
11805         __Pyx_RefNannyFinishContext();
11806         return __pyx_r;
11807     }
11808
11809     static PyObject *__pyx_pf_8PyClical_8clifford_hidden_doctests(CYTHON_UNUSED PyObject
*__pyx_self) {
11810         PyObject *__pyx_r = NULL;
11811         __Pyx_RefNannyDeclarations
11812         __Pyx_RefNannySetupContext("clifford_hidden_doctests", 0);
11813
11814         /* "PyClical.pyx":1335
11815 *     True

```



```

11816 *      """
11817 *      return          # ««««««««
11818 *
11819 * cpdef inline error_squared_tol(obj):
11820 */
11821         __Pyx_XDECREF(__pyx_r);
11822         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
11823         goto __pyx_L0;
11824
11825         /* "PyClical.pyx":1253
11826 *         return clifford_to_str( self.unwrap() ).decode()
11827 *
11828 * def clifford_hidden_doctests():          # ««««««««
11829 *      """
11830 *      Tests for functions that Doctest cannot see.
11831 */
11832
11833         /* function exit code */
11834         __pyx_L0:;
11835         __Pyx_XGIVEREF(__pyx_r);
11836         __Pyx_RefNannyFinishContext();
11837         return __pyx_r;
11838     }
11839
11840     /* "PyClical.pyx":1337
11841 *     return
11842 *
11843 * cpdef inline error_squared_tol(obj):          # ««««««««
11844 *      """
11845 *      Quadratic norm error tolerance relative to a specific multivector.
11846 */
11847
11848     static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
11849 *__pyx_v_obj); /*proto*/
11850     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared_tol(PyObject
11851 *__pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
11852     PyObject *__pyx_r = NULL;
11853     __Pyx_RefNannyDeclarations
11854     PyObject *__pyx_t_1 = NULL;
11855     int __pyx_lineno = 0;
11856     const char *__pyx_filename = NULL;
11857     int __pyx_clineno = 0;
11858     __Pyx_RefNannySetupContext("error_squared_tol", 0);
11859
11860     /* "PyClical.pyx":1344
11861 *     0.0
11862 *     """
11863 *     return glucat.error_squared_tol(toClifford(obj))          # ««««««««
11864 *
11865 * cpdef inline error_squared(lhs, rhs, threshold):
11866 */
11867         __Pyx_XDECREF(__pyx_r);
11868         __pyx_t_1 =
11869     PyFloat_FromDouble(error_squared_tol(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
11870     (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1344, __pyx_L1_error)
11871         __Pyx_GOTREF(__pyx_t_1);
11872         __pyx_r = __pyx_t_1;
11873         __pyx_t_1 = 0;
11874         goto __pyx_L0;
11875
11876     /* "PyClical.pyx":1337
11877 *     return
11878 *
11879 * cpdef inline error_squared_tol(obj):          # ««««««««
11880 *      """
11881 *      Quadratic norm error tolerance relative to a specific multivector.
11882 */
11883
11884         /* function exit code */
11885         __pyx_L1_error:;
11886         __Pyx_XDECREF(__pyx_t_1);
11887         __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
11888         __pyx_filename);
11889         __pyx_r = 0;
11890         __pyx_L0:;
11891         __Pyx_XGIVEREF(__pyx_r);
11892         __Pyx_RefNannyFinishContext();
11893         return __pyx_r;
11894     }
11895
11896     /* Python wrapper */
11897     static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
11898 *__pyx_v_obj); /*proto*/
11899     static char __pyx_doc_8PyClical_10error_squared_tol[] = "\n    Quadratic norm error
11900 tolerance relative to a specific multivector.\n\n    >> print(error_squared_tol(clifford(\"{1}\") *
11901 3.0 - error_squared_tol(clifford(\"{1}{1}-2{2}+3{3}\")\n    0.0\n    \"");
11902     static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject

```

```

11895 *__pyx_v_obj) {
11896     PyObject *__pyx_r = 0;
11897     __Pyx_RefNannyDeclarations
11898     __Pyx_RefNannySetupContext("error_squared_tol (wrapper)", 0);
11899     __pyx_r = __pyx_pf_8PyClical_10error_squared_tol(__pyx_self, ((PyObject
11900 *)__pyx_v_obj));
11901
11902     /* function exit code */
11903     __Pyx_RefNannyFinishContext();
11904     return __pyx_r;
11905 }
11906
11907 static PyObject *__pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject
11908 *__pyx_self, PyObject *__pyx_v_obj) {
11909     PyObject *__pyx_r = NULL;
11910     __Pyx_RefNannyDeclarations
11911     PyObject *__pyx_t_1 = NULL;
11912     int __pyx_lineno = 0;
11913     const char *__pyx_filename = NULL;
11914     int __pyx_clineno = 0;
11915     __Pyx_RefNannySetupContext("error_squared_tol", 0);
11916     __Pyx_XDECREF(__pyx_r);
11917     __pyx_t_1 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_obj, 0); if
11918 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1337, __pyx_L1_error)
11919     __Pyx_GOTREF(__pyx_t_1);
11920     __pyx_r = __pyx_t_1;
11921     __pyx_t_1 = 0;
11922     goto __pyx_L0;
11923
11924     /* function exit code */
11925     __pyx_L1_error++;
11926     __Pyx_XDECREF(__pyx_t_1);
11927     __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
11928 __pyx_filename);
11929     __pyx_r = NULL;
11930     __pyx_L0++;
11931     __Pyx_XGIVEREF(__pyx_r);
11932     __Pyx_RefNannyFinishContext();
11933     return __pyx_r;
11934 }
11935
11936 /* "PyClical.pyx":1346
11937 *      return glucat.error_squared_tol(toClifford(obj))
11938 *
11939 * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11940 *     """
11941 *     Relative or absolute error using the quadratic norm.
11942 */
11943
11944 static PyObject *__pyx_pw_8PyClical_13error_squared(PyObject *__pyx_self, PyObject
11945 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
11946 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared(PyObject *__pyx_v_lhs,
11947 PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, CYTHON_UNUSED int __pyx_skip_dispatch) {
11948     PyObject *__pyx_r = NULL;
11949     __Pyx_RefNannyDeclarations
11950     scalar_t __pyx_t_1;
11951     PyObject *__pyx_t_2 = NULL;
11952     int __pyx_lineno = 0;
11953     const char *__pyx_filename = NULL;
11954     int __pyx_clineno = 0;
11955     __Pyx_RefNannySetupContext("error_squared", 0);
11956
11957     /* "PyClical.pyx":1357
11958 *      25.0
11959 *      """
11960 *      return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
11961 # ««««««««
11962 *
11963 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
11964 */
11965     __Pyx_XDECREF(__pyx_r);
11966     __pyx_t_1 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_1 ==
11967 ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1357, __pyx_L1_error)
11968     __pyx_t_2 =
11969 PyFloat_FromDouble(error_squared(__pyx_f_8PyClical_toClifford(__pyx_v_lhs),
11970 __pyx_f_8PyClical_toClifford(__pyx_v_rhs), ((scalar_t)__pyx_t_1))); if (unlikely(!__pyx_t_2))
11971 __PYX_ERR(0, 1357, __pyx_L1_error)
11972     __Pyx_GOTREF(__pyx_t_2);
11973     __pyx_r = __pyx_t_2;
11974     __pyx_t_2 = 0;
11975     goto __pyx_L0;
11976
11977     /* "PyClical.pyx":1346
11978 *      return glucat.error_squared_tol(toClifford(obj))
11979 *
11980 * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11981 *     """

```

```

11970 *      Relative or absolute error using the quadratic norm.
11971 */
11972
11973         /* function exit code */
11974         __pyx_L1_error;
11975         __Pyx_XDECREF(__pyx_t_2);
11976         __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11977         __pyx_r = 0;
11978         __pyx_L0;
11979         __Pyx_XGIVEREF(__pyx_r);
11980         __Pyx_RefNannyFinishContext();
11981         return __pyx_r;
11982     }
11983
11984     /* Python wrapper */
11985     static PyObject *__pyx_pw_8PyClical_13error_squared(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
11986     static char __pyx_doc_8PyClical_12error_squared[] = "\n      Relative or absolute error
using the quadratic norm.\n\n      >> err2=scalar_epsilon*scalar_epsilon\n\n      >>
print(error_squared(clifford(\"1{1}\"), clifford(\"1{1}\"), err2))\n      0.0\n      >>
print(error_squared(clifford(\"1{1}-3{2}+4{3}\"), clifford(\"1{1}\"), err2))\n      25.0\n      ";
11987     static PyObject *__pyx_pw_8PyClical_13error_squared(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
11988         PyObject *__pyx_v_lhs = 0;
11989         PyObject *__pyx_v_rhs = 0;
11990         PyObject *__pyx_v_threshold = 0;
11991         int __pyx_lineno = 0;
11992         const char *__pyx_filename = NULL;
11993         int __pyx_clineno = 0;
11994         PyObject *__pyx_r = 0;
11995         __Pyx_RefNannyDeclarations
11996         __Pyx_RefNannySetupContext("error_squared (wrapper)", 0);
11997         {
11998             static PyObject **__pyx_pyargnames[] =
{&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,0};
11999             PyObject* values[3] = {0,0,0};
12000             if (unlikely(__pyx_kwds)) {
12001                 Py_ssize_t kw_args;
12002                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12003                 switch (pos_args) {
12004                     case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
CYTHON_FALLTHROUGH;
12005                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
12006                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
12007                     case 0: break;
12008                     default: goto __pyx_L5_argtuple_error;
12009                 }
12010                 kw_args = PyDict_Size(__pyx_kwds);
12011                 switch (pos_args) {
12012                     case 0:
12013                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
0)) kw_args--;
12014                     else goto __pyx_L5_argtuple_error;
12015                     case 1:
12016                     if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
0)) kw_args--;
12017                     else {
12018                         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 1); __PYX_ERR(0, 1346,
__pyx_L3_error)
12019                     }
12020                     case 2:
12021                     if (likely((values[2] = __Pyx_PyDict_GetItemStr(__pyx_kwds,
__pyx_n_s_threshold)) != 0)) kw_args--;
12022                     else {
12023                         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 2); __PYX_ERR(0, 1346,
__pyx_L3_error)
12024                     }
12025                     if (unlikely(kw_args > 0)) {
12026                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "error_squared") < 0)) __PYX_ERR(0, 1346, __pyx_L3_error)
12027                     }
12028                     else if (PyTuple_GET_SIZE(__pyx_args) != 3) {
12029                         goto __pyx_L5_argtuple_error;
12030                     }
12031                     else {
12032                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12033                         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12034                         values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12035                     }
12036                     __pyx_v_lhs = values[0];
12037                     __pyx_v_rhs = values[1];
12038                     __pyx_v_threshold = values[2];

```

```

12044         }
12045         goto __pyx_L4_argument_unpacking_done;
12046         __pyx_L5_argtuple_error++;
12047         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, PyTuple_GET_SIZE(__pyx_args));
12048     __PYX_ERR(0, 1346, __pyx_L3_error)
12049     __pyx_L3_error++;
12049     __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12050     __Pyx_RefNannyFinishContext();
12051     return NULL;
12052     __pyx_L4_argument_unpacking_done:;
12053     __pyx_r = __pyx_pf_8PyClical_12error_squared(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
__pyx_v_threshold);
12054
12055     /* function exit code */
12056     __Pyx_RefNannyFinishContext();
12057     return __pyx_r;
12058 }
12059
12060 static PyObject *__pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject
*__pyx_self, PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold) {
12061     PyObject *__pyx_r = NULL;
12062     __Pyx_RefNannyDeclarations
12063     PyObject *__pyx_t_1 = NULL;
12064     int __pyx_lineno = 0;
12065     const char *__pyx_filename = NULL;
12066     int __pyx_clineno = 0;
12067     __Pyx_RefNannySetupContext("error_squared", 0);
12068     __Pyx_XDECREF(__pyx_r);
12069     __pyx_t_1 = __pyx_f_8PyClical_error_squared(__pyx_v_lhs, __pyx_v_rhs,
__pyx_v_threshold, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1346, __pyx_L1_error)
12070     __Pyx_GOTREF(__pyx_t_1);
12071     __pyx_r = __pyx_t_1;
12072     __pyx_t_1 = 0;
12073     goto __pyx_L0;
12074
12075     /* function exit code */
12076     __pyx_L1_error:;
12077     __Pyx_XDECREF(__pyx_t_1);
12078     __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12079     __pyx_r = NULL;
12080     __pyx_L0:;
12081     __Pyx_XGIVEREF(__pyx_r);
12082     __Pyx_RefNannyFinishContext();
12083     return __pyx_r;
12084 }
12085
12086     /* "PyClical.pyx":1359
12087 *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12088 *
12089 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):           # ««««««««
12090 *     """
12091 *     Test for approximate equality of multivectors.
12092 */
12093
12094 static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
12095 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_approx_equal(PyObject *__pyx_v_lhs,
PyObject *__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_approx_equal *__pyx_optional_args) {
12096     PyObject *__pyx_v_threshold = ((PyObject *)Py_None);
12097     PyObject *__pyx_v_tol = ((PyObject *)Py_None);
12098     PyObject *__pyx_r = NULL;
12099     __Pyx_RefNannyDeclarations
12100     PyObject *__pyx_t_1 = NULL;
12101     int __pyx_t_2;
12102     PyObject *__pyx_t_3 = NULL;
12103     scalar_t __pyx_t_4;
12104     scalar_t __pyx_t_5;
12105     int __pyx_lineno = 0;
12106     const char *__pyx_filename = NULL;
12107     int __pyx_clineno = 0;
12108     __Pyx_RefNannySetupContext("approx_equal", 0);
12109     if (__pyx_optional_args) {
12110         if (__pyx_optional_args->__pyx_n > 0) {
12111             __pyx_v_threshold = __pyx_optional_args->threshold;
12112             if (__pyx_optional_args->__pyx_n > 1) {
12113                 __pyx_v_tol = __pyx_optional_args->tol;
12114             }
12115         }
12116     }
12117     __Pyx_INCREF(__pyx_v_threshold);
12118     __Pyx_INCREF(__pyx_v_tol);
12119
12120     /* "PyClical.pyx":1374
12121 *     True

```

```

12122 *      """
12123 *      threshold = error_squared_tol(rhs) if threshold is None else threshold      # ««««««««
12124 *      tol      = error_squared_tol(rhs) if tol      is None else tol
12125 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12126 */
12127         __pyx_t_2 = (__pyx_v_threshold == Py_None);
12128         if ((__pyx_t_2 != 0)) {
12129             __pyx_t_3 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_rhs, 0); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1374, __pyx_L1_error)
12130             __Pyx_GOTREF(__pyx_t_3);
12131             __pyx_t_1 = __pyx_t_3;
12132             __pyx_t_3 = 0;
12133         } else {
12134             __Pyx_INCREF(__pyx_v_threshold);
12135             __pyx_t_1 = __pyx_v_threshold;
12136         }
12137         __Pyx_DECREF_SET(__pyx_v_threshold, __pyx_t_1);
12138         __pyx_t_1 = 0;
12139
12140         /* "PyClical.pyx":1375
12141 *      """
12142 *      threshold = error_squared_tol(rhs) if threshold is None else threshold
12143 *      tol      = error_squared_tol(rhs) if tol      is None else tol      # ««««««««
12144 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12145 *
12146 */
12147         __pyx_t_2 = (__pyx_v_tol == Py_None);
12148         if ((__pyx_t_2 != 0)) {
12149             __pyx_t_3 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_rhs, 0); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1375, __pyx_L1_error)
12150             __Pyx_GOTREF(__pyx_t_3);
12151             __pyx_t_1 = __pyx_t_3;
12152             __pyx_t_3 = 0;
12153         } else {
12154             __Pyx_INCREF(__pyx_v_tol);
12155             __pyx_t_1 = __pyx_v_tol;
12156         }
12157         __Pyx_DECREF_SET(__pyx_v_tol, __pyx_t_1);
12158         __pyx_t_1 = 0;
12159
12160         /* "PyClical.pyx":1376
12161 *      threshold = error_squared_tol(rhs) if threshold is None else threshold
12162 *      tol      = error_squared_tol(rhs) if tol      is None else tol
12163 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)      # ««««««««
12164 *
12165 * cpdef inline inv(obj):
12166 */
12167         __Pyx_XDECREF(__pyx_r);
12168         __pyx_t_4 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_4 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12169         __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_tol); if (unlikely((__pyx_t_5 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12170         __pyx_t_1 =
__Pyx_PyBool_FromLong(approx_equal(__pyx_f_8PyClical_toClifford(__pyx_v_lhs),
__pyx_f_8PyClical_toClifford(__pyx_v_rhs), ((scalar_t)__pyx_t_4), ((scalar_t)__pyx_t_5))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1376, __pyx_L1_error)
12171         __Pyx_GOTREF(__pyx_t_1);
12172         __pyx_r = __pyx_t_1;
12173         __pyx_t_1 = 0;
12174         goto __pyx_L0;
12175
12176         /* "PyClical.pyx":1359
12177 *      return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12178 *
12179 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):      # ««««««««
12180 *      """
12181 *      Test for approximate equality of multivectors.
12182 */
12183
12184         /* function exit code */
12185         __pyx_L1_error:;
12186         __Pyx_XDECREF(__pyx_t_1);
12187         __Pyx_XDECREF(__pyx_t_3);
12188         __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12189         __pyx_r = 0;
12190         __pyx_L0:;
12191         __Pyx_XDECREF(__pyx_v_threshold);
12192         __Pyx_XDECREF(__pyx_v_tol);
12193         __Pyx_XGIVEREF(__pyx_r);
12194         __Pyx_RefNannyFinishContext();
12195         return __pyx_r;
12196     }
12197

```

```

12198         /* Python wrapper */
12199         static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
12200 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
12201         static char __pyx_doc_8PyClical_14approx_equal[] = "\n    Test for approximate
equality of multivectors.\n\n    >> err2=scalar_epsilon*scalar_epsilon\n\n    >>
print(approx_equal(clifford(\"1{1}\"), clifford(\"1{1}\")))\n    True\n    >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}\"), clifford(\"1{1}\")))\n    False\n    >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}+0.001\"), clifford(\"1{1}-3{2}+4{3}\"), err2, err2))\n
False\n    >> print(approx_equal(clifford(\"1{1}-3{2}+4{3}+1.0e-30\"), clifford(\"1{1}-3{2}+4{3}\"),
err2, err2))\n    True\n    ";
12202         static PyObject *__pyx_pw_8PyClical_15approx_equal(PyObject *__pyx_self, PyObject
12203 *__pyx_args, PyObject *__pyx_kwds) {
12204             PyObject *__pyx_v_lhs = 0;
12205             PyObject *__pyx_v_rhs = 0;
12206             PyObject *__pyx_v_threshold = 0;
12207             PyObject *__pyx_v_tol = 0;
12208             int __pyx_lineno = 0;
12209             const char *__pyx_filename = NULL;
12210             int __pyx_clineno = 0;
12211             PyObject *__pyx_r = 0;
12212             __Pyx_RefNannyDeclarations
12213             __Pyx_RefNannySetupContext("approx_equal (wrapper)", 0);
12214             static PyObject **__pyx_pyargnames[] =
12215             {&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,&__pyx_n_s_tol,0};
12216             PyObject* values[4] = {0,0,0,0};
12217             values[2] = ((PyObject *)Py_None);
12218             values[3] = ((PyObject *)Py_None);
12219             if (unlikely(__pyx_kwds)) {
12220                 Py_ssize_t kw_args;
12221                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12222                 switch (pos_args) {
12223                     case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
CYTHON_FALLTHROUGH;
12224                     case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
CYTHON_FALLTHROUGH;
12225                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
12226                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
12227                     case 0: break;
12228                     default: goto __pyx_L5_argtuple_error;
12229                 }
12230                 kw_args = PyDict_Size(__pyx_kwds);
12231                 switch (pos_args) {
12232                     case 0:
12233                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
12234 0)) kw_args--;
12235                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
12236                     case 1:
12237                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
12238 0)) kw_args--;
12239                         else {
12240                             __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, 1); __PYX_ERR(0, 1359,
__pyx_L3_error)
12241                         }
CYTHON_FALLTHROUGH;
12242                     case 2:
12243                         if (kw_args > 0) {
12244                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_threshold);
12245                             if (value) { values[2] = value; kw_args--; }
12246                         }
CYTHON_FALLTHROUGH;
12247                     case 3:
12248                         if (kw_args > 0) {
12249                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_tol);
12250                             if (value) { values[3] = value; kw_args--; }
12251                         }
12252                     }
12253                     if (unlikely(kw_args > 0)) {
12254                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "approx_equal") < 0)) __PYX_ERR(0, 1359, __pyx_L3_error)
12255                     }
12256                     } else {
12257                         switch (PyTuple_GET_SIZE(__pyx_args)) {
12258                             case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
CYTHON_FALLTHROUGH;
12259                             case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
CYTHON_FALLTHROUGH;
12260                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
12261                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
12262                             case 0: break;
12263                             default: goto __pyx_L5_argtuple_error;
12264                         }
12265                     }
12266                 }
12267                 __pyx_v_lhs = values[0];
12268             }

```

```

12272         __pyx_v_rhs = values[1];
12273         __pyx_v_threshold = values[2];
12274         __pyx_v_tol = values[3];
12275     }
12276     goto __pyx_L4_argument_unpacking_done;
12277     __pyx_L5_argtuple_error:;
12278     __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, PyTuple_GET_SIZE(__pyx_args));
12279 __PYX_ERR(0, 1359, __pyx_L3_error)
12279     __pyx_L3_error:;
12280     __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12281     __Pyx_RefNannyFinishContext();
12282     return NULL;
12283     __pyx_L4_argument_unpacking_done:;
12284     __pyx_r = __pyx_pf_8PyClical_14approx_equal(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
__pyx_v_threshold, __pyx_v_tol);
12285
12286     /* function exit code */
12287     __Pyx_RefNannyFinishContext();
12288     return __pyx_r;
12289 }
12290
12291 static PyObject *__pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol) {
12292     PyObject *__pyx_r = NULL;
12293     __Pyx_RefNannyDeclarations
12294     PyObject *__pyx_t_1 = NULL;
12295     struct __pyx_opt_args_8PyClical_approx_equal __pyx_t_2;
12296     int __pyx_lineno = 0;
12297     const char *__pyx_filename = NULL;
12298     int __pyx_clineno = 0;
12299     __Pyx_RefNannySetupContext("approx_equal", 0);
12300     __Pyx_XDECREF(__pyx_r);
12301     __pyx_t_2.__pyx_n = 2;
12302     __pyx_t_2.threshold = __pyx_v_threshold;
12303     __pyx_t_2.tol = __pyx_v_tol;
12304     __pyx_t_1 = __pyx_f_8PyClical_approx_equal(__pyx_v_lhs, __pyx_v_rhs, 0, &__pyx_t_2);
12305     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1359, __pyx_L1_error)
12306     __Pyx_GOTREF(__pyx_t_1);
12307     __pyx_r = __pyx_t_1;
12308     __pyx_t_1 = 0;
12309     goto __pyx_L0;
12310
12311     /* function exit code */
12312     __pyx_L1_error:;
12313     __Pyx_XDECREF(__pyx_t_1);
12314     __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12315     __pyx_r = NULL;
12316     __pyx_L0:;
12317     __Pyx_XGIVEREF(__pyx_r);
12318     __Pyx_RefNannyFinishContext();
12319     return __pyx_r;
12320 }
12321
12322 /* "PyClical.pyx":1378
*   return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12323
*   cpdef inline inv(obj):                # ««««««««
12324
*   """
12325
*   Geometric multiplicative inverse.
12326
*/
12327
12328
12329 static PyObject *__pyx_pw_8PyClical_17inv(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12330 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_inv(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12331     PyObject *__pyx_r = NULL;
12332     __Pyx_RefNannyDeclarations
12333     PyObject *__pyx_t_1 = NULL;
12334     PyObject *__pyx_t_2 = NULL;
12335     PyObject *__pyx_t_3 = NULL;
12336     int __pyx_lineno = 0;
12337     const char *__pyx_filename = NULL;
12338     int __pyx_clineno = 0;
12339     __Pyx_RefNannySetupContext("inv", 0);
12340
12341     /* "PyClical.pyx":1391
12342
*   nan
12343
*   """
12344
*   return clifford(obj).inv()                # ««««««««
12345
*   cpdef inline scalar(obj):
12346
*/
12347
12348     __Pyx_XDECREF(__pyx_r);
12349     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),

```

```

__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1391, __pyx_L1_error)
12350     __Pyx_GOTREF(__pyx_t_2);
12351     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_inv); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1391, __pyx_L1_error)
12352     __Pyx_GOTREF(__pyx_t_3);
12353     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12354     __pyx_t_2 = NULL;
12355     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12356         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12357         if (likely(__pyx_t_2)) {
12358             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12359             __Pyx_INCREF(__pyx_t_2);
12360             __Pyx_INCREF(function);
12361             __Pyx_DECREF_SET(__pyx_t_3, function);
12362         }
12363     }
12364     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12365     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12366     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1391, __pyx_L1_error)
12367     __Pyx_GOTREF(__pyx_t_1);
12368     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12369     __pyx_r = __pyx_t_1;
12370     __pyx_t_1 = 0;
12371     goto __pyx_L0;
12372
12373     /* "PyClical.pyx":1378
12374 *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12375 *
12376 * cpdef inline inv(obj):          # ««««««««
12377 *     """
12378 *     Geometric multiplicative inverse.
12379 */
12380
12381     /* function exit code */
12382     __pyx_L1_error;;
12383     __Pyx_XDECREF(__pyx_t_1);
12384     __Pyx_XDECREF(__pyx_t_2);
12385     __Pyx_XDECREF(__pyx_t_3);
12386     __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
12387     __pyx_r = 0;
12388     __pyx_L0;;
12389     __Pyx_XGIVEREF(__pyx_r);
12390     __Pyx_RefNannyFinishContext();
12391     return __pyx_r;
12392 }
12393
12394     /* Python wrapper */
12395     static PyObject * __pyx_pw_8PyClical_17inv(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12396     static char __pyx_doc_8PyClical_16inv[] = "\n    Geometric multiplicative inverse.\n\n
>> print(inv(clifford(\"{1}\")))\n    {1}\n    >> print(inv(clifford(\"{-1}\")))\n    -{-1}\n
>> print(inv(clifford(\"{-2,-1}\")))\n    -{-2,-1}\n    >> print(inv(clifford(\"{-1}+{1}\")))\n
nan\n    ";
12397     static PyObject * __pyx_pw_8PyClical_17inv(PyObject * __pyx_self, PyObject * __pyx_v_obj)
{
12398         PyObject * __pyx_r = 0;
12399         __Pyx_RefNannyDeclarations
12400         __Pyx_RefNannySetupContext("inv (wrapper)", 0);
12401         __pyx_r = __pyx_pf_8PyClical_16inv(__pyx_self, ((PyObject *) __pyx_v_obj));
12402
12403         /* function exit code */
12404         __Pyx_RefNannyFinishContext();
12405         return __pyx_r;
12406     }
12407
12408     static PyObject * __pyx_pf_8PyClical_16inv(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12409         PyObject * __pyx_r = NULL;
12410         __Pyx_RefNannyDeclarations
12411         PyObject * __pyx_t_1 = NULL;
12412         int __pyx_lineno = 0;
12413         const char * __pyx_filename = NULL;
12414         int __pyx_clineno = 0;
12415         __Pyx_RefNannySetupContext("inv", 0);
12416         __Pyx_XDECREF(__pyx_r);
12417         __pyx_t_1 = __pyx_f_8PyClical_inv(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1378, __pyx_L1_error)
12418         __Pyx_GOTREF(__pyx_t_1);
12419         __pyx_r = __pyx_t_1;
12420         __pyx_t_1 = 0;
12421         goto __pyx_L0;
12422
12423         /* function exit code */
12424         __pyx_L1_error;;
12425         __Pyx_XDECREF(__pyx_t_1);

```



```

12426         __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
12427         __pyx_r = NULL;
12428         __pyx_L0;
12429         __Pyx_XGIVEREF(__pyx_r);
12430         __Pyx_RefNannyFinishContext();
12431         return __pyx_r;
12432     }
12433
12434     /* "PyClical.pyx":1393
12435  *     return clifford(obj).inv()
12436  *
12437  * cpdef inline scalar(obj):          # ««««««««
12438  *     """
12439  *     Scalar part.
12440  */
12441
12442     static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
12443 *__pyx_v_obj); /*proto*/
12444     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_scalar(PyObject *__pyx_v_obj,
12445 CYTHON_UNUSED int __pyx_skip_dispatch) {
12446         PyObject *__pyx_r = NULL;
12447         __Pyx_RefNannyDeclarations
12448         PyObject *__pyx_t_1 = NULL;
12449         PyObject *__pyx_t_2 = NULL;
12450         PyObject *__pyx_t_3 = NULL;
12451         int __pyx_lineno = 0;
12452         const char *__pyx_filename = NULL;
12453         int __pyx_clineno = 0;
12454         __Pyx_RefNannySetupContext("scalar", 0);
12455
12456         /* "PyClical.pyx":1402
12457  *     0.0
12458  *     """
12459  *     return clifford(obj).scalar()          # ««««««««
12460  *
12461  * cpdef inline real(obj):
12462  */
12463         __Pyx_XDECREF(__pyx_r);
12464         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
12465 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1402, __pyx_L1_error)
12466         __Pyx_GOTREF(__pyx_t_2);
12467         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
12468 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1402, __pyx_L1_error)
12469         __Pyx_GOTREF(__pyx_t_3);
12470         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12471         __pyx_t_2 = NULL;
12472         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12473             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12474             if (likely(__pyx_t_2)) {
12475                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12476                 __Pyx_INCREF(__pyx_t_2);
12477                 __Pyx_INCREF(function);
12478                 __Pyx_DECREF_SET(__pyx_t_3, function);
12479             }
12480         }
12481         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
12482 __Pyx_PyObject_CallNoArg(__pyx_t_3);
12483         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12484         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1402, __pyx_L1_error)
12485         __Pyx_GOTREF(__pyx_t_1);
12486         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12487         __pyx_r = __pyx_t_1;
12488         __pyx_t_1 = 0;
12489         goto __pyx_L0;
12490
12491         /* "PyClical.pyx":1393
12492  *     return clifford(obj).inv()
12493  *
12494  * cpdef inline scalar(obj):          # ««««««««
12495  *     """
12496  *     Scalar part.
12497  */
12498
12499         /* function exit code */
12500         __pyx_L1_error;
12501         __Pyx_XDECREF(__pyx_t_1);
12502         __Pyx_XDECREF(__pyx_t_2);
12503         __Pyx_XDECREF(__pyx_t_3);
12504         __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12505         __pyx_r = 0;
12506         __pyx_L0;
12507         __Pyx_XGIVEREF(__pyx_r);
12508         __Pyx_RefNannyFinishContext();
12509         return __pyx_r;
12510     }
12511
12512     /* Python wrapper */

```

```

12508         static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
12509 *__pyx_v_obj); /*proto*/
12509         static char __pyx_doc_8PyClical_18scalar[] = "\n      Scalar part.\n\n      >>
12510 scalar(clifford(\"1+{1}+{1,2}\"))\n      1.0\n      >> scalar(clifford(\"{1,2}\"))\n      0.0\n      ";
12510         static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
12511 *__pyx_v_obj) {
12511             PyObject *__pyx_r = 0;
12512             __Pyx_RefNannyDeclarations
12513             __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
12514             __pyx_r = __pyx_pf_8PyClical_18scalar(__pyx_self, ((PyObject *)__pyx_v_obj));
12515
12516             /* function exit code */
12517             __Pyx_RefNannyFinishContext();
12518             return __pyx_r;
12519         }
12520
12521         static PyObject *__pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject *__pyx_self,
12522 PyObject *__pyx_v_obj) {
12522             PyObject *__pyx_r = NULL;
12523             __Pyx_RefNannyDeclarations
12524             PyObject *__pyx_t_1 = NULL;
12525             int __pyx_lineno = 0;
12526             const char *__pyx_filename = NULL;
12527             int __pyx_clineno = 0;
12528             __Pyx_RefNannySetupContext("scalar", 0);
12529             __Pyx_XDECREF(__pyx_r);
12530             __pyx_t_1 = __pyx_f_8PyClical_scalar(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12531 __PYX_ERR(0, 1393, __pyx_L1_error)
12532             __Pyx_GOTREF(__pyx_t_1);
12533             __pyx_r = __pyx_t_1;
12534             __pyx_t_1 = 0;
12535             goto __pyx_L0;
12536
12537             /* function exit code */
12538             __Pyx_XDECREF(__pyx_r);
12539             __Pyx_XDECREF(__pyx_t_1);
12540             __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12541             __pyx_r = NULL;
12542             __pyx_L0:;
12543             __Pyx_XGIVEREF(__pyx_r);
12544             __Pyx_RefNannyFinishContext();
12545             return __pyx_r;
12546         }
12547
12548         /* "PyClical.pyx":1404
12549 *      return clifford(obj).scalar()
12550 *
12551 *      cpdef inline real(obj):
12552 *          # ««««««««
12553 *          """
12554 *          Real part: synonym for scalar part.
12555 */
12555         static PyObject *__pyx_pw_8PyClical_21real(PyObject *__pyx_self, PyObject
12556 *__pyx_v_obj); /*proto*/
12556         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_real(PyObject *__pyx_v_obj,
12557 CYTHON_UNUSED int __pyx_skip_dispatch) {
12558             PyObject *__pyx_r = NULL;
12559             __Pyx_RefNannyDeclarations
12560             PyObject *__pyx_t_1 = NULL;
12561             PyObject *__pyx_t_2 = NULL;
12562             PyObject *__pyx_t_3 = NULL;
12563             int __pyx_lineno = 0;
12564             const char *__pyx_filename = NULL;
12565             int __pyx_clineno = 0;
12566             __Pyx_RefNannySetupContext("real", 0);
12567
12568             /* "PyClical.pyx":1413
12569 *      0.0
12570 *      """
12571 *      return clifford(obj).scalar()
12572 *          # ««««««««
12573 *
12574 *      cpdef inline imag(obj):
12575 */
12576             __Pyx_XDECREF(__pyx_r);
12577             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
12578 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1413, __pyx_L1_error)
12579             __Pyx_GOTREF(__pyx_t_2);
12580             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
12581 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1413, __pyx_L1_error)
12582             __Pyx_GOTREF(__pyx_t_3);
12583             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12584             __pyx_t_2 = NULL;
12585             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12586                 __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12587                 if (likely(__pyx_t_2)) {
12588                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12589                     __Pyx_INCREF(__pyx_t_2);

```

```

12586         __Pyx_INCREF(function);
12587         __Pyx_DECREF_SET(__pyx_t_3, function);
12588     }
12589 }
12590 __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12591 __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12592 if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1413, __pyx_L1_error)
12593 __Pyx_GOTREF(__pyx_t_1);
12594 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12595 __pyx_r = __pyx_t_1;
12596 __pyx_t_1 = 0;
12597 goto __pyx_L0;
12598
12599 /* "PyClical.pyx":1404
12600 *     return clifford(obj).scalar()
12601 *
12602 * cpdef inline real(obj):                # ««««««««
12603 *     """
12604 *     Real part: synonym for scalar part.
12605 */
12606
12607     /* function exit code */
12608     __pyx_L1_error;
12609     __Pyx_XDECREF(__pyx_t_1);
12610     __Pyx_XDECREF(__pyx_t_2);
12611     __Pyx_XDECREF(__pyx_t_3);
12612     __Pyx_AddTraceback("PyClical.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12613     __pyx_r = 0;
12614     __pyx_L0;
12615     __Pyx_XGIVEREF(__pyx_r);
12616     __Pyx_RefNannyFinishContext();
12617     return __pyx_r;
12618 }
12619
12620 /* Python wrapper */
12621 static PyObject * __pyx_pw_8PyClical_21real(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12622 static char __pyx_doc_8PyClical_20real[] = "\n    Real part: synonym for scalar
part.\n\n    >> real(clifford(\"1+{1}+{1,2}\"))\n        1.0\n    >> real(clifford(\"{1,2}\"))\n        0.0\n
";
12623 static PyObject * __pyx_pw_8PyClical_21real(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12624     PyObject * __pyx_r = 0;
12625     __Pyx_RefNannyDeclarations
12626     __Pyx_RefNannySetupContext("real (wrapper)", 0);
12627     __pyx_r = __pyx_pf_8PyClical_20real(__pyx_self, ((PyObject *) __pyx_v_obj));
12628
12629     /* function exit code */
12630     __Pyx_RefNannyFinishContext();
12631     return __pyx_r;
12632 }
12633
12634 static PyObject * __pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12635     PyObject * __pyx_r = NULL;
12636     __Pyx_RefNannyDeclarations
12637     PyObject * __pyx_t_1 = NULL;
12638     int __pyx_lineno = 0;
12639     const char * __pyx_filename = NULL;
12640     int __pyx_clineno = 0;
12641     __Pyx_RefNannySetupContext("real", 0);
12642     __Pyx_XDECREF(__pyx_r);
12643     __pyx_t_1 = __pyx_f_8PyClical_real(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1404, __pyx_L1_error)
12644     __Pyx_GOTREF(__pyx_t_1);
12645     __pyx_r = __pyx_t_1;
12646     __pyx_t_1 = 0;
12647     goto __pyx_L0;
12648
12649     /* function exit code */
12650     __pyx_L1_error;
12651     __Pyx_XDECREF(__pyx_t_1);
12652     __Pyx_AddTraceback("PyClical.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12653     __pyx_r = NULL;
12654     __pyx_L0;
12655     __Pyx_XGIVEREF(__pyx_r);
12656     __Pyx_RefNannyFinishContext();
12657     return __pyx_r;
12658 }
12659
12660 /* "PyClical.pyx":1415
12661 *     return clifford(obj).scalar()
12662 *
12663 * cpdef inline imag(obj):                # ««««««««
12664 *     """
12665 *     Imaginary part: deprecated (always 0).

```

```

12666 */
12667
12668     static PyObject * __pyx_pw_8PyClical_23imag(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
12669     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_imag(CYTHON_UNUSED PyObject
* __pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
12670         PyObject * __pyx_r = NULL;
12671         __Pyx_RefNannyDeclarations
12672         __Pyx_RefNannySetupContext("imag", 0);
12673
12674         /* "PyClical.pyx":1424
12675         *      0.0
12676         *      """
12677         *      return 0.0          # ««««««««
12678         *
12679         * cpdef inline pure(obj):
12680         */
12681         __Pyx_XDECREF(__pyx_r);
12682         __Pyx_INCREF(__pyx_float_0_0);
12683         __pyx_r = __pyx_float_0_0;
12684         goto __pyx_L0;
12685
12686         /* "PyClical.pyx":1415
12687         *      return clifford(obj).scalar()
12688         *
12689         * cpdef inline imag(obj):          # ««««««««
12690         *      """
12691         *      Imaginary part: deprecated (always 0).
12692         */
12693
12694         /* function exit code */
12695         __pyx_L0:;
12696         __Pyx_XGIVEREF(__pyx_r);
12697         __Pyx_RefNannyFinishContext();
12698         return __pyx_r;
12699     }
12700
12701     /* Python wrapper */
12702     static PyObject * __pyx_pw_8PyClical_23imag(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
12703     static char __pyx_doc_8PyClical_22imag[] = "\n    Imaginary part: deprecated (always
0).\n\n    >> imag(clifford(\"1+{1}+{1,2}\"))\n    0.0\n    >> imag(clifford(\"{1,2}\"))\n    0.0\n";
12704     static PyObject * __pyx_pw_8PyClical_23imag(PyObject * __pyx_self, PyObject
* __pyx_v_obj) {
12705         PyObject * __pyx_r = 0;
12706         __Pyx_RefNannyDeclarations
12707         __Pyx_RefNannySetupContext("imag (wrapper)", 0);
12708         __pyx_r = __pyx_pf_8PyClical_22imag(__pyx_self, (PyObject *) __pyx_v_obj));
12709
12710         /* function exit code */
12711         __Pyx_RefNannyFinishContext();
12712         return __pyx_r;
12713     }
12714
12715     static PyObject * __pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12716         PyObject * __pyx_r = NULL;
12717         __Pyx_RefNannyDeclarations
12718         PyObject * __pyx_t_1 = NULL;
12719         int __pyx_lineno = 0;
12720         const char * __pyx_filename = NULL;
12721         int __pyx_clineno = 0;
12722         __Pyx_RefNannySetupContext("imag", 0);
12723         __Pyx_XDECREF(__pyx_r);
12724         __pyx_t_1 = __pyx_f_8PyClical_imag(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1415, __pyx_L1_error)
12725         __Pyx_GOTREF(__pyx_t_1);
12726         __pyx_r = __pyx_t_1;
12727         __pyx_t_1 = 0;
12728         goto __pyx_L0;
12729
12730         /* function exit code */
12731         __pyx_L1_error:;
12732         __Pyx_XDECREF(__pyx_t_1);
12733         __Pyx_AddTraceback("PyClical.imag", __pyx_clineno, __pyx_lineno, __pyx_filename);
12734         __pyx_r = NULL;
12735         __pyx_L0:;
12736         __Pyx_XGIVEREF(__pyx_r);
12737         __Pyx_RefNannyFinishContext();
12738         return __pyx_r;
12739     }
12740
12741     /* "PyClical.pyx":1426
12742     *      return 0.0
12743     *
12744     * cpdef inline pure(obj):          # ««««««««

```

```

12745 *      """
12746 *      Pure part
12747 */
12748
12749 static PyObject *__pyx_pw_8PyClical_25pure(PyObject *__pyx_self, PyObject
12750 *__pyx_v_obj); /*proto*/
12751 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pure(PyObject *__pyx_v_obj,
12752 CYTHON_UNUSED int __pyx_skip_dispatch) {
12753     PyObject *__pyx_r = NULL;
12754     __Pyx_RefNannyDeclarations
12755     PyObject *__pyx_t_1 = NULL;
12756     PyObject *__pyx_t_2 = NULL;
12757     PyObject *__pyx_t_3 = NULL;
12758     int __pyx_lineno = 0;
12759     const char *__pyx_filename = NULL;
12760     int __pyx_clineno = 0;
12761     __Pyx_RefNannySetupContext("pure", 0);
12762
12763     /* "PyClical.pyx":1435
12764     *      {1,2}
12765     *      """
12766     *      return clifford(obj).pure() # ««««««««
12767     *
12768     * cpdef inline even(obj):
12769     */
12770     __Pyx_XDECREF(__pyx_r);
12771     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
12772 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1435, __pyx_L1_error)
12773     __Pyx_GOTREF(__pyx_t_2);
12774     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_pure); if
12775 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1435, __pyx_L1_error)
12776     __Pyx_GOTREF(__pyx_t_3);
12777     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12778     __pyx_t_2 = NULL;
12779     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12780         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12781         if (likely(__pyx_t_2)) {
12782             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12783             __Pyx_INCREF(__pyx_t_2);
12784             __Pyx_INCREF(function);
12785             __Pyx_DECREF_SET(__pyx_t_3, function);
12786         }
12787     }
12788     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
12789 __Pyx_PyObject_CallNoArg(__pyx_t_3);
12790     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12791     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1435, __pyx_L1_error)
12792     __Pyx_GOTREF(__pyx_t_1);
12793     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12794     __pyx_r = __pyx_t_1;
12795     __pyx_t_1 = 0;
12796     goto __pyx_L0;
12797
12798     /* "PyClical.pyx":1426
12799     *      return 0.0
12800     *
12801     * cpdef inline pure(obj): # ««««««««
12802     *      """
12803     *      Pure part
12804     */
12805
12806     /* function exit code */
12807     __pyx_L1_error:;
12808     __Pyx_XDECREF(__pyx_t_1);
12809     __Pyx_XDECREF(__pyx_t_2);
12810     __Pyx_XDECREF(__pyx_t_3);
12811     __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12812     __pyx_r = 0;
12813     __pyx_L0:;
12814     __Pyx_XGIVEREF(__pyx_r);
12815     __Pyx_RefNannyFinishContext();
12816     return __pyx_r;
12817 }
12818
12819 /* Python wrapper */
12820 static PyObject *__pyx_pw_8PyClical_25pure(PyObject *__pyx_self, PyObject
12821 *__pyx_v_obj); /*proto*/
12822 static char __pyx_doc_8PyClical_24pure[] = "\n    Pure part\n\n    >>
12823 print(pure(clifford(\"1+{1}+{1,2}\"))\n    {1}+{1,2}\n    >> print(pure(clifford(\"{1,2}\"))\n
12824 {1,2}\n    ";
12825 static PyObject *__pyx_pw_8PyClical_25pure(PyObject *__pyx_self, PyObject
12826 *__pyx_v_obj) {
12827     PyObject *__pyx_r = 0;
12828     __Pyx_RefNannyDeclarations
12829     __Pyx_RefNannySetupContext("pure (wrapper)", 0);
12830     __pyx_r = __pyx_pf_8PyClical_24pure(__pyx_self, ((PyObject *)__pyx_v_obj));
12831

```

```

12823         /* function exit code */
12824         __Pyx_RefNannyFinishContext();
12825         return __pyx_r;
12826     }
12827
12828     static PyObject *__pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
12829         PyObject *__pyx_r = NULL;
12830         __Pyx_RefNannyDeclarations
12831         PyObject *__pyx_t_1 = NULL;
12832         int __pyx_lineno = 0;
12833         const char *__pyx_filename = NULL;
12834         int __pyx_clineno = 0;
12835         __Pyx_RefNannySetupContext("pure", 0);
12836         __Pyx_XDECREF(__pyx_r);
12837         __pyx_t_1 = __pyx_f_8PyClical_pure(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1426, __pyx_L1_error)
12838         __Pyx_GOTREF(__pyx_t_1);
12839         __pyx_r = __pyx_t_1;
12840         __pyx_t_1 = 0;
12841         goto __pyx_L0;
12842
12843         /* function exit code */
12844         __pyx_L1_error:;
12845         __Pyx_XDECREF(__pyx_t_1);
12846         __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12847         __pyx_r = NULL;
12848         __pyx_L0:;
12849         __Pyx_XGIVEREF(__pyx_r);
12850         __Pyx_RefNannyFinishContext();
12851         return __pyx_r;
12852     }
12853
12854     /* "PyClical.pyx":1437
12855     *     return clifford(obj).pure()
12856     *
12857     * cpdef inline even(obj):
12858     *     """
12859     *     Even part of multivector, sum of even grade terms.
12860     */
12861
12862     static PyObject *__pyx_pw_8PyClical_27even(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12863     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_even(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12864         PyObject *__pyx_r = NULL;
12865         __Pyx_RefNannyDeclarations
12866         PyObject *__pyx_t_1 = NULL;
12867         PyObject *__pyx_t_2 = NULL;
12868         PyObject *__pyx_t_3 = NULL;
12869         int __pyx_lineno = 0;
12870         const char *__pyx_filename = NULL;
12871         int __pyx_clineno = 0;
12872         __Pyx_RefNannySetupContext("even", 0);
12873
12874         /* "PyClical.pyx":1444
12875     *     1+{1,2}
12876     *     """
12877     *     return clifford(obj).even()
12878     *
12879     * cpdef inline odd(obj):
12880     */
12881         __Pyx_XDECREF(__pyx_r);
12882         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1444, __pyx_L1_error)
12883         __Pyx_GOTREF(__pyx_t_2);
12884         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_even); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1444, __pyx_L1_error)
12885         __Pyx_GOTREF(__pyx_t_3);
12886         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12887         __pyx_t_2 = NULL;
12888         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12889             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12890             if (likely(__pyx_t_2)) {
12891                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12892                 __Pyx_INCREF(__pyx_t_2);
12893                 __Pyx_INCREF(function);
12894                 __Pyx_DECREF_SET(__pyx_t_3, function);
12895             }
12896         }
12897         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12898         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12899         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1444, __pyx_L1_error)
12900         __Pyx_GOTREF(__pyx_t_1);
12901         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12902         __pyx_r = __pyx_t_1;

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```

12903         __pyx_t_1 = 0;
12904         goto __pyx_L0;
12905
12906         /* "PyClical.pyx":1437
12907  *      return clifford(obj).pure()
12908  *
12909  * cpdef inline even(obj):          # ««««««««
12910  *      """
12911  *      Even part of multivector, sum of even grade terms.
12912  */
12913
12914         /* function exit code */
12915         __pyx_L1_error++;
12916         __Pyx_XDECREF(__pyx_t_1);
12917         __Pyx_XDECREF(__pyx_t_2);
12918         __Pyx_XDECREF(__pyx_t_3);
12919         __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12920         __pyx_r = 0;
12921         __pyx_L0++;
12922         __Pyx_XGIVEREF(__pyx_r);
12923         __Pyx_RefNannyFinishContext();
12924         return __pyx_r;
12925     }
12926
12927     /* Python wrapper */
12928     static PyObject * __pyx_pw_8PyClical_27even(PyObject * __pyx_self, PyObject
12929 * __pyx_v_obj); /*proto*/
12930     static char __pyx_doc_8PyClical_26even[] = "\n      Even part of multivector, sum of
12931 even grade terms.\n\n      >> print(even(clifford(\"1+{1}+{1,2}\")))\n      1+{1,2}\n      ";
12932     static PyObject * __pyx_pf_8PyClical_27even(PyObject * __pyx_self, PyObject
12933 * __pyx_v_obj) {
12934         PyObject * __pyx_r = 0;
12935         __Pyx_RefNannyDeclarations
12936         __Pyx_RefNannySetupContext("even (wrapper)", 0);
12937         __pyx_r = __pyx_pf_8PyClical_26even(__pyx_self, ((PyObject *) __pyx_v_obj));
12938
12939         /* function exit code */
12940         __Pyx_RefNannyFinishContext();
12941         return __pyx_r;
12942     }
12943
12944     static PyObject * __pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject * __pyx_self,
12945 PyObject * __pyx_v_obj) {
12946         PyObject * __pyx_r = NULL;
12947         __Pyx_RefNannyDeclarations
12948         PyObject * __pyx_t_1 = NULL;
12949         int __pyx_lineno = 0;
12950         const char * __pyx_filename = NULL;
12951         int __pyx_clineno = 0;
12952         __Pyx_RefNannySetupContext("even", 0);
12953         __Pyx_XDECREF(__pyx_r);
12954         __pyx_t_1 = __pyx_f_8PyClical_even(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12955         __PYX_ERR(0, 1437, __pyx_L1_error)
12956         __Pyx_GOTREF(__pyx_t_1);
12957         __pyx_r = __pyx_t_1;
12958         __pyx_t_1 = 0;
12959         goto __pyx_L0;
12960
12961         /* function exit code */
12962         __pyx_L1_error++;
12963         __Pyx_XDECREF(__pyx_t_1);
12964         __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12965         __pyx_r = NULL;
12966         __pyx_L0++;
12967         __Pyx_XGIVEREF(__pyx_r);
12968         __Pyx_RefNannyFinishContext();
12969         return __pyx_r;
12970     }
12971
12972     /* "PyClical.pyx":1446
12973  *      return clifford(obj).even()
12974  *
12975  * cpdef inline odd(obj):          # ««««««««
12976  *      """
12977  *      Odd part of multivector, sum of odd grade terms.
12978  */
12979
12980     static PyObject * __pyx_pw_8PyClical_29odd(PyObject * __pyx_self, PyObject
12981 * __pyx_v_obj); /*proto*/
12982     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_odd(PyObject * __pyx_v_obj,
12983 CYTHON_UNUSED int __pyx_skip_dispatch) {
12984         PyObject * __pyx_r = NULL;
12985         __Pyx_RefNannyDeclarations
12986         PyObject * __pyx_t_1 = NULL;
12987         PyObject * __pyx_t_2 = NULL;
12988         PyObject * __pyx_t_3 = NULL;
12989         int __pyx_lineno = 0;

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```

12983         const char *__pyx_filename = NULL;
12984         int __pyx_clineno = 0;
12985         __Pyx_RefNannySetupContext("odd", 0);
12986
12987         /* "PyCliclal.pyx":1453
12988         *      {1}
12989         *      """
12990         *      return clifford(obj).odd()          # ««««««««
12991         *
12992         * cpdef inline involute(obj):
12993         */
12994         __Pyx_XDECREF(__pyx_r);
12995         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyCliclal_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1453, __pyx_L1_error)
12996         __Pyx_GOTREF(__pyx_t_2);
12997         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_odd); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1453, __pyx_L1_error)
12998         __Pyx_GOTREF(__pyx_t_3);
12999         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13000         __pyx_t_2 = NULL;
13001         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13002             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13003             if (likely(__pyx_t_2)) {
13004                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13005                 __Pyx_INCREF(__pyx_t_2);
13006                 __Pyx_INCREF(function);
13007                 __Pyx_DECREF_SET(__pyx_t_3, function);
13008             }
13009         }
13010         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13011         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13012         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1453, __pyx_L1_error)
13013         __Pyx_GOTREF(__pyx_t_1);
13014         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13015         __pyx_r = __pyx_t_1;
13016         __pyx_t_1 = 0;
13017         goto __pyx_L0;
13018
13019         /* "PyCliclal.pyx":1446
13020         *      return clifford(obj).even()
13021         *
13022         * cpdef inline odd(obj):          # ««««««««
13023         *      """
13024         *      Odd part of multivector, sum of odd grade terms.
13025         */
13026
13027         /* function exit code */
13028         __pyx_L1_error;
13029         __Pyx_XDECREF(__pyx_t_1);
13030         __Pyx_XDECREF(__pyx_t_2);
13031         __Pyx_XDECREF(__pyx_t_3);
13032         __Pyx_AddTraceback("PyCliclal.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13033         __pyx_r = 0;
13034         __pyx_L0;
13035         __Pyx_XGIVEREF(__pyx_r);
13036         __Pyx_RefNannyFinishContext();
13037         return __pyx_r;
13038     }
13039
13040     /* Python wrapper */
13041     static PyObject *__pyx_pw_8PyCliclal_29odd(PyObject *__pyx_self, PyObject
__pyx_v_obj); /*proto*/
13042     static char __pyx_doc_8PyCliclal_28odd[] = "\n    Odd part of multivector, sum of odd
grade terms.\n\n    >> print(odd(clifford(\\"1+{1}+{1,2}\\")))\n    {1}\n    ";
13043     static PyObject *__pyx_pw_8PyCliclal_29odd(PyObject *__pyx_self, PyObject *__pyx_v_obj)
{
13044         PyObject *__pyx_r = 0;
13045         __Pyx_RefNannyDeclarations
13046         __Pyx_RefNannySetupContext("odd (wrapper)", 0);
13047         __pyx_r = __pyx_pf_8PyCliclal_28odd(__pyx_self, ((PyObject *)__pyx_v_obj));
13048
13049         /* function exit code */
13050         __Pyx_RefNannyFinishContext();
13051         return __pyx_r;
13052     }
13053
13054     static PyObject *__pyx_pf_8PyCliclal_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj) {
13055         PyObject *__pyx_r = NULL;
13056         __Pyx_RefNannyDeclarations
13057         PyObject *__pyx_t_1 = NULL;
13058         int __pyx_lineno = 0;
13059         const char *__pyx_filename = NULL;
13060         int __pyx_clineno = 0;
13061         __Pyx_RefNannySetupContext("odd", 0);
13062         __Pyx_XDECREF(__pyx_r);

```



```

13063     __pyx_t_1 = __pyx_f_8PyClical_odd(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
13064     __PYX_ERR(0, 1446, __pyx_L1_error)
13065     __Pyx_GOTREF(__pyx_t_1);
13066     __pyx_r = __pyx_t_1;
13067     __pyx_t_1 = 0;
13068     goto __pyx_L0;
13069
13070     /* function exit code */
13071     __pyx_L1_error:;
13072     __Pyx_XDECREF(__pyx_t_1);
13073     __Pyx_AddTraceback("PyClical.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13074     __pyx_r = NULL;
13075     __pyx_L0:;
13076     __Pyx_XGIVEREF(__pyx_r);
13077     __Pyx_RefNannyFinishContext();
13078     return __pyx_r;
13079 }
13080
13081 /* "PyClical.pyx":1455
13082 *
13083 * cpdef inline involute(obj):
13084 *     """
13085 *     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13086 */
13087
13088 static PyObject * __pyx_pw_8PyClical_31involute(PyObject * __pyx_self, PyObject
13089 * __pyx_v_obj); /*proto*/
13090 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_involute(PyObject * __pyx_v_obj,
13091 CYTHON_UNUSED int __pyx_skip_dispatch) {
13092     PyObject * __pyx_r = NULL;
13093     __Pyx_RefNannyDeclarations
13094     PyObject * __pyx_t_1 = NULL;
13095     PyObject * __pyx_t_2 = NULL;
13096     PyObject * __pyx_t_3 = NULL;
13097     int __pyx_lineno = 0;
13098     const char * __pyx_filename = NULL;
13099     int __pyx_clineno = 0;
13100     __Pyx_RefNannySetupContext("involute", 0);
13101
13102     /* "PyClical.pyx":1468
13103 *     1-{1}+{1,2}
13104 *
13105 *     return clifford(obj).involute()
13106 *     """
13107 *
13108 * cpdef inline reverse(obj):
13109 */
13110     __Pyx_XDECREF(__pyx_r);
13111     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClical_clifford),
13112     __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1468, __pyx_L1_error)
13113     __Pyx_GOTREF(__pyx_t_2);
13114     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_involute); if
13115     (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1468, __pyx_L1_error)
13116     __Pyx_GOTREF(__pyx_t_3);
13117     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13118     __pyx_t_2 = NULL;
13119     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13120         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13121         if (likely(__pyx_t_2)) {
13122             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13123             __Pyx_INCREF(__pyx_t_2);
13124             __Pyx_INCREF(function);
13125             __Pyx_DECREF_SET(__pyx_t_3, function);
13126         }
13127     }
13128     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
13129     __Pyx_PyObject_CallNoArg(__pyx_t_3);
13130     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13131     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1468, __pyx_L1_error)
13132     __Pyx_GOTREF(__pyx_t_1);
13133     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13134     __pyx_r = __pyx_t_1;
13135     __pyx_t_1 = 0;
13136     goto __pyx_L0;
13137
13138     /* "PyClical.pyx":1455
13139 *
13140 *     return clifford(obj).odd()
13141 *
13142 * cpdef inline involute(obj):
13143 *     """
13144 *     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13145 */
13146
13147     /* function exit code */
13148     __pyx_L1_error:;
13149     __Pyx_XDECREF(__pyx_t_1);
13150     __Pyx_XDECREF(__pyx_t_2);

```

```

13144         __Pyx_XDECREF(__pyx_t_3);
13145         __Pyx_AddTraceback("PyClicl.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
13146         __pyx_r = 0;
13147         __pyx_L0;
13148         __Pyx_XGIVEREF(__pyx_r);
13149         __Pyx_RefNannyFinishContext();
13150         return __pyx_r;
13151     }
13152
13153     /* Python wrapper */
13154     static PyObject *__pyx_pw_8PyClicl_31involute(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13155     static char __pyx_doc_8PyClicl_30involute[] = "\n    Main involution, each {i} is
replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})\n\n    >>
print(involute(clifford(\"{1}\")))\n    -{1}\n    >> print(involute(clifford(\"{2}\")
*
clifford(\"{1}\")))\n    -{1,2}\n    >> print(involute(clifford(\"{1}\") * clifford(
\"{2}\")))\n
{1,2}\n    >> print(involute(clifford(\"1+{1}+{1,2}\")))\n    1-{1}+{1,2}\n    ";
13156     static PyObject *__pyx_pw_8PyClicl_31involute(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13157         PyObject *__pyx_r = 0;
13158         __Pyx_RefNannyDeclarations
13159         __Pyx_RefNannySetupContext("involute (wrapper)", 0);
13160         __pyx_r = __pyx_pf_8PyClicl_30involute(__pyx_self, ((PyObject *)__pyx_v_obj));
13161
13162         /* function exit code */
13163         __Pyx_RefNannyFinishContext();
13164         return __pyx_r;
13165     }
13166
13167     static PyObject *__pyx_pf_8PyClicl_30involute(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13168         PyObject *__pyx_r = NULL;
13169         __Pyx_RefNannyDeclarations
13170         PyObject *__pyx_t_1 = NULL;
13171         int __pyx_lineno = 0;
13172         const char *__pyx_filename = NULL;
13173         int __pyx_clineno = 0;
13174         __Pyx_RefNannySetupContext("involute", 0);
13175         __Pyx_XDECREF(__pyx_r);
13176         __pyx_t_1 = __pyx_f_8PyClicl_involute(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1455, __pyx_L1_error)
13177         __Pyx_GOTREF(__pyx_t_1);
13178         __pyx_r = __pyx_t_1;
13179         __pyx_t_1 = 0;
13180         goto __pyx_L0;
13181
13182         /* function exit code */
13183         __pyx_L1_error;
13184         __Pyx_XDECREF(__pyx_t_1);
13185         __Pyx_AddTraceback("PyClicl.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
13186         __pyx_r = NULL;
13187         __pyx_L0;
13188         __Pyx_XGIVEREF(__pyx_r);
13189         __Pyx_RefNannyFinishContext();
13190         return __pyx_r;
13191     }
13192
13193     /* "PyClicl.pyx":1470
13194     *     return clifford(obj).involute()
13195     *
13196     * cpdef inline reverse(obj):                # ««««««««
13197     *     """
13198     *     Reversion, eg. {1}*{2} -> {2}*{1}
13199     */
13200
13201     static PyObject *__pyx_pw_8PyClicl_33reverse(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13202     static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_reverse(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13203         PyObject *__pyx_r = NULL;
13204         __Pyx_RefNannyDeclarations
13205         PyObject *__pyx_t_1 = NULL;
13206         PyObject *__pyx_t_2 = NULL;
13207         PyObject *__pyx_t_3 = NULL;
13208         int __pyx_lineno = 0;
13209         const char *__pyx_filename = NULL;
13210         int __pyx_clineno = 0;
13211         __Pyx_RefNannySetupContext("reverse", 0);
13212
13213         /* "PyClicl.pyx":1483
13214     *     1+{1}-{1,2}
13215     *     """
13216     *     return clifford(obj).reverse()        # ««««««««
13217     *
13218     * cpdef inline conj(obj):

```

```

13219 */
13220     __Pyx_XDECREF(__pyx_r);
13221     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1483, __pyx_L1_error)
13222     __Pyx_GOTREF(__pyx_t_2);
13223     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_reverse); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1483, __pyx_L1_error)
13224     __Pyx_GOTREF(__pyx_t_3);
13225     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13226     __pyx_t_2 = NULL;
13227     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13228         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13229         if (likely(__pyx_t_2)) {
13230             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13231             __Pyx_INCREF(__pyx_t_2);
13232             __Pyx_INCREF(function);
13233             __Pyx_DECREF_SET(__pyx_t_3, function);
13234         }
13235     }
13236     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13237     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13238     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1483, __pyx_L1_error)
13239     __Pyx_GOTREF(__pyx_t_1);
13240     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13241     __pyx_r = __pyx_t_1;
13242     __pyx_t_1 = 0;
13243     goto __pyx_L0;
13244
13245     /* "PyClical.pyx":1470
13246 *     return clifford(obj).involute()
13247 *
13248 * cpdef inline reverse(obj):                # <<<<<<<<
13249 *     """
13250 *     Reversion, eg. {1}*{2} -> {2}*{1}
13251 */
13252
13253     /* function exit code */
13254     __pyx_L1_error:;
13255     __Pyx_XDECREF(__pyx_t_1);
13256     __Pyx_XDECREF(__pyx_t_2);
13257     __Pyx_XDECREF(__pyx_t_3);
13258     __Pyx_AddTraceback("PyClical.reverse", __pyx_clineno, __pyx_lineno, __pyx_filename);
13259     __pyx_r = 0;
13260     __pyx_L0:;
13261     __Pyx_XGIVEREF(__pyx_r);
13262     __Pyx_RefNannyFinishContext();
13263     return __pyx_r;
13264 }
13265
13266     /* Python wrapper */
13267     static PyObject * __pyx_pw_8PyClical_33reverse(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13268     static char __pyx_doc_8PyClical_32reverse[] = "\n    Reversion, eg. {1}*{2} ->
{2}*{1}\n\n    >> print(reverse(clifford(\"{1}\")))\n    {1}\n    >> print(reverse(clifford(\"{2}\")
* clifford(\"{1}\")))\n    {1,2}\n    >> print(reverse(clifford(\"{1}\") * clifford(\"{2}\")))\n
-{1,2}\n    >> print(reverse(clifford(\"1+{1}+{1,2}\")))\n    1+{1}-{1,2}\n    ";
13269     static PyObject * __pyx_pw_8PyClical_33reverse(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
13270         PyObject * __pyx_r = 0;
13271         __Pyx_RefNannyDeclarations
13272         __Pyx_RefNannySetupContext("reverse (wrapper)", 0);
13273         __pyx_r = __pyx_pf_8PyClical_32reverse(__pyx_self, ((PyObject *) __pyx_v_obj));
13274
13275         /* function exit code */
13276         __Pyx_RefNannyFinishContext();
13277         return __pyx_r;
13278     }
13279
13280     static PyObject * __pyx_pf_8PyClical_32reverse(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
13281         PyObject * __pyx_r = NULL;
13282         __Pyx_RefNannyDeclarations
13283         PyObject * __pyx_t_1 = NULL;
13284         int __pyx_lineno = 0;
13285         const char * __pyx_filename = NULL;
13286         int __pyx_clineno = 0;
13287         __Pyx_RefNannySetupContext("reverse", 0);
13288         __Pyx_XDECREF(__pyx_r);
13289         __pyx_t_1 = __pyx_f_8PyClical_reverse(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1470, __pyx_L1_error)
13290         __Pyx_GOTREF(__pyx_t_1);
13291         __pyx_r = __pyx_t_1;
13292         __pyx_t_1 = 0;
13293         goto __pyx_L0;
13294
13295         /* function exit code */

```

```

13296         __pyx_L1_error;;
13297         __Pyx_XDECREF(__pyx_t_1);
13298         __Pyx_AddTraceback("PyClicl.reverse", __pyx_clineno, __pyx_lineno, __pyx_filename);
13299         __pyx_r = NULL;
13300         __pyx_L0;
13301         __Pyx_XGIVEREF(__pyx_r);
13302         __Pyx_RefNannyFinishContext();
13303         return __pyx_r;
13304     }
13305
13306     /* "PyClicl.pyx":1485
13307     *     return clifford(obj).reverse()
13308     *
13309     * cpdef inline conj(obj):
13310     *     """
13311     *     Conjugation, reverse o involute == involute o reverse.
13312     */
13313
13314     static PyObject *__pyx_pw_8PyClicl_35conj(PyObject *__pyx_self, PyObject
13315 *__pyx_v_obj); /*proto*/
13316     static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_conj(PyObject *__pyx_v_obj,
13317 CYTHON_UNUSED int __pyx_skip_dispatch) {
13318     PyObject *__pyx_r = NULL;
13319     __Pyx_RefNannyDeclarations
13320     PyObject *__pyx_t_1 = NULL;
13321     PyObject *__pyx_t_2 = NULL;
13322     PyObject *__pyx_t_3 = NULL;
13323     int __pyx_lineno = 0;
13324     const char *__pyx_filename = NULL;
13325     int __pyx_clineno = 0;
13326     __Pyx_RefNannySetupContext("conj", 0);
13327
13328     /* "PyClicl.pyx":1498
13329     *     1-{1}-{1,2}
13330     *     """
13331     *     return clifford(obj).conj()
13332     *     # ««««««««
13333     *
13334     * cpdef inline quad(obj):
13335     */
13336     __Pyx_XDECREF(__pyx_r);
13337     __pyx_t_2 = __PyxPyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClicl_clifford),
13338 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1498, __pyx_L1_error)
13339     __Pyx_GOTREF(__pyx_t_2);
13340     __pyx_t_3 = __PyxPyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_conj); if
13341 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1498, __pyx_L1_error)
13342     __Pyx_GOTREF(__pyx_t_3);
13343     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13344     __pyx_t_2 = NULL;
13345     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13346         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13347         if (likely(__pyx_t_2)) {
13348             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13349             __Pyx_INCREF(__pyx_t_2);
13350             __Pyx_INCREF(function);
13351             __Pyx_DECREF_SET(__pyx_t_3, function);
13352         }
13353     }
13354     __pyx_t_1 = (__pyx_t_2) ? __PyxPyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
13355 __PyxPyObject_CallNoArg(__pyx_t_3);
13356     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13357     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1498, __pyx_L1_error)
13358     __Pyx_GOTREF(__pyx_t_1);
13359     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13360     __pyx_r = __pyx_t_1;
13361     __pyx_t_1 = 0;
13362     goto __pyx_L0;
13363
13364     /* "PyClicl.pyx":1485
13365     *     return clifford(obj).reverse()
13366     *
13367     * cpdef inline conj(obj):
13368     *     """
13369     *     Conjugation, reverse o involute == involute o reverse.
13370     */
13371
13372     /* function exit code */
13373     __pyx_L1_error;;
13374     __Pyx_XDECREF(__pyx_t_1);
13375     __Pyx_XDECREF(__pyx_t_2);
13376     __Pyx_XDECREF(__pyx_t_3);
13377     __Pyx_AddTraceback("PyClicl.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13378     __pyx_r = 0;
13379     __pyx_L0;
13380     __Pyx_XGIVEREF(__pyx_r);
13381     __Pyx_RefNannyFinishContext();
13382     return __pyx_r;
13383 }

```

```

13378
13379         /* Python wrapper */
13380         static PyObject *__pyx_pw_8PyClical_35conj(PyObject *__pyx_self, PyObject
13381         *__pyx_v_obj); /*proto*/
13382         static char __pyx_doc_8PyClical_34conj[] = "\n      Conjugation, reverse o involute ==
13383         involute o reverse.\n\n      >> print(conj(clifford(\"{1}\"))\n      -{1}\n      >>
13384         print(conj(clifford(\"{2}\") * clifford(\"{1}\"))\n      {1,2}\n      >> print(conj(clifford(\"{1}\") *
13385         clifford(\"{2}\"))\n      -{1,2}\n      >> print(conj(clifford(\"1+{1}+{1,2}\"))\n      1-{1}-{1,2}\n
13386         ";
13387         static PyObject *__pyx_pw_8PyClical_35conj(PyObject *__pyx_self, PyObject
13388         *__pyx_v_obj) {
13389             PyObject *__pyx_r = 0;
13390             __Pyx_RefNannyDeclarations
13391             __Pyx_RefNannySetupContext("conj (wrapper)", 0);
13392             __pyx_r = __pyx_pf_8PyClical_34conj(__pyx_self, ((PyObject *)__pyx_v_obj));
13393
13394             /* function exit code */
13395             __Pyx_RefNannyFinishContext();
13396             return __pyx_r;
13397         }
13398
13399         static PyObject *__pyx_pf_8PyClical_34conj(CYTHON_UNUSED PyObject *__pyx_self,
13400         PyObject *__pyx_v_obj) {
13401             PyObject *__pyx_r = NULL;
13402             __Pyx_RefNannyDeclarations
13403             PyObject *__pyx_t_1 = NULL;
13404             int __pyx_lineno = 0;
13405             const char *__pyx_filename = NULL;
13406             int __pyx_clineno = 0;
13407             __Pyx_RefNannySetupContext("conj", 0);
13408             __Pyx_XDECREF(__pyx_r);
13409             __pyx_t_1 = __pyx_f_8PyClical_conj(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
13410             __PYX_ERR(0, 1485, __pyx_L1_error)
13411             __Pyx_GOTREF(__pyx_t_1);
13412             __pyx_r = __pyx_t_1;
13413             __pyx_t_1 = 0;
13414             goto __pyx_L0;
13415
13416             /* function exit code */
13417             __pyx_L1_error:;
13418             __Pyx_XDECREF(__pyx_t_1);
13419             __Pyx_AddTraceback("PyClical.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13420             __pyx_r = NULL;
13421             __pyx_L0:;
13422             __Pyx_XGIVEREF(__pyx_r);
13423             __Pyx_RefNannyFinishContext();
13424             return __pyx_r;
13425         }
13426
13427         /* "PyClical.pyx":1500
13428         *      return clifford(obj).conj()
13429         *
13430         * cpdef inline quad(obj):          # ««««««««
13431         *      """
13432         *      Quadratic form == (rev(x)*x)(0).
13433         */
13434
13435         static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
13436         *__pyx_v_obj); /*proto*/
13437         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_quad(PyObject *__pyx_v_obj,
13438         CYTHON_UNUSED int __pyx_skip_dispatch) {
13439             PyObject *__pyx_r = NULL;
13440             __Pyx_RefNannyDeclarations
13441             PyObject *__pyx_t_1 = NULL;
13442             PyObject *__pyx_t_2 = NULL;
13443             PyObject *__pyx_t_3 = NULL;
13444             int __pyx_lineno = 0;
13445             const char *__pyx_filename = NULL;
13446             int __pyx_clineno = 0;
13447             __Pyx_RefNannySetupContext("quad", 0);
13448
13449             /* "PyClical.pyx":1509
13450             *      2.0
13451             *      """
13452             *      return clifford(obj).quad()          # ««««««««
13453             *
13454             * cpdef inline norm(obj):
13455             */
13456             __Pyx_XDECREF(__pyx_r);
13457             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
13458             __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1509, __pyx_L1_error)
13459             __Pyx_GOTREF(__pyx_t_2);
13460             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_quad); if
13461             (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1509, __pyx_L1_error)
13462             __Pyx_GOTREF(__pyx_t_3);
13463             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13464             __pyx_t_2 = NULL;

```

```

13453         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13454             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13455             if (likely(__pyx_t_2)) {
13456                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13457                 __Pyx_INCREF(__pyx_t_2);
13458                 __Pyx_INCREF(function);
13459                 __Pyx_DECREF_SET(__pyx_t_3, function);
13460             }
13461         }
13462         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13463         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13464         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1509, __pyx_L1_error)
13465         __Pyx_GOTREF(__pyx_t_1);
13466         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13467         __pyx_r = __pyx_t_1;
13468         __pyx_t_1 = 0;
13469         goto __pyx_L0;
13470
13471         /* "PyClical.pyx":1500
13472  *     return clifford(obj).conj()
13473  *
13474  * cpdef inline quad(obj):                # ««««««««
13475  *     """
13476  *     Quadratic form == (rev(x)*x)(0).
13477  */
13478
13479         /* function exit code */
13480         __pyx_L1_error;
13481         __Pyx_XDECREF(__pyx_t_1);
13482         __Pyx_XDECREF(__pyx_t_2);
13483         __Pyx_XDECREF(__pyx_t_3);
13484         __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13485         __pyx_r = 0;
13486         __pyx_L0;
13487         __Pyx_XGIVEREF(__pyx_r);
13488         __Pyx_RefNannyFinishContext();
13489         return __pyx_r;
13490     }
13491
13492     /* Python wrapper */
13493     static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13494     static char __pyx_doc_8PyClical_36quad[] = "\n    Quadratic form == (rev(x)*x)(0).\n\n
    >> print(quad(clifford(\"1+{1}+{1,2}\"))\n    3.0\n    >>
    print(quad(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n    2.0\n    ";
13495     static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13496         PyObject *__pyx_r = 0;
13497         __Pyx_RefNannyDeclarations
13498         __Pyx_RefNannySetupContext("quad (wrapper)", 0);
13499         __pyx_r = __pyx_pf_8PyClical_36quad(__pyx_self, ((PyObject *)__pyx_v_obj));
13500
13501         /* function exit code */
13502         __Pyx_RefNannyFinishContext();
13503         return __pyx_r;
13504     }
13505
13506     static PyObject *__pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13507         PyObject *__pyx_r = NULL;
13508         __Pyx_RefNannyDeclarations
13509         PyObject *__pyx_t_1 = NULL;
13510         int __pyx_lineno = 0;
13511         const char *__pyx_filename = NULL;
13512         int __pyx_clineno = 0;
13513         __Pyx_RefNannySetupContext("quad", 0);
13514         __Pyx_XDECREF(__pyx_r);
13515         __pyx_t_1 = __pyx_pf_8PyClical_quad(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1500, __pyx_L1_error)
13516         __Pyx_GOTREF(__pyx_t_1);
13517         __pyx_r = __pyx_t_1;
13518         __pyx_t_1 = 0;
13519         goto __pyx_L0;
13520
13521         /* function exit code */
13522         __pyx_L1_error;
13523         __Pyx_XDECREF(__pyx_t_1);
13524         __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13525         __pyx_r = NULL;
13526         __pyx_L0;
13527         __Pyx_XGIVEREF(__pyx_r);
13528         __Pyx_RefNannyFinishContext();
13529         return __pyx_r;
13530     }
13531
13532     /* "PyClical.pyx":1511

```

```

13533 *      return clifford(obj).quad()
13534 *
13535 * cpdef inline norm(obj):          # ««««««««
13536 *      """
13537 *      norm == sum of squares of coordinates.
13538 */
13539
13540 static PyObject *__pyx_pw_8PyClical_39norm(PyObject *__pyx_self, PyObject
13541 *__pyx_v_obj); /*proto*/
13542 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_norm(PyObject *__pyx_v_obj,
13543 CYTHON_UNUSED int __pyx_skip_dispatch) {
13544     PyObject *__pyx_r = NULL;
13545     __Pyx_RefNannyDeclarations
13546     PyObject *__pyx_t_1 = NULL;
13547     PyObject *__pyx_t_2 = NULL;
13548     PyObject *__pyx_t_3 = NULL;
13549     int __pyx_lineno = 0;
13550     const char *__pyx_filename = NULL;
13551     int __pyx_clineno = 0;
13552     __Pyx_RefNannySetupContext("norm", 0);
13553
13554     /* "PyClical.pyx":1520
13555     *      4.0
13556     *      """
13557     *      return clifford(obj).norm()          # ««««««««
13558 */
13559     __Pyx_XDECREF(__pyx_r);
13560     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
13561     __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1520, __pyx_L1_error)
13562     __Pyx_GOTREF(__pyx_t_2);
13563     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_norm); if
13564     (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1520, __pyx_L1_error)
13565     __Pyx_GOTREF(__pyx_t_3);
13566     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13567     __pyx_t_2 = NULL;
13568     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13569         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13570         if (likely(__pyx_t_2)) {
13571             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13572             __Pyx_INCREF(__pyx_t_2);
13573             __Pyx_INCREF(function);
13574             __Pyx_DECREF_SET(__pyx_t_3, function);
13575         }
13576     }
13577     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
13578     __Pyx_PyObject_CallNoArg(__pyx_t_3);
13579     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13580     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1520, __pyx_L1_error)
13581     __Pyx_GOTREF(__pyx_t_1);
13582     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13583     __pyx_r = __pyx_t_1;
13584     __pyx_t_1 = 0;
13585     goto __pyx_L0;
13586
13587     /* "PyClical.pyx":1511
13588     *      return clifford(obj).quad()
13589     *
13590     * cpdef inline norm(obj):          # ««««««««
13591     *      """
13592     *      norm == sum of squares of coordinates.
13593     */
13594
13595     /* function exit code */
13596     __pyx_L1_error:;
13597     __Pyx_XDECREF(__pyx_t_1);
13598     __Pyx_XDECREF(__pyx_t_2);
13599     __Pyx_XDECREF(__pyx_t_3);
13600     __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13601     __pyx_r = 0;
13602     __pyx_L0:;
13603     __Pyx_XGIVEREF(__pyx_r);
13604     __Pyx_RefNannyFinishContext();
13605     return __pyx_r;
13606 }
13607
13608 /* Python wrapper */
13609 static PyObject *__pyx_pw_8PyClical_39norm(PyObject *__pyx_self, PyObject
13610 *__pyx_v_obj); /*proto*/
13611 static char __pyx_doc_8PyClical_38norm[] = "\n      norm == sum of squares of
13612 coordinates.\n\n      >> norm(clifford(\"1+{1}+{1,2}\"))\n      3.0\n      >>
13613 norm(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n      4.0\n      ";
13614 static PyObject *__pyx_pw_8PyClical_39norm(PyObject *__pyx_self, PyObject
13615 *__pyx_v_obj) {
13616     PyObject *__pyx_r = 0;
13617     __Pyx_RefNannyDeclarations

```

```

13611         __Pyx_RefNannySetupContext("norm (wrapper)", 0);
13612         __pyx_r = __pyx_pf_8PyClical_38norm(__pyx_self, ((PyObject *)__pyx_v_obj));
13613
13614         /* function exit code */
13615         __Pyx_RefNannyFinishContext();
13616         return __pyx_r;
13617     }
13618
13619     static PyObject *__pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13620         PyObject *__pyx_r = NULL;
13621         __Pyx_RefNannyDeclarations
13622         PyObject *__pyx_t_1 = NULL;
13623         int __pyx_lineno = 0;
13624         const char *__pyx_filename = NULL;
13625         int __pyx_clineno = 0;
13626         __Pyx_RefNannySetupContext("norm", 0);
13627         __Pyx_XDECREF(__pyx_r);
13628         __pyx_t_1 = __pyx_f_8PyClical_norm(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1511, __pyx_L1_error)
13629         __Pyx_GOTREF(__pyx_t_1);
13630         __pyx_r = __pyx_t_1;
13631         __pyx_t_1 = 0;
13632         goto __pyx_L0;
13633
13634         /* function exit code */
13635         __pyx_L1_error:;
13636         __Pyx_XDECREF(__pyx_t_1);
13637         __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13638         __pyx_r = NULL;
13639         __pyx_L0:;
13640         __Pyx_XGIVEREF(__pyx_r);
13641         __Pyx_RefNannyFinishContext();
13642         return __pyx_r;
13643     }
13644
13645     /* "PyClical.pyx":1522
13646     *     return clifford(obj).norm()
13647     *
13648     * cpdef inline abs(obj):          # ««««««««
13649     *     """
13650     *     Absolute value of multivector: multivector 2-norm.
13651     */
13652
13653     static PyObject *__pyx_pw_8PyClical_41abs(PyObject *__pyx_self, PyObject
__pyx_v_obj); /*proto*/
13654     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_abs(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13655         PyObject *__pyx_r = NULL;
13656         __Pyx_RefNannyDeclarations
13657         PyObject *__pyx_t_1 = NULL;
13658         int __pyx_lineno = 0;
13659         const char *__pyx_filename = NULL;
13660         int __pyx_clineno = 0;
13661         __Pyx_RefNannySetupContext("abs", 0);
13662
13663         /* "PyClical.pyx":1529
13664     *     2.0
13665     *     """
13666     *     return glucat.abs(toClifford(obj))          # ««««««««
13667     *
13668     * cpdef inline max_abs(obj):
13669     */
13670         __Pyx_XDECREF(__pyx_r);
13671         __pyx_t_1 = PyFloat_FromDouble(abs(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1529, __pyx_L1_error)
13672         __Pyx_GOTREF(__pyx_t_1);
13673         __pyx_r = __pyx_t_1;
13674         __pyx_t_1 = 0;
13675         goto __pyx_L0;
13676
13677         /* "PyClical.pyx":1522
13678     *     return clifford(obj).norm()
13679     *
13680     * cpdef inline abs(obj):          # ««««««««
13681     *     """
13682     *     Absolute value of multivector: multivector 2-norm.
13683     */
13684
13685         /* function exit code */
13686         __pyx_L1_error:;
13687         __Pyx_XDECREF(__pyx_t_1);
13688         __Pyx_AddTraceback("PyClical.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13689         __pyx_r = 0;
13690         __pyx_L0:;
13691         __Pyx_XGIVEREF(__pyx_r);
13692         __Pyx_RefNannyFinishContext();

```



```

13693         return __pyx_r;
13694     }
13695
13696     /* Python wrapper */
13697     static PyObject * __pyx_pw_8PyClicl_41abs(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13698     static char __pyx_doc_8PyClicl_40abs[] = "\n    Absolute value of multivector:
multivector 2-norm.\n\n    >> abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n    2.0\n    ";
13699     static PyObject * __pyx_pw_8PyClicl_41abs(PyObject * __pyx_self, PyObject * __pyx_v_obj)
{
13700         PyObject * __pyx_r = 0;
13701         __Pyx_RefNannyDeclarations
13702         __Pyx_RefNannySetupContext("abs (wrapper)", 0);
13703         __pyx_r = __pyx_pf_8PyClicl_40abs(__pyx_self, ((PyObject *) __pyx_v_obj));
13704
13705         /* function exit code */
13706         __Pyx_RefNannyFinishContext();
13707         return __pyx_r;
13708     }
13709
13710     static PyObject * __pyx_pf_8PyClicl_40abs(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
13711         PyObject * __pyx_r = NULL;
13712         __Pyx_RefNannyDeclarations
13713         PyObject * __pyx_t_1 = NULL;
13714         int __pyx_lineno = 0;
13715         const char * __pyx_filename = NULL;
13716         int __pyx_clineno = 0;
13717         __Pyx_RefNannySetupContext("abs", 0);
13718         __Pyx_XDECREF(__pyx_r);
13719         __pyx_t_1 = __pyx_f_8PyClicl_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1522, __pyx_L1_error)
13720         __Pyx_GOTREF(__pyx_t_1);
13721         __pyx_r = __pyx_t_1;
13722         __pyx_t_1 = 0;
13723         goto __pyx_L0;
13724
13725         /* function exit code */
13726         __pyx_L1_error:;
13727         __Pyx_XDECREF(__pyx_t_1);
13728         __Pyx_AddTraceback("PyClicl.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13729         __pyx_r = NULL;
13730         __pyx_L0:;
13731         __Pyx_XGIVEREF(__pyx_r);
13732         __Pyx_RefNannyFinishContext();
13733         return __pyx_r;
13734     }
13735
13736     /* "PyClicl.pyx":1531
13737     *     return glucat.abs(toClifford(obj))
13738     *
13739     * cpdef inline max_abs(obj): # <<<<<<<<
13740     *     """
13741     *     Maximum absolute value of coordinates multivector: multivector infinity-norm.
13742     */
13743
13744     static PyObject * __pyx_pw_8PyClicl_43max_abs(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13745     static CYTHON_INLINE PyObject * __pyx_f_8PyClicl_max_abs(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13746         PyObject * __pyx_r = NULL;
13747         __Pyx_RefNannyDeclarations
13748         PyObject * __pyx_t_1 = NULL;
13749         int __pyx_lineno = 0;
13750         const char * __pyx_filename = NULL;
13751         int __pyx_clineno = 0;
13752         __Pyx_RefNannySetupContext("max_abs", 0);
13753
13754         /* "PyClicl.pyx":1541
13755     *
13756     *     """
13757     *     return glucat.max_abs(toClifford(obj)) # <<<<<<<<
13758     *
13759     * cpdef inline pow(obj, m):
13760     */
13761         __Pyx_XDECREF(__pyx_r);
13762         __pyx_t_1 = PyFloat_FromDouble(max_abs(__pyx_f_8PyClicl_toClifford(__pyx_v_obj)));
if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1541, __pyx_L1_error)
13763         __Pyx_GOTREF(__pyx_t_1);
13764         __pyx_r = __pyx_t_1;
13765         __pyx_t_1 = 0;
13766         goto __pyx_L0;
13767
13768         /* "PyClicl.pyx":1531
13769     *     return glucat.abs(toClifford(obj))
13770     *
13771     * cpdef inline max_abs(obj): # <<<<<<<<

```

```

13772 *      """
13773 *      Maximum absolute value of coordinates multivector: multivector infinity-norm.
13774 */
13775
13776         /* function exit code */
13777         __pyx_L1_error;;
13778         __Pyx_XDECREF(__pyx_t_1);
13779         __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13780         __pyx_r = 0;
13781         __pyx_L0;;
13782         __Pyx_XGIVEREF(__pyx_r);
13783         __Pyx_RefNannyFinishContext();
13784         return __pyx_r;
13785     }
13786
13787     /* Python wrapper */
13788     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
13789 *__pyx_v_obj); /*proto*/
13790     static char __pyx_doc_8PyClical_42max_abs[] = "\n      Maximum absolute value of
coordinates multivector: multivector infinity-norm.\n\n      >>
max_abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n      1.0\n      >> max_abs(clifford(\"3+2{1}+{1,2}\"))\n
3.0\n\n      ";
13791     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
13792 *__pyx_v_obj) {
13793         PyObject *__pyx_r = 0;
13794         __Pyx_RefNannyDeclarations
13795         __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
13796         __pyx_r = __pyx_pf_8PyClical_42max_abs(__pyx_self, ((PyObject *)__pyx_v_obj));
13797
13798         /* function exit code */
13799         __Pyx_RefNannyFinishContext();
13800         return __pyx_r;
13801     }
13802     static PyObject *__pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13803         PyObject *__pyx_r = NULL;
13804         __Pyx_RefNannyDeclarations
13805         PyObject *__pyx_t_1 = NULL;
13806         int __pyx_lineno = 0;
13807         const char *__pyx_filename = NULL;
13808         int __pyx_clineno = 0;
13809         __Pyx_RefNannySetupContext("max_abs", 0);
13810         __Pyx_XDECREF(__pyx_r);
13811         __pyx_t_1 = __pyx_f_8PyClical_max_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1531, __pyx_L1_error)
13812         __Pyx_GOTREF(__pyx_t_1);
13813         __pyx_r = __pyx_t_1;
13814         __pyx_t_1 = 0;
13815         goto __pyx_L0;
13816
13817         /* function exit code */
13818         __pyx_L1_error;;
13819         __Pyx_XDECREF(__pyx_t_1);
13820         __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13821         __pyx_r = NULL;
13822         __pyx_L0;;
13823         __Pyx_XGIVEREF(__pyx_r);
13824         __Pyx_RefNannyFinishContext();
13825         return __pyx_r;
13826     }
13827
13828     /* "PyClical.pyx":1543
13829 *      return glucat.max_abs(toClifford(obj))
13830 *
13831 *      cpdef inline pow(obj, m):          # ««««««««
13832 *      """
13833 *      Integer power of multivector: obj to the m.
13834 */
13835
13836     static PyObject *__pyx_pw_8PyClical_45pow(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
13837     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pow(PyObject *__pyx_v_obj, PyObject
13838 *__pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
13839         PyObject *__pyx_r = NULL;
13840         __Pyx_RefNannyDeclarations
13841         PyObject *__pyx_t_1 = NULL;
13842         PyObject *__pyx_t_2 = NULL;
13843         PyObject *__pyx_t_3 = NULL;
13844         PyObject *__pyx_t_4 = NULL;
13845         PyObject *__pyx_t_5 = NULL;
13846         PyObject *__pyx_t_6 = NULL;
13847         int __pyx_t_7;
13848         PyObject *__pyx_t_8 = NULL;
13849         PyObject *__pyx_t_9 = NULL;
13850         PyObject *__pyx_t_10 = NULL;
13851         int __pyx_lineno = 0;

```

```

13850         const char *__pyx_filename = NULL;
13851         int __pyx_clineno = 0;
13852         __Pyx_RefNannySetupContext("pow", 0);
13853
13854         /* "PyClical.pyx":1562
13855         *      1
13856         *      """
13857         *      try:          # ««««««««
13858         *          math.pow(obj, m)
13859         *      except:
13860         */
13861         {
13862             __Pyx_PyThreadState_declare
13863             __Pyx_PyThreadState_assign
13864             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
13865             __Pyx_XGOTREF(__pyx_t_1);
13866             __Pyx_XGOTREF(__pyx_t_2);
13867             __Pyx_XGOTREF(__pyx_t_3);
13868             /*try:*/ {
13869
13870             /* "PyClical.pyx":1563
13871             *      """
13872             *      try:
13873             *          math.pow(obj, m)          # ««««««««
13874             *      except:
13875             *          return clifford(obj).pow(m)
13876             */
13877             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
13878             __PYX_ERR(0, 1563, __pyx_L3_error)
13879             __Pyx_GOTREF(__pyx_t_5);
13880             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_pow); if
13881             (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1563, __pyx_L3_error)
13882             __Pyx_GOTREF(__pyx_t_6);
13883             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
13884             __pyx_t_5 = NULL;
13885             __pyx_t_7 = 0;
13886             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
13887                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
13888                 if (likely(__pyx_t_5)) {
13889                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
13890                     __Pyx_INCREF(__pyx_t_5);
13891                     __Pyx_INCREF(function);
13892                     __Pyx_DECREF_SET(__pyx_t_6, function);
13893                     __pyx_t_7 = 1;
13894                 }
13895             }
13896             #if CYTHON_FAST_PYCALL
13897             if (PyFunction_Check(__pyx_t_6)) {
13898                 PyObject *__pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13899                 __pyx_t_4 = __Pyx_PyFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
13900                 2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13901                 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13902                 __Pyx_GOTREF(__pyx_t_4);
13903             } else
13904             #endif
13905             #if CYTHON_FAST_PYCCALL
13906             if (__Pyx_PyFastCFunction_Check(__pyx_t_6)) {
13907                 PyObject *__pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13908                 __pyx_t_4 = __Pyx_PyCFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
13909                 2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13910                 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13911                 __Pyx_GOTREF(__pyx_t_4);
13912             } else
13913             #endif
13914             {
13915                 __pyx_t_8 = PyTuple_New(2+__pyx_t_7); if (unlikely(!__pyx_t_8)) __PYX_ERR(0,
13916                 1563, __pyx_L3_error)
13917                 __Pyx_GOTREF(__pyx_t_8);
13918                 if (__pyx_t_5) {
13919                     __Pyx_GIVEREF(__pyx_t_5); PyTuple_SET_ITEM(__pyx_t_8, 0, __pyx_t_5);
13920                     __pyx_t_5 = NULL;
13921                 }
13922                 __Pyx_INCREF(__pyx_v_obj);
13923                 __Pyx_GIVEREF(__pyx_v_obj);
13924                 PyTuple_SET_ITEM(__pyx_t_8, 0+__pyx_t_7, __pyx_v_obj);
13925                 __Pyx_INCREF(__pyx_v_m);
13926                 __Pyx_GIVEREF(__pyx_v_m);
13927                 PyTuple_SET_ITEM(__pyx_t_8, 1+__pyx_t_7, __pyx_v_m);
13928                 __pyx_t_4 = __Pyx_PyObject_Call(__pyx_t_6, __pyx_t_8, NULL); if
13929                 (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13930                 __Pyx_GOTREF(__pyx_t_4);
13931                 __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13932             }
13933             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13934             __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13935
13936             /* "PyClical.pyx":1562

```

```

13930 *      1
13931 *      """
13932 *      try:          # ««««««««
13933 *          math.pow(obj, m)
13934 *      except:
13935 */
13936
13937         __Pyx_XDECREF(__pyx_t_1); __pyx_t_1 = 0;
13938         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13939         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
13940         goto __pyx_L8_try_end;
13941         __pyx_L3_error;;
13942         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
13943         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13944         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
13945         __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
13946
13947         /* "PyClicl.pyx":1564
13948 *      try:
13949 *          math.pow(obj, m)
13950 *      except:      # ««««««««
13951 *          return clifford(obj).pow(m)
13952 *
13953 */
13954
13955         /*except:*/ {
13956             __Pyx_AddTraceback("PyClicl.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
13957             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_8) < 0) __PYX_ERR(0,
1564, __pyx_L5_except_error)
13958             __Pyx_GOTREF(__pyx_t_4);
13959             __Pyx_GOTREF(__pyx_t_6);
13960             __Pyx_GOTREF(__pyx_t_8);
13961
13962             /* "PyClicl.pyx":1565
13963 *          math.pow(obj, m)
13964 *      except:
13965 *          return clifford(obj).pow(m)          # ««««««««
13966 *
13967 *      cpdef inline outer_pow(obj, m):
13968 */
13969             __Pyx_XDECREF(__pyx_r);
13970             __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject
*)__pyx_ptype_8PyClicl_clifford, __pyx_v_obj); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1565,
__pyx_L5_except_error)
13971             __Pyx_GOTREF(__pyx_t_9);
13972             __pyx_t_10 = __Pyx_PyObject_GetAttrStr(__pyx_t_9, __pyx_n_s_pow); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13973             __Pyx_GOTREF(__pyx_t_10);
13974             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
13975             __pyx_t_9 = NULL;
13976             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_10))) {
13977                 __pyx_t_9 = PyMethod_GET_SELF(__pyx_t_10);
13978                 if (likely(__pyx_t_9)) {
13979                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_10);
13980                     __Pyx_INCREF(__pyx_t_9);
13981                     __Pyx_INCREF(function);
13982                     __Pyx_DECREF_SET(__pyx_t_10, function);
13983                 }
13984             }
13985             __pyx_t_5 = (__pyx_t_9) ? __Pyx_PyObject_Call2Args(__pyx_t_10, __pyx_t_9,
__pyx_v_m) : __Pyx_PyObject_CallOneArg(__pyx_t_10, __pyx_v_m);
13986             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
13987             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13988             __Pyx_GOTREF(__pyx_t_5);
13989             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
13990             __pyx_r = __pyx_t_5;
13991             __pyx_t_5 = 0;
13992             __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13993             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13994             __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13995             goto __pyx_L6_except_return;
13996         }
13997         __pyx_L5_except_error;;
13998
13999         /* "PyClicl.pyx":1562
14000 *      1
14001 *      """
14002 *      try:          # ««««««««
14003 *          math.pow(obj, m)
14004 *      except:
14005 */
14006
14007         __Pyx_XGIVEREF(__pyx_t_1);
14008         __Pyx_XGIVEREF(__pyx_t_2);
14009         __Pyx_XGIVEREF(__pyx_t_3);
14010         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14011         goto __pyx_L1_error;
14012         __pyx_L6_except_return;;
14013         __Pyx_XGIVEREF(__pyx_t_1);

```

```

14012         __Pyx_XGIVEREF(__pyx_t_2);
14013         __Pyx_XGIVEREF(__pyx_t_3);
14014         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14015         goto __pyx_L0;
14016         __pyx_L8_try_end;
14017     }
14018
14019     /* "PyClical.pyx":1543
14020 *     return glucat.max_abs(toClifford(obj))
14021 *
14022 * cpdef inline pow(obj, m):          # ««««««««
14023 *     """
14024 *     Integer power of multivector: obj to the m.
14025 */
14026
14027     /* function exit code */
14028     __pyx_r = Py_None; __Pyx_INCREF(Py_None);
14029     goto __pyx_L0;
14030     __pyx_L1_error;
14031     __Pyx_XDECREF(__pyx_t_4);
14032     __Pyx_XDECREF(__pyx_t_5);
14033     __Pyx_XDECREF(__pyx_t_6);
14034     __Pyx_XDECREF(__pyx_t_8);
14035     __Pyx_XDECREF(__pyx_t_9);
14036     __Pyx_XDECREF(__pyx_t_10);
14037     __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14038     __pyx_r = 0;
14039     __pyx_L0;
14040     __Pyx_XGIVEREF(__pyx_r);
14041     __Pyx_RefNannyFinishContext();
14042     return __pyx_r;
14043 }
14044
14045     /* Python wrapper */
14046     static PyObject* __pyx_pw_8PyClical_45pow(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds); /*proto*/
14047     static char __pyx_doc_8PyClical_44pow[] = "\n Integer power of multivector: obj to
the m.\n\n >> x=clifford(\"{1}\"); print(pow(x,2))\n 1\n >> x=clifford(\"2\");
print(pow(x,2))\n 4\n >> x=clifford(\"2+{1}\"); print(pow(x,0))\n 1\n >>
x=clifford(\"2+{1}\"); print(pow(x,1))\n 2+{1}\n >> x=clifford(\"2+{1}\"); print(pow(x,2))\n
5+4{1}\n >> print(pow(clifford(\"1+{1}+{1,2}\"),3))\n 1+3{1}+3{1,2}\n >>
i=clifford(\"{1,2}\"); print(exp(pi/2) * pow(i, i))\n 1\n ";
14048     static PyObject* __pyx_pw_8PyClical_45pow(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds) {
14049         PyObject* __pyx_v_obj = 0;
14050         PyObject* __pyx_v_m = 0;
14051         int __pyx_lineno = 0;
14052         const char* __pyx_filename = NULL;
14053         int __pyx_clineno = 0;
14054         PyObject* __pyx_r = 0;
14055         __Pyx_RefNannyDeclarations
14056         __Pyx_RefNannySetupContext("pow (wrapper)", 0);
14057         {
14058             static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14059             PyObject* values[2] = {0,0};
14060             if (unlikely(__pyx_kwds)) {
14061                 Py_ssize_t kw_args;
14062                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14063                 switch (pos_args) {
14064                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14065                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
14066                     case 0: break;
14067                     default: goto __pyx_L5_argtuple_error;
14068                 }
14069                 kw_args = PyDict_Size(__pyx_kwds);
14070                 switch (pos_args) {
14071                     case 0:
14072                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14073                         else goto __pyx_L5_argtuple_error;
14074                         CYTHON_FALLTHROUGH;
14075                     case 1:
14076                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14077                         else {
14078                             __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, 1); __PYX_ERR(0, 1543,
__pyx_L3_error)
14079                         }
14080                     }
14081                 if (unlikely(kw_args > 0)) {
14082                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "pow") < 0)) __PYX_ERR(0, 1543, __pyx_L3_error)
14083                 }
14084                 } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14085                     goto __pyx_L5_argtuple_error;
14086                 }
14087             }

```

```

14088         } else {
14089             values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14090             values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14091         }
14092         __pyx_v_obj = values[0];
14093         __pyx_v_m = values[1];
14094     }
14095     goto __pyx_L4_argument_unpacking_done;
14096     __pyx_L5_argtuple_error:;
14097     __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14098     __PYX_ERR(0, 1543, __pyx_L3_error)
14099     __pyx_L3_error:;
14100     __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14101     __Pyx_RefNannyFinishContext();
14102     return NULL;
14103     __pyx_L4_argument_unpacking_done:;
14104     __pyx_r = __pyx_pf_8PyClical_44pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14105
14106     /* function exit code */
14107     __Pyx_RefNannyFinishContext();
14108     return __pyx_r;
14109 }
14110
14111 static PyObject * __pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj, PyObject * __pyx_v_m) {
14112     PyObject * __pyx_r = NULL;
14113     __Pyx_RefNannyDeclarations
14114     PyObject * __pyx_t_1 = NULL;
14115     int __pyx_lineno = 0;
14116     const char * __pyx_filename = NULL;
14117     int __pyx_clineno = 0;
14118     __Pyx_RefNannySetupContext("pow", 0);
14119     __Pyx_XDECREF(__pyx_r);
14120     __pyx_t_1 = __pyx_f_8PyClical_pow(__pyx_v_obj, __pyx_v_m, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1543, __pyx_L1_error)
14121     __Pyx_GOTREF(__pyx_t_1);
14122     __pyx_r = __pyx_t_1;
14123     __pyx_t_1 = 0;
14124     goto __pyx_L0;
14125
14126     /* function exit code */
14127     __pyx_L1_error:;
14128     __Pyx_XDECREF(__pyx_t_1);
14129     __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14130     __pyx_r = NULL;
14131     __Pyx_L0:;
14132     __Pyx_XGIVEREF(__pyx_r);
14133     __Pyx_RefNannyFinishContext();
14134     return __pyx_r;
14135 }
14136
14137 /* "PyClical.pyx":1567
14138 *
14139 * cpdef inline outer_pow(obj, m): # ««««««
14140 *     """
14141 *     Outer product power of multivector.
14142 */
14143
14144 static PyObject * __pyx_pw_8PyClical_47outer_pow(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds); /*proto*/
14145 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_outer_pow(PyObject * __pyx_v_obj,
PyObject * __pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
14146     PyObject * __pyx_r = NULL;
14147     __Pyx_RefNannyDeclarations
14148     PyObject * __pyx_t_1 = NULL;
14149     PyObject * __pyx_t_2 = NULL;
14150     PyObject * __pyx_t_3 = NULL;
14151     int __pyx_lineno = 0;
14152     const char * __pyx_filename = NULL;
14153     int __pyx_clineno = 0;
14154     __Pyx_RefNannySetupContext("outer_pow", 0);
14155
14156     /* "PyClical.pyx":1574
14157 *     1+3{1}+3{1,2}
14158 *     """
14159 *     return clifford(obj).outer_pow(m) # ««««««
14160 *
14161 * cpdef inline complexifier(obj):
14162 */
14163     __Pyx_XDECREF(__pyx_r);
14164     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1574, __pyx_L1_error)
14165     __Pyx_GOTREF(__pyx_t_2);
14166     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_outer_pow); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1574, __pyx_L1_error)
14167     __Pyx_GOTREF(__pyx_t_3);

```

```

14168         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
14169         __pyx_t_2 = NULL;
14170         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
14171             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
14172             if (likely(__pyx_t_2)) {
14173                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
14174                 __Pyx_INCREF(__pyx_t_2);
14175                 __Pyx_INCREF(function);
14176                 __Pyx_DECREF_SET(__pyx_t_3, function);
14177             }
14178         }
14179         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_Call2Args(__pyx_t_3, __pyx_t_2, __pyx_v_m)
: __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_v_m);
14180         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
14181         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1574, __pyx_L1_error)
14182         __Pyx_GOTREF(__pyx_t_1);
14183         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14184         __pyx_r = __pyx_t_1;
14185         __pyx_t_1 = 0;
14186         goto __pyx_L0;
14187
14188         /* "PyClical.pyx":1567
14189         *         return clifford(obj).pow(m)
14190         *
14191         * cpdef inline outer_pow(obj, m):          # ««««««««
14192         *         """
14193         *         Outer product power of multivector.
14194         */
14195
14196         /* function exit code */
14197         __pyx_L1_error;;
14198         __Pyx_XDECREF(__pyx_t_1);
14199         __Pyx_XDECREF(__pyx_t_2);
14200         __Pyx_XDECREF(__pyx_t_3);
14201         __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14202         __pyx_r = 0;
14203         __pyx_L0;;
14204         __Pyx_XGIVEREF(__pyx_r);
14205         __Pyx_RefNannyFinishContext();
14206         return __pyx_r;
14207     }
14208
14209     /* Python wrapper */
14210     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
14211     static char __pyx_doc_8PyClical_46outer_pow[] = "\n    Outer product power of
multivector.\n\n    >> print(outer_pow(clifford(\"1+{1}+{1,2}\",3))\n    1+3{1}+3{1,2}\n    ";
14212     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
14213         PyObject *__pyx_v_obj = 0;
14214         PyObject *__pyx_v_m = 0;
14215         int __pyx_lineno = 0;
14216         const char *__pyx_filename = NULL;
14217         int __pyx_clineno = 0;
14218         PyObject *__pyx_r = 0;
14219         __Pyx_RefNannyDeclarations
14220         __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
14221         {
14222             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14223             PyObject* values[2] = {0,0};
14224             if (unlikely(__pyx_kwds)) {
14225                 Py_ssize_t kw_args;
14226                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14227                 switch (pos_args) {
14228                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14229                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
14230                     case 0: break;
14231                     default: goto __pyx_L5_argtuple_error;
14232                 }
14233                 kw_args = PyDict_Size(__pyx_kwds);
14234                 switch (pos_args) {
14235                     case 0:
14236                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14237
14238                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
14239                     case 1:
14240                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14241
14242                         else {
14243                             __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, 1); __PYX_ERR(0, 1567,
__pyx_L3_error)
14244                         }
14245                 }
14246             }

```

```

14247         if (unlikely(kw_args > 0)) {
14248             if (unlikely(!Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "outer_pow") < 0)) __PYX_ERR(0, 1567, __pyx_L3_error)
14249         }
14250     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14251         goto __pyx_L5_argtuple_error;
14252     } else {
14253         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14254         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14255     }
14256     __pyx_v_obj = values[0];
14257     __pyx_v_m = values[1];
14258 }
14259 goto __pyx_L4_argument_unpacking_done;
14260 __pyx_L5_argtuple_error:;
14261 __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14262 __PYX_ERR(0, 1567, __pyx_L3_error)
14263 __pyx_L3_error:;
14264 __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14265 __Pyx_RefNannyFinishContext();
14266 return NULL;
14267 __pyx_L4_argument_unpacking_done:;
14268 __pyx_r = __pyx_pf_8PyClical_46outer_pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14269
14270 /* function exit code */
14271 __Pyx_RefNannyFinishContext();
14272 return __pyx_r;
14273 }
14274
14275 static PyObject *__pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_m) {
14276     PyObject *__pyx_r = NULL;
14277     __Pyx_RefNannyDeclarations
14278     PyObject *__pyx_t_1 = NULL;
14279     int __pyx_lineno = 0;
14280     const char *__pyx_filename = NULL;
14281     int __pyx_clineno = 0;
14282     __Pyx_RefNannySetupContext("outer_pow", 0);
14283     __Pyx_XDECREF(__pyx_r);
14284     __pyx_t_1 = __pyx_f_8PyClical_outer_pow(__pyx_v_obj, __pyx_v_m, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1567, __pyx_L1_error)
14285     __Pyx_GOTREF(__pyx_t_1);
14286     __pyx_r = __pyx_t_1;
14287     __pyx_t_1 = 0;
14288     goto __pyx_L0;
14289
14290 /* function exit code */
14291 __pyx_L1_error:;
14292 __Pyx_XDECREF(__pyx_t_1);
14293 __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14294 __pyx_r = NULL;
14295 __pyx_L0:;
14296 __Pyx_XGIVEREF(__pyx_r);
14297 __Pyx_RefNannyFinishContext();
14298 return __pyx_r;
14299 }
14300
14301 /* "PyClical.pyx":1576
14302 *     return clifford(obj).outer_pow(m)
14303 *
14304 * cpdef inline complexifier(obj): # ««««««
14305 *     Square root of -1 which commutes with all members of the frame of the given multivector.
14306 */
14307
14308 static PyObject *__pyx_pw_8PyClical_49complexifier(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
14309 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_complexifier(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
14310     PyObject *__pyx_r = NULL;
14311     __Pyx_RefNannyDeclarations
14312     PyObject *__pyx_t_1 = NULL;
14313     PyObject *__pyx_t_2 = NULL;
14314     int __pyx_lineno = 0;
14315     const char *__pyx_filename = NULL;
14316     int __pyx_clineno = 0;
14317     __Pyx_RefNannySetupContext("complexifier", 0);
14318
14319 /* "PyClical.pyx":1589
14320 *     {-1}
14321 *     """
14322 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) ) # ««««««
14323 *
14324 * cpdef inline sqrt(obj, i = None):
14325 */

```



```

14326         __Pyx_XDECREF(__pyx_r);
14327         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
14328         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1589, __pyx_L1_error)
14329         __Pyx_GOTREF(__pyx_t_1);
14330         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
14331         *)__pyx_t_1), complexifier(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
14332         __PYX_ERR(0, 1589, __pyx_L1_error)
14333         __Pyx_GOTREF(__pyx_t_2);
14334         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
14335         __pyx_r = __pyx_t_2;
14336         __pyx_t_2 = 0;
14337         goto __pyx_L0;
14338
14339         /* "PyClical.pyx":1576
14340         *
14341         * cpdef inline complexifier(obj):
14342         *     """
14343         *     Square root of -1 which commutes with all members of the frame of the given multivector.
14344         */
14345
14346         /* function exit code */
14347         __pyx_L1_error;
14348         __Pyx_XDECREF(__pyx_t_1);
14349         __Pyx_XDECREF(__pyx_t_2);
14350         __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
14351         __pyx_filename);
14352         __pyx_r = 0;
14353         __pyx_L0;
14354         __Pyx_XGIVEREF(__pyx_r);
14355         __Pyx_RefNannyFinishContext();
14356         return __pyx_r;
14357     }
14358
14359     /* Python wrapper */
14360     static PyObject *__pyx_pw_8PyClical_49complexifier(PyObject *__pyx_self, PyObject
14361     *__pyx_v_obj); /*proto*/
14362     static char __pyx_doc_8PyClical_48complexifier[] = "\n    Square root of -1 which
14363     commutes with all members of the frame of the given multivector.\n\n    >>
14364     print(complexifier(clifford(index_set({1})))\n    {1,2,3}\n    >>
14365     print(complexifier(clifford(index_set({-1})))\n    {-1}\n    >>
14366     print(complexifier(index_set({1})))\n    {1,2,3}\n    >> print(complexifier(index_set({-1})))\n
14367     {-1}\n    ";
14368     static PyObject *__pyx_pw_8PyClical_49complexifier(PyObject *__pyx_self, PyObject
14369     *__pyx_v_obj) {
14370         PyObject *__pyx_r = 0;
14371         __Pyx_RefNannyDeclarations
14372         __Pyx_RefNannySetupContext("complexifier (wrapper)", 0);
14373         __pyx_r = __pyx_pf_8PyClical_48complexifier(__pyx_self, ((PyObject *)__pyx_v_obj));
14374
14375         /* function exit code */
14376         __Pyx_RefNannyFinishContext();
14377         return __pyx_r;
14378     }
14379
14380     static PyObject *__pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject *__pyx_self,
14381     PyObject *__pyx_v_obj) {
14382         PyObject *__pyx_r = NULL;
14383         __Pyx_RefNannyDeclarations
14384         PyObject *__pyx_t_1 = NULL;
14385         int __pyx_lineno = 0;
14386         const char *__pyx_filename = NULL;
14387         int __pyx_clineno = 0;
14388         __Pyx_RefNannySetupContext("complexifier", 0);
14389         __Pyx_XDECREF(__pyx_r);
14390         __pyx_t_1 = __pyx_f_8PyClical_complexifier(__pyx_v_obj, 0); if
14391         (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1576, __pyx_L1_error)
14392         __Pyx_GOTREF(__pyx_t_1);
14393         __pyx_r = __pyx_t_1;
14394         __pyx_t_1 = 0;
14395         goto __pyx_L0;
14396
14397         /* function exit code */
14398         __pyx_L1_error;
14399         __Pyx_XDECREF(__pyx_t_1);
14400         __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
14401         __pyx_filename);
14402         __pyx_r = NULL;
14403         __pyx_L0;
14404         __Pyx_XGIVEREF(__pyx_r);
14405         __Pyx_RefNannyFinishContext();
14406         return __pyx_r;
14407     }
14408
14409     /* "PyClical.pyx":1591
14410     *
14411     * return clifford().wrap( glucat.complexifier(toClifford(obj)) )
14412     */

```

```

14399 * cpdef inline sqrt(obj, i = None):          # ««««««««
14400 *      """
14401 *      Square root of multivector with optional complexifier.
14402 */
14403
14404     static PyObject *__pyx_pw_8PyClical_51sqrt(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
14405     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sqrt(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_sqrt *__pyx_optional_args) {
14406         PyObject *__pyx_v_i = ((PyObject *)Py_None);
14407         PyObject *__pyx_r = NULL;
14408         __Pyx_RefNannyDeclarations
14409         int __pyx_t_1;
14410         int __pyx_t_2;
14411         PyObject *__pyx_t_3 = NULL;
14412         Clifford __pyx_t_4;
14413         PyObject *__pyx_t_5 = NULL;
14414         PyObject *__pyx_t_6 = NULL;
14415         PyObject *__pyx_t_7 = NULL;
14416         PyObject *__pyx_t_8 = NULL;
14417         PyObject *__pyx_t_9 = NULL;
14418         PyObject *__pyx_t_10 = NULL;
14419         PyObject *__pyx_t_11 = NULL;
14420         int __pyx_lineno = 0;
14421         const char *__pyx_filename = NULL;
14422         int __pyx_clineno = 0;
14423         __Pyx_RefNannySetupContext("sqrt", 0);
14424         if (__pyx_optional_args) {
14425             if (__pyx_optional_args->__pyx_n > 0) {
14426                 __pyx_v_i = __pyx_optional_args->i;
14427             }
14428         }
14429
14430         /* "PyClical.pyx":1606
14431 *      -1
14432 *      """
14433 *      if not (i is None):          # ««««««««
14434 *          return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14435 *      else:
14436 */
14437         __pyx_t_1 = (__pyx_v_i != Py_None);
14438         __pyx_t_2 = (__pyx_t_1 != 0);
14439         if (__pyx_t_2) {
14440
14441             /* "PyClical.pyx":1607
14442 *      """
14443 *      if not (i is None):
14444 *          return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )          # ««««««««
14445 *      else:
14446 *          try:
14447 */
14448         __Pyx_XDECREF(__pyx_r);
14449         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1607, __pyx_L1_error)
14450         __Pyx_GOTREF(__pyx_t_3);
14451         try {
14452             __pyx_t_4 = sqrt(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
14453         } catch (...) {
14454             __Pyx_CppExn2PyErr();
14455             __PYX_ERR(0, 1607, __pyx_L1_error)
14456         }
14457         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1607, __pyx_L1_error)
14458         __Pyx_GOTREF(__pyx_t_5);
14459         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14460         __pyx_r = __pyx_t_5;
14461         __pyx_t_5 = 0;
14462         goto __pyx_L0;
14463
14464         /* "PyClical.pyx":1606
14465 *      -1
14466 *      """
14467 *      if not (i is None):          # ««««««««
14468 *          return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14469 *      else:
14470 */
14471         }
14472
14473         /* "PyClical.pyx":1609
14474 *      return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14475 *      else:
14476 *          try:          # ««««««««
14477 *              return math.sqrt(obj)
14478 *          except:
14479 */
14480         /*else*/ {

```

```

14481         {
14482             __Pyx_PyThreadState_declare
14483             __Pyx_PyThreadState_assign
14484             __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
14485             __Pyx_XGOTREF(__pyx_t_6);
14486             __Pyx_XGOTREF(__pyx_t_7);
14487             __Pyx_XGOTREF(__pyx_t_8);
14488             /*try:*/ {
14489
14490                 /* "PyClical.pyx":1610
14491 *
14492 *         try:
14493 *             return math.sqrt(obj)                # ««««««««
14494 *         except:
14495 *             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14496 */
14497             __Pyx_XDECREF(__pyx_r);
14498             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
14499 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1610, __pyx_L4_error)
14500             __Pyx_GOTREF(__pyx_t_3);
14501             __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sqrt); if
14502 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1610, __pyx_L4_error)
14503             __Pyx_GOTREF(__pyx_t_9);
14504             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14505             __pyx_t_3 = NULL;
14506             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
14507                 __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
14508                 if (likely(__pyx_t_3)) {
14509                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
14510                     __Pyx_INCREF(__pyx_t_3);
14511                     __Pyx_INCREF(function);
14512                     __Pyx_DECREF_SET(__pyx_t_9, function);
14513                 }
14514             }
14515             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
14516 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
14517             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14518             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1610, __pyx_L4_error)
14519             __Pyx_GOTREF(__pyx_t_5);
14520             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14521             __pyx_r = __pyx_t_5;
14522             __pyx_t_5 = 0;
14523             goto __pyx_L8_try_return;
14524
14525             /* "PyClical.pyx":1609
14526 *
14527 *         return clifford().wrap( glucat.sqrt(toClifford(obj)), toClifford(i)) )
14528 *
14529 *         else:
14530 *             try:
14531 *                 # ««««««««
14532 *                 return math.sqrt(obj)
14533 *             except:
14534 *
14535 */
14536             }
14537             __pyx_L4_error:;
14538             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14539             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14540             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
14541
14542             /* "PyClical.pyx":1611
14543 *
14544 *         try:
14545 *             return math.sqrt(obj)
14546 *         except:
14547 *             # ««««««««
14548 *             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14549 *
14550 */
14551             }
14552             /*except:*/ {
14553                 __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno,
14554 __pyx_filename);
14555                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
14556 1611, __pyx_L6_except_error)
14557                 __Pyx_GOTREF(__pyx_t_5);
14558                 __Pyx_GOTREF(__pyx_t_9);
14559                 __Pyx_GOTREF(__pyx_t_3);
14560
14561                 /* "PyClical.pyx":1612
14562 *
14563 *         return math.sqrt(obj)
14564 *         except:
14565 *             return clifford().wrap( glucat.sqrt(toClifford(obj)) )                # ««««««««
14566 *
14567 *         cpdef inline exp(obj):
14568 */
14569                 __Pyx_XDECREF(__pyx_r);
14570                 __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
14571 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1612,
14572 __pyx_L6_except_error)
14573                 __Pyx_GOTREF(__pyx_t_10);
14574                 __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
14575 __pyx_obj_8PyClical_clifford *)__pyx_t_10), sqrt(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if

```

```

(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1612, __pyx_L6_except_error)
14560     __Pyx_GOTREF(__pyx_t_11);
14561     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
14562     __pyx_r = __pyx_t_11;
14563     __pyx_t_11 = 0;
14564     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14565     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14566     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14567     goto __pyx_L7_except_return;
14568 }
14569 __pyx_L6_except_error;;
14570
14571 /* "PyClical.pyx":1609
14572 *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14573 * else:
14574 *     try:
14575 *         # ««««««««
14576 *         return math.sqrt(obj)
14577 *     except:
14578 */
14578     __Pyx_XGIVEREF(__pyx_t_6);
14579     __Pyx_XGIVEREF(__pyx_t_7);
14580     __Pyx_XGIVEREF(__pyx_t_8);
14581     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14582     goto __pyx_L1_error;
14583     __pyx_L8_try_return;;
14584     __Pyx_XGIVEREF(__pyx_t_6);
14585     __Pyx_XGIVEREF(__pyx_t_7);
14586     __Pyx_XGIVEREF(__pyx_t_8);
14587     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14588     goto __pyx_L0;
14589     __pyx_L7_except_return;;
14590     __Pyx_XGIVEREF(__pyx_t_6);
14591     __Pyx_XGIVEREF(__pyx_t_7);
14592     __Pyx_XGIVEREF(__pyx_t_8);
14593     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14594     goto __pyx_L0;
14595 }
14596 }
14597
14598 /* "PyClical.pyx":1591
14599 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
14600 *
14601 * cpdef inline sqrt(obj, i = None):
14602 *     # ««««««««
14603 *     """
14604 *     Square root of multivector with optional complexifier.
14605 */
14606
14606     /* function exit code */
14607     __pyx_L1_error;;
14608     __Pyx_XDECREF(__pyx_t_3);
14609     __Pyx_XDECREF(__pyx_t_5);
14610     __Pyx_XDECREF(__pyx_t_9);
14611     __Pyx_XDECREF(__pyx_t_10);
14612     __Pyx_XDECREF(__pyx_t_11);
14613     __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14614     __pyx_r = 0;
14615     __pyx_L0;;
14616     __Pyx_XGIVEREF(__pyx_r);
14617     __Pyx_RefNannyFinishContext();
14618     return __pyx_r;
14619 }
14620
14621 /* Python wrapper */
14622 static PyObject* __pyx_pw_8PyClical_51sqrt(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds); /*proto*/
14623 static char __pyx_doc_8PyClical_50sqrt[] = "\n    Square root of multivector with\noptional complexifier.\n\n    >> print(sqrt(-1))\n    {-1}\n    >> print(sqrt(clifford(\"2{-1}\")))\n    1+{-1}\n    >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)\n    {1,2,3}\n-1\n    >> j=sqrt(-1,\"{1,2,3}\" ); print(j); print(j*j)\n    {1,2,3}\n    -1\n    ";
14624 static PyObject* __pyx_pw_8PyClical_51sqrt(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kwds) {
14625     PyObject* __pyx_v_obj = 0;
14626     PyObject* __pyx_v_i = 0;
14627     int __pyx_lineno = 0;
14628     const char* __pyx_filename = NULL;
14629     int __pyx_clineno = 0;
14630     PyObject* __pyx_r = 0;
14631     __Pyx_RefNannyDeclarations
14632     __Pyx_RefNannySetupContext("sqrt (wrapper)", 0);
14633     {
14634         static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
14635         PyObject* values[2] = {0,0};
14636         values[1] = ((PyObject*)Py_None);
14637         if (unlikely(__pyx_kwds)) {
14638             Py_ssize_t kw_args;
14639             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14640             switch (pos_args) {

```

```

14641         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14642         CYTHON_FALLTHROUGH;
14643         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14644         CYTHON_FALLTHROUGH;
14645         case 0: break;
14646         default: goto __pyx_L5_argtuple_error;
14647     }
14648     kw_args = PyDict_Size(__pyx_kwds);
14649     switch (pos_args) {
14650     case 0:
14651         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14652         else goto __pyx_L5_argtuple_error;
14653         CYTHON_FALLTHROUGH;
14654         case 1:
14655             if (kw_args > 0) {
14656                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
14657                 if (value) { values[1] = value; kw_args--; }
14658             }
14659             if (unlikely(kw_args > 0)) {
14660                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "sqrt") < 0)) __PYX_ERR(0, 1591, __pyx_L3_error)
14661             }
14662             } else {
14663                 switch (PyTuple_GET_SIZE(__pyx_args)) {
14664                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14665                 CYTHON_FALLTHROUGH;
14666                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14667                 break;
14668                 default: goto __pyx_L5_argtuple_error;
14669                 }
14670             }
14671             __pyx_v_obj = values[0];
14672             __pyx_v_i = values[1];
14673         }
14674         goto __pyx_L4_argument_unpacking_done;
14675         __pyx_L5_argtuple_error:;
14676         __Pyx_RaiseArgtupleInvalid("sqrt", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
14677         __PYX_ERR(0, 1591, __pyx_L3_error)
14678         __pyx_L3_error:;
14679         __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14680         __Pyx_RefNannyFinishContext();
14681         return NULL;
14682         __pyx_L4_argument_unpacking_done:;
14683         __pyx_r = __pyx_pf_8PyClical_50sqrt(__pyx_self, __pyx_v_obj, __pyx_v_i);
14684
14685         /* function exit code */
14686         __Pyx_RefNannyFinishContext();
14687         return __pyx_r;
14688     }
14689
14690     static PyObject *__pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
14691         PyObject *__pyx_r = NULL;
14692         __Pyx_RefNannyDeclarations
14693         PyObject *__pyx_t_1 = NULL;
14694         struct __pyx_opt_args_8PyClical_sqrt __pyx_t_2;
14695         int __pyx_lineno = 0;
14696         const char *__pyx_filename = NULL;
14697         int __pyx_clineno = 0;
14698         __Pyx_RefNannySetupContext("sqrt", 0);
14699         __Pyx_XDECREF(__pyx_r);
14700         __pyx_t_2.__pyx_n = 1;
14701         __pyx_t_2.i = __pyx_v_i;
14702         __pyx_t_1 = __pyx_f_8PyClical_sqrt(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1591, __pyx_L1_error)
14703         __Pyx_GOTREF(__pyx_t_1);
14704         __pyx_r = __pyx_t_1;
14705         __pyx_t_1 = 0;
14706         goto __pyx_L0;
14707
14708         /* function exit code */
14709         __pyx_L1_error:;
14710         __Pyx_XDECREF(__pyx_t_1);
14711         __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14712         __pyx_r = NULL;
14713         __pyx_L0:;
14714         __Pyx_XGIVEREF(__pyx_r);
14715         __Pyx_RefNannyFinishContext();
14716         return __pyx_r;
14717     }
14718
14719     /* "PyClical.pyx":1614
14720     *         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14721     *
14722     * cpdef inline exp(obj): # ««««««

```

```

14723 *      """
14724 *      Exponential of multivector.
14725 */
14726
14727 static PyObject *__pyx_pw_8PyClical_53exp(PyObject *__pyx_self, PyObject
14728 *__pyx_v_obj); /*proto*/
14729 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_exp(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
14730     PyObject *__pyx_r = NULL;
14731     __Pyx_RefNannyDeclarations
14732     PyObject *__pyx_t_1 = NULL;
14733     PyObject *__pyx_t_2 = NULL;
14734     PyObject *__pyx_t_3 = NULL;
14735     PyObject *__pyx_t_4 = NULL;
14736     PyObject *__pyx_t_5 = NULL;
14737     PyObject *__pyx_t_6 = NULL;
14738     PyObject *__pyx_t_7 = NULL;
14739     PyObject *__pyx_t_8 = NULL;
14740     int __pyx_lineno = 0;
14741     const char *__pyx_filename = NULL;
14742     int __pyx_clineno = 0;
14743     __Pyx_RefNannySetupContext("exp", 0);
14744
14745     /* "PyClical.pyx":1623
14746     *
14747     *      try:
14748     *          return math.exp(obj)
14749     *      except:
14750     */
14751     {
14752         __Pyx_PyThreadState_declare
14753         __Pyx_PyThreadState_assign
14754         __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
14755         __Pyx_XGOTREF(__pyx_t_1);
14756         __Pyx_XGOTREF(__pyx_t_2);
14757         __Pyx_XGOTREF(__pyx_t_3);
14758         /*try:*/ {
14759
14760             /* "PyClical.pyx":1624
14761             *
14762             *      try:
14763             *          return math.exp(obj)
14764             *      except:
14765             *          return clifford().wrap( glucat.exp(toClifford(obj)) )
14766             */
14767             __Pyx_XDECREF(__pyx_r);
14768             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1624, __pyx_L3_error)
14769             __Pyx_GOTREF(__pyx_t_5);
14770             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_exp); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1624, __pyx_L3_error)
14771             __Pyx_GOTREF(__pyx_t_6);
14772             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14773             __pyx_t_5 = NULL;
14774             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
14775                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
14776                 if (likely(__pyx_t_5)) {
14777                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
14778                     __Pyx_INCREF(__pyx_t_5);
14779                     __Pyx_INCREF(function);
14780                     __Pyx_DECREF_SET(__pyx_t_6, function);
14781                 }
14782             }
14783             __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
14784             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14785             if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1624, __pyx_L3_error)
14786             __Pyx_GOTREF(__pyx_t_4);
14787             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14788             __pyx_r = __pyx_t_4;
14789             __pyx_t_4 = 0;
14790             goto __pyx_L7_try_return;
14791
14792             /* "PyClical.pyx":1623
14793             *
14794             *      try:
14795             *          return math.exp(obj)
14796             *      except:
14797             */
14798             */
14799         }
14800         __pyx_L3_error:;
14801         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
14802         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14803         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
14804

```

```

14805             /* "PyClical.pyx":1625
14806 *      try:
14807 *          return math.exp(obj)
14808 *      except: # ««««««««
14809 *          return clifford().wrap( glucat.exp(toClifford(obj)) )
14810 *
14811 */
14812             /*except:*/ {
14813                 __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14814                 if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1625, __pyx_L5_except_error)
14815                 __Pyx_GOTREF(__pyx_t_4);
14816                 __Pyx_GOTREF(__pyx_t_6);
14817                 __Pyx_GOTREF(__pyx_t_5);
14818
14819             /* "PyClical.pyx":1626
14820 *      return math.exp(obj)
14821 *      except:
14822 *          return clifford().wrap( glucat.exp(toClifford(obj)) ) # ««««««««
14823 *
14824 *      cpdef inline log(obj,i = None):
14825 */
14826                 __Pyx_XDECREF(__pyx_r);
14827                 __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1626,
__pyx_L5_except_error)
14828                 __Pyx_GOTREF(__pyx_t_7);
14829                 __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), exp(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1626, __pyx_L5_except_error)
14830                 __Pyx_GOTREF(__pyx_t_8);
14831                 __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
14832                 __pyx_r = __pyx_t_8;
14833                 __pyx_t_8 = 0;
14834                 __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
14835                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14836                 __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14837                 goto __pyx_L6_except_return;
14838             }
14839             __pyx_L5_except_error;;
14840
14841             /* "PyClical.pyx":1623
14842 *      {1,2}
14843 *      """
14844 *      try: # ««««««««
14845 *          return math.exp(obj)
14846 *      except:
14847 */
14848                 __Pyx_XGIVEREF(__pyx_t_1);
14849                 __Pyx_XGIVEREF(__pyx_t_2);
14850                 __Pyx_XGIVEREF(__pyx_t_3);
14851                 __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14852                 goto __pyx_L1_error;
14853                 __pyx_L7_try_return;
14854                 __Pyx_XGIVEREF(__pyx_t_1);
14855                 __Pyx_XGIVEREF(__pyx_t_2);
14856                 __Pyx_XGIVEREF(__pyx_t_3);
14857                 __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14858                 goto __pyx_L0;
14859                 __pyx_L6_except_return;
14860                 __Pyx_XGIVEREF(__pyx_t_1);
14861                 __Pyx_XGIVEREF(__pyx_t_2);
14862                 __Pyx_XGIVEREF(__pyx_t_3);
14863                 __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14864                 goto __pyx_L0;
14865             }
14866
14867             /* "PyClical.pyx":1614
14868 *      return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14869 *
14870 *      cpdef inline exp(obj): # ««««««««
14871 *      """
14872 *      Exponential of multivector.
14873 */
14874
14875             /* function exit code */
14876             __pyx_L1_error;;
14877             __Pyx_XDECREF(__pyx_t_4);
14878             __Pyx_XDECREF(__pyx_t_5);
14879             __Pyx_XDECREF(__pyx_t_6);
14880             __Pyx_XDECREF(__pyx_t_7);
14881             __Pyx_XDECREF(__pyx_t_8);
14882             __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14883             __pyx_r = 0;
14884             __pyx_L0;
14885             __Pyx_XGIVEREF(__pyx_r);
14886             __Pyx_RefNannyFinishContext();

```

```

14887         return __pyx_r;
14888     }
14889
14890     /* Python wrapper */
14891     static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject
14892 * __pyx_v_obj); /*proto*/
14893     static char __pyx_doc_8PyClical_52exp[] = "\n    Exponential of multivector.\n\n    >>
x=clifford(\\"{1,2}\\") * pi/4; print(exp(x))\n    0.7071+0.7071{1,2}\n    >> x=clifford(\\"{1,2}\\") *
pi/2; print(exp(x))\n    {1,2}\n    ";
14894     static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject * __pyx_v_obj)
14895 {
14896     PyObject * __pyx_r = 0;
14897     __Pyx_RefNannyDeclarations
14898     __Pyx_RefNannySetupContext("exp (wrapper)", 0);
14899     __pyx_r = __pyx_pf_8PyClical_52exp(__pyx_self, ((PyObject *) __pyx_v_obj));
14900
14901     /* function exit code */
14902     __Pyx_RefNannyFinishContext();
14903     return __pyx_r;
14904 }
14905
14906 static PyObject * __pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject * __pyx_self, PyObject
14907 * __pyx_v_obj) {
14908     PyObject * __pyx_r = NULL;
14909     __Pyx_RefNannyDeclarations
14910     PyObject * __pyx_t_1 = NULL;
14911     int __pyx_lineno = 0;
14912     const char * __pyx_filename = NULL;
14913     int __pyx_clineno = 0;
14914     __Pyx_RefNannySetupContext("exp", 0);
14915     __Pyx_XDECREF(__pyx_r);
14916     __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
14917     __PYX_ERR(0, 1614, __pyx_L1_error)
14918     __Pyx_GOTREF(__pyx_t_1);
14919     __pyx_r = __pyx_t_1;
14920     __pyx_t_1 = 0;
14921     goto __pyx_L0;
14922
14923     /* function exit code */
14924     __pyx_L1_error:;
14925     __Pyx_XDECREF(__pyx_t_1);
14926     __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14927     __pyx_r = NULL;
14928     __pyx_L0:;
14929     __Pyx_XGIVEREF(__pyx_r);
14930     __Pyx_RefNannyFinishContext();
14931     return __pyx_r;
14932 }
14933
14934 /* "PyClical.pyx":1628
14935 *
14936 *     return clifford().wrap( glucat.exp(toClifford(obj)) )
14937 *
14938 * cpdef inline log(obj,i = None): # ««««««««
14939 *     """
14940 *     Natural logarithm of multivector with optional complexifier.
14941 */
14942
14943     static PyObject * __pyx_pw_8PyClical_55log(PyObject * __pyx_self, PyObject * __pyx_args,
14944 PyObject * __pyx_kwds); /*proto*/
14945     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_log(PyObject * __pyx_v_obj,
14946 CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_log * __pyx_optional_args) {
14947     PyObject * __pyx_v_i = ((PyObject *) Py_None);
14948     PyObject * __pyx_r = NULL;
14949     __Pyx_RefNannyDeclarations
14950     int __pyx_t_1;
14951     int __pyx_t_2;
14952     PyObject * __pyx_t_3 = NULL;
14953     Clifford __pyx_t_4;
14954     PyObject * __pyx_t_5 = NULL;
14955     PyObject * __pyx_t_6 = NULL;
14956     PyObject * __pyx_t_7 = NULL;
14957     PyObject * __pyx_t_8 = NULL;
14958     PyObject * __pyx_t_9 = NULL;
14959     PyObject * __pyx_t_10 = NULL;
14960     PyObject * __pyx_t_11 = NULL;
14961     int __pyx_lineno = 0;
14962     const char * __pyx_filename = NULL;
14963     int __pyx_clineno = 0;
14964     __Pyx_RefNannySetupContext("log", 0);
14965     if (__pyx_optional_args) {
14966         if (__pyx_optional_args->__pyx_n > 0) {
14967             __pyx_v_i = __pyx_optional_args->i;
14968         }
14969     }
14970
14971     /* "PyClical.pyx":1643
14972 *     RuntimeError: check_complex(val, i): i is not a valid complexifier for val

```



```

14966 *      """
14967 *      if not (i is None):          # ««««««««
14968 *          return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
14969 *      else:
14970 */
14971         __pyx_t_1 = (__pyx_v_i != Py_None);
14972         __pyx_t_2 = (__pyx_t_1 != 0);
14973         if (__pyx_t_2) {
14974
14975             /* "PyClical.pyx":1644
14976 *      """
14977 *      if not (i is None):
14978 *          return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )          # ««««««««
14979 *      else:
14980 *          try:
14981 */
14982         __Pyx_XDECREF(__pyx_r);
14983         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1644, __pyx_L1_error)
14984         __Pyx_GOTREF(__pyx_t_3);
14985         try {
14986             __pyx_t_4 = log(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
14987         } catch (...) {
14988             __Pyx_CppExn2PyErr();
14989             __PYX_ERR(0, 1644, __pyx_L1_error)
14990         }
14991         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1644, __pyx_L1_error)
14992         __Pyx_GOTREF(__pyx_t_5);
14993         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14994         __pyx_r = __pyx_t_5;
14995         __pyx_t_5 = 0;
14996         goto __pyx_L0;
14997
14998         /* "PyClical.pyx":1643
14999 *      RuntimeError: check_complex(val, i): i is not a valid complexifier for val
15000 *      """
15001 *      if not (i is None):          # ««««««««
15002 *          return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15003 *      else:
15004 */
15005         }
15006
15007         /* "PyClical.pyx":1646
15008 *          return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15009 *      else:
15010 *          try:          # ««««««««
15011 *              return math.log(obj)
15012 *          except:
15013 */
15014         /*else*/ {
15015             {
15016                 __Pyx_PyThreadState_declare
15017                 __Pyx_PyThreadState_assign
15018                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15019                 __Pyx_XGOTREF(__pyx_t_6);
15020                 __Pyx_XGOTREF(__pyx_t_7);
15021                 __Pyx_XGOTREF(__pyx_t_8);
15022                 /*try:*/ {
15023
15024                     /* "PyClical.pyx":1647
15025 *          else:
15026 *              try:
15027 *                  return math.log(obj)          # ««««««««
15028 *              except:
15029 *                  return clifford().wrap( glucat.log(toClifford(obj)) )
15030 */
15031                 __Pyx_XDECREF(__pyx_r);
15032                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1647, __pyx_L4_error)
15033                 __Pyx_GOTREF(__pyx_t_3);
15034                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_log); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1647, __pyx_L4_error)
15035                 __Pyx_GOTREF(__pyx_t_9);
15036                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15037                 __pyx_t_3 = NULL;
15038                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15039                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15040                     if (likely(__pyx_t_3)) {
15041                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15042                         __Pyx_INCREF(__pyx_t_3);
15043                         __Pyx_INCREF(function);
15044                         __Pyx_DECREF_SET(__pyx_t_9, function);
15045                     }
15046                 }
15047                 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,

```

```

__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15048     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15049     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1647, __pyx_L4_error)
15050     __Pyx_GOTREF(__pyx_t_5);
15051     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15052     __pyx_r = __pyx_t_5;
15053     __pyx_t_5 = 0;
15054     goto __pyx_L8_try_return;
15055
15056     /* "PyClical.pyx":1646
15057     *     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15058     * else:
15059     *     try:
15060     *         return math.log(obj)
15061     *     except:
15062 */
15063     }
15064     __pyx_L4_error:;
15065     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15066     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15067     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15068
15069     /* "PyClical.pyx":1648
15070     *     try:
15071     *         return math.log(obj)
15072     *     except:
15073     *         return clifford().wrap( glucat.log(toClifford(obj)) )
15074     *
15075 */
15076     /*except:*/ {
15077     __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15078     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1648, __pyx_L6_except_error)
15079     __Pyx_GOTREF(__pyx_t_5);
15080     __Pyx_GOTREF(__pyx_t_9);
15081     __Pyx_GOTREF(__pyx_t_3);
15082
15083     /* "PyClical.pyx":1649
15084     *     return math.log(obj)
15085     * except:
15086     *     return clifford().wrap( glucat.log(toClifford(obj)) )
15087     *
15088     * cpdef inline cos(obj,i = None):
15089 */
15090     __Pyx_XDECREF(__pyx_r);
15091     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1649,
__pyx_L6_except_error)
15092     __Pyx_GOTREF(__pyx_t_10);
15093     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), log(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1649, __pyx_L6_except_error)
15094     __Pyx_GOTREF(__pyx_t_11);
15095     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15096     __pyx_r = __pyx_t_11;
15097     __pyx_t_11 = 0;
15098     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15099     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15100     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15101     goto __pyx_L7_except_return;
15102     }
15103     __pyx_L6_except_error:;
15104
15105     /* "PyClical.pyx":1646
15106     *     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15107     * else:
15108     *     try:
15109     *         return math.log(obj)
15110     *     except:
15111 */
15112     __Pyx_XGIVEREF(__pyx_t_6);
15113     __Pyx_XGIVEREF(__pyx_t_7);
15114     __Pyx_XGIVEREF(__pyx_t_8);
15115     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15116     goto __pyx_L1_error;
15117     __pyx_L8_try_return:;
15118     __Pyx_XGIVEREF(__pyx_t_6);
15119     __Pyx_XGIVEREF(__pyx_t_7);
15120     __Pyx_XGIVEREF(__pyx_t_8);
15121     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15122     goto __pyx_L0;
15123     __pyx_L7_except_return:;
15124     __Pyx_XGIVEREF(__pyx_t_6);
15125     __Pyx_XGIVEREF(__pyx_t_7);
15126     __Pyx_XGIVEREF(__pyx_t_8);
15127     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);

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15128         goto __pyx_L0;
15129     }
15130 }
15131
15132     /* "PyClical.pyx":1628
15133 *         return clifford().wrap( glucat.exp(toClifford(obj)) )
15134 *
15135 * cpdef inline log(obj,i = None):          # ««««««««
15136 *     """
15137 *     Natural logarithm of multivector with optional complexifier.
15138 */
15139
15140     /* function exit code */
15141     __pyx_L1_error;
15142     __Pyx_XDECREF(__pyx_t_3);
15143     __Pyx_XDECREF(__pyx_t_5);
15144     __Pyx_XDECREF(__pyx_t_9);
15145     __Pyx_XDECREF(__pyx_t_10);
15146     __Pyx_XDECREF(__pyx_t_11);
15147     __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15148     __pyx_r = 0;
15149     __pyx_L0;
15150     __Pyx_XGIVEREF(__pyx_r);
15151     __Pyx_RefNannyFinishContext();
15152     return __pyx_r;
15153 }
15154
15155     /* Python wrapper */
15156     static PyObject *__pyx_pw_8PyClical_55log(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15157     static char __pyx_doc_8PyClical_54log[] = "\n    Natural logarithm of multivector with
optional complexifier.\n\n    >> x=clifford(\"{-1}\"); print((log(x,\"{-1}\") * 2/pi))\n    {-1}\n
>> x=clifford(\"{1,2}\"); print((log(x,\"{1,2,3}\") * 2/pi))\n    {1,2}\n    >>
x=clifford(\"{1,2}\"); print((log(x) * 2/pi))\n    {1,2}\n    >> x=clifford(\"{1,2}\");
print((log(x,\"{1,2}\") * 2/pi))\n    Traceback (most recent call last):\n    ...\n    RuntimeError:
check_complex(val, i): i is not a valid complexifier for val\n    ";
15158     static PyObject *__pyx_pw_8PyClical_55log(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
15159         PyObject *__pyx_v_obj = 0;
15160         PyObject *__pyx_v_i = 0;
15161         int __pyx_lineno = 0;
15162         const char *__pyx_filename = NULL;
15163         int __pyx_clineno = 0;
15164         PyObject *__pyx_r = 0;
15165         __Pyx_RefNannyDeclarations
15166         __Pyx_RefNannySetupContext("log (wrapper)", 0);
15167         {
15168             static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15169             PyObject* values[2] = {0,0};
15170             values[1] = ((PyObject *)__Py_None);
15171             if (unlikely(__pyx_kwds)) {
15172                 Py_ssize_t kw_args;
15173                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15174                 switch (pos_args) {
15175                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15176                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15177                     case 0: break;
15178                     default: goto __pyx_L5_argtuple_error;
15179                 }
15180                 kw_args = PyDict_Size(__pyx_kwds);
15181                 switch (pos_args) {
15182                     case 0:
15183                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15184                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
15185                     case 1:
15186                         if (kw_args > 0) {
15187                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15188                             if (value) { values[1] = value; kw_args--; }
15189                         }
15190                     }
15191                 if (unlikely(kw_args > 0)) {
15192                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "log") < 0)) __PYX_ERR(0, 1628, __pyx_L3_error)
15193                 }
15194             } else {
15195                 switch (PyTuple_GET_SIZE(__pyx_args)) {
15196                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15197                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15198                     case 0: break;
15199                     default: goto __pyx_L5_argtuple_error;
15200                 }
15201             }
15202         }
15203     }
15204 }
15205

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15206         __pyx_v_obj = values[0];
15207         __pyx_v_i = values[1];
15208     }
15209     goto __pyx_L4_argument_unpacking_done;
15210     __pyx_L5_argtuple_error:;
15211     __Pyx_RaiseArgtupleInvalid("log", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1628, __pyx_L3_error)
15212     __pyx_L3_error:;
15213     __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15214     __Pyx_RefNannyFinishContext();
15215     return NULL;
15216     __pyx_L4_argument_unpacking_done:;
15217     __pyx_r = __pyx_pf_8PyClical_54log(__pyx_self, __pyx_v_obj, __pyx_v_i);
15218
15219     /* function exit code */
15220     __Pyx_RefNannyFinishContext();
15221     return __pyx_r;
15222 }
15223
15224 static PyObject *__pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_i) {
15225     PyObject *__pyx_r = NULL;
15226     __Pyx_RefNannyDeclarations
15227     PyObject *__pyx_t_1 = NULL;
15228     struct __pyx_opt_args_8PyClical_log __pyx_t_2;
15229     int __pyx_lineno = 0;
15230     const char *__pyx_filename = NULL;
15231     int __pyx_clineno = 0;
15232     __Pyx_RefNannySetupContext("log", 0);
15233     __Pyx_XDECREF(__pyx_r);
15234     __pyx_t_2.__pyx_n = 1;
15235     __pyx_t_2.i = __pyx_v_i;
15236     __pyx_t_1 = __pyx_f_8PyClical_log(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1628, __pyx_L1_error)
15237     __Pyx_GOTREF(__pyx_t_1);
15238     __pyx_r = __pyx_t_1;
15239     __pyx_t_1 = 0;
15240     goto __pyx_L0;
15241
15242     /* function exit code */
15243     __pyx_L1_error:;
15244     __Pyx_XDECREF(__pyx_t_1);
15245     __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15246     __pyx_r = NULL;
15247     __pyx_L0:;
15248     __Pyx_XGIVEREF(__pyx_r);
15249     __Pyx_RefNannyFinishContext();
15250     return __pyx_r;
15251 }
15252
15253 /* "PyClical.pyx":1651
15254 *         return clifford().wrap( glucat.log(toClifford(obj)) )
15255 *
15256 * cpdef inline cos(obj,i = None):                # ««««««««
15257 *     """
15258 *     Cosine of multivector with optional complexifier.
15259 */
15260
15261 static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15262 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cos(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_cos *__pyx_optional_args) {
15263     PyObject *__pyx_v_i = ((PyObject *)Py_None);
15264     PyObject *__pyx_r = NULL;
15265     __Pyx_RefNannyDeclarations
15266     int __pyx_t_1;
15267     int __pyx_t_2;
15268     PyObject *__pyx_t_3 = NULL;
15269     Clifford __pyx_t_4;
15270     PyObject *__pyx_t_5 = NULL;
15271     PyObject *__pyx_t_6 = NULL;
15272     PyObject *__pyx_t_7 = NULL;
15273     PyObject *__pyx_t_8 = NULL;
15274     PyObject *__pyx_t_9 = NULL;
15275     PyObject *__pyx_t_10 = NULL;
15276     PyObject *__pyx_t_11 = NULL;
15277     int __pyx_lineno = 0;
15278     const char *__pyx_filename = NULL;
15279     int __pyx_clineno = 0;
15280     __Pyx_RefNannySetupContext("cos", 0);
15281     if (__pyx_optional_args) {
15282         if (__pyx_optional_args->__pyx_n > 0) {
15283             __pyx_v_i = __pyx_optional_args->i;
15284         }
15285     }
15286
15287     /* "PyClical.pyx":1660

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```

15288 *      {1,2}
15289 *      """
15290 *      if not (i is None):          # ««««««««
15291 *          return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15292 *      else:
15293 */
15294         __pyx_t_1 = (__pyx_v_i != Py_None);
15295         __pyx_t_2 = (__pyx_t_1 != 0);
15296         if (__pyx_t_2) {
15297
15298             /* "PyClical.pyx":1661
15299 *      """
15300 *      if not (i is None):
15301 *          return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )          # ««««««««
15302 *      else:
15303 *          try:
15304 */
15305         __Pyx_XDECREF(__pyx_r);
15306         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
15307 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1661, __pyx_L1_error)
15308         __Pyx_GOTREF(__pyx_t_3);
15309         try {
15310             __pyx_t_4 = cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
15311 __pyx_f_8PyClical_toClifford(__pyx_v_i));
15312         } catch (...) {
15313             __Pyx_CppExn2PyErr();
15314             __PYX_ERR(0, 1661, __pyx_L1_error)
15315         }
15316         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
15317 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1661, __pyx_L1_error)
15318         __Pyx_GOTREF(__pyx_t_5);
15319         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15320         __pyx_r = __pyx_t_5;
15321         __pyx_t_5 = 0;
15322         goto __pyx_L0;
15323
15324         /* "PyClical.pyx":1660
15325 *      {1,2}
15326 *      """
15327 *      if not (i is None):          # ««««««««
15328 *          return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15329 *      else:
15330 */
15331         }
15332
15333         /* "PyClical.pyx":1663
15334 *      return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15335 *      else:
15336 *          try:          # ««««««««
15337 *              return math.cos(obj)
15338 *          except:
15339 */
15340         /*else*/ {
15341             {
15342                 __Pyx_PyThreadState_declare
15343                 __Pyx_PyThreadState_assign
15344                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15345                 __Pyx_XGOTREF(__pyx_t_6);
15346                 __Pyx_XGOTREF(__pyx_t_7);
15347                 __Pyx_XGOTREF(__pyx_t_8);
15348             }
15349             /*try:*/ {
15350
15351                 /* "PyClical.pyx":1664
15352 *      else:
15353 *          try:
15354 *              return math.cos(obj)          # ««««««««
15355 *          except:
15356 *              return clifford().wrap( glucat.cos(toClifford(obj)) )
15357 */
15358         __Pyx_XDECREF(__pyx_r);
15359         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
15360 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1664, __pyx_L4_error)
15361         __Pyx_GOTREF(__pyx_t_3);
15362         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_cos); if
15363 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1664, __pyx_L4_error)
15364         __Pyx_GOTREF(__pyx_t_9);
15365         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15366         __pyx_t_3 = NULL;
15367         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15368             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15369             if (likely(__pyx_t_3)) {
15370                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15371                 __Pyx_INCREF(__pyx_t_3);
15372                 __Pyx_INCREF(function);
15373                 __Pyx_DECREF_SET(__pyx_t_9, function);
15374             }
15375         }
15376     }
15377 }

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15370         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15371         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15372         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1664, __pyx_L4_error)
15373         __Pyx_GOTREF(__pyx_t_5);
15374         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15375         __pyx_r = __pyx_t_5;
15376         __pyx_t_5 = 0;
15377         goto __pyx_L8_try_return;
15378
15379         /* "PyClical.pyx":1663
15380  *         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15381  *     else:
15382  *         try:
15383  *             return math.cos(obj)
15384  *         except:
15385  */
15386         }
15387         __pyx_L4_error:;
15388         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15389         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15390         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15391
15392         /* "PyClical.pyx":1665
15393  *         try:
15394  *             return math.cos(obj)
15395  *         except:
15396  *             return clifford().wrap( glucat.cos(toClifford(obj)) )
15397  *
15398  */
15399         /*except:*/ {
15400         __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15401         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1665, __pyx_L6_except_error)
15402         __Pyx_GOTREF(__pyx_t_5);
15403         __Pyx_GOTREF(__pyx_t_9);
15404         __Pyx_GOTREF(__pyx_t_3);
15405
15406         /* "PyClical.pyx":1666
15407  *         return math.cos(obj)
15408  *     except:
15409  *         return clifford().wrap( glucat.cos(toClifford(obj)) )
15410  *
15411  * cpdef inline acos(obj, i = None):
15412  */
15413         __Pyx_XDECREF(__pyx_r);
15414         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1666,
__pyx_L6_except_error)
15415         __Pyx_GOTREF(__pyx_t_10);
15416         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1666, __pyx_L6_except_error)
15417         __Pyx_GOTREF(__pyx_t_11);
15418         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15419         __pyx_r = __pyx_t_11;
15420         __pyx_t_11 = 0;
15421         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15422         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15423         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15424         goto __pyx_L7_except_return;
15425     }
15426     __pyx_L6_except_error:;
15427
15428     /* "PyClical.pyx":1663
15429  *         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15430  *     else:
15431  *         try:
15432  *             return math.cos(obj)
15433  *         except:
15434  */
15435         __Pyx_XGIVEREF(__pyx_t_6);
15436         __Pyx_XGIVEREF(__pyx_t_7);
15437         __Pyx_XGIVEREF(__pyx_t_8);
15438         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15439         goto __pyx_L1_error;
15440         __pyx_L8_try_return:;
15441         __Pyx_XGIVEREF(__pyx_t_6);
15442         __Pyx_XGIVEREF(__pyx_t_7);
15443         __Pyx_XGIVEREF(__pyx_t_8);
15444         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15445         goto __pyx_L0;
15446         __pyx_L7_except_return:;
15447         __Pyx_XGIVEREF(__pyx_t_6);
15448         __Pyx_XGIVEREF(__pyx_t_7);
15449         __Pyx_XGIVEREF(__pyx_t_8);

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```

15450         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15451         goto __pyx_L0;
15452     }
15453 }
15454
15455     /* "PyClical.pyx":1651
15456 *         return clifford().wrap( glucat.log(toClifford(obj)) )
15457 *
15458 * cpdef inline cos(obj,i = None):          # ««««««««
15459 *     """
15460 *     Cosine of multivector with optional complexifier.
15461 */
15462
15463     /* function exit code */
15464     __pyx_L1_error:;
15465     __Pyx_XDECREF(__pyx_t_3);
15466     __Pyx_XDECREF(__pyx_t_5);
15467     __Pyx_XDECREF(__pyx_t_9);
15468     __Pyx_XDECREF(__pyx_t_10);
15469     __Pyx_XDECREF(__pyx_t_11);
15470     __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15471     __pyx_r = 0;
15472     __pyx_L0:;
15473     __Pyx_XGIVEREF(__pyx_r);
15474     __Pyx_RefNannyFinishContext();
15475     return __pyx_r;
15476 }
15477
15478     /* Python wrapper */
15479     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15480     static char __pyx_doc_8PyClical_56cos[] = "\n    Cosine of multivector with optional
complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{1,2,3}\")\n    {1,2}\n    >>
x=clifford(\"{1,2}\"); print(cos(acos(x))\n    {1,2}\n    ";
15481     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
15482         PyObject *__pyx_v_obj = 0;
15483         PyObject *__pyx_v_i = 0;
15484         int __pyx_lineno = 0;
15485         const char *__pyx_filename = NULL;
15486         int __pyx_clineno = 0;
15487         PyObject *__pyx_r = 0;
15488         __Pyx_RefNannyDeclarations
15489         __Pyx_RefNannySetupContext("cos (wrapper)", 0);
15490         {
15491             static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15492             PyObject* values[2] = {0,0};
15493             values[1] = ((PyObject *)Py_None);
15494             if (unlikely(__pyx_kwds)) {
15495                 Py_ssize_t kw_args;
15496                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15497                 switch (pos_args) {
15498                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15499                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15500                     case 0: break;
15501                     default: goto __pyx_L5_argtuple_error;
15502                 }
15503                 kw_args = PyDict_Size(__pyx_kwds);
15504                 switch (pos_args) {
15505                     case 0:
15506                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15509                     else goto __pyx_L5_argtuple_error;
15510                     case 1:
15511                         if (kw_args > 0) {
15512                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15513                             if (value) { values[1] = value; kw_args--; }
15514                         }
15515                     }
15516                     if (unlikely(kw_args > 0)) {
15517                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
15518 values, pos_args, "cos") < 0)) __PYX_ERR(0, 1651, __pyx_L3_error)
15519                     }
15520                     else {
15521                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15522                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15523                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15524                             break;
15525                             default: goto __pyx_L5_argtuple_error;
15526                         }
15527                     }
15528                 }
15529                 __pyx_v_obj = values[0];
15530                 __pyx_v_i = values[1];

```

```

15531         }
15532         goto __pyx_L4_argument_unpacking_done;
15533         __pyx_L5_argtuple_error;;
15534         __Pyx_RaiseArgtupleInvalid("cos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15535     __PYX_ERR(0, 1651, __pyx_L3_error)
15536     __pyx_L3_error;;
15537     __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15538     __Pyx_RefNannyFinishContext();
15539     return NULL;
15540     __pyx_L4_argument_unpacking_done;;
15541     __pyx_r = __pyx_pf_8PyClical_56cos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15542
15543     /* function exit code */
15544     __Pyx_RefNannyFinishContext();
15545     return __pyx_r;
15546 }
15547
15548 static PyObject * __pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj, PyObject * __pyx_v_i) {
15549     PyObject * __pyx_r = NULL;
15550     __Pyx_RefNannyDeclarations
15551     PyObject * __pyx_t_1 = NULL;
15552     struct __pyx_opt_args_8PyClical_cos __pyx_t_2;
15553     int __pyx_lineno = 0;
15554     const char * __pyx_filename = NULL;
15555     int __pyx_clineno = 0;
15556     __Pyx_RefNannySetupContext("cos", 0);
15557     __Pyx_XDECREF(__pyx_r);
15558     __pyx_t_2.__pyx_n = 1;
15559     __pyx_t_2.i = __pyx_v_i;
15560     __pyx_t_1 = __pyx_f_8PyClical_cos(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1651, __pyx_L1_error)
15561     __Pyx_GOTREF(__pyx_t_1);
15562     __pyx_r = __pyx_t_1;
15563     __pyx_t_1 = 0;
15564     goto __pyx_L0;
15565
15566     /* function exit code */
15567     __pyx_L1_error;;
15568     __Pyx_XDECREF(__pyx_t_1);
15569     __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15570     __pyx_r = NULL;
15571     __pyx_L0;;
15572     __Pyx_XGIVEREF(__pyx_r);
15573     __Pyx_RefNannyFinishContext();
15574     return __pyx_r;
15575 }
15576
15577 /* "PyClical.pyx":1668
15578 *
15579 * cpdef inline acos(obj, i = None):          # ««««««««
15580 *     """
15581 *     Inverse cosine of multivector with optional complexifier.
15582 */
15583
15584 static PyObject * __pyx_pw_8PyClical_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
15585 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_acos(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_acos * __pyx_optional_args) {
15586     PyObject * __pyx_v_i = (PyObject *)Py_None;
15587     PyObject * __pyx_r = NULL;
15588     __Pyx_RefNannyDeclarations
15589     int __pyx_t_1;
15590     int __pyx_t_2;
15591     PyObject * __pyx_t_3 = NULL;
15592     Clifford __pyx_t_4;
15593     PyObject * __pyx_t_5 = NULL;
15594     PyObject * __pyx_t_6 = NULL;
15595     PyObject * __pyx_t_7 = NULL;
15596     PyObject * __pyx_t_8 = NULL;
15597     PyObject * __pyx_t_9 = NULL;
15598     PyObject * __pyx_t_10 = NULL;
15599     PyObject * __pyx_t_11 = NULL;
15600     int __pyx_lineno = 0;
15601     const char * __pyx_filename = NULL;
15602     int __pyx_clineno = 0;
15603     __Pyx_RefNannySetupContext("acos", 0);
15604     if (__pyx_optional_args) {
15605         if (__pyx_optional_args->__pyx_n > 0) {
15606             __pyx_v_i = __pyx_optional_args->i;
15607         }
15608     }
15609
15610     /* "PyClical.pyx":1681
15611 *     {1,2}
15612 *     """

```



```

15613 *      if not (i is None):          # ««««««««
15614 *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15615 *      else:
15616 */
15617         __pyx_t_1 = (__pyx_v_i != Py_None);
15618         __pyx_t_2 = (__pyx_t_1 != 0);
15619         if (__pyx_t_2) {
15620
15621             /* "PyClical.pyx":1682
15622 *      """
15623 *      if not (i is None):
15624 *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )          # ««««««««
15625 *      else:
15626 *          try:
15627 */
15628         __Pyx_XDECREF(__pyx_r);
15629         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
*)__pyx_ptype_8PyClical_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1682, __pyx_L1_error)
15630         __Pyx_GOTREF(__pyx_t_3);
15631         try {
15632             __pyx_t_4 = acos(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
15633         } catch (...) {
15634             __Pyx_CppExn2PyErr();
15635             __PYX_ERR(0, 1682, __pyx_L1_error)
15636         }
15637         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1682, __pyx_L1_error)
15638         __Pyx_GOTREF(__pyx_t_5);
15639         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15640         __pyx_r = __pyx_t_5;
15641         __pyx_t_5 = 0;
15642         goto __pyx_L0;
15643
15644             /* "PyClical.pyx":1681
15645 *      {1,2}
15646 *      """
15647 *      if not (i is None):          # ««««««««
15648 *          return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15649 *      else:
15650 */
15651         }
15652
15653         /* "PyClical.pyx":1684
15654 *      return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15655 *      else:
15656 *          try:          # ««««««««
15657 *              return math.acos(obj)
15658 *          except:
15659 */
15660         /*else*/ {
15661             {
15662                 __Pyx_PyThreadState_declare
15663                 __Pyx_PyThreadState_assign
15664                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15665                 __Pyx_XGOTREF(__pyx_t_6);
15666                 __Pyx_XGOTREF(__pyx_t_7);
15667                 __Pyx_XGOTREF(__pyx_t_8);
15668                 /*try:*/ {
15669
15670                     /* "PyClical.pyx":1685
15671 *      else:
15672 *          try:
15673 *              return math.acos(obj)          # ««««««««
15674 *          except:
15675 *              return clifford().wrap( glucat.acos(toClifford(obj)) )
15676 */
15677                 __Pyx_XDECREF(__pyx_r);
15678                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1685, __pyx_L4_error)
15679                 __Pyx_GOTREF(__pyx_t_3);
15680                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acos); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1685, __pyx_L4_error)
15681                 __Pyx_GOTREF(__pyx_t_9);
15682                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15683                 __pyx_t_3 = NULL;
15684                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15685                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15686                     if (likely(__pyx_t_3)) {
15687                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15688                         __Pyx_INCREF(__pyx_t_3);
15689                         __Pyx_INCREF(function);
15690                         __Pyx_DECREF_SET(__pyx_t_9, function);
15691                     }
15692                 }
15693                 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);

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15694         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15695         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1685, __pyx_L4_error)
15696         __Pyx_GOTREF(__pyx_t_5);
15697         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15698         __pyx_r = __pyx_t_5;
15699         __pyx_t_5 = 0;
15700         goto __pyx_L8_try_return;
15701
15702         /* "PyClical.pyx":1684
15703  *         return clifford().wrap( glucat.acos(toClifford(obj)), toClifford(i)) )
15704  *     else:
15705  *         try:
15706  *             # ««««««««
15707  *             return math.acos(obj)
15708  *         except:
15709  */
15710     }
15711     __pyx_L4_error:;
15712     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15713     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15714     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15715     /* "PyClical.pyx":1686
15716  *     try:
15717  *         return math.acos(obj)
15718  *     except:
15719  *         # ««««««««
15720  *         return clifford().wrap( glucat.acos(toClifford(obj)) )
15721  */
15722     /*except:*/ {
15723         __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15724         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1686, __pyx_L6_except_error)
15725         __Pyx_GOTREF(__pyx_t_5);
15726         __Pyx_GOTREF(__pyx_t_9);
15727         __Pyx_GOTREF(__pyx_t_3);
15728
15729         /* "PyClical.pyx":1687
15730  *         return math.acos(obj)
15731  *     except:
15732  *         return clifford().wrap( glucat.acos(toClifford(obj)) )
15733  *         # ««««««««
15734  * cpdef inline cosh(obj):
15735  */
15736         __Pyx_XDECREF(__pyx_r);
15737         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1687,
__pyx_L6_except_error)
15738         __Pyx_GOTREF(__pyx_t_10);
15739         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), acos(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1687, __pyx_L6_except_error)
15740         __Pyx_GOTREF(__pyx_t_11);
15741         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15742         __pyx_r = __pyx_t_11;
15743         __pyx_t_11 = 0;
15744         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15745         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15746         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15747         goto __pyx_L7_except_return;
15748     }
15749     __pyx_L6_except_error:;
15750
15751     /* "PyClical.pyx":1684
15752  *         return clifford().wrap( glucat.acos(toClifford(obj)), toClifford(i)) )
15753  *     else:
15754  *         try:
15755  *             # ««««««««
15756  *             return math.acos(obj)
15757  *         except:
15758  */
15759         __Pyx_XGIVEREF(__pyx_t_6);
15760         __Pyx_XGIVEREF(__pyx_t_7);
15761         __Pyx_XGIVEREF(__pyx_t_8);
15762         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15763         goto __pyx_L1_error;
15764         __pyx_L8_try_return:;
15765         __Pyx_XGIVEREF(__pyx_t_6);
15766         __Pyx_XGIVEREF(__pyx_t_7);
15767         __Pyx_XGIVEREF(__pyx_t_8);
15768         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15769         goto __pyx_L0;
15770         __pyx_L7_except_return:;
15771         __Pyx_XGIVEREF(__pyx_t_6);
15772         __Pyx_XGIVEREF(__pyx_t_7);
15773         __Pyx_XGIVEREF(__pyx_t_8);
15774         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15775         goto __pyx_L0;

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```

15775         }
15776     }
15777
15778     /* "PyClical.pyx":1668
15779     * return clifford().wrap( glucat.cos(toClifford(obj)) )
15780     *
15781     * cpdef inline acos(obj,i = None):          # ««««««««
15782     *     """
15783     *     Inverse cosine of multivector with optional complexifier.
15784     */
15785
15786     /* function exit code */
15787     __pyx_L1_error++;
15788     __Pyx_XDECREF(__pyx_t_3);
15789     __Pyx_XDECREF(__pyx_t_5);
15790     __Pyx_XDECREF(__pyx_t_9);
15791     __Pyx_XDECREF(__pyx_t_10);
15792     __Pyx_XDECREF(__pyx_t_11);
15793     __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15794     __pyx_r = 0;
15795     __pyx_L0:;
15796     __Pyx_XGIVEREF(__pyx_r);
15797     __Pyx_RefNannyFinishContext();
15798     return __pyx_r;
15799 }
15800
15801 /* Python wrapper */
15802 static PyObject * __pyx_pw_8PyClical_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
15803 static char __pyx_doc_8PyClical_58acos[] = "\n    Inverse cosine of multivector with
optional complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{1,2,3}\"))\n    {1,2}\n
>> x=clifford(\"{1,2}\"); print(cos(acos(x),\"{-1,1,2,3,4}\"))\n    {1,2}\n    >> print(acos(0) /
pi)\n    0.5\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x))\n    {1,2}\n    ";
15804 static PyObject * __pyx_pw_8PyClical_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
15805     PyObject * __pyx_v_obj = 0;
15806     PyObject * __pyx_v_i = 0;
15807     int __pyx_lineno = 0;
15808     const char * __pyx_filename = NULL;
15809     int __pyx_clineno = 0;
15810     PyObject * __pyx_r = 0;
15811     __Pyx_RefNannyDeclarations
15812     __Pyx_RefNannySetupContext("acos (wrapper)", 0);
15813     {
15814         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
15815         PyObject* values[2] = {0,0};
15816         values[1] = ((PyObject *)Py_None);
15817         if (unlikely(__pyx_kwds)) {
15818             Py_ssize_t kw_args;
15819             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15820             switch (pos_args) {
15821                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15822                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
15823                 case 0: break;
15824                 default: goto __pyx_L5_argtuple_error;
15825             }
15826             kw_args = PyDict_Size(__pyx_kwds);
15827             switch (pos_args) {
15828                 case 0:
15829                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15830                     else goto __pyx_L5_argtuple_error;
15831                     CYTHON_FALLTHROUGH;
15832                     case 1:
15833                         if (kw_args > 0) {
15834                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15835                             if (value) { values[1] = value; kw_args--; }
15836                         }
15837                     if (unlikely(kw_args > 0)) {
15838                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "acos") < 0)) __PYX_ERR(0, 1668, __pyx_L3_error)
15839                     }
15840                     } else {
15841                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15842                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
15843                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
break;
15844                             default: goto __pyx_L5_argtuple_error;
15845                         }
15846                     }
15847                     __pyx_v_obj = values[0];
15848                     __pyx_v_i = values[1];
15849                 }

```

```

15855         goto __pyx_L4_argument_unpacking_done;
15856         __pyx_L5_argtuple_error:;
15857         __Pyx_RaiseArgtupleInvalid("acos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1668, __pyx_L3_error)
15858         __pyx_L3_error:;
15859         __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15860         __Pyx_RefNannyFinishContext();
15861         return NULL;
15862         __pyx_L4_argument_unpacking_done:;
15863         __pyx_r = __pyx_pf_8PyClical_58acos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15864
15865         /* function exit code */
15866         __Pyx_RefNannyFinishContext();
15867         return __pyx_r;
15868     }
15869
15870     static PyObject *__pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
15871         PyObject *__pyx_r = NULL;
15872         __Pyx_RefNannyDeclarations
15873         PyObject *__pyx_t_1 = NULL;
15874         struct __pyx_opt_args_8PyClical_acos __pyx_t_2;
15875         int __pyx_lineno = 0;
15876         const char *__pyx_filename = NULL;
15877         int __pyx_clineno = 0;
15878         __Pyx_RefNannySetupContext("acos", 0);
15879         __Pyx_XDECREF(__pyx_r);
15880         __pyx_t_2.__pyx_n = 1;
15881         __pyx_t_2.i = __pyx_v_i;
15882         __pyx_t_1 = __pyx_f_8PyClical_acos(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1668, __pyx_L1_error)
15883         __Pyx_GOTREF(__pyx_t_1);
15884         __pyx_r = __pyx_t_1;
15885         __pyx_t_1 = 0;
15886         goto __pyx_L0;
15887
15888         /* function exit code */
15889         __pyx_L1_error:;
15890         __Pyx_XDECREF(__pyx_t_1);
15891         __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15892         __pyx_r = NULL;
15893         __pyx_L0:;
15894         __Pyx_XGIVEREF(__pyx_r);
15895         __Pyx_RefNannyFinishContext();
15896         return __pyx_r;
15897     }
15898
15899     /* "PyClical.pyx":1689
15900     *
15901     * return clifford().wrap( glucat.acos(toClifford(obj)) )
15902     *
15903     * cpdef inline cosh(obj): # ««««««
15904     *     """
15905     *     Hyperbolic cosine of multivector.
15906     */
15907     static PyObject *__pyx_pw_8PyClical_61cosh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
15908     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cosh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
15909         PyObject *__pyx_r = NULL;
15910         __Pyx_RefNannyDeclarations
15911         PyObject *__pyx_t_1 = NULL;
15912         PyObject *__pyx_t_2 = NULL;
15913         PyObject *__pyx_t_3 = NULL;
15914         PyObject *__pyx_t_4 = NULL;
15915         PyObject *__pyx_t_5 = NULL;
15916         PyObject *__pyx_t_6 = NULL;
15917         PyObject *__pyx_t_7 = NULL;
15918         PyObject *__pyx_t_8 = NULL;
15919         int __pyx_lineno = 0;
15920         const char *__pyx_filename = NULL;
15921         int __pyx_clineno = 0;
15922         __Pyx_RefNannySetupContext("cosh", 0);
15923
15924         /* "PyClical.pyx":1700
15925         *     {1,2}
15926         *     """
15927         *     try: # ««««««
15928         *         return math.cosh(obj)
15929         *     except:
15930         */
15931         {
15932             __Pyx_PyThreadState_declare
15933             __Pyx_PyThreadState_assign
15934             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
15935             __Pyx_XGOTREF(__pyx_t_1);
15936             __Pyx_XGOTREF(__pyx_t_2);

```

```

15937         __Pyx_XGOTREF(__pyx_t_3);
15938         /*try:*/ {
15939
15940             /* "PyClical.pyx":1701
15941 *         """
15942 *         try:
15943 *             return math.cosh(obj)          # ««««««««
15944 *         except:
15945 *             return clifford().wrap( glucat.cosh(toClifford(obj)) )
15946 */
15947         __Pyx_XDECREF(__pyx_r);
15948         __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1701, __pyx_L3_error)
15949         __Pyx_GOTREF(__pyx_t_5);
15950         __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_cosh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1701, __pyx_L3_error)
15951         __Pyx_GOTREF(__pyx_t_6);
15952         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15953         __pyx_t_5 = NULL;
15954         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
15955             __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
15956             if (likely(__pyx_t_5)) {
15957                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
15958                 __Pyx_INCREF(__pyx_t_5);
15959                 __Pyx_INCREF(function);
15960                 __Pyx_DECREF_SET(__pyx_t_6, function);
15961             }
15962         }
15963         __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
15964         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15965         if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1701, __pyx_L3_error)
15966         __Pyx_GOTREF(__pyx_t_4);
15967         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
15968         __pyx_r = __pyx_t_4;
15969         __pyx_t_4 = 0;
15970         goto __pyx_L7_try_return;
15971
15972         /* "PyClical.pyx":1700
15973 *         {1,2}
15974 *         """
15975 *         try:          # ««««««««
15976 *             return math.cosh(obj)
15977 *         except:
15978 */
15979         }
15980         __pyx_L3_error:;
15981         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
15982         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15983         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
15984
15985         /* "PyClical.pyx":1702
15986 *         try:
15987 *             return math.cosh(obj)
15988 *         except:          # ««««««««
15989 *             return clifford().wrap( glucat.cosh(toClifford(obj)) )
15990 */
15991         /*except:*/ {
15992             __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15993
15994             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1702, __pyx_L5_except_error)
15995             __Pyx_GOTREF(__pyx_t_4);
15996             __Pyx_GOTREF(__pyx_t_6);
15997             __Pyx_GOTREF(__pyx_t_5);
15998
15999             /* "PyClical.pyx":1703
16000 *             return math.cosh(obj)
16001 *         except:
16002 *             return clifford().wrap( glucat.cosh(toClifford(obj)) )          # ««««««««
16003 *
16004 * cpdef inline acosh(obj,i = None):
16005 */
16006         __Pyx_XDECREF(__pyx_r);
16007         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1703,
__pyx_L5_except_error)
16008         __Pyx_GOTREF(__pyx_t_7);
16009         __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), cosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1703, __pyx_L5_except_error)
16010         __Pyx_GOTREF(__pyx_t_8);
16011         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
16012         __pyx_r = __pyx_t_8;
16013         __pyx_t_8 = 0;
16014         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;

```

```

16015         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16016         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
16017         goto __pyx_L6_except_return;
16018     }
16019     __pyx_L5_except_error:;
16020
16021     /* "PyClicl.pyx":1700
16022     *     {1,2}
16023     *     """
16024     *     try:                # ««««««««
16025     *         return math.cosh(obj)
16026     *     except:
16027     */
16028         __Pyx_XGIVEREF(__pyx_t_1);
16029         __Pyx_XGIVEREF(__pyx_t_2);
16030         __Pyx_XGIVEREF(__pyx_t_3);
16031         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16032         goto __pyx_L1_error;
16033         __pyx_L7_try_return:;
16034         __Pyx_XGIVEREF(__pyx_t_1);
16035         __Pyx_XGIVEREF(__pyx_t_2);
16036         __Pyx_XGIVEREF(__pyx_t_3);
16037         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16038         goto __pyx_L0;
16039         __pyx_L6_except_return:;
16040         __Pyx_XGIVEREF(__pyx_t_1);
16041         __Pyx_XGIVEREF(__pyx_t_2);
16042         __Pyx_XGIVEREF(__pyx_t_3);
16043         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16044         goto __pyx_L0;
16045     }
16046
16047     /* "PyClicl.pyx":1689
16048     *     return clifford().wrap( glucat.acos(toClifford(obj)) )
16049     *
16050     * cpdef inline cosh(obj):                # ««««««««
16051     *     """
16052     *     Hyperbolic cosine of multivector.
16053     */
16054
16055     /* function exit code */
16056     __pyx_L1_error:;
16057     __Pyx_XDECREF(__pyx_t_4);
16058     __Pyx_XDECREF(__pyx_t_5);
16059     __Pyx_XDECREF(__pyx_t_6);
16060     __Pyx_XDECREF(__pyx_t_7);
16061     __Pyx_XDECREF(__pyx_t_8);
16062     __Pyx_AddTraceback("PyClicl.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16063     __pyx_r = 0;
16064     __pyx_L0:;
16065     __Pyx_XGIVEREF(__pyx_r);
16066     __Pyx_RefNannyFinishContext();
16067     return __pyx_r;
16068 }
16069
16070 /* Python wrapper */
16071 static PyObject * __pyx_pw_8PyClicl_61cosh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
16072 static char __pyx_doc_8PyClicl_60cosh[] = "\n    Hyperbolic cosine of
multivector.\n\n    >> x=clifford(\"{1,2}\") * pi; print(cosh(x))\n    -1\n    >>
x=clifford(\"{1,2,3}\"); print(cosh(acosh(x)))\n    {1,2,3}\n    >> x=clifford(\"{1,2}\");
print(cosh(acosh(x)))\n    {1,2}\n    ";
16073 static PyObject * __pyx_pw_8PyClicl_61cosh(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
16074     PyObject * __pyx_r = 0;
16075     __Pyx_RefNannyDeclarations
16076     __Pyx_RefNannySetupContext("cosh (wrapper)", 0);
16077     __pyx_r = __pyx_pf_8PyClicl_60cosh(__pyx_self, ((PyObject *) __pyx_v_obj));
16078
16079     /* function exit code */
16080     __Pyx_RefNannyFinishContext();
16081     return __pyx_r;
16082 }
16083
16084 static PyObject * __pyx_pf_8PyClicl_60cosh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
16085     PyObject * __pyx_r = NULL;
16086     __Pyx_RefNannyDeclarations
16087     PyObject * __pyx_t_1 = NULL;
16088     int __pyx_lineno = 0;
16089     const char * __pyx_filename = NULL;
16090     int __pyx_clineno = 0;
16091     __Pyx_RefNannySetupContext("cosh", 0);
16092     __Pyx_XDECREF(__pyx_r);
16093     __pyx_t_1 = __pyx_f_8PyClicl_cosh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1689, __pyx_L1_error)
16094     __Pyx_GOTREF(__pyx_t_1);

```

```

16095         __pyx_r = __pyx_t_1;
16096         __pyx_t_1 = 0;
16097         goto __pyx_L0;
16098
16099         /* function exit code */
16100         __pyx_L1_error++;
16101         __Pyx_XDECREF(__pyx_t_1);
16102         __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16103         __pyx_r = NULL;
16104         __pyx_L0++;
16105         __Pyx_XGIVEREF(__pyx_r);
16106         __Pyx_RefNannyFinishContext();
16107         return __pyx_r;
16108     }
16109
16110     /* "PyClical.pyx":1705
16111     *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16112     *
16113     * cpdef inline acosh(obj,i = None):          # ««««««««
16114     *     """
16115     *         Inverse hyperbolic cosine of multivector with optional complexifier.
16116     */
16117
16118     static PyObject * __pyx_pw_8PyClical_63acosh(PyObject * __pyx_self, PyObject
16119 * __pyx_args, PyObject * __pyx_kwds); /*proto*/
16120     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_acosh(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_acosh * __pyx_optional_args) {
16121     PyObject * __pyx_v_i = ((PyObject *)Py_None);
16122     PyObject * __pyx_r = NULL;
16123     __Pyx_RefNannyDeclarations
16124     int __pyx_t_1;
16125     int __pyx_t_2;
16126     PyObject * __pyx_t_3 = NULL;
16127     Clifford __pyx_t_4;
16128     PyObject * __pyx_t_5 = NULL;
16129     PyObject * __pyx_t_6 = NULL;
16130     PyObject * __pyx_t_7 = NULL;
16131     PyObject * __pyx_t_8 = NULL;
16132     PyObject * __pyx_t_9 = NULL;
16133     PyObject * __pyx_t_10 = NULL;
16134     PyObject * __pyx_t_11 = NULL;
16135     int __pyx_lineno = 0;
16136     const char * __pyx_filename = NULL;
16137     int __pyx_clineno = 0;
16138     __Pyx_RefNannySetupContext("acosh", 0);
16139     if (__pyx_optional_args) {
16140         if (__pyx_optional_args->__pyx_n > 0) {
16141             __pyx_v_i = __pyx_optional_args->i;
16142         }
16143     }
16144     /* "PyClical.pyx":1720
16145     *     {1,2}
16146     *     """
16147     *     if not (i is None):          # ««««««««
16148     *         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16149     *     else:
16150     */
16151     __pyx_t_1 = (__pyx_v_i != Py_None);
16152     __pyx_t_2 = (__pyx_t_1 != 0);
16153     if (__pyx_t_2) {
16154         /* "PyClical.pyx":1721
16155     *     """
16156     *     if not (i is None):
16157     *         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16158     *     else:
16159     *         try:
16160     */
16161     /*
16162         __Pyx_XDECREF(__pyx_r);
16163         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
16164 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1721, __pyx_L1_error)
16165         __Pyx_GOTREF(__pyx_t_3);
16166         try {
16167             __pyx_t_4 = acosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
16168 __pyx_f_8PyClical_toClifford(__pyx_v_i));
16169         } catch (...) {
16170             __Pyx_CppExn2PyErr();
16171             __PYX_ERR(0, 1721, __pyx_L1_error)
16172         }
16173         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
16174 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1721, __pyx_L1_error)
16175         __Pyx_GOTREF(__pyx_t_5);
16176         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16177         __pyx_r = __pyx_t_5;
16178         __pyx_t_5 = 0;

```

```

16176         goto __pyx_L0;
16177
16178         /* "PyClicl.pyx":1720
16179  *      {1,2}
16180  *      """
16181  *      if not (i is None):                # ««««««««
16182  *          return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16183  *      else:
16184  */
16185         }
16186
16187         /* "PyClicl.pyx":1723
16188  *      return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16189  *      else:
16190  *          try:                # ««««««««
16191  *              return math.acosh(obj)
16192  *          except:
16193  */
16194         /*else*/ {
16195             {
16196                 __Pyx_PyThreadState_declare
16197                 __Pyx_PyThreadState_assign
16198                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16199                 __Pyx_XGOTREF(__pyx_t_6);
16200                 __Pyx_XGOTREF(__pyx_t_7);
16201                 __Pyx_XGOTREF(__pyx_t_8);
16202                 /*try:*/ {
16203
16204                     /* "PyClicl.pyx":1724
16205  *          else:
16206  *              try:
16207  *                  return math.acosh(obj)                # ««««««««
16208  *              except:
16209  *                  return clifford().wrap( glucat.acosh(toClifford(obj)) )
16210  */
16211                     __Pyx_XDECREF(__pyx_r);
16212                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16213 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1724, __pyx_L4_error)
16214                     __Pyx_GOTREF(__pyx_t_3);
16215                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acosh); if
16216 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1724, __pyx_L4_error)
16217                     __Pyx_GOTREF(__pyx_t_9);
16218                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16219                     __pyx_t_3 = NULL;
16220                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16221                         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16222                         if (likely(__pyx_t_3)) {
16223                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16224                             __Pyx_INCREF(__pyx_t_3);
16225                             __Pyx_INCREF(function);
16226                             __Pyx_DECREF_SET(__pyx_t_9, function);
16227                         }
16228                     }
16229                     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
16230 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16231                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16232                     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1724, __pyx_L4_error)
16233                     __Pyx_GOTREF(__pyx_t_5);
16234                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16235                     __pyx_r = __pyx_t_5;
16236                     __pyx_t_5 = 0;
16237                     goto __pyx_L8_try_return;
16238
16239                     /* "PyClicl.pyx":1723
16240  *      return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16241  *      else:
16242  *          try:                # ««««««««
16243  *              return math.acosh(obj)
16244  *          except:
16245  */
16246                     }
16247                     __pyx_L4_error:;
16248                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16249                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16250                     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16251
16252                     /* "PyClicl.pyx":1725
16253  *          try:
16254  *              return math.acosh(obj)
16255  *          except:                # ««««««««
16256  *              return clifford().wrap( glucat.acosh(toClifford(obj)) )
16257  */
16258                     /*except:*/ {
16259                         __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno,
16260 __pyx_filename);
16261                         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,

```



```

1725, __pyx_L6_except_error)
16259     __Pyx_GOTREF(__pyx_t_5);
16260     __Pyx_GOTREF(__pyx_t_9);
16261     __Pyx_GOTREF(__pyx_t_3);
16262
16263     /* "PyClical.pyx":1726
16264     *         return math.acosh(obj)
16265     *     except:
16266     *         return clifford().wrap( glucat.acosh(toClifford(obj)) )           # ««««««««
16267     *
16268     * cpdef inline sin(obj,i = None):
16269     */
16270     __Pyx_XDECREF(__pyx_r);
16271     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1726,
__pyx_L6_except_error)
16272     __Pyx_GOTREF(__pyx_t_10);
16273     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), acosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1726, __pyx_L6_except_error)
16274     __Pyx_GOTREF(__pyx_t_11);
16275     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16276     __pyx_r = __pyx_t_11;
16277     __pyx_t_11 = 0;
16278     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16279     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16280     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16281     goto __pyx_L7_except_return;
16282 }
16283 __pyx_L6_except_error:;
16284
16285     /* "PyClical.pyx":1723
16286     *         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16287     *     else:
16288     *         try:           # ««««««««
16289     *             return math.acosh(obj)
16290     *         except:
16291     */
16292     __Pyx_XGIVEREF(__pyx_t_6);
16293     __Pyx_XGIVEREF(__pyx_t_7);
16294     __Pyx_XGIVEREF(__pyx_t_8);
16295     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16296     goto __pyx_L1_error;
16297     __pyx_L8_try_return:;
16298     __Pyx_XGIVEREF(__pyx_t_6);
16299     __Pyx_XGIVEREF(__pyx_t_7);
16300     __Pyx_XGIVEREF(__pyx_t_8);
16301     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16302     goto __pyx_L0;
16303     __pyx_L7_except_return:;
16304     __Pyx_XGIVEREF(__pyx_t_6);
16305     __Pyx_XGIVEREF(__pyx_t_7);
16306     __Pyx_XGIVEREF(__pyx_t_8);
16307     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16308     goto __pyx_L0;
16309 }
16310 }
16311
16312     /* "PyClical.pyx":1705
16313     *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16314     *
16315     * cpdef inline acosh(obj,i = None):           # ««««««««
16316     *     """
16317     *     Inverse hyperbolic cosine of multivector with optional complexifier.
16318     */
16319
16320     /* function exit code */
16321     __pyx_L1_error:;
16322     __Pyx_XDECREF(__pyx_t_3);
16323     __Pyx_XDECREF(__pyx_t_5);
16324     __Pyx_XDECREF(__pyx_t_9);
16325     __Pyx_XDECREF(__pyx_t_10);
16326     __Pyx_XDECREF(__pyx_t_11);
16327     __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16328     __pyx_r = 0;
16329     __pyx_L0:;
16330     __Pyx_XGIVEREF(__pyx_r);
16331     __Pyx_RefNannyFinishContext();
16332     return __pyx_r;
16333 }
16334
16335     /* Python wrapper */
16336     static PyObject *__pyx_pw_8PyClical_63acosh(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
16337     static char __pyx_doc_8PyClical_62acosh[] = "\n    Inverse hyperbolic cosine of
multivector with optional complexifier.\n\n    >> print(acosh(0,\"{-2,-1,1}\"))\n    1.571{-2,-1,1}\n
    >> x=clifford(\"{1,2,3}\"); print(cosh(acosh(x,\"{-1,1,2,3,4}\")))\n    {1,2,3}\n    >>";

```

```

print(acosh(0))\n    1.571{-1}\n    >> x=clifford("\{1,2,3}\n"); print(cosh(acosh(x)))\n    {1,2,3}\n
>> x=clifford("\{1,2}\n"); print(cosh(acosh(x)))\n    {1,2}\n    ";
16338     static PyObject *__pyx_pw_8PyClical_63acosh(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
16339         PyObject *__pyx_v_obj = 0;
16340         PyObject *__pyx_v_i = 0;
16341         int __pyx_lineno = 0;
16342         const char *__pyx_filename = NULL;
16343         int __pyx_clineno = 0;
16344         PyObject *__pyx_r = 0;
16345         __Pyx_RefNannyDeclarations
16346         __Pyx_RefNannySetupContext("acosh (wrapper)", 0);
16347         {
16348             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16349             PyObject* values[2] = {0,0};
16350             values[1] = (PyObject *)Py_None;
16351             if (unlikely(__pyx_kwds)) {
16352                 Py_ssize_t kw_args;
16353                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16354                 switch (pos_args) {
16355                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16356                         CYTHON_FALLTHROUGH;
16357                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16358                         CYTHON_FALLTHROUGH;
16359                     case 0: break;
16360                     default: goto __pyx_L5_argtuple_error;
16361                 }
16362                 kw_args = PyDict_Size(__pyx_kwds);
16363                 switch (pos_args) {
16364                     case 0:
16365                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
16366                         else goto __pyx_L5_argtuple_error;
16367                         CYTHON_FALLTHROUGH;
16368                     case 1:
16369                         if (kw_args > 0) {
16370                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16371                             if (value) { values[1] = value; kw_args--; }
16372                         }
16373                 }
16374                 if (unlikely(kw_args > 0)) {
16375                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "acosh") < 0)) __PYX_ERR(0, 1705, __pyx_L3_error)
16376                 }
16377                 else {
16378                     switch (PyTuple_GET_SIZE(__pyx_args)) {
16379                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16380                         CYTHON_FALLTHROUGH;
16381                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16382                         break;
16383                         default: goto __pyx_L5_argtuple_error;
16384                     }
16385                 }
16386                 __pyx_v_obj = values[0];
16387                 __pyx_v_i = values[1];
16388             }
16389             goto __pyx_L4_argument_unpacking_done;
16390             __pyx_L5_argtuple_error:;
16391             __Pyx_RaiseArgtupleInvalid("acosh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
16392             __PYX_ERR(0, 1705, __pyx_L3_error)
16393             __pyx_L3_error:;
16394             __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16395             __Pyx_RefNannyFinishContext();
16396             return NULL;
16397             __pyx_L4_argument_unpacking_done:;
16398             __pyx_r = __pyx_pf_8PyClical_62acosh(__pyx_self, __pyx_v_obj, __pyx_v_i);
16399
16400             /* function exit code */
16401             __Pyx_RefNannyFinishContext();
16402             return __pyx_r;
16403         }
16404         static PyObject *__pyx_pf_8PyClical_62acosh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
16405             PyObject *__pyx_r = NULL;
16406             __Pyx_RefNannyDeclarations
16407             PyObject *__pyx_t_1 = NULL;
16408             struct __pyx_opt_args_8PyClical_acosh __pyx_t_2;
16409             int __pyx_lineno = 0;
16410             const char *__pyx_filename = NULL;
16411             int __pyx_clineno = 0;
16412             __Pyx_RefNannySetupContext("acosh", 0);
16413             __Pyx_XDECREF(__pyx_r);
16414             __pyx_t_2.__pyx_n = 1;
16415             __pyx_t_2.i = __pyx_v_i;
16416             __pyx_t_1 = __pyx_pf_8PyClical_acosh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1705, __pyx_L1_error)

```

```

16417         __Pyx_GOTREF(__pyx_t_1);
16418         __pyx_r = __pyx_t_1;
16419         __pyx_t_1 = 0;
16420         goto __pyx_L0;
16421
16422         /* function exit code */
16423         __pyx_L1_error++;
16424         __Pyx_XDECREF(__pyx_t_1);
16425         __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16426         __pyx_r = NULL;
16427         __pyx_L0:;
16428         __Pyx_XGIVEREF(__pyx_r);
16429         __Pyx_RefNannyFinishContext();
16430         return __pyx_r;
16431     }
16432
16433     /* "PyClical.pyx":1728
16434     *
16435     * cpdef inline sin(obj,i = None):
16436     *     """
16437     *     Sine of multivector with optional complexifier.
16438     */
16439
16440     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
16441     PyObject *__pyx_kwds); /*proto*/
16442     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sin(PyObject *__pyx_v_obj,
16443     CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_sin *__pyx_optional_args) {
16444         PyObject *__pyx_v_i = ((PyObject *)Py_None);
16445         PyObject *__pyx_r = NULL;
16446         __Pyx_RefNannyDeclarations
16447         int __pyx_t_1;
16448         int __pyx_t_2;
16449         PyObject *__pyx_t_3 = NULL;
16450         Clifford __pyx_t_4;
16451         PyObject *__pyx_t_5 = NULL;
16452         PyObject *__pyx_t_6 = NULL;
16453         PyObject *__pyx_t_7 = NULL;
16454         PyObject *__pyx_t_8 = NULL;
16455         PyObject *__pyx_t_9 = NULL;
16456         PyObject *__pyx_t_10 = NULL;
16457         PyObject *__pyx_t_11 = NULL;
16458         int __pyx_lineno = 0;
16459         const char *__pyx_filename = NULL;
16460         int __pyx_clineno = 0;
16461         __Pyx_RefNannySetupContext("sin", 0);
16462         if (__pyx_optional_args) {
16463             if (__pyx_optional_args->__pyx_n > 0) {
16464                 __pyx_v_i = __pyx_optional_args->i;
16465             }
16466         }
16467         /* "PyClical.pyx":1739
16468         *     {1,2,3}
16469         *
16470         * if not (i is None):
16471         *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16472         * else:
16473         */
16474         __pyx_t_1 = (__pyx_v_i != Py_None);
16475         __pyx_t_2 = (__pyx_t_1 != 0);
16476         if (__pyx_t_2) {
16477             /* "PyClical.pyx":1740
16478             *
16479             * if not (i is None):
16480             *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16481             * else:
16482             *     try:
16483             */
16484             __Pyx_XDECREF(__pyx_r);
16485             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
16486             *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1740, __pyx_L1_error)
16487             __Pyx_GOTREF(__pyx_t_3);
16488             try {
16489                 __pyx_t_4 = sin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
16490                 __pyx_f_8PyClical_toClifford(__pyx_v_i));
16491             } catch (...) {
16492                 __Pyx_CppExn2PyErr();
16493                 __PYX_ERR(0, 1740, __pyx_L1_error)
16494             }
16495             __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
16496             *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1740, __pyx_L1_error)
16497             __Pyx_GOTREF(__pyx_t_5);
16498             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16499             __pyx_r = __pyx_t_5;
16500             __pyx_t_5 = 0;

```

```

16499         goto __pyx_L0;
16500
16501         /* "PyClical.pyx":1739
16502  *      {1,2,3}
16503  *      """
16504  *      if not (i is None):                # ««««««««
16505  *          return clifford().wrap( glucat.sin(toClifford(obj)), toClifford(i)) )
16506  *      else:
16507  */
16508         }
16509
16510         /* "PyClical.pyx":1742
16511  *      return clifford().wrap( glucat.sin(toClifford(obj)), toClifford(i)) )
16512  *      else:
16513  *          try:                # ««««««««
16514  *              return math.sin(obj)
16515  *          except:
16516  */
16517         /*else*/ {
16518             {
16519                 __Pyx_PyThreadState_declare
16520                 __Pyx_PyThreadState_assign
16521                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16522                 __Pyx_XGOTREF(__pyx_t_6);
16523                 __Pyx_XGOTREF(__pyx_t_7);
16524                 __Pyx_XGOTREF(__pyx_t_8);
16525                 /*try*/ {
16526
16527                     /* "PyClical.pyx":1743
16528  *          else:
16529  *              try:
16530  *                  return math.sin(obj)                # ««««««««
16531  *              except:
16532  *                  return clifford().wrap( glucat.sin(toClifford(obj)) )
16533  */
16534                     __Pyx_XDECREF(__pyx_r);
16535                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16536 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1743, __pyx_L4_error)
16537                     __Pyx_GOTREF(__pyx_t_3);
16538                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sin); if
16539 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1743, __pyx_L4_error)
16540                     __Pyx_GOTREF(__pyx_t_9);
16541                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16542                     __pyx_t_3 = NULL;
16543                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16544                         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16545                         if (likely(__pyx_t_3)) {
16546                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16547                             __Pyx_INCREF(__pyx_t_3);
16548                             __Pyx_INCREF(function);
16549                             __Pyx_DECREF_SET(__pyx_t_9, function);
16550                         }
16551                     }
16552                     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
16553 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16554                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16555                     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1743, __pyx_L4_error)
16556                     __Pyx_GOTREF(__pyx_t_5);
16557                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16558                     __pyx_r = __pyx_t_5;
16559                     __pyx_t_5 = 0;
16560                     goto __pyx_L8_try_return;
16561
16562                     /* "PyClical.pyx":1742
16563  *      return clifford().wrap( glucat.sin(toClifford(obj)), toClifford(i)) )
16564  *      else:
16565  *          try:                # ««««««««
16566  *              return math.sin(obj)
16567  *          except:
16568  */
16569                     }
16570                     __pyx_L4_error:;
16571                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16572                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16573                     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16574
16575                     /* "PyClical.pyx":1744
16576  *          try:
16577  *              return math.sin(obj)
16578  *          except:                # ««««««««
16579  *              return clifford().wrap( glucat.sin(toClifford(obj)) )
16580  */
16581                     /*except*/ {
16582                         __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno,
16583 __pyx_filename);
16584                     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,

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1744, __pyx_L6_except_error)
16582     __Pyx_GOTREF(__pyx_t_5);
16583     __Pyx_GOTREF(__pyx_t_9);
16584     __Pyx_GOTREF(__pyx_t_3);
16585
16586     /* "PyClical.pyx":1745
16587  *         return math.sin(obj)
16588  *     except:
16589  *         return clifford().wrap( glucat.sin(toClifford(obj)) )           # ««««««««
16590  *
16591  * cpdef inline asin(obj,i = None):
16592  */
16593     __Pyx_XDECREF(__pyx_r);
16594     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1745,
__pyx_L6_except_error)
16595     __Pyx_GOTREF(__pyx_t_10);
16596     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), sin(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1745, __pyx_L6_except_error)
16597     __Pyx_GOTREF(__pyx_t_11);
16598     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16599     __pyx_r = __pyx_t_11;
16600     __pyx_t_11 = 0;
16601     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16602     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16603     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16604     goto __pyx_L7_except_return;
16605 }
16606 __pyx_L6_except_error:;
16607
16608     /* "PyClical.pyx":1742
16609  *         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16610  *     else:
16611  *         try:                       # ««««««««
16612  *             return math.sin(obj)
16613  *         except:
16614  */
16615     __Pyx_XGIVEREF(__pyx_t_6);
16616     __Pyx_XGIVEREF(__pyx_t_7);
16617     __Pyx_XGIVEREF(__pyx_t_8);
16618     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16619     goto __pyx_L1_error;
16620     __pyx_L8_try_return:;
16621     __Pyx_XGIVEREF(__pyx_t_6);
16622     __Pyx_XGIVEREF(__pyx_t_7);
16623     __Pyx_XGIVEREF(__pyx_t_8);
16624     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16625     goto __pyx_L0;
16626     __pyx_L7_except_return:;
16627     __Pyx_XGIVEREF(__pyx_t_6);
16628     __Pyx_XGIVEREF(__pyx_t_7);
16629     __Pyx_XGIVEREF(__pyx_t_8);
16630     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16631     goto __pyx_L0;
16632 }
16633 }
16634
16635     /* "PyClical.pyx":1728
16636  *         return clifford().wrap( glucat.acosh(toClifford(obj)) )
16637  *
16638  * cpdef inline sin(obj,i = None):           # ««««««««
16639  *     """
16640  *     Sine of multivector with optional complexifier.
16641  */
16642
16643     /* function exit code */
16644     __pyx_L1_error:;
16645     __Pyx_XDECREF(__pyx_t_3);
16646     __Pyx_XDECREF(__pyx_t_5);
16647     __Pyx_XDECREF(__pyx_t_9);
16648     __Pyx_XDECREF(__pyx_t_10);
16649     __Pyx_XDECREF(__pyx_t_11);
16650     __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16651     __pyx_r = 0;
16652     __pyx_L0:;
16653     __Pyx_XGIVEREF(__pyx_r);
16654     __Pyx_RefNannyFinishContext();
16655     return __pyx_r;
16656 }
16657
16658     /* Python wrapper */
16659     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
16660     static char __pyx_doc_8PyClical_64sin[] = "\n    Sine of multivector with optional
complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>
s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >>"

```

```

x=clifford("\{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    ";
16661     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
16662         PyObject *__pyx_v_obj = 0;
16663         PyObject *__pyx_v_i = 0;
16664         int __pyx_lineno = 0;
16665         const char *__pyx_filename = NULL;
16666         int __pyx_clineno = 0;
16667         PyObject *__pyx_r = 0;
16668         __Pyx_RefNannyDeclarations
16669         __Pyx_RefNannySetupContext("sin (wrapper)", 0);
16670         {
16671             static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16672             PyObject* values[2] = {0,0};
16673             values[1] = ((PyObject *)Py_None);
16674             if (unlikely(__pyx_kwds)) {
16675                 Py_ssize_t kw_args;
16676                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16677                 switch (pos_args) {
16678                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16679                         CYTHON_FALLTHROUGH;
16680                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16681                         CYTHON_FALLTHROUGH;
16682                     case 0: break;
16683                     default: goto __pyx_L5_argtuple_error;
16684                 }
16685                 kw_args = PyDict_Size(__pyx_kwds);
16686                 switch (pos_args) {
16687                     case 0:
16688                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
16689                         else goto __pyx_L5_argtuple_error;
16690                         CYTHON_FALLTHROUGH;
16691                     case 1:
16692                         if (kw_args > 0) {
16693                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16694                             if (value) { values[1] = value; kw_args--; }
16695                         }
16696                     }
16697                 if (unlikely(kw_args > 0)) {
16698                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "sin") < 0)) __PYX_ERR(0, 1728, __pyx_L3_error)
16699                 }
16700                 } else {
16701                     switch (PyTuple_GET_SIZE(__pyx_args)) {
16702                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16703                             CYTHON_FALLTHROUGH;
16704                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16705                             break;
16706                         default: goto __pyx_L5_argtuple_error;
16707                     }
16708                 }
16709                 __pyx_v_obj = values[0];
16710                 __pyx_v_i = values[1];
16711             }
16712             goto __pyx_L4_argument_unpacking_done;
16713             __pyx_L5_argtuple_error:;
16714             __Pyx_RaiseArgtupleInvalid("sin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
16715             __PYX_ERR(0, 1728, __pyx_L3_error)
16716             __pyx_L3_error:;
16717             __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16718             __Pyx_RefNannyFinishContext();
16719             return NULL;
16720             __pyx_L4_argument_unpacking_done:;
16721             __pyx_r = __pyx_pf_8PyClical_64sin(__pyx_self, __pyx_v_obj, __pyx_v_i);
16722
16723             /* function exit code */
16724             __Pyx_RefNannyFinishContext();
16725             return __pyx_r;
16726         }
16727     static PyObject *__pyx_pf_8PyClical_64sin(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_i) {
16728         PyObject *__pyx_r = NULL;
16729         __Pyx_RefNannyDeclarations
16730         PyObject *__pyx_t_1 = NULL;
16731         struct __pyx_opt_args_8PyClical_sin __pyx_t_2;
16732         int __pyx_lineno = 0;
16733         const char *__pyx_filename = NULL;
16734         int __pyx_clineno = 0;
16735         __Pyx_RefNannySetupContext("sin", 0);
16736         __Pyx_XDECREF(__pyx_r);
16737         __pyx_t_2.__pyx_n = 1;
16738         __pyx_t_2.i = __pyx_v_i;
16739         __pyx_t_1 = __pyx_f_8PyClical_sin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1728, __pyx_L1_error)
16740         __Pyx_GOTREF(__pyx_t_1);

```

```

16741         __pyx_r = __pyx_t_1;
16742         __pyx_t_1 = 0;
16743         goto __pyx_L0;
16744
16745         /* function exit code */
16746         __pyx_L1_error++;
16747         __Pyx_XDECREF(__pyx_t_1);
16748         __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16749         __pyx_r = NULL;
16750         __pyx_L0++;
16751         __Pyx_XGIVEREF(__pyx_r);
16752         __Pyx_RefNannyFinishContext();
16753         return __pyx_r;
16754     }
16755
16756     /* "PyClical.pyx":1747
16757     *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16758     *
16759     * cpdef inline asin(obj,i = None):          # ««««««««
16760     *     """
16761     *         Inverse sine of multivector with optional complexifier.
16762     */
16763
16764     static PyObject * __pyx_pw_8PyClical_67asin(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
16765     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_asin(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_asin * __pyx_optional_args) {
16766         PyObject * __pyx_v_i = ((PyObject *)Py_None);
16767         PyObject * __pyx_r = NULL;
16768         __Pyx_RefNannyDeclarations
16769         int __pyx_t_1;
16770         int __pyx_t_2;
16771         PyObject * __pyx_t_3 = NULL;
16772         Clifford __pyx_t_4;
16773         PyObject * __pyx_t_5 = NULL;
16774         PyObject * __pyx_t_6 = NULL;
16775         PyObject * __pyx_t_7 = NULL;
16776         PyObject * __pyx_t_8 = NULL;
16777         PyObject * __pyx_t_9 = NULL;
16778         PyObject * __pyx_t_10 = NULL;
16779         PyObject * __pyx_t_11 = NULL;
16780         int __pyx_lineno = 0;
16781         const char * __pyx_filename = NULL;
16782         int __pyx_clineno = 0;
16783         __Pyx_RefNannySetupContext("asin", 0);
16784         if (__pyx_optional_args) {
16785             if (__pyx_optional_args->__pyx_n > 0) {
16786                 __pyx_v_i = __pyx_optional_args->i;
16787             }
16788         }
16789
16790         /* "PyClical.pyx":1760
16791     *     {1,2,3}
16792     *     """
16793     *     if not (i is None):          # ««««««««
16794     *         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16795     *     else:
16796     */
16797         __pyx_t_1 = (__pyx_v_i != Py_None);
16798         __pyx_t_2 = (__pyx_t_1 != 0);
16799         if (__pyx_t_2) {
16800
16801             /* "PyClical.pyx":1761
16802     *     """
16803     *     if not (i is None):
16804     *         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )          # ««««««««
16805     *     else:
16806     *         try:
16807     */
16808         __Pyx_XDECREF(__pyx_r);
16809         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1761, __pyx_L1_error)
16810         __Pyx_GOTREF(__pyx_t_3);
16811         try {
16812             __pyx_t_4 = asin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
16813         } catch (...) {
16814             __Pyx_CppExn2PyErr();
16815             __PYX_ERR(0, 1761, __pyx_L1_error)
16816         }
16817         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1761, __pyx_L1_error)
16818         __Pyx_GOTREF(__pyx_t_5);
16819         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16820         __pyx_r = __pyx_t_5;
16821         __pyx_t_5 = 0;
16822         goto __pyx_L0;

```

```

16823
16824                                     /* "PyCliclcal.pyx":1760
16825 *      {1,2,3}
16826 *      ""
16827 *      if not (i is None):          # ««««««««
16828 *          return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16829 *      else:
16830 */
16831     }
16832
16833                                     /* "PyCliclcal.pyx":1763
16834 *      return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16835 *      else:
16836 *          try:          # ««««««««
16837 *              return math.asin(obj)
16838 *          except:
16839 */
16840                                     /*else*/ {
16841     {
16842         __Pyx_PyThreadState_declare
16843         __Pyx_PyThreadState_assign
16844         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16845         __Pyx_XGOTREF(__pyx_t_6);
16846         __Pyx_XGOTREF(__pyx_t_7);
16847         __Pyx_XGOTREF(__pyx_t_8);
16848         /*try:*/ {
16849
16850                                     /* "PyCliclcal.pyx":1764
16851 *      else:
16852 *          try:
16853 *              return math.asin(obj)          # ««««««««
16854 *          except:
16855 *              return clifford().wrap( glucat.asin(toClifford(obj)) )
16856 */
16857         __Pyx_XDECREF(__pyx_r);
16858         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16859 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1764, __pyx_L4_error)
16860         __Pyx_GOTREF(__pyx_t_3);
16861         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asin); if
16862 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1764, __pyx_L4_error)
16863         __Pyx_GOTREF(__pyx_t_9);
16864         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16865         __pyx_t_3 = NULL;
16866         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16867             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16868             if (likely(__pyx_t_3)) {
16869                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16870                 __Pyx_INCREF(__pyx_t_3);
16871                 __Pyx_INCREF(function);
16872                 __Pyx_DECREF_SET(__pyx_t_9, function);
16873             }
16874         }
16875         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
16876 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16877         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16878         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1764, __pyx_L4_error)
16879         __Pyx_GOTREF(__pyx_t_5);
16880         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16881         __pyx_r = __pyx_t_5;
16882         __pyx_t_5 = 0;
16883         goto __pyx_L8_try_return;
16884
16885                                     /* "PyCliclcal.pyx":1763
16886 *      return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16887 *      else:
16888 *          try:          # ««««««««
16889 *              return math.asin(obj)
16890 *          except:
16891 */
16892     }
16893     __pyx_L4_error:;
16894     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16895     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16896     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16897
16898                                     /* "PyCliclcal.pyx":1765
16899 *      try:
16900 *          return math.asin(obj)
16901 *      except:          # ««««««««
16902 *          return clifford().wrap( glucat.asin(toClifford(obj)) )
16903 */
16904                                     /*except:*/ {
16905         __Pyx_AddTraceback("PyCliclcal.asin", __pyx_clineno, __pyx_lineno,
16906 __pyx_filename);
16907         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
16908 1765, __pyx_L6_except_error)

```



```

16905         __Pyx_GOTREF(__pyx_t_5);
16906         __Pyx_GOTREF(__pyx_t_9);
16907         __Pyx_GOTREF(__pyx_t_3);
16908
16909         /* "PyClical.pyx":1766
16910  *         return math.asin(obj)
16911  *     except:
16912  *         return clifford().wrap( glucat.asin(toClifford(obj)) )           # ««««««««
16913  *
16914  * cpdef inline sinh(obj):
16915  */
16916         __Pyx_XDECREF(__pyx_r);
16917         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
16918 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1766,
16919         __pyx_L6_except_error)
16918         __Pyx_GOTREF(__pyx_t_10);
16919         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
16920 __pyx_obj_8PyClical_clifford *)__pyx_t_10), asin(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
16921 (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1766, __pyx_L6_except_error)
16922         __Pyx_GOTREF(__pyx_t_11);
16923         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16924         __pyx_r = __pyx_t_11;
16925         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16926         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16927         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16928         goto __pyx_L7_except_return;
16929     }
16930     __pyx_L6_except_error;
16931
16932     /* "PyClical.pyx":1763
16933  *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16934  * else:
16935  *     try:
16936  *         # ««««««««
16937  *         return math.asin(obj)
16938  *     except:
16939  */
16940         __Pyx_XDECREF(__pyx_t_6);
16941         __Pyx_XDECREF(__pyx_t_7);
16942         __Pyx_XDECREF(__pyx_t_8);
16943         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16944         goto __pyx_L1_error;
16945         __pyx_L8_try_return:;
16946         __Pyx_XDECREF(__pyx_t_6);
16947         __Pyx_XDECREF(__pyx_t_7);
16948         __Pyx_XDECREF(__pyx_t_8);
16949         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16950         goto __pyx_L0;
16951         __pyx_L7_except_return:;
16952         __Pyx_XDECREF(__pyx_t_6);
16953         __Pyx_XDECREF(__pyx_t_7);
16954         __Pyx_XDECREF(__pyx_t_8);
16955         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16956         goto __pyx_L0;
16957     }
16958
16959     /* "PyClical.pyx":1747
16960  *     return clifford().wrap( glucat.sin(toClifford(obj)) )
16961  *
16962  * cpdef inline asin(obj,i = None):
16963  *     # ««««««««
16964  *     """
16965  *     Inverse sine of multivector with optional complexifier.
16966  */
16967
16968     /* function exit code */
16969     __pyx_L1_error:;
16970     __Pyx_XDECREF(__pyx_t_3);
16971     __Pyx_XDECREF(__pyx_t_5);
16972     __Pyx_XDECREF(__pyx_t_9);
16973     __Pyx_XDECREF(__pyx_t_10);
16974     __Pyx_XDECREF(__pyx_t_11);
16975     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16976     __pyx_r = 0;
16977     __pyx_L0:;
16978     __Pyx_XDECREF(__pyx_r);
16979     __Pyx_RefNannyFinishContext();
16980     return __pyx_r;
16981 }
16982
16983 /* Python wrapper */
16984 static PyObject *__pyx_pw_8PyClical_67asin(PyObject *__pyx_self, PyObject *__pyx_args,
16985 PyObject *__pyx_kws); /*proto*/
16986 static char __pyx_doc_8PyClical_66asin[] = "\n    Inverse sine of multivector with
16987 optional complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>
16988 s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >> print(asin(1) /
16989 pi)\n    0.5\n    >> x=clifford(\"{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    ";

```

```

16984         static PyObject *__pyx_pw_8PyClical_67asin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
16985             PyObject *__pyx_v_obj = 0;
16986             PyObject *__pyx_v_i = 0;
16987             int __pyx_lineno = 0;
16988             const char *__pyx_filename = NULL;
16989             int __pyx_clineno = 0;
16990             PyObject *__pyx_r = 0;
16991             __Pyx_RefNannyDeclarations
16992             __Pyx_RefNannySetupContext("asin (wrapper)", 0);
16993             {
16994                 static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16995                 PyObject* values[2] = {0,0};
16996                 values[1] = ((PyObject *)Py_None);
16997                 if (unlikely(__pyx_kwds)) {
16998                     Py_ssize_t kw_args;
16999                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17000                     switch (pos_args) {
17001                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17002                             CYTHON_FALLTHROUGH;
17003                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17004                             CYTHON_FALLTHROUGH;
17005                         case 0: break;
17006                         default: goto __pyx_L5_argtuple_error;
17007                     }
17008                     kw_args = PyDict_Size(__pyx_kwds);
17009                     switch (pos_args) {
17010                         case 0:
17011                             if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
17012                             else goto __pyx_L5_argtuple_error;
17013                             CYTHON_FALLTHROUGH;
17014                         case 1:
17015                             if (kw_args > 0) {
17016                                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17017                                 if (value) { values[1] = value; kw_args--; }
17018                             }
17019                             if (unlikely(kw_args > 0)) {
17020                                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "asin") < 0)) __PYX_ERR(0, 1747, __pyx_L3_error)
17021                             }
17022                         } else {
17023                             switch (PyTuple_GET_SIZE(__pyx_args)) {
17024                                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17025                                     CYTHON_FALLTHROUGH;
17026                                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17027                                     break;
17028                                 default: goto __pyx_L5_argtuple_error;
17029                             }
17030                         }
17031                     }
17032                     __pyx_v_obj = values[0];
17033                     __pyx_v_i = values[1];
17034                 }
17035                 goto __pyx_L4_argument_unpacking_done;
17036                 __pyx_L5_argtuple_error:;
17037                 __Pyx_RaiseArgtupleInvalid("asin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17038                 __PYX_ERR(0, 1747, __pyx_L3_error)
17039                 __pyx_L3_error:;
17040                 __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17041                 __Pyx_RefNannyFinishContext();
17042                 return NULL;
17043                 __pyx_L4_argument_unpacking_done:;
17044                 __pyx_r = __pyx_pf_8PyClical_66asin(__pyx_self, __pyx_v_obj, __pyx_v_i);
17045
17046                 /* function exit code */
17047                 __Pyx_RefNannyFinishContext();
17048                 return __pyx_r;
17049             }
17050             static PyObject *__pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
17051                 PyObject *__pyx_r = NULL;
17052                 __Pyx_RefNannyDeclarations
17053                 PyObject *__pyx_t_1 = NULL;
17054                 struct __pyx_opt_args_8PyClical_asin __pyx_t_2;
17055                 int __pyx_lineno = 0;
17056                 const char *__pyx_filename = NULL;
17057                 int __pyx_clineno = 0;
17058                 __Pyx_RefNannySetupContext("asin", 0);
17059                 __Pyx_XDECREF(__pyx_r);
17060                 __pyx_t_2.__pyx_n = 1;
17061                 __pyx_t_2.i = __pyx_v_i;
17062                 __pyx_t_1 = __pyx_f_8PyClical_asin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1747, __pyx_L1_error)
17063                 __Pyx_GOTREF(__pyx_t_1);
17064                 __pyx_r = __pyx_t_1;

```

```

17065         __pyx_t_1 = 0;
17066         goto __pyx_L0;
17067
17068         /* function exit code */
17069         __pyx_L1_error:;
17070         __Pyx_XDECREF(__pyx_t_1);
17071         __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17072         __pyx_r = NULL;
17073         __pyx_L0:;
17074         __Pyx_XGIVEREF(__pyx_r);
17075         __Pyx_RefNannyFinishContext();
17076         return __pyx_r;
17077     }
17078
17079     /* "PyClical.pyx":1768
17080     *
17081     * return clifford().wrap( glucat.asin(toClifford(obj)) )
17082     *
17083     * cpdef inline sinh(obj):
17084     *     """
17085     *     Hyperbolic sine of multivector.
17086     */
17087
17088     static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject
17089 *__pyx_v_obj); /*proto*/
17090     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sinh(PyObject *__pyx_v_obj,
17091 CYTHON_UNUSED int __pyx_skip_dispatch) {
17092     PyObject *__pyx_r = NULL;
17093     __Pyx_RefNannyDeclarations
17094     PyObject *__pyx_t_1 = NULL;
17095     PyObject *__pyx_t_2 = NULL;
17096     PyObject *__pyx_t_3 = NULL;
17097     PyObject *__pyx_t_4 = NULL;
17098     PyObject *__pyx_t_5 = NULL;
17099     PyObject *__pyx_t_6 = NULL;
17100     PyObject *__pyx_t_7 = NULL;
17101     PyObject *__pyx_t_8 = NULL;
17102     int __pyx_lineno = 0;
17103     const char *__pyx_filename = NULL;
17104     int __pyx_clineno = 0;
17105     __Pyx_RefNannySetupContext("sinh", 0);
17106
17107     /* "PyClical.pyx":1777
17108     *
17109     * 0.5{1,2}
17110     *
17111     * try:
17112     *     # ««««««««
17113     *     return math.sinh(obj)
17114     * except:
17115     *
17116     */
17117     {
17118         __Pyx_PyThreadState_declare
17119         __Pyx_PyThreadState_assign
17120         __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
17121         __Pyx_XGOTREF(__pyx_t_1);
17122         __Pyx_XGOTREF(__pyx_t_2);
17123         __Pyx_XGOTREF(__pyx_t_3);
17124         /*try:*/ {
17125
17126             /* "PyClical.pyx":1778
17127             *
17128             * try:
17129             *     return math.sinh(obj)
17130             *     # ««««««««
17131             * except:
17132             *     return clifford().wrap( glucat.sinh(toClifford(obj)) )
17133             */
17134
17135             __Pyx_XDECREF(__pyx_r);
17136             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
17137             __PYX_ERR(0, 1778, __pyx_L3_error)
17138             __Pyx_GOTREF(__pyx_t_5);
17139             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_sinh); if
17140             (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1778, __pyx_L3_error)
17141             __Pyx_GOTREF(__pyx_t_6);
17142             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17143             __pyx_t_5 = NULL;
17144             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
17145                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
17146                 if (likely(__pyx_t_5)) {
17147                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
17148                     __Pyx_INCREF(__pyx_t_5);
17149                     __Pyx_INCREF(function);
17150                     __Pyx_DECREF_SET(__pyx_t_6, function);
17151                 }
17152             }
17153             __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
17154             __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
17155             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17156             if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1778, __pyx_L3_error)
17157             __Pyx_GOTREF(__pyx_t_4);

```

```

17147         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17148         __pyx_r = __pyx_t_4;
17149         __pyx_t_4 = 0;
17150         goto __pyx_L7_try_return;
17151
17152         /* "PyClical.pyx":1777
17153  *      0.5{1,2}
17154  *      """
17155  *      try:                # ««««««««
17156  *          return math.sinh(obj)
17157  *      except:
17158  */
17159     }
17160     __Pyx_L3_error::
17161     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
17162     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17163     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
17164
17165     /* "PyClical.pyx":1779
17166  *      try:
17167  *          return math.sinh(obj)
17168  *      except:            # ««««««««
17169  *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17170  *
17171  */
17172     /*except:*/ {
17173         __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17174         if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
17175 1779, __pyx_L5_except_error)
17176         __Pyx_GOTREF(__pyx_t_4);
17177         __Pyx_GOTREF(__pyx_t_6);
17178         __Pyx_GOTREF(__pyx_t_5);
17179
17180         /* "PyClical.pyx":1780
17181  *          return math.sinh(obj)
17182  *      except:
17183  *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17184  *
17185  * cpdef inline asinh(obj,i = None):
17186  */
17187         __Pyx_XDECREF(__pyx_r);
17188         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1780,
__pyx_L5_except_error)
17189         __Pyx_GOTREF(__pyx_t_7);
17190         __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), sinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1780, __pyx_L5_except_error)
17191         __Pyx_GOTREF(__pyx_t_8);
17192         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
17193         __pyx_r = __pyx_t_8;
17194         __pyx_t_8 = 0;
17195         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
17196         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17197         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17198         goto __pyx_L6_except_return;
17199     }
17200     __pyx_L5_except_error::
17201
17202     /* "PyClical.pyx":1777
17203  *      0.5{1,2}
17204  *      """
17205  *      try:                # ««««««««
17206  *          return math.sinh(obj)
17207  *      except:
17208  */
17209     __Pyx_XGIVEREF(__pyx_t_1);
17210     __Pyx_XGIVEREF(__pyx_t_2);
17211     __Pyx_XGIVEREF(__pyx_t_3);
17212     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17213     goto __pyx_L1_error;
17214     __pyx_L7_try_return:
17215     __Pyx_XGIVEREF(__pyx_t_1);
17216     __Pyx_XGIVEREF(__pyx_t_2);
17217     __Pyx_XGIVEREF(__pyx_t_3);
17218     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17219     goto __pyx_L0;
17220     __pyx_L6_except_return:
17221     __Pyx_XGIVEREF(__pyx_t_1);
17222     __Pyx_XGIVEREF(__pyx_t_2);
17223     __Pyx_XGIVEREF(__pyx_t_3);
17224     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17225     goto __pyx_L0;
17226 }
17227
17228     /* "PyClical.pyx":1768

```

```

17228 *         return clifford().wrap( glucat.asin(toClifford(obj)) )
17229 *
17230 * cpdef inline sinh(obj):           # ««««««««
17231 *     """
17232 *     Hyperbolic sine of multivector.
17233 */
17234
17235     /* function exit code */
17236     __pyx_L1_error:;
17237     __Pyx_XDECREF(__pyx_t_4);
17238     __Pyx_XDECREF(__pyx_t_5);
17239     __Pyx_XDECREF(__pyx_t_6);
17240     __Pyx_XDECREF(__pyx_t_7);
17241     __Pyx_XDECREF(__pyx_t_8);
17242     __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17243     __pyx_r = 0;
17244     __pyx_L0:;
17245     __Pyx_XGIVEREF(__pyx_r);
17246     __Pyx_RefNannyFinishContext();
17247     return __pyx_r;
17248 }
17249
17250     /* Python wrapper */
17251     static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject
17252 *__pyx_v_obj); /*proto*/
17253     static char __pyx_doc_8PyClical_68sinh[] = "\n    Hyperbolic sine of multivector.\n\n
    >> x=clifford(\"{1,2}\") * pi/2; print(sinh(x))\n    {1,2}\n    >> x=clifford(\"{1,2}\") * pi/6;
    print(sinh(x))\n    0.5{1,2}\n    ";
17254     static PyObject *__pyx_pf_8PyClical_69sinh(PyObject *__pyx_self, PyObject
17255 *__pyx_v_obj) {
17256     PyObject *__pyx_r = 0;
17257     __Pyx_RefNannyDeclarations
17258     __Pyx_RefNannySetupContext("sinh (wrapper)", 0);
17259     __pyx_r = __pyx_pf_8PyClical_68sinh(__pyx_self, ((PyObject *)__pyx_v_obj));
17260
17261     /* function exit code */
17262     __Pyx_RefNannyFinishContext();
17263     return __pyx_r;
17264 }
17265
17266     static PyObject *__pyx_pf_8PyClical_68sinh(CYTHON_UNUSED PyObject *__pyx_self,
17267 PyObject *__pyx_v_obj) {
17268     PyObject *__pyx_r = NULL;
17269     __Pyx_RefNannyDeclarations
17270     PyObject *__pyx_t_1 = NULL;
17271     int __pyx_lineno = 0;
17272     const char *__pyx_filename = NULL;
17273     int __pyx_clineno = 0;
17274     __Pyx_RefNannySetupContext("sinh", 0);
17275     __Pyx_XDECREF(__pyx_r);
17276     __pyx_t_1 = __pyx_f_8PyClical_sinh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
17277     __PYX_ERR(0, 1768, __pyx_L1_error)
17278     __Pyx_GOTREF(__pyx_t_1);
17279     __pyx_r = __pyx_t_1;
17280     __pyx_t_1 = 0;
17281     goto __pyx_L0;
17282
17283     /* function exit code */
17284     __pyx_L1_error:;
17285     __Pyx_XDECREF(__pyx_t_1);
17286     __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17287     __pyx_r = NULL;
17288     __pyx_L0:;
17289     __Pyx_XGIVEREF(__pyx_r);
17290     __Pyx_RefNannyFinishContext();
17291     return __pyx_r;
17292 }
17293
17294     /* "PyClical.pyx":1782
17295     return clifford().wrap( glucat.sinh(toClifford(obj)) )
17296     */
17297
17298 * cpdef inline asinh(obj,i = None):           # ««««««««
17299 *     """
17300 *     Inverse hyperbolic sine of multivector with optional complexifier.
17301 */
17302
17303     static PyObject *__pyx_pw_8PyClical_71asinh(PyObject *__pyx_self, PyObject
17304 *__pyx_args, PyObject *__pyx_kws); /*proto*/
17305     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asinh(PyObject *__pyx_v_obj,
17306 CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_asinh *__pyx_optional_args) {
17307     PyObject *__pyx_v_i = ((PyObject *)__Py_None);
17308     PyObject *__pyx_r = NULL;
17309     __Pyx_RefNannyDeclarations
17310     int __pyx_t_1;
17311     int __pyx_t_2;
17312     PyObject *__pyx_t_3 = NULL;
17313     Clifford __pyx_t_4;

```

```

17307         PyObject *__pyx_t_5 = NULL;
17308         PyObject *__pyx_t_6 = NULL;
17309         PyObject *__pyx_t_7 = NULL;
17310         PyObject *__pyx_t_8 = NULL;
17311         PyObject *__pyx_t_9 = NULL;
17312         PyObject *__pyx_t_10 = NULL;
17313         PyObject *__pyx_t_11 = NULL;
17314         int __pyx_lineno = 0;
17315         const char *__pyx_filename = NULL;
17316         int __pyx_clineno = 0;
17317         __Pyx_RefNannySetupContext("asinh", 0);
17318         if (__pyx_optional_args) {
17319             if (__pyx_optional_args->__pyx_n > 0) {
17320                 __pyx_v_i = __pyx_optional_args->i;
17321             }
17322         }
17323
17324         /* "PyClicl.pyx":1793
17325  *      {1,2}
17326  *      """
17327  *      if not (i is None):                # ««««««««
17328  *          return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17329  *      else:
17330  */
17331         __pyx_t_1 = (__pyx_v_i != Py_None);
17332         __pyx_t_2 = (__pyx_t_1 != 0);
17333         if (__pyx_t_2) {
17334
17335             /* "PyClicl.pyx":1794
17336  *      """
17337  *      if not (i is None):
17338  *          return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )      #
17339  *      else:
17340  *          try:
17341  */
17342         __Pyx_XDECREF(__pyx_r);
17343         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
17344 *)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1794, __pyx_L1_error)
17345         __Pyx_GOTREF(__pyx_t_3);
17346         try {
17347             __pyx_t_4 = asinh(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
17348 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
17349         } catch (...) {
17350             __Pyx_CppExn2PyErr();
17351             __PYX_ERR(0, 1794, __pyx_L1_error)
17352         }
17353         __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
17354 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1794, __pyx_L1_error)
17355         __Pyx_GOTREF(__pyx_t_5);
17356         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17357         __pyx_r = __pyx_t_5;
17358         __pyx_t_5 = 0;
17359         goto __pyx_L0;
17360
17361         /* "PyClicl.pyx":1793
17362  *      {1,2}
17363  *      """
17364  *      if not (i is None):                # ««««««««
17365  *          return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17366  *      else:
17367  */
17368         }
17369
17370         /* "PyClicl.pyx":1796
17371  *      return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17372  *      else:
17373  *          try:                # ««««««««
17374  *              return math.asinh(obj)
17375  *          except:
17376  */
17377         /*else*/ {
17378             {
17379                 __Pyx_PyThreadState_declare
17380                 __Pyx_PyThreadState_assign
17381                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17382                 __Pyx_XGOTREF(__pyx_t_6);
17383                 __Pyx_XGOTREF(__pyx_t_7);
17384                 __Pyx_XGOTREF(__pyx_t_8);
17385             }
17386             /*try:*/ {
17387
17388                 /* "PyClicl.pyx":1797
17389  *      else:
17390  *          try:
17391  *              return math.asinh(obj)                # ««««««««
17392  *          except:
17393  *              return clifford().wrap( glucat.asinh(toClifford(obj)) )

```

```

17390 */
17391
17392     __Pyx_XDECREF(__pyx_r);
17393     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
17394 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1797, __pyx_L4_error)
17395     __Pyx_GOTREF(__pyx_t_3);
17396     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asinh); if
17397 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1797, __pyx_L4_error)
17398     __Pyx_GOTREF(__pyx_t_9);
17399     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17400     __pyx_t_3 = NULL;
17401     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17402         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17403         if (likely(__pyx_t_3)) {
17404             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17405             __Pyx_INCREF(__pyx_t_3);
17406             __Pyx_INCREF(function);
17407             __Pyx_DECREF_SET(__pyx_t_9, function);
17408         }
17409     }
17410     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
17411 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17412     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17413     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1797, __pyx_L4_error)
17414     __Pyx_GOTREF(__pyx_t_5);
17415     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17416     __pyx_r = __pyx_t_5;
17417     __pyx_t_5 = 0;
17418     goto __pyx_L8_try_return;
17419
17420     /* "PyClical.pyx":1796
17421 *         return clifford().wrap( glucat.asinh(toClifford(obj)), toClifford(i)) )
17422 *     else:
17423 *         try:
17424 *             # ««««««««
17425 *             return math.asinh(obj)
17426 *         except:
17427 */
17428     }
17429     __pyx_L4_error:;
17430     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17431     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17432     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17433
17434     /* "PyClical.pyx":1798
17435 *         try:
17436 *             return math.asinh(obj)
17437 *         except:
17438 *             # ««««««««
17439 *             return clifford().wrap( glucat.asinh(toClifford(obj)) )
17440 */
17441     /*except:*/ {
17442         __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno,
17443 __pyx_filename);
17444         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
17445 1798, __pyx_L6_except_error)
17446         __Pyx_GOTREF(__pyx_t_5);
17447         __Pyx_GOTREF(__pyx_t_9);
17448         __Pyx_GOTREF(__pyx_t_3);
17449
17450         /* "PyClical.pyx":1799
17451 *         return math.asinh(obj)
17452 *     except:
17453 *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17454 *             # ««««««««
17455 * cpdef inline tan(obj,i = None):
17456 */
17457         __Pyx_XDECREF(__pyx_r);
17458         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
17459 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1799,
17460 __pyx_L6_except_error)
17461         __Pyx_GOTREF(__pyx_t_10);
17462         __pyx_t_11 = __Pyx_f_8PyClical_8clifford_wrap(((struct
17463 __pyx_obj_8PyClical_clifford *)__pyx_t_10), asinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
17464 (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1799, __pyx_L6_except_error)
17465         __Pyx_GOTREF(__pyx_t_11);
17466         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17467         __pyx_r = __pyx_t_11;
17468         __pyx_t_11 = 0;
17469         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17470         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17471         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17472         goto __pyx_L7_except_return;
17473     }
17474     __pyx_L6_except_error:;
17475
17476     /* "PyClical.pyx":1796
17477 *         return clifford().wrap( glucat.asinh(toClifford(obj)), toClifford(i)) )
17478 *     else:

```

```

17468 *         try:             # ««««««««
17469 *             return math.asinh(obj)
17470 *         except:
17471 */
17472         __Pyx_XGIVEREF(__pyx_t_6);
17473         __Pyx_XGIVEREF(__pyx_t_7);
17474         __Pyx_XGIVEREF(__pyx_t_8);
17475         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17476         goto __pyx_L1_error;
17477         __pyx_L8_try_return:;
17478         __Pyx_XGIVEREF(__pyx_t_6);
17479         __Pyx_XGIVEREF(__pyx_t_7);
17480         __Pyx_XGIVEREF(__pyx_t_8);
17481         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17482         goto __pyx_L0;
17483         __pyx_L7_except_return:;
17484         __Pyx_XGIVEREF(__pyx_t_6);
17485         __Pyx_XGIVEREF(__pyx_t_7);
17486         __Pyx_XGIVEREF(__pyx_t_8);
17487         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17488         goto __pyx_L0;
17489     }
17490 }
17491
17492 /* "PyClicl.pyx":1782
17493 *     return clifford().wrap( glucat.sinh(toClifford(obj)) )
17494 *
17495 * cpdef inline asinh(obj,i = None):             # ««««««««
17496 *     """
17497 *     Inverse hyperbolic sine of multivector with optional complexifier.
17498 */
17499
17500 /* function exit code */
17501 __pyx_L1_error:;
17502 __Pyx_XDECREF(__pyx_t_3);
17503 __Pyx_XDECREF(__pyx_t_5);
17504 __Pyx_XDECREF(__pyx_t_9);
17505 __Pyx_XDECREF(__pyx_t_10);
17506 __Pyx_XDECREF(__pyx_t_11);
17507 __Pyx_AddTraceback("PyClicl.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17508 __pyx_r = 0;
17509 __pyx_L0:;
17510 __Pyx_XGIVEREF(__pyx_r);
17511 __Pyx_RefNannyFinishContext();
17512 return __pyx_r;
17513 }
17514
17515 /* Python wrapper */
17516 static PyObject * __pyx_pw_8PyClicl_7lasinh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds); /*proto*/
17517 static char __pyx_doc_8PyClicl_70asinh[] = "\n    Inverse hyperbolic sine of
multivector with optional complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(asinh(x,\"{1,2,3}\") *
2/pi)\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print(asinh(x) * 2/pi)\n    {1,2}\n    >>
x=clifford(\"{1,2}\") / 2; print(asinh(x) * 6/pi)\n    {1,2}\n    ";
17518 static PyObject * __pyx_pw_8PyClicl_7lasinh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds) {
17519     PyObject * __pyx_v_obj = 0;
17520     PyObject * __pyx_v_i = 0;
17521     int __pyx_lineno = 0;
17522     const char * __pyx_filename = NULL;
17523     int __pyx_clineno = 0;
17524     PyObject * __pyx_r = 0;
17525     __Pyx_RefNannyDeclarations
17526     __Pyx_RefNannySetupContext("asinh (wrapper)", 0);
17527     {
17528         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17529         PyObject* values[2] = {0,0};
17530         values[1] = ((PyObject *)Py_None);
17531         if (unlikely(__pyx_kwds)) {
17532             Py_ssize_t kw_args;
17533             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17534             switch (pos_args) {
17535                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17536                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
17537                 case 0: break;
17538                 default: goto __pyx_L5_argtuple_error;
17539             }
17540             kw_args = PyDict_Size(__pyx_kwds);
17541             switch (pos_args) {
17542                 case 0:
17543                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
17544                     else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
17545                 case 1:

```



```

17549         if (kw_args > 0) {
17550             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17551             if (value) { values[1] = value; kw_args--; }
17552         }
17553     }
17554     if (unlikely(kw_args > 0)) {
17555         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "asinh") < 0)) __PYX_ERR(0, 1782, __pyx_L3_error)
17556     }
17557     } else {
17558         switch (PyTuple_GET_SIZE(__pyx_args)) {
17559             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17560             CYTHON_FALLTHROUGH;
17561             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17562             break;
17563             default: goto __pyx_L5_argtuple_error;
17564         }
17565     }
17566     __pyx_v_obj = values[0];
17567     __pyx_v_i = values[1];
17568 }
17569 goto __pyx_L4_argument_unpacking_done;
17570 __pyx_L5_argtuple_error:;
17571 __Pyx_RaiseArgtupleInvalid("asinh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1782, __pyx_L3_error)
17572 __pyx_L3_error:;
17573 __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17574 __Pyx_RefNannyFinishContext();
17575 return NULL;
17576 __pyx_L4_argument_unpacking_done:;
17577 __pyx_r = __pyx_pf_8PyClical_70asinh(__pyx_self, __pyx_v_obj, __pyx_v_i);
17578
17579 /* function exit code */
17580 __Pyx_RefNannyFinishContext();
17581 return __pyx_r;
17582 }
17583
17584 static PyObject *__pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
17585     PyObject *__pyx_r = NULL;
17586     __Pyx_RefNannyDeclarations
17587     PyObject *__pyx_t_1 = NULL;
17588     struct __pyx_opt_args_8PyClical_asinh __pyx_t_2;
17589     int __pyx_lineno = 0;
17590     const char *__pyx_filename = NULL;
17591     int __pyx_clineno = 0;
17592     __Pyx_RefNannySetupContext("asinh", 0);
17593     __Pyx_XDECREF(__pyx_r);
17594     __pyx_t_2.__pyx_n = 1;
17595     __pyx_t_2.i = __pyx_v_i;
17596     __pyx_t_1 = __pyx_f_8PyClical_asinh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1782, __pyx_L1_error)
17597     __Pyx_GOTREF(__pyx_t_1);
17598     __pyx_r = __pyx_t_1;
17599     __pyx_t_1 = 0;
17600     goto __pyx_L0;
17601
17602     /* function exit code */
17603     __pyx_L1_error:;
17604     __Pyx_XDECREF(__pyx_t_1);
17605     __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17606     __pyx_r = NULL;
17607     __pyx_L0:;
17608     __Pyx_XGIVEREF(__pyx_r);
17609     __Pyx_RefNannyFinishContext();
17610     return __pyx_r;
17611 }
17612
17613 /* "PyClical.pyx":1801
17614 *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17615 *
17616 * cpdef inline tan(obj,i = None): # ««««««««
17617 *     """
17618 *     Tangent of multivector with optional complexifier.
17619 */
17620
17621 static PyObject *__pyx_pw_8PyClical_73tan(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
17622 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tan(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_tan *__pyx_optional_args) {
17623     PyObject *__pyx_v_i = ((PyObject *)Py_None);
17624     PyObject *__pyx_r = NULL;
17625     __Pyx_RefNannyDeclarations
17626     int __pyx_t_1;
17627     int __pyx_t_2;
17628     PyObject *__pyx_t_3 = NULL;
17629     Clifford __pyx_t_4;

```

```

17630         PyObject *__pyx_t_5 = NULL;
17631         PyObject *__pyx_t_6 = NULL;
17632         PyObject *__pyx_t_7 = NULL;
17633         PyObject *__pyx_t_8 = NULL;
17634         PyObject *__pyx_t_9 = NULL;
17635         PyObject *__pyx_t_10 = NULL;
17636         PyObject *__pyx_t_11 = NULL;
17637         int __pyx_lineno = 0;
17638         const char *__pyx_filename = NULL;
17639         int __pyx_clineno = 0;
17640         __Pyx_RefNannySetupContext("tan", 0);
17641         if (__pyx_optional_args) {
17642             if (__pyx_optional_args->__pyx_n > 0) {
17643                 __pyx_v_i = __pyx_optional_args->i;
17644             }
17645         }
17646
17647         /* "PyClicl.pyx":1810
17648  *      0.7616{1,2}
17649  *      """
17650  *      if not (i is None):                # <<<<<<<<<
17651  *          return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17652  *      else:
17653  */
17654         __pyx_t_1 = (__pyx_v_i != Py_None);
17655         __pyx_t_2 = (__pyx_t_1 != 0);
17656         if (__pyx_t_2) {
17657
17658             /* "PyClicl.pyx":1811
17659  *      """
17660  *      if not (i is None):
17661  *          return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )           # <<<<<<<<<
17662  *      else:
17663  *          try:
17664  */
17665             __Pyx_XDECREF(__pyx_r);
17666             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
17667 *)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1811, __pyx_L1_error)
17668             __Pyx_GOTREF(__pyx_t_3);
17669             try {
17670                 __pyx_t_4 = tan(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
17671 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
17672             } catch (...) {
17673                 __Pyx_CppExn2PyErr();
17674                 __PYX_ERR(0, 1811, __pyx_L1_error)
17675             }
17676             __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
17677 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1811, __pyx_L1_error)
17678             __Pyx_GOTREF(__pyx_t_5);
17679             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17680             __pyx_r = __pyx_t_5;
17681             __pyx_t_5 = 0;
17682             goto __pyx_L0;
17683
17684             /* "PyClicl.pyx":1810
17685  *      0.7616{1,2}
17686  *      """
17687  *      if not (i is None):                # <<<<<<<<<
17688  *          return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17689  *      else:
17690  */
17691             }
17692
17693             /* "PyClicl.pyx":1813
17694  *      return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17695  *      else:
17696  *          try:                # <<<<<<<<<
17697  *              return math.tan(obj)
17698  *          except:
17699  */
17700             /*else*/ {
17701                 {
17702                     __Pyx_PyThreadState_declare
17703                     __Pyx_PyThreadState_assign
17704                     __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17705                     __Pyx_XGOTREF(__pyx_t_6);
17706                     __Pyx_XGOTREF(__pyx_t_7);
17707                     __Pyx_XGOTREF(__pyx_t_8);
17708                 }
17709                 /*try:*/ {
17710
17711                     /* "PyClicl.pyx":1814
17712  *      else:
17713  *          try:
17714  *              return math.tan(obj)                # <<<<<<<<<
17715  *          except:
17716  *              return clifford().wrap( glucat.tan(toClifford(obj)) )
17717  */

```

```

17714         __Pyx_XDECREF(__pyx_r);
17715         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3))
17716             __PYX_ERR(0, 1814, __pyx_L4_error)
17717         __Pyx_GOTREF(__pyx_t_3);
17718         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_tan); if
(unlikely(!__pyx_t_9))
17719             __PYX_ERR(0, 1814, __pyx_L4_error)
17720         __Pyx_GOTREF(__pyx_t_9);
17721         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17722         __pyx_t_3 = NULL;
17723         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17724             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17725             if (likely(__pyx_t_3)) {
17726                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17727                 __Pyx_INCREF(__pyx_t_3);
17728                 __Pyx_INCREF(function);
17729                 __Pyx_DECREF_SET(__pyx_t_9, function);
17730             }
17731             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17732             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17733             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1814, __pyx_L4_error)
17734             __Pyx_GOTREF(__pyx_t_5);
17735             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17736             __pyx_r = __pyx_t_5;
17737             __pyx_t_5 = 0;
17738             goto __pyx_L8_try_return;
17739
17740             /* "PyClical.pyx":1813
17741             *         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17742             *     else:
17743             *         try:
17744             *             # ««««««««
17745             *             return math.tan(obj)
17746             *         except:
17747             */
17748             }
17749             __pyx_L4_error:;
17750             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17751             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17752             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17753
17754             /* "PyClical.pyx":1815
17755             *         try:
17756             *             return math.tan(obj)
17757             *         except:
17758             *             # ««««««««
17759             *             return clifford().wrap( glucat.tan(toClifford(obj)) )
17760             */
17761             /*except:*/ {
17762                 __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17763                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1815, __pyx_L6_except_error)
17764                 __Pyx_GOTREF(__pyx_t_5);
17765                 __Pyx_GOTREF(__pyx_t_9);
17766                 __Pyx_GOTREF(__pyx_t_3);
17767
17768                 /* "PyClical.pyx":1816
17769                 *         return math.tan(obj)
17770                 *     except:
17771                 *         return clifford().wrap( glucat.tan(toClifford(obj)) )
17772                 *             # ««««««««
17773                 * cpdef inline atan(obj,i = None):
17774                 */
17775                 __Pyx_XDECREF(__pyx_r);
17776                 __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1816,
__pyx_L6_except_error)
17777                 __Pyx_GOTREF(__pyx_t_10);
17778                 __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), tan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11))
17779                     __PYX_ERR(0, 1816, __pyx_L6_except_error)
17780                 __Pyx_GOTREF(__pyx_t_11);
17781                 __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17782                 __pyx_r = __pyx_t_11;
17783                 __pyx_t_11 = 0;
17784                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17785                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17786                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17787                 goto __pyx_L7_except_return;
17788             }
17789             __pyx_L6_except_error:;
17790
17791             /* "PyClical.pyx":1813
17792             *         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17793             *     else:
17794             *         try:
17795             *             # ««««««««

```

```

17792 *         return math.tan(obj)
17793 *     except:
17794 */
17795         __Pyx_XGIVEREF(__pyx_t_6);
17796         __Pyx_XGIVEREF(__pyx_t_7);
17797         __Pyx_XGIVEREF(__pyx_t_8);
17798         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17799         goto __pyx_L1_error;
17800         __pyx_L8_try_return:;
17801         __Pyx_XGIVEREF(__pyx_t_6);
17802         __Pyx_XGIVEREF(__pyx_t_7);
17803         __Pyx_XGIVEREF(__pyx_t_8);
17804         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17805         goto __pyx_L0;
17806         __pyx_L7_except_return:;
17807         __Pyx_XGIVEREF(__pyx_t_6);
17808         __Pyx_XGIVEREF(__pyx_t_7);
17809         __Pyx_XGIVEREF(__pyx_t_8);
17810         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17811         goto __pyx_L0;
17812     }
17813 }
17814
17815 /* "PyClical.pyx":1801
17816 *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
17817 *
17818 * cpdef inline tan(obj,i = None):          # ««««««««
17819 *     """
17820 *     Tangent of multivector with optional complexifier.
17821 */
17822
17823 /* function exit code */
17824 __pyx_L1_error:;
17825 __Pyx_XDECREF(__pyx_t_3);
17826 __Pyx_XDECREF(__pyx_t_5);
17827 __Pyx_XDECREF(__pyx_t_9);
17828 __Pyx_XDECREF(__pyx_t_10);
17829 __Pyx_XDECREF(__pyx_t_11);
17830 __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17831 __pyx_r = 0;
17832 __pyx_L0:;
17833 __Pyx_XGIVEREF(__pyx_r);
17834 __Pyx_RefNannyFinishContext();
17835 return __pyx_r;
17836 }
17837
17838 /* Python wrapper */
17839 static PyObject * __pyx_pw_8PyClical_73tan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
17840 static char __pyx_doc_8PyClical_72tan[] = "\n    Tangent of multivector with optional
complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(tan(x,\"{1,2,3}\"))\n    0.7616{1,2}\n    >>
x=clifford(\"{1,2}\"); print(tan(x))\n    0.7616{1,2}\n    ";
17841 static PyObject * __pyx_pw_8PyClical_73tan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
17842     PyObject * __pyx_v_obj = 0;
17843     PyObject * __pyx_v_i = 0;
17844     int __pyx_lineno = 0;
17845     const char * __pyx_filename = NULL;
17846     int __pyx_clineno = 0;
17847     PyObject * __pyx_r = 0;
17848     __Pyx_RefNannyDeclarations
17849     __Pyx_RefNannySetupContext("tan (wrapper)", 0);
17850     {
17851         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17852         PyObject* values[2] = {0,0};
17853         values[1] = ((PyObject *)Py_None);
17854         if (unlikely(__pyx_kwds)) {
17855             Py_ssize_t kw_args;
17856             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17857             switch (pos_args) {
17858                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17859                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
17860                 case 0: break;
17861                 default: goto __pyx_L5_argtuple_error;
17862             }
17863             kw_args = PyDict_Size(__pyx_kwds);
17864             switch (pos_args) {
17865                 case 0:
17866                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
17869                     else goto __pyx_L5_argtuple_error;
17870                 case 1:
17871                     if (kw_args > 0) {
17872                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17873

```

```

17874         if (value) { values[1] = value; kw_args--; }
17875     }
17876 }
17877 if (unlikely(kw_args > 0)) {
17878     if (unlikely(!__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "tan") < 0)) __PYX_ERR(0, 1801, __pyx_L3_error)
17879 }
17880 } else {
17881     switch (PyTuple_GET_SIZE(__pyx_args)) {
17882     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17883             CYTHON_FALLTHROUGH;
17884     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17885             break;
17886     default: goto __pyx_L5_argtuple_error;
17887     }
17888 }
17889 __pyx_v_obj = values[0];
17890 __pyx_v_i = values[1];
17891 }
17892 goto __pyx_L4_argument_unpacking_done;
17893 __pyx_L5_argtuple_error:
17894     __Pyx_RaiseArgtupleInvalid("tan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1801, __pyx_L3_error)
17895     __pyx_L3_error:
17896     __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17897     __Pyx_RefNannyFinishContext();
17898     return NULL;
17899     __pyx_L4_argument_unpacking_done:
17900     __pyx_r = __pyx_pf_8PyClical_72tan(__pyx_self, __pyx_v_obj, __pyx_v_i);
17901
17902     /* function exit code */
17903     __Pyx_RefNannyFinishContext();
17904     return __pyx_r;
17905 }
17906
17907 static PyObject * __pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject * __pyx_self, PyObject
* __pyx_v_obj, PyObject * __pyx_v_i) {
17908     PyObject * __pyx_r = NULL;
17909     __Pyx_RefNannyDeclarations
17910     PyObject * __pyx_t_1 = NULL;
17911     struct __pyx_opt_args_8PyClical_tan __pyx_t_2;
17912     int __pyx_lineno = 0;
17913     const char * __pyx_filename = NULL;
17914     int __pyx_clineno = 0;
17915     __Pyx_RefNannySetupContext("tan", 0);
17916     __Pyx_XDECREF(__pyx_r);
17917     __pyx_t_2.__pyx_n = 1;
17918     __pyx_t_2.i = __pyx_v_i;
17919     __pyx_t_1 = __pyx_f_8PyClical_tan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1801, __pyx_L1_error)
17920     __Pyx_GOTREF(__pyx_t_1);
17921     __pyx_r = __pyx_t_1;
17922     __pyx_t_1 = 0;
17923     goto __pyx_L0;
17924
17925     /* function exit code */
17926     __pyx_L1_error:
17927     __Pyx_XDECREF(__pyx_t_1);
17928     __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17929     __pyx_r = NULL;
17930     __pyx_L0:
17931     __Pyx_XGIVEREF(__pyx_r);
17932     __Pyx_RefNannyFinishContext();
17933     return __pyx_r;
17934 }
17935
17936     /* "PyClical.pyx":1818
17937     *
17938     *
17939     * cpdef inline atan(obj,i = None):                                # ««««««««
17940     *     """
17941     *     Inverse tangent of multivector with optional complexifier.
17942     */
17943
17944     static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
17945     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atan(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_atan * __pyx_optional_args) {
17946     PyObject * __pyx_v_i = ((PyObject *)Py_None);
17947     PyObject * __pyx_r = NULL;
17948     __Pyx_RefNannyDeclarations
17949     int __pyx_t_1;
17950     int __pyx_t_2;
17951     PyObject * __pyx_t_3 = NULL;
17952     Clifford __pyx_t_4;
17953     PyObject * __pyx_t_5 = NULL;
17954     PyObject * __pyx_t_6 = NULL;

```

```

17955         PyObject *__pyx_t_7 = NULL;
17956         PyObject *__pyx_t_8 = NULL;
17957         PyObject *__pyx_t_9 = NULL;
17958         PyObject *__pyx_t_10 = NULL;
17959         PyObject *__pyx_t_11 = NULL;
17960         int __pyx_lineno = 0;
17961         const char *__pyx_filename = NULL;
17962         int __pyx_clineno = 0;
17963         __Pyx_RefNannySetupContext("atan", 0);
17964         if (__pyx_optional_args) {
17965             if (__pyx_optional_args->__pyx_n > 0) {
17966                 __pyx_v_i = __pyx_optional_args->i;
17967             }
17968         }
17969
17970         /* "PyClicl.pyx":1827
17971         {
17972         """
17973         if not (i is None):
17974             # ««««««««
17975             return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
17976         else:
17977             __pyx_t_1 = (__pyx_v_i != Py_None);
17978             __pyx_t_2 = (__pyx_t_1 != 0);
17979             if (__pyx_t_2) {
17980
17981                 /* "PyClicl.pyx":1828
17982                 """
17983                 if not (i is None):
17984                     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
17985                 else:
17986                     try:
17987
17988                         __Pyx_XDECREF(__pyx_r);
17989                         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
17990 *)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1828, __pyx_L1_error)
17991                         __Pyx_GOTREF(__pyx_t_3);
17992                         try {
17993                             __pyx_t_4 = atan(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
17994 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
17995                         } catch (...) {
17996                             __Pyx_CppExn2PyErr();
17997                             __PYX_ERR(0, 1828, __pyx_L1_error)
17998                         }
17999                         __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
2000 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1828, __pyx_L1_error)
2001                         __Pyx_GOTREF(__pyx_t_5);
2002                         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
2003                         __pyx_r = __pyx_t_5;
2004                         __pyx_t_5 = 0;
2005                         goto __pyx_L0;
2006
2007                 /* "PyClicl.pyx":1827
2008                 {
2009                 """
2010                 if not (i is None):
2011                     # ««««««««
2012                     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
2013                 else:
2014                     try:
2015                         # ««««««««
2016                         return math.atan(obj)
2017                     except:
2018
2019                 */
2020
2021                 /*else*/ {
2022                     {
2023                         __Pyx_PyThreadState_declare
2024                         __Pyx_PyThreadState_assign
2025                         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
2026                         __Pyx_XGOTREF(__pyx_t_6);
2027                         __Pyx_XGOTREF(__pyx_t_7);
2028                         __Pyx_XGOTREF(__pyx_t_8);
2029                         /*try:*/ {
2030
2031                             /* "PyClicl.pyx":1831
2032                             {
2033                             try:
2034                                 return math.atan(obj)
2035                                 # ««««««««
2036                             except:
2037                                 return clifford().wrap( glucat.atan(toClifford(obj)) )
2038                             */
2039
2040                             __Pyx_XDECREF(__pyx_r);
2041                             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if

```

```

(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1831, __pyx_L4_error)
18039 __Pyx_GOTREF(__pyx_t_3);
18040 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_atan); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1831, __pyx_L4_error)
18041 __Pyx_GOTREF(__pyx_t_9);
18042 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18043 __pyx_t_3 = NULL;
18044 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
18045     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
18046     if (likely(__pyx_t_3)) {
18047         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
18048         __Pyx_INCREF(__pyx_t_3);
18049         __Pyx_INCREF(function);
18050         __Pyx_DECREF_SET(__pyx_t_9, function);
18051     }
18052 }
18053 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
18054 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18055 if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1831, __pyx_L4_error)
18056 __Pyx_GOTREF(__pyx_t_5);
18057 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18058 __pyx_r = __pyx_t_5;
18059 __pyx_t_5 = 0;
18060 goto __pyx_L8_try_return;
18061
18062 /* "PyClical.pyx":1830
18063 *     return clifford().wrap( glucat.atan(toClifford(obj)), toClifford(i)) )
18064 * else:
18065 *     try:
18066 *         # ««««««««
18067 *         return math.atan(obj)
18068 *     except:
18069 */
18070     __pyx_L4_error;
18071 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18072 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18073 __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
18074
18075 /* "PyClical.pyx":1832
18076 *     try:
18077 *         return math.atan(obj)
18078 *     except:
18079 *         # ««««««««
18080 *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18081 */
18082 /*except:*/ {
18083     __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18084     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1832, __pyx_L6_except_error)
18085     __Pyx_GOTREF(__pyx_t_5);
18086     __Pyx_GOTREF(__pyx_t_9);
18087     __Pyx_GOTREF(__pyx_t_3);
18088
18089     /* "PyClical.pyx":1833
18090 *     return math.atan(obj)
18091 *     except:
18092 *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18093 *         # ««««««««
18094 * cpdef inline tanh(obj):
18095 */
18096     __Pyx_XDECREF(__pyx_r);
18097     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1833,
__pyx_L6_except_error)
18098     __Pyx_GOTREF(__pyx_t_10);
18099     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), atan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1833, __pyx_L6_except_error)
18100     __Pyx_GOTREF(__pyx_t_11);
18101     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18102     __pyx_r = __pyx_t_11;
18103     __pyx_t_11 = 0;
18104     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18105     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18106     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18107     goto __pyx_L7_except_return;
18108 }
18109 __pyx_L6_except_error;
18110
18111 /* "PyClical.pyx":1830
18112 *     return clifford().wrap( glucat.atan(toClifford(obj)), toClifford(i)) )
18113 * else:
18114 *     try:
18115 *         # ««««««««
18116 *         return math.atan(obj)
18117 *     except:

```

```

18117 */
18118     __Pyx_XGIVEREF(__pyx_t_6);
18119     __Pyx_XGIVEREF(__pyx_t_7);
18120     __Pyx_XGIVEREF(__pyx_t_8);
18121     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18122     goto __pyx_L1_error;
18123     __pyx_L8_try_return:;
18124     __Pyx_XGIVEREF(__pyx_t_6);
18125     __Pyx_XGIVEREF(__pyx_t_7);
18126     __Pyx_XGIVEREF(__pyx_t_8);
18127     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18128     goto __pyx_L0;
18129     __pyx_L7_except_return:;
18130     __Pyx_XGIVEREF(__pyx_t_6);
18131     __Pyx_XGIVEREF(__pyx_t_7);
18132     __Pyx_XGIVEREF(__pyx_t_8);
18133     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18134     goto __pyx_L0;
18135 }
18136 }
18137
18138 /* "PyClicl.pyx":1818
18139 *         return clifford().wrap( glucat.tan(toClifford(obj)) )
18140 *
18141 * cpdef inline atan(obj,i = None):          # ««««««««
18142 *     """
18143 *         Inverse tangent of multivector with optional complexifier.
18144 */
18145
18146     /* function exit code */
18147     __pyx_L1_error:;
18148     __Pyx_XDECREF(__pyx_t_3);
18149     __Pyx_XDECREF(__pyx_t_5);
18150     __Pyx_XDECREF(__pyx_t_9);
18151     __Pyx_XDECREF(__pyx_t_10);
18152     __Pyx_XDECREF(__pyx_t_11);
18153     __Pyx_AddTraceback("PyClicl.ATAN", __pyx_clineno, __pyx_lineno, __pyx_filename);
18154     __pyx_r = 0;
18155     __pyx_L0:;
18156     __Pyx_XGIVEREF(__pyx_r);
18157     __Pyx_RefNannyFinishContext();
18158     return __pyx_r;
18159 }
18160
18161     /* Python wrapper */
18162     static PyObject* __pyx_pw_8PyClicl_75atan(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kws); /*proto*/
18163     static char __pyx_doc_8PyClicl_74atan[] = "\n    Inverse tangent of multivector with\noptional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford(\"{1}\"); print(tan(atan(x,s),s))\n\n    {1}\n    >> x=clifford(\"{1}\"); print(tan(atan(x)))\n    {1}\n    ";
18164     static PyObject* __pyx_pw_8PyClicl_75atan(PyObject* __pyx_self, PyObject* __pyx_args,
PyObject* __pyx_kws) {
18165         PyObject* __pyx_v_obj = 0;
18166         PyObject* __pyx_v_i = 0;
18167         int __pyx_lineno = 0;
18168         const char* __pyx_filename = NULL;
18169         int __pyx_clineno = 0;
18170         PyObject* __pyx_r = 0;
18171         __Pyx_RefNannyDeclarations
18172         __Pyx_RefNannySetupContext("atan (wrapper)", 0);
18173         {
18174             static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
18175             PyObject* values[2] = {0,0};
18176             values[1] = ((PyObject*)Py_None);
18177             if (unlikely(__pyx_kws)) {
18178                 Py_ssize_t kw_args;
18179                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18180                 switch (pos_args) {
18181                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18182                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18183                     case 0: break;
18184                     default: goto __pyx_L5_argtuple_error;
18185                 }
18186                 kw_args = PyDict_Size(__pyx_kws);
18187                 switch (pos_args) {
18188                     case 0:
18189                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kws, __pyx_n_s_obj)) !=
18190 0)) kw_args--;
18191
18192                     else goto __pyx_L5_argtuple_error;
18193                 }
18194                 case 1:
18195                     if (kw_args > 0) {
18196                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kws, __pyx_n_s_i);
18197                         if (value) { values[1] = value; kw_args--; }
18198                     }

```



```

18199         }
18200         if (unlikely(kw_args > 0)) {
18201             if (unlikely(!Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "atan") < 0)) __PYX_ERR(0, 1818, __pyx_L3_error)
18202         }
18203     } else {
18204         switch (PyTuple_GET_SIZE(__pyx_args)) {
18205             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18206                 CYTHON_FALLTHROUGH;
18207             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18208                 break;
18209             default: goto __pyx_L5_argtuple_error;
18210         }
18211     }
18212     __pyx_v_obj = values[0];
18213     __pyx_v_i = values[1];
18214 }
18215 goto __pyx_L4_argument_unpacking_done;
18216 __pyx_L5_argtuple_error:;
18217 __Pyx_RaiseArgtupleInvalid("atan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1818, __pyx_L3_error)
18218 __pyx_L3_error:;
18219 __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18220 __Pyx_RefNannyFinishContext();
18221 return NULL;
18222 __pyx_L4_argument_unpacking_done:;
18223 __pyx_r = __pyx_pf_8PyClical_74atan(__pyx_self, __pyx_v_obj, __pyx_v_i);
18224
18225 /* function exit code */
18226 __Pyx_RefNannyFinishContext();
18227 return __pyx_r;
18228 }
18229
18230 static PyObject * __pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
18231     PyObject *__pyx_r = NULL;
18232     __Pyx_RefNannyDeclarations
18233     PyObject *__pyx_t_1 = NULL;
18234     struct __pyx_opt_args_8PyClical_atan __pyx_t_2;
18235     int __pyx_lineno = 0;
18236     const char *__pyx_filename = NULL;
18237     int __pyx_clineno = 0;
18238     __Pyx_RefNannySetupContext("atan", 0);
18239     __Pyx_XDECREF(__pyx_r);
18240     __pyx_t_2.__pyx_n = 1;
18241     __pyx_t_2.i = __pyx_v_i;
18242     __pyx_t_1 = __pyx_f_8PyClical_atan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1818, __pyx_L1_error)
18243     __Pyx_GOTREF(__pyx_t_1);
18244     __pyx_r = __pyx_t_1;
18245     __pyx_t_1 = 0;
18246     goto __pyx_L0;
18247
18248 /* function exit code */
18249 __pyx_L1_error:;
18250 __Pyx_XDECREF(__pyx_t_1);
18251 __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18252 __pyx_r = NULL;
18253 __pyx_L0:;
18254 __Pyx_XGIVEREF(__pyx_r);
18255 __Pyx_RefNannyFinishContext();
18256 return __pyx_r;
18257 }
18258
18259 /* "PyClical.pyx":1835
18260 *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18261 *
18262 * cpdef inline tanh(obj): # ««««««««
18263 *     """
18264 *     Hyperbolic tangent of multivector.
18265 */
18266
18267 static PyObject * __pyx_pw_8PyClical_77tanh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18268 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_tanh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
18269     PyObject *__pyx_r = NULL;
18270     __Pyx_RefNannyDeclarations
18271     PyObject *__pyx_t_1 = NULL;
18272     PyObject *__pyx_t_2 = NULL;
18273     PyObject *__pyx_t_3 = NULL;
18274     PyObject *__pyx_t_4 = NULL;
18275     PyObject *__pyx_t_5 = NULL;
18276     PyObject *__pyx_t_6 = NULL;
18277     PyObject *__pyx_t_7 = NULL;
18278     PyObject *__pyx_t_8 = NULL;
18279     int __pyx_lineno = 0;

```

```

18280         const char *__pyx_filename = NULL;
18281         int __pyx_clineno = 0;
18282         __Pyx_RefNannySetupContext("tanh", 0);
18283
18284         /* "PyClicl.pyx":1842
18285  *         {1,2}
18286  *         """
18287  *         try:             # ««««««««
18288  *             return math.tanh(obj)
18289  *         except:
18290  */
18291
18292         {
18293             __Pyx_PyThreadState_declare
18294             __Pyx_PyThreadState_assign
18295             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
18296             __Pyx_XGOTREF(__pyx_t_1);
18297             __Pyx_XGOTREF(__pyx_t_2);
18298             __Pyx_XGOTREF(__pyx_t_3);
18299             /*try:*/ {
18300
18301                 /* "PyClicl.pyx":1843
18302  *         try:
18303  *             return math.tanh(obj)             # ««««««««
18304  *         except:
18305  *             return clifford().wrap( glucat.tanh(toClifford(obj)) )
18306  */
18307
18308             __Pyx_XDECREF(__pyx_r);
18309             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
18310             __PYX_ERR(0, 1843, __pyx_L3_error)
18311             __Pyx_GOTREF(__pyx_t_5);
18312             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_tanh); if
18313             (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1843, __pyx_L3_error)
18314             __Pyx_GOTREF(__pyx_t_6);
18315             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18316             __pyx_t_5 = NULL;
18317             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
18318                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
18319                 if (likely(__pyx_t_5)) {
18320                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
18321                     __Pyx_INCREF(__pyx_t_5);
18322                     __Pyx_INCREF(function);
18323                     __Pyx_DECREF_SET(__pyx_t_6, function);
18324                 }
18325             }
18326             __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
18327             __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
18328             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18329             if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1843, __pyx_L3_error)
18330             __Pyx_GOTREF(__pyx_t_4);
18331             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18332             __pyx_r = __pyx_t_4;
18333             __pyx_t_4 = 0;
18334             goto __pyx_L7_try_return;
18335
18336             /* "PyClicl.pyx":1842
18337  *         {1,2}
18338  *         """
18339  *         try:             # ««««««««
18340  *             return math.tanh(obj)
18341  *         except:
18342  */
18343
18344             }
18345             __pyx_L3_error:;
18346             __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
18347             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18348             __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
18349
18350             /* "PyClicl.pyx":1844
18351  *         try:
18352  *             return math.tanh(obj)
18353  *         except:             # ««««««««
18354  *             return clifford().wrap( glucat.tanh(toClifford(obj)) )
18355  */
18356
18357             /*except:*/ {
18358                 __Pyx_AddTraceback("PyClicl.tanh", __pyx_clineno, __pyx_lineno,
18359             __pyx_filename);
18360                 if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
18361             1844, __pyx_L5_except_error)
18362                 __Pyx_GOTREF(__pyx_t_4);
18363                 __Pyx_GOTREF(__pyx_t_6);
18364                 __Pyx_GOTREF(__pyx_t_5);
18365
18366                 /* "PyClicl.pyx":1845
18367  *         return math.tanh(obj)
18368  *         except:

```

```

18362 *         return clifford().wrap( glucat.tanh(toClifford(obj)) )           # ««««««««
18363 *
18364 * cpdef inline atanh(obj,i = None):
18365 */
18366         __Pyx_XDECREF(__pyx_r);
18367         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1845,
__pyx_L5_except_error)
18368         __Pyx_GOTREF(__pyx_t_7);
18369         __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), tanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1845, __pyx_L5_except_error)
18370         __Pyx_GOTREF(__pyx_t_8);
18371         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
18372         __pyx_r = __pyx_t_8;
18373         __pyx_t_8 = 0;
18374         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
18375         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18376         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18377         goto __pyx_L6_except_return;
18378     }
18379     __pyx_L5_except_error;;
18380
18381     /* "PyClical.pyx":1842
18382 *         {1,2}
18383 *         """
18384 *         try:                       # ««««««««
18385 *             return math.tanh(obj)
18386 *         except:
18387 */
18388         __Pyx_XGIVEREF(__pyx_t_1);
18389         __Pyx_XGIVEREF(__pyx_t_2);
18390         __Pyx_XGIVEREF(__pyx_t_3);
18391         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18392         goto __pyx_L1_error;
18393         __pyx_L7_try_return;
18394         __Pyx_XGIVEREF(__pyx_t_1);
18395         __Pyx_XGIVEREF(__pyx_t_2);
18396         __Pyx_XGIVEREF(__pyx_t_3);
18397         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18398         goto __pyx_L0;
18399         __pyx_L6_except_return;
18400         __Pyx_XGIVEREF(__pyx_t_1);
18401         __Pyx_XGIVEREF(__pyx_t_2);
18402         __Pyx_XGIVEREF(__pyx_t_3);
18403         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18404         goto __pyx_L0;
18405     }
18406
18407     /* "PyClical.pyx":1835
18408 *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
18409 *
18410 * cpdef inline tanh(obj):           # ««««««««
18411 *         """
18412 *         Hyperbolic tangent of multivector.
18413 */
18414
18415         /* function exit code */
18416         __pyx_L1_error;;
18417         __Pyx_XDECREF(__pyx_t_4);
18418         __Pyx_XDECREF(__pyx_t_5);
18419         __Pyx_XDECREF(__pyx_t_6);
18420         __Pyx_XDECREF(__pyx_t_7);
18421         __Pyx_XDECREF(__pyx_t_8);
18422         __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18423         __pyx_r = 0;
18424         __pyx_L0;
18425         __Pyx_XGIVEREF(__pyx_r);
18426         __Pyx_RefNannyFinishContext();
18427         return __pyx_r;
18428     }
18429
18430     /* Python wrapper */
18431     static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18432     static char __pyx_doc_8PyClical_76tanh[] = "\n    Hyperbolic tangent of
multivector.\n\n    >> x=clifford('{1,2}') * pi/4; print(tanh(x))\n    {1,2}\n    ";
18433     static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
18434         PyObject * __pyx_r = 0;
18435         __Pyx_RefNannyDeclarations
18436         __Pyx_RefNannySetupContext("tanh (wrapper)", 0);
18437         __pyx_r = __pyx_pf_8PyClical_76tanh(__pyx_self, ((PyObject *) __pyx_v_obj));
18438
18439         /* function exit code */
18440         __Pyx_RefNannyFinishContext();
18441         return __pyx_r;

```

```

18442     }
18443
18444     static PyObject * __pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
18445         PyObject *__pyx_r = NULL;
18446         __Pyx_RefNannyDeclarations
18447         PyObject *__pyx_t_1 = NULL;
18448         int __pyx_lineno = 0;
18449         const char *__pyx_filename = NULL;
18450         int __pyx_clineno = 0;
18451         __Pyx_RefNannySetupContext("tanh", 0);
18452         __Pyx_XDECREF(__pyx_r);
18453         __pyx_t_1 = __pyx_f_8PyClical_tanh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1835, __pyx_L1_error)
18454         __Pyx_GOTREF(__pyx_t_1);
18455         __pyx_r = __pyx_t_1;
18456         __pyx_t_1 = 0;
18457         goto __pyx_L0;
18458
18459         /* function exit code */
18460         __pyx_L1_error:;
18461         __Pyx_XDECREF(__pyx_t_1);
18462         __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18463         __pyx_r = NULL;
18464         __pyx_L0:;
18465         __Pyx_XGIVEREF(__pyx_r);
18466         __Pyx_RefNannyFinishContext();
18467         return __pyx_r;
18468     }
18469
18470     /* "PyClical.pyx":1847
18471     *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
18472     *
18473     * cpdef inline atanh(obj,i = None):          # ««««««««
18474     *     """
18475     *     Inverse hyperbolic tangent of multivector with optional complexifier.
18476     */
18477
18478     static PyObject * __pyx_pw_8PyClical_79atanh(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
18479     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atanh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_atanh *__pyx_optional_args) {
18480         PyObject *__pyx_v_i = (PyObject *)Py_None;
18481         PyObject *__pyx_r = NULL;
18482         __Pyx_RefNannyDeclarations
18483         int __pyx_t_1;
18484         int __pyx_t_2;
18485         PyObject *__pyx_t_3 = NULL;
18486         Clifford __pyx_t_4;
18487         PyObject *__pyx_t_5 = NULL;
18488         PyObject *__pyx_t_6 = NULL;
18489         PyObject *__pyx_t_7 = NULL;
18490         PyObject *__pyx_t_8 = NULL;
18491         PyObject *__pyx_t_9 = NULL;
18492         PyObject *__pyx_t_10 = NULL;
18493         PyObject *__pyx_t_11 = NULL;
18494         int __pyx_lineno = 0;
18495         const char *__pyx_filename = NULL;
18496         int __pyx_clineno = 0;
18497         __Pyx_RefNannySetupContext("atanh", 0);
18498         if (__pyx_optional_args) {
18499             if (__pyx_optional_args->__pyx_n > 0) {
18500                 __pyx_v_i = __pyx_optional_args->i;
18501             }
18502         }
18503
18504         /* "PyClical.pyx":1856
18505     *     {1,2}
18506     *     """
18507     *     if not (i is None):          # ««««««««
18508     *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18509     *     else:
18510     */
18511         __pyx_t_1 = (__pyx_v_i != Py_None);
18512         __pyx_t_2 = (__pyx_t_1 != 0);
18513         if (__pyx_t_2) {
18514
18515             /* "PyClical.pyx":1857
18516     *     """
18517     *     if not (i is None):
18518     *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18519     *     else:
18520     *         try:
18521     */
18522         __Pyx_XDECREF(__pyx_r);
18523         __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject

```

```

*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1857, __pyx_L1_error)
18524     __Pyx_GOTREF(__pyx_t_3);
18525     try {
18526         __pyx_t_4 = atanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
18527     } catch (...) {
18528         __Pyx_CppExn2PyErr();
18529         __PYX_ERR(0, 1857, __pyx_L1_error)
18530     }
18531     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1857, __pyx_L1_error)
18532     __Pyx_GOTREF(__pyx_t_5);
18533     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18534     __pyx_r = __pyx_t_5;
18535     __pyx_t_5 = 0;
18536     goto __pyx_L0;
18537
18538     /* "PyClical.pyx":1856
18539     *     {1,2}
18540     *     """
18541     *     if not (i is None):                # ««««««««
18542     *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18543     *     else:
18544     */
18545     }
18546
18547     /* "PyClical.pyx":1859
18548     *     return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18549     *     else:
18550     *         try:                # ««««««««
18551     *             return math.atanh(obj)
18552     *         except:
18553     */
18554     /*else*/ {
18555     {
18556         __Pyx_PyThreadState_declare
18557         __Pyx_PyThreadState_assign
18558         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
18559         __Pyx_XGOTREF(__pyx_t_6);
18560         __Pyx_XGOTREF(__pyx_t_7);
18561         __Pyx_XGOTREF(__pyx_t_8);
18562         /*try:*/ {
18563
18564         /* "PyClical.pyx":1860
18565     *     else:
18566     *         try:
18567     *             return math.atanh(obj)                # ««««««««
18568     *         except:
18569     *             return clifford().wrap( glucat.atanh(toClifford(obj)) )
18570     */
18571         __Pyx_XDECREF(__pyx_r);
18572         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1860, __pyx_L4_error)
18573         __Pyx_GOTREF(__pyx_t_3);
18574         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_atanh); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1860, __pyx_L4_error)
18575         __Pyx_GOTREF(__pyx_t_9);
18576         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18577         __pyx_t_3 = NULL;
18578         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
18579             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
18580             if (likely(__pyx_t_3)) {
18581                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
18582                 __Pyx_INCREF(__pyx_t_3);
18583                 __Pyx_INCREF(function);
18584                 __Pyx_DECREF_SET(__pyx_t_9, function);
18585             }
18586         }
18587         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
18588         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18589         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1860, __pyx_L4_error)
18590         __Pyx_GOTREF(__pyx_t_5);
18591         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18592         __pyx_r = __pyx_t_5;
18593         __pyx_t_5 = 0;
18594         goto __pyx_L8_try_return;
18595
18596         /* "PyClical.pyx":1859
18597     *     return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18598     *     else:
18599     *         try:                # ««««««««
18600     *             return math.atanh(obj)
18601     *         except:
18602     */
18603     }
18604     __pyx_L4_error:;

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```

18605         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18606         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18607         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
18608
18609         /* "PyClical.pyx":1861
18610 *         try:
18611 *             return math.atanh(obj)
18612 *         except:
18613 *             # ««««««««
18614 *             return clifford().wrap( glucat.atanh(toClifford(obj)) )
18615 */
18616
18617         /*except:*/ {
18618             __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno,
18619 __pyx_filename);
18620             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
18621 1861, __pyx_L6_except_error)
18622             __Pyx_GOTREF(__pyx_t_5);
18623             __Pyx_GOTREF(__pyx_t_9);
18624             __Pyx_GOTREF(__pyx_t_3);
18625
18626             /* "PyClical.pyx":1862
18627 *             return math.atanh(obj)
18628 *         except:
18629 *             return clifford().wrap( glucat.atanh(toClifford(obj)) )
18630 *             # ««««««««
18631 * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18632 */
18633             __Pyx_XDECREF(__pyx_r);
18634             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
18635 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1862,
18636 __pyx_L6_except_error)
18637             __Pyx_GOTREF(__pyx_t_10);
18638             __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
18639 __pyx_obj_8PyClical_clifford *)__pyx_t_10), atanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
18640 (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1862, __pyx_L6_except_error)
18641             __Pyx_GOTREF(__pyx_t_11);
18642             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18643             __pyx_r = __pyx_t_11;
18644             __pyx_t_11 = 0;
18645             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18646             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18647             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18648             goto __pyx_L7_except_return;
18649         }
18650         __pyx_L6_except_error:;
18651
18652         /* "PyClical.pyx":1859
18653 *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18654 *     else:
18655 *         try:
18656 *             # ««««««««
18657 *             return math.atanh(obj)
18658 *         except:
18659 */
18660             __Pyx_XGIVEREF(__pyx_t_6);
18661             __Pyx_XGIVEREF(__pyx_t_7);
18662             __Pyx_XGIVEREF(__pyx_t_8);
18663             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18664             goto __pyx_L1_error;
18665             __pyx_L8_try_return:;
18666             __Pyx_XGIVEREF(__pyx_t_6);
18667             __Pyx_XGIVEREF(__pyx_t_7);
18668             __Pyx_XGIVEREF(__pyx_t_8);
18669             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18670             goto __pyx_L0;
18671             __pyx_L7_except_return:;
18672             __Pyx_XGIVEREF(__pyx_t_6);
18673             __Pyx_XGIVEREF(__pyx_t_7);
18674             __Pyx_XGIVEREF(__pyx_t_8);
18675             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18676             goto __pyx_L0;
18677         }
18678     }
18679
18680     /* "PyClical.pyx":1847
18681 *     return clifford().wrap( glucat.tanh(toClifford(obj)) )
18682 *
18683 * cpdef inline atanh(obj,i = None):
18684 *     """
18685 *     Inverse hyperbolic tangent of multivector with optional complexifier.
18686 */
18687
18688     /* function exit code */
18689     __pyx_L1_error:;
18690     __Pyx_XDECREF(__pyx_t_3);
18691     __Pyx_XDECREF(__pyx_t_5);
18692     __Pyx_XDECREF(__pyx_t_9);
18693     __Pyx_XDECREF(__pyx_t_10);

```

```

18686         __Pyx_XDECREF(__pyx_t_11);
18687         __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18688         __pyx_r = 0;
18689         __pyx_L0:;
18690         __Pyx_XGIVEREF(__pyx_r);
18691         __Pyx_RefNannyFinishContext();
18692         return __pyx_r;
18693     }
18694
18695     /* Python wrapper */
18696     static PyObject *__pyx_pw_8PyClical_79atanh(PyObject *__pyx_self, PyObject
18697 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
18698     static char __pyx_doc_8PyClical_78atanh[] = "\n    Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford(\"{1,2}\");
print(tanh(atanh(x,s)))\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print(tanh(atanh(x)))\n    {1,2}\n
";
18699     static PyObject *__pyx_pw_8PyClical_79atanh(PyObject *__pyx_self, PyObject
18700 *__pyx_args, PyObject *__pyx_kwds) {
18701         PyObject *__pyx_v_obj = 0;
18702         PyObject *__pyx_v_i = 0;
18703         int __pyx_lineno = 0;
18704         const char *__pyx_filename = NULL;
18705         int __pyx_clineno = 0;
18706         PyObject *__pyx_r = 0;
18707         __Pyx_RefNannyDeclarations
18708         __Pyx_RefNannySetupContext("atanh (wrapper)", 0);
18709         {
18710             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
18711             PyObject* values[2] = {0,0};
18712             values[1] = (PyObject *)Py_None;
18713             if (unlikely(__pyx_kwds)) {
18714                 Py_ssize_t kw_args;
18715                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18716                 switch (pos_args) {
18717                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18718                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18719                     case 0: break;
18720                     default: goto __pyx_L5_argtuple_error;
18721                 }
18722                 kw_args = PyDict_Size(__pyx_kwds);
18723                 switch (pos_args) {
18724                     case 0:
18725                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
18726                         else goto __pyx_L5_argtuple_error;
18727                         CYTHON_FALLTHROUGH;
18728                     case 1:
18729                         if (kw_args > 0) {
18730                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
18731                             if (value) { values[1] = value; kw_args--; }
18732                         }
18733                     if (unlikely(kw_args > 0)) {
18734                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
18735 values, pos_args, "atanh") < 0)) __PYX_ERR(0, 1847, __pyx_L3_error)
18736                     }
18737                     else {
18738                         switch (PyTuple_GET_SIZE(__pyx_args)) {
18739                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18740                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18741                             break;
18742                             default: goto __pyx_L5_argtuple_error;
18743                         }
18744                     }
18745                     __pyx_v_obj = values[0];
18746                     __pyx_v_i = values[1];
18747                 }
18748                 goto __pyx_L4_argument_unpacking_done;
18749                 __pyx_L5_argtuple_error:;
18750                 __Pyx_RaiseArgtupleInvalid("atanh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
18751                 __PYX_ERR(0, 1847, __pyx_L3_error)
18752                 __pyx_L3_error:;
18753                 __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18754                 __Pyx_RefNannyFinishContext();
18755                 return NULL;
18756                 __pyx_L4_argument_unpacking_done:;
18757                 __pyx_r = __pyx_pf_8PyClical_78atanh(__pyx_self, __pyx_v_obj, __pyx_v_i);
18758
18759                 /* function exit code */
18760                 __Pyx_RefNannyFinishContext();
18761                 return __pyx_r;
18762             }
18763
18764             static PyObject *__pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject *__pyx_self,

```

```

PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
18765     PyObject *__pyx_r = NULL;
18766     __Pyx_RefNannyDeclarations
18767     PyObject *__pyx_t_1 = NULL;
18768     struct __pyx_opt_args_8PyClical_atanh __pyx_t_2;
18769     int __pyx_lineno = 0;
18770     const char *__pyx_filename = NULL;
18771     int __pyx_clineno = 0;
18772     __Pyx_RefNannySetupContext("atanh", 0);
18773     __Pyx_XDECREF(__pyx_r);
18774     __pyx_t_2.__pyx_n = 1;
18775     __pyx_t_2.i = __pyx_v_i;
18776     __pyx_t_1 = __pyx_f_8PyClical_atanh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1847, __pyx_L1_error)
18777     __Pyx_GOTREF(__pyx_t_1);
18778     __pyx_r = __pyx_t_1;
18779     __pyx_t_1 = 0;
18780     goto __pyx_L0;
18781
18782     /* function exit code */
18783     __pyx_L1_error:;
18784     __Pyx_XDECREF(__pyx_t_1);
18785     __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18786     __pyx_r = NULL;
18787     __pyx_L0:;
18788     __Pyx_XGIVEREF(__pyx_r);
18789     __Pyx_RefNannyFinishContext();
18790     return __pyx_r;
18791 }
18792
18793     /* "PyClical.pyx":1864
18794     *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
18795     *
18796     * cpdef inline random_clifford(index_set ixt, fill = 1.0):           # ««««««««
18797     *     """
18798     *         Random multivector within a frame.
18799     */
18800
18801     static PyObject *__pyx_pw_8PyClical_8lrandom_clifford(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
18802     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_random_clifford(struct
__pyx_obj_8PyClical_index_set *__pyx_v_ixt, CYTHON_UNUSED int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_random_clifford *__pyx_optional_args) {
18803         PyObject *__pyx_v_fill = ((PyObject *)__pyx_float_1_0);
18804         PyObject *__pyx_r = NULL;
18805         __Pyx_RefNannyDeclarations
18806         PyObject *__pyx_t_1 = NULL;
18807         PyObject *__pyx_t_2 = NULL;
18808         scalar_t __pyx_t_3;
18809         PyObject *__pyx_t_4 = NULL;
18810         int __pyx_lineno = 0;
18811         const char *__pyx_filename = NULL;
18812         int __pyx_clineno = 0;
18813         __Pyx_RefNannySetupContext("random_clifford", 0);
18814         if (__pyx_optional_args) {
18815             if (__pyx_optional_args->__pyx_n > 0) {
18816                 __pyx_v_fill = __pyx_optional_args->fill;
18817             }
18818         }
18819
18820         /* "PyClical.pyx":1871
18821         *     {-3,-1,2}
18822         *     """
18823         *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18824         *     # ««««««««
18825         * cpdef inline cga3(obj):
18826         */
18827         __Pyx_XDECREF(__pyx_r);
18828         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
18829         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1871, __pyx_L1_error)
18830         __Pyx_GOTREF(__pyx_t_1);
18831         __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
18832         if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1871, __pyx_L1_error)
18833         __Pyx_GOTREF(__pyx_t_2);
18834         __pyx_t_3 = __pyx_PyFloat_AsDouble(__pyx_v_fill); if (unlikely((__pyx_t_3 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1871, __pyx_L1_error)
18835         __pyx_t_4 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), ((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_2)->instance->random(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_ixt),
((scalar_t)__pyx_t_3)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1871, __pyx_L1_error)
18836         __Pyx_GOTREF(__pyx_t_4);
18837         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18838         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
18839         __pyx_r = __pyx_t_4;
18840         __pyx_t_4 = 0;
18841         goto __pyx_L0;

```



```

18840
18841      /* "PyClical.pyx":1864
18842      *      return clifford().wrap( glucat.atanh(toClifford(obj)) )
18843      *
18844      * cpdef inline random_clifford(index_set ixt, fill = 1.0):          # ««««««««
18845      *      """
18846      *      Random multivector within a frame.
18847      */
18848
18849      /* function exit code */
18850      __pyx_L1_error;
18851      __Pyx_XDECREF(__pyx_t_1);
18852      __Pyx_XDECREF(__pyx_t_2);
18853      __Pyx_XDECREF(__pyx_t_4);
18854      __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18855      __pyx_r = 0;
18856      __pyx_L0;
18857      __Pyx_XGIVEREF(__pyx_r);
18858      __Pyx_RefNannyFinishContext();
18859      return __pyx_r;
18860  }
18861
18862      /* Python wrapper */
18863      static PyObject * __pyx_pw_8PyClical_81random_clifford(PyObject * __pyx_self, PyObject
* __pyx_args, PyObject * __pyx_kwds); /*proto*/
18864      static char __pyx_doc_8PyClical_80random_clifford[] = "\n      Random multivector within
a frame.\n\n      >> print(random_clifford(index_set((-3,-1,2))).frame())\n      {-3,-1,2}\n      ";
18865      static PyObject * __pyx_pw_8PyClical_81random_clifford(PyObject * __pyx_self, PyObject
* __pyx_args, PyObject * __pyx_kwds) {
18866          struct __pyx_obj_8PyClical_index_set * __pyx_v_ixt = 0;
18867          PyObject * __pyx_v_fill = 0;
18868          int __pyx_lineno = 0;
18869          const char * __pyx_filename = NULL;
18870          int __pyx_clineno = 0;
18871          PyObject * __pyx_r = 0;
18872          __Pyx_RefNannyDeclarations
18873          __Pyx_RefNannySetupContext("random_clifford (wrapper)", 0);
18874          {
18875              static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_ixt,&__pyx_n_s_fill,0};
18876              PyObject* values[2] = {0,0};
18877              values[1] = ((PyObject *) __pyx_float_1_0);
18878              if (unlikely(__pyx_kwds)) {
18879                  Py_ssize_t kw_args;
18880                  const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18881                  switch (pos_args) {
18882                      case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18883                      case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18884                      case 0: break;
18885                      default: goto __pyx_L5_argtuple_error;
18886                  }
18887                  kw_args = PyDict_Size(__pyx_kwds);
18888                  switch (pos_args) {
18889                      case 0:
18890                          if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt)) !=
0)) kw_args--;
18891                          else goto __pyx_L5_argtuple_error;
18892                          CYTHON_FALLTHROUGH;
18893                          case 1:
18894                              if (kw_args > 0) {
18895                                  PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_fill);
18896                                  if (value) { values[1] = value; kw_args--; }
18897                              }
18898                              if (unlikely(kw_args > 0)) {
18899                                  if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "random_clifford") < 0)) __PYX_ERR(0, 1864, __pyx_L3_error)
18900                              }
18901                              else {
18902                                  switch (PyTuple_GET_SIZE(__pyx_args)) {
18903                                      case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18904                                      case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
break;
18905                                      default: goto __pyx_L5_argtuple_error;
18906                                  }
18907                                  __pyx_v_ixt = ((struct __pyx_obj_8PyClical_index_set *)values[0]);
18908                                  __pyx_v_fill = values[1];
18909                              }
18910                              goto __pyx_L4_argument_unpacking_done;
18911                          __pyx_L5_argtuple_error;
18912                          __Pyx_RaiseArgtupleInvalid("random_clifford", 0, 1, 2,
PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 1864, __pyx_L3_error)
18913                          __pyx_L3_error;
18914

```

```

18920     __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
18921     __pyx_filename);
18922     __Pyx_RefNannyFinishContext();
18923     return NULL;
18924     __pyx_L4_argument_unpacking_done:;
18925     if (unlikely(!__Pyx_ArgTypeTest(((PyObject *)__pyx_v_ixt),
18926     __pyx_ptype_8PyClical_index_set, 1, "ixt", 0))) __PYX_ERR(0, 1864, __pyx_L1_error)
18927     __pyx_r = __pyx_pf_8PyClical_80random_clifford(__pyx_self, __pyx_v_ixt,
18928     __pyx_v_fill);
18929
18930     /* function exit code */
18931     goto __pyx_L0;
18932     __pyx_L1_error:;
18933     __pyx_r = NULL;
18934     __pyx_L0:;
18935     __Pyx_RefNannyFinishContext();
18936     return __pyx_r;
18937 }
18938
18939 static PyObject *__pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject
18940 *__pyx_self, struct __pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill) {
18941     PyObject *__pyx_r = NULL;
18942     __Pyx_RefNannyDeclarations
18943     PyObject *__pyx_t_1 = NULL;
18944     struct __pyx_opt_args_8PyClical_random_clifford __pyx_t_2;
18945     int __pyx_lineno = 0;
18946     const char *__pyx_filename = NULL;
18947     int __pyx_clineno = 0;
18948     __Pyx_RefNannySetupContext("random_clifford", 0);
18949     __Pyx_XDECREF(__pyx_r);
18950     __pyx_t_2.__pyx_n = 1;
18951     __pyx_t_2.fill = __pyx_v_fill;
18952     __pyx_t_1 = __pyx_f_8PyClical_random_clifford(__pyx_v_ixt, 0, &__pyx_t_2); if
18953     (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1864, __pyx_L1_error)
18954     __Pyx_GOTREF(__pyx_t_1);
18955     __pyx_r = __pyx_t_1;
18956     __pyx_t_1 = 0;
18957     goto __pyx_L0;
18958
18959     /* function exit code */
18960     __pyx_L1_error:;
18961     __Pyx_XDECREF(__pyx_t_1);
18962     __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
18963     __pyx_filename);
18964     __pyx_r = NULL;
18965     __pyx_L0:;
18966     __Pyx_XGIVEREF(__pyx_r);
18967     __Pyx_RefNannyFinishContext();
18968     return __pyx_r;
18969 }
18970
18971 /* "PyClical.pyx":1873
18972 *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18973 *
18974 * cpdef inline cga3(obj): # ««««««««
18975 *     """
18976 *     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
18977 *     definition.
18978 */
18979
18980 static PyObject *__pyx_pw_8PyClical_83cga3(PyObject *__pyx_self, PyObject
18981 *__pyx_v_obj); /*proto*/
18982 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cga3(PyObject *__pyx_v_obj,
18983 CYTHON_UNUSED int __pyx_skip_dispatch) {
18984     PyObject *__pyx_r = NULL;
18985     __Pyx_RefNannyDeclarations
18986     PyObject *__pyx_t_1 = NULL;
18987     PyObject *__pyx_t_2 = NULL;
18988     int __pyx_lineno = 0;
18989     const char *__pyx_filename = NULL;
18990     int __pyx_clineno = 0;
18991     __Pyx_RefNannySetupContext("cga3", 0);
18992
18993     /* "PyClical.pyx":1880
18994 *     87{-1}+4{1}+18{2}+2{3}+85{4}
18995 *     """
18996 *     return clifford().wrap( glucat.cga3(toClifford(obj)) ) # ««««««««
18997 *
18998 * cpdef inline cga3std(obj):
18999 */
19000     __Pyx_XDECREF(__pyx_r);
19001     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
19002     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1880, __pyx_L1_error)
19003     __Pyx_GOTREF(__pyx_t_1);
19004     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
19005 *)__pyx_t_1), cga3: cga3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
19006     __PYX_ERR(0, 1880, __pyx_L1_error)

```

```

18995         __Pyx_GOTREF(__pyx_t_2);
18996         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18997         __pyx_r = __pyx_t_2;
18998         __pyx_t_2 = 0;
18999         goto __pyx_L0;
19000
19001         /* "PyClical.pyx":1873
19002  *      return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
19003  *
19004  * cpdef inline cga3(obj):
19005  *      """
19006  *          Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
19007  *          definition.
19008  */
19009
19010         /* function exit code */
19011         __pyx_L1_error:;
19012         __Pyx_XDECREF(__pyx_t_1);
19013         __Pyx_XDECREF(__pyx_t_2);
19014         __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19015         __pyx_r = 0;
19016         __pyx_L0:;
19017         __Pyx_XGIVEREF(__pyx_r);
19018         __Pyx_RefNannyFinishContext();
19019         return __pyx_r;
19020     }
19021
19022     /* Python wrapper */
19023     static PyObject * __pyx_pw_8PyClical_83cga3(PyObject * __pyx_self, PyObject
19024 * __pyx_v_obj); /*proto*/
19025     static char __pyx_doc_8PyClical_82cga3[] = "\n      Convert Euclidean 3D multivector to
19026 Conformal Geometric Algebra using Doran and Lasenby definition.\n\n      >>
19027 x=clifford(\"2{1}+9{2}+{3}\"); print(cga3(x))\n      87{-1}+4{1}+18{2}+2{3}+85{4}\n      ";
19028     static PyObject * __pyx_pf_8PyClical_83cga3(PyObject * __pyx_self, PyObject
19029 * __pyx_v_obj) {
19030         PyObject * __pyx_r = 0;
19031         __Pyx_RefNannyDeclarations
19032         __Pyx_RefNannySetupContext("cga3 (wrapper)", 0);
19033         __pyx_r = __pyx_pf_8PyClical_82cga3(__pyx_self, ((PyObject *) __pyx_v_obj));
19034
19035         /* function exit code */
19036         __Pyx_RefNannyFinishContext();
19037         return __pyx_r;
19038     }
19039
19040     static PyObject * __pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject * __pyx_self,
19041 PyObject * __pyx_v_obj) {
19042         PyObject * __pyx_r = NULL;
19043         __Pyx_RefNannyDeclarations
19044         PyObject * __pyx_t_1 = NULL;
19045         int __pyx_lineno = 0;
19046         const char * __pyx_filename = NULL;
19047         int __pyx_clineno = 0;
19048         __Pyx_RefNannySetupContext("cga3", 0);
19049         __Pyx_XDECREF(__pyx_r);
19050         __pyx_t_1 = __pyx_f_8PyClical_cga3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
19051         __PYX_ERR(0, 1873, __pyx_L1_error)
19052         __Pyx_GOTREF(__pyx_t_1);
19053         __pyx_r = __pyx_t_1;
19054         __pyx_t_1 = 0;
19055         goto __pyx_L0;
19056
19057         /* function exit code */
19058         __pyx_L1_error:;
19059         __Pyx_XDECREF(__pyx_t_1);
19060         __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19061         __pyx_r = NULL;
19062         __pyx_L0:;
19063         __Pyx_XGIVEREF(__pyx_r);
19064         __Pyx_RefNannyFinishContext();
19065         return __pyx_r;
19066     }
19067
19068     /* "PyClical.pyx":1882
19069  *      return clifford().wrap( glucat.cga3(toClifford(obj)) )
19070  *
19071  * cpdef inline cga3std(obj):
19072  *      """
19073  *          Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19074  */
19075
19076     static PyObject * __pyx_pw_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
19077 * __pyx_v_obj); /*proto*/
19078     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cga3std(PyObject * __pyx_v_obj,
19079 CYTHON_UNUSED int __pyx_skip_dispatch) {
19080         PyObject * __pyx_r = NULL;
19081         __Pyx_RefNannyDeclarations

```

```

19073         PyObject *__pyx_t_1 = NULL;
19074         PyObject *__pyx_t_2 = NULL;
19075         int __pyx_lineno = 0;
19076         const char *__pyx_filename = NULL;
19077         int __pyx_clineno = 0;
19078         __Pyx_RefNannySetupContext("cga3std", 0);
19079
19080         /* "PyClicl.pyx":1891
19081  *      0
19082  *      ""
19083  *      return clifford().wrap( glucat.cga3std(toClifford(obj)) )          # ««««««««
19084  *
19085  * cpdef inline agc3(obj):
19086  */
19087         __Pyx_XDECREF(__pyx_r);
19088         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClicl_clifford));
19089         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1891, __pyx_L1_error)
19090         __Pyx_GOTREF(__pyx_t_1);
19091         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
19092 *)__pyx_t_1), cga3:cga3std(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
19093         __PYX_ERR(0, 1891, __pyx_L1_error)
19094         __Pyx_GOTREF(__pyx_t_2);
19095         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19096         __pyx_r = __pyx_t_2;
19097         __pyx_t_2 = 0;
19098         goto __pyx_L0;
19099
19100         /* "PyClicl.pyx":1882
19101  *      return clifford().wrap( glucat.cga3(toClifford(obj)) )
19102  *
19103  * cpdef inline cga3std(obj):          # ««««««««
19104  *      ""
19105  *      Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19106  */
19107         /* function exit code */
19108         __pyx_L1_error;
19109         __Pyx_XDECREF(__pyx_t_1);
19110         __Pyx_XDECREF(__pyx_t_2);
19111         __Pyx_AddTraceback("PyClicl.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);
19112         __pyx_r = 0;
19113         __pyx_L0;
19114         __Pyx_XGIVEREF(__pyx_r);
19115         __Pyx_RefNannyFinishContext();
19116         return __pyx_r;
19117     }
19118
19119     /* Python wrapper */
19120     static PyObject *__pyx_pw_8PyClicl_85cga3std(PyObject *__pyx_self, PyObject
19121 *__pyx_v_obj); /*proto*/
19122     static char __pyx_doc_8PyClicl_84cga3std[] = "\n    Convert CGA3 null vector to
19123 standard conformal null vector using Doran and Lasenby definition.\n\n    >>
19124 x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x)))\n    87{-1}+4{1}+18{2}+2{3}+85{4}\n    >>
19125 x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x))-cga3(x))\n    0\n    ";
19126     static PyObject *__pyx_pw_8PyClicl_85cga3std(PyObject *__pyx_self, PyObject
19127 *__pyx_v_obj) {
19128         PyObject *__pyx_r = 0;
19129         __Pyx_RefNannyDeclarations
19130         __Pyx_RefNannySetupContext("cga3std (wrapper)", 0);
19131         __pyx_r = __pyx_f_8PyClicl_84cga3std(__pyx_self, ((PyObject *)__pyx_v_obj));
19132
19133         /* function exit code */
19134         __Pyx_RefNannyFinishContext();
19135         return __pyx_r;
19136     }
19137
19138     static PyObject *__pyx_pf_8PyClicl_84cga3std(CYTHON_UNUSED PyObject *__pyx_self,
19139 PyObject *__pyx_v_obj) {
19140         PyObject *__pyx_r = NULL;
19141         __Pyx_RefNannyDeclarations
19142         PyObject *__pyx_t_1 = NULL;
19143         int __pyx_lineno = 0;
19144         const char *__pyx_filename = NULL;
19145         int __pyx_clineno = 0;
19146         __Pyx_RefNannySetupContext("cga3std", 0);
19147         __Pyx_XDECREF(__pyx_r);
19148         __pyx_t_1 = __pyx_f_8PyClicl_cga3std(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
19149         __PYX_ERR(0, 1882, __pyx_L1_error)
19150         __Pyx_GOTREF(__pyx_t_1);
19151         __pyx_r = __pyx_t_1;
19152         __pyx_t_1 = 0;
19153         goto __pyx_L0;
19154
19155         /* function exit code */
19156         __pyx_L1_error;
19157         __Pyx_XDECREF(__pyx_t_1);
19158         __Pyx_AddTraceback("PyClicl.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);

```

```

19150         __pyx_r = NULL;
19151         __pyx_L0:;
19152         __Pyx_XGIVEREF(__pyx_r);
19153         __Pyx_RefNannyFinishContext();
19154         return __pyx_r;
19155     }
19156
19157     /* "PyClical.pyx":1893
19158     *     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19159     *
19160     * cpdef inline agc3(obj):
19161     *     """
19162     *     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19163     */
19164
19165     static PyObject *__pyx_pw_8PyClical_87agc3(PyObject *__pyx_self, PyObject
19166 *__pyx_v_obj); /*proto*/
19167     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_agc3(PyObject *__pyx_v_obj,
19168 CYTHON_UNUSED int __pyx_skip_dispatch) {
19169     PyObject *__pyx_r = NULL;
19170     __Pyx_RefNannyDeclarations
19171     PyObject *__pyx_t_1 = NULL;
19172     PyObject *__pyx_t_2 = NULL;
19173     int __pyx_lineno = 0;
19174     const char *__pyx_filename = NULL;
19175     int __pyx_clineno = 0;
19176     __Pyx_RefNannySetupContext("agc3", 0);
19177
19178     /* "PyClical.pyx":1902
19179     *     0
19180     *     """
19181     *     return clifford().wrap( glucat.agc3(toClifford(obj)) )
19182     *
19183     * # Some abbreviations.
19184     */
19185     __Pyx_XDECREF(__pyx_r);
19186     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
19187     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1902, __pyx_L1_error)
19188     __Pyx_GOTREF(__pyx_t_1);
19189     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
19190 *)__pyx_t_1), cga3:agc3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
19191     __PYX_ERR(0, 1902, __pyx_L1_error)
19192     __Pyx_GOTREF(__pyx_t_2);
19193     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19194     __pyx_r = __pyx_t_2;
19195     __pyx_t_2 = 0;
19196     goto __pyx_L0;
19197
19198     /* "PyClical.pyx":1893
19199     *     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19200     *
19201     * cpdef inline agc3(obj):
19202     *     """
19203     *     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19204     */
19205
19206     /* function exit code */
19207     __pyx_L1_error:;
19208     __Pyx_XDECREF(__pyx_t_1);
19209     __Pyx_XDECREF(__pyx_t_2);
19210     __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19211     __pyx_r = 0;
19212     __pyx_L0:;
19213     __Pyx_XGIVEREF(__pyx_r);
19214     __Pyx_RefNannyFinishContext();
19215     return __pyx_r;
19216 }
19217
19218 /* Python wrapper */
19219 static PyObject *__pyx_pw_8PyClical_87agc3(PyObject *__pyx_self, PyObject
19220 *__pyx_v_obj); /*proto*/
19221 static char __pyx_doc_8PyClical_86agc3[] = "\n    Convert CGA3 null vector to
19222 Euclidean 3D vector using Doran and Lasenby definition.\n\n    >> x=clifford(\"2{1}+9{2}+{3}\");
19223 print(agc3(cga3(x)))\n    2{1}+9{2}+{3}\n    >> x=clifford(\"2{1}+9{2}+{3}\");
19224 print(agc3(cga3(x))-x)\n    0\n    ";
19225 static PyObject *__pyx_pw_8PyClical_87agc3(PyObject *__pyx_self, PyObject
19226 *__pyx_v_obj) {
19227     PyObject *__pyx_r = 0;
19228     __Pyx_RefNannyDeclarations
19229     __Pyx_RefNannySetupContext("agc3 (wrapper)", 0);
19230     __pyx_r = __pyx_f_8PyClical_86agc3(__pyx_self, ((PyObject *)__pyx_v_obj));
19231
19232     /* function exit code */
19233     __Pyx_RefNannyFinishContext();
19234     return __pyx_r;
19235 }
19236

```

```

19227         static PyObject *__pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject *__pyx_self,
19228 PyObject *__pyx_v_obj) {
19229     PyObject *__pyx_r = NULL;
19229     __Pyx_RefNannyDeclarations
19230     PyObject *__pyx_t_1 = NULL;
19231     int __pyx_lineno = 0;
19232     const char *__pyx_filename = NULL;
19233     int __pyx_clineno = 0;
19234     __Pyx_RefNannySetupContext("agc3", 0);
19235     __Pyx_XDECREF(__pyx_r);
19236     __pyx_t_1 = __pyx_f_8PyClical_agc3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1893, __pyx_L1_error)
19237     __Pyx_GOTREF(__pyx_t_1);
19238     __pyx_r = __pyx_t_1;
19239     __pyx_t_1 = 0;
19240     goto __pyx_L0;
19241
19242     /* function exit code */
19243     __pyx_L1_error:;
19244     __Pyx_XDECREF(__pyx_t_1);
19245     __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19246     __pyx_r = NULL;
19247     __pyx_L0:;
19248     __Pyx_XGIVEREF(__pyx_r);
19249     __Pyx_RefNannyFinishContext();
19250     return __pyx_r;
19251 }
19252
19253     /* "PyClical.pyx":1936
19254     * """
19255     *
19256     * def e(obj):          # ««««««««
19257     *     """
19258     *     Abbreviation for clifford(index_set(obj)).
19259     */
19260
19261     /* Python wrapper */
19262     static PyObject *__pyx_pw_8PyClical_89e(PyObject *__pyx_self, PyObject *__pyx_v_obj);
19263
19264     /*proto*/
19265     static char __pyx_doc_8PyClical_88e[] = "\n    Abbreviation for
clifford(index_set(obj)).\n\n    >> print(e(1))\n    {1}\n    >> print(e(-1))\n    {-1}\n    >>
print(e(0))\n    1\n    ";
19266     static PyMethodDef __pyx_mdef_8PyClical_89e = {"e",
(PyCFunction) __pyx_pw_8PyClical_89e, METH_O, __pyx_doc_8PyClical_88e};
19267     static PyObject *__pyx_pw_8PyClical_89e(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
19268     PyObject *__pyx_r = 0;
19269     __Pyx_RefNannyDeclarations
19270     __Pyx_RefNannySetupContext("e (wrapper)", 0);
19271     __pyx_r = __pyx_pf_8PyClical_88e(__pyx_self, ((PyObject *) __pyx_v_obj));
19272
19273     /* function exit code */
19274     __Pyx_RefNannyFinishContext();
19275     return __pyx_r;
19276 }
19277
19278     static PyObject *__pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
19279     PyObject *__pyx_r = NULL;
19280     __Pyx_RefNannyDeclarations
19281     PyObject *__pyx_t_1 = NULL;
19282     PyObject *__pyx_t_2 = NULL;
19283     int __pyx_lineno = 0;
19284     const char *__pyx_filename = NULL;
19285     int __pyx_clineno = 0;
19286     __Pyx_RefNannySetupContext("e", 0);
19287
19288     /* "PyClical.pyx":1947
19289     * 1
19290     * """
19291     * return clifford(index_set(obj))          # ««««««««
19292     *
19293     * def istpq(p, q):
19294     */
19295     __Pyx_XDECREF(__pyx_r);
19296     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_index_set),
__pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1947, __pyx_L1_error)
19297     __Pyx_GOTREF(__pyx_t_1);
19298     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1947, __pyx_L1_error)
19299     __Pyx_GOTREF(__pyx_t_2);
19300     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19301     __pyx_r = __pyx_t_2;
19302     __pyx_t_2 = 0;
19303     goto __pyx_L0;
19304
19305     /* "PyClical.pyx":1936
19306     * """

```

```

19305 *
19306 * def e(obj):          # ««««««««
19307 *     """
19308 *     Abbreviation for clifford(index_set(obj)).
19309 */
19310
19311     /* function exit code */
19312     __pyx_L1_error;
19313     __Pyx_XDECREF(__pyx_t_1);
19314     __Pyx_XDECREF(__pyx_t_2);
19315     __Pyx_AddTraceback("PyClical.e", __pyx_clineno, __pyx_lineno, __pyx_filename);
19316     __pyx_r = NULL;
19317     __pyx_L0;
19318     __Pyx_XGIVEREF(__pyx_r);
19319     __Pyx_RefNannyFinishContext();
19320     return __pyx_r;
19321 }
19322
19323     /* "PyClical.pyx":1949
19324 *     return clifford(index_set(obj))
19325 *
19326 * def istpq(p, q):      # ««««««««
19327 *     """
19328 *     Abbreviation for index_set({-q,...p}).
19329 */
19330
19331     /* Python wrapper */
19332     static PyObject* __pyx_pw_8PyClical_9listpq(PyObject* __pyx_self, PyObject
19333 *__pyx_args, PyObject* __pyx_kwds); /*proto*/
19334     static char __pyx_doc_8PyClical_9listpq[] = "\n    Abbreviation for
index_set({-q,...p}).\n\n    >> print(istpq(2,3))\n    {-3,-2,-1,1,2}\n    ";
19335     static PyMethodDef __pyx_mdef_8PyClical_9listpq = {"istpq",
(PyCFunction)(void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_9listpq, METH_VARARGS|METH_KEYWORDS,
__pyx_doc_8PyClical_9listpq);
19336     static PyObject* __pyx_pw_8PyClical_9listpq(PyObject* __pyx_self, PyObject
*__pyx_args, PyObject* __pyx_kwds) {
19337         PyObject* __pyx_v_p = 0;
19338         PyObject* __pyx_v_q = 0;
19339         int __pyx_lineno = 0;
19340         const char* __pyx_filename = NULL;
19341         int __pyx_clineno = 0;
19342         PyObject* __pyx_r = 0;
19343         __Pyx_RefNannyDeclarations
19344         __Pyx_RefNannySetupContext("istpq (wrapper)", 0);
19345         {
19346             static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_p,&__pyx_n_s_q,0};
19347             PyObject* values[2] = {0,0};
19348             if (unlikely(__pyx_kwds)) {
19349                 Py_ssize_t kw_args;
19350                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
19351                 switch (pos_args) {
19352                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
19353                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
19354                     case 0: break;
19355                     default: goto __pyx_L5_argtuple_error;
19356                 }
19357                 kw_args = PyDict_Size(__pyx_kwds);
19358                 switch (pos_args) {
19359                     case 0:
19360                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_p)) !=
0)) kw_args--;
19361
19362                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
19363                     case 1:
19364                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_q)) !=
0)) kw_args--;
19365
19366                         else {
19367                             __Pyx_RaiseArgtupleInvalid("istpq", 1, 2, 2, 1); __PYX_ERR(0, 1949,
__pyx_L3_error)
19368                         }
19369                 }
19370                 if (unlikely(kw_args > 0)) {
19371                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "istpq") < 0)) __PYX_ERR(0, 1949, __pyx_L3_error)
19372                 }
19373                 else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
19374                     goto __pyx_L5_argtuple_error;
19375                 } else {
19376                     values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
19377                     values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
19378                 }
19379                 __pyx_v_p = values[0];
19380                 __pyx_v_q = values[1];
19381             }
19382             goto __pyx_L4_argument_unpacking_done;

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```

19383         __pyx_L5_argtuple_error;;
19384         __Pyx_RaiseArgtupleInvalid("istpq", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
19385     __PYX_ERR(0, 1949, __pyx_L3_error)
19386     __pyx_L3_error;;
19387     __Pyx_AddTraceback("PyClical.istpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19388     __Pyx_RefNannyFinishContext();
19389     return NULL;
19390     __pyx_L4_argument_unpacking_done;;
19391     __pyx_r = __pyx_pf_8PyClical_90istpq(__pyx_self, __pyx_v_p, __pyx_v_q);
19392
19393     /* function exit code */
19394     __Pyx_RefNannyFinishContext();
19395     return __pyx_r;
19396 }
19397
19398 static PyObject *__pyx_pf_8PyClical_90istpq(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_p, PyObject *__pyx_v_q) {
19399     PyObject *__pyx_r = NULL;
19400     __Pyx_RefNannyDeclarations
19401     PyObject *__pyx_t_1 = NULL;
19402     PyObject *__pyx_t_2 = NULL;
19403     PyObject *__pyx_t_3 = NULL;
19404     int __pyx_lineno = 0;
19405     const char *__pyx_filename = NULL;
19406     int __pyx_clineno = 0;
19407     __Pyx_RefNannySetupContext("istpq", 0);
19408
19409     /* "PyClical.pyx":1956
19410     *
19411     *     return index_set(set(range(-q,p+1)))          # ««««««««
19412     *
19413     * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
19414     */
19415     __Pyx_XDECREF(__pyx_r);
19416     __pyx_t_1 = PyNumber_Negative(__pyx_v_q); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1956, __pyx_L1_error)
19417     __Pyx_GOTREF(__pyx_t_1);
19418     __pyx_t_2 = __Pyx_PyInt_AddObjC(__pyx_v_p, __pyx_t_1, 1, 0, 0); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19419     __Pyx_GOTREF(__pyx_t_2);
19420     __pyx_t_3 = PyTuple_New(2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,
__pyx_L1_error)
19421     __Pyx_GOTREF(__pyx_t_3);
19422     __Pyx_GIVEREF(__pyx_t_1);
19423     PyTuple_SET_ITEM(__pyx_t_3, 0, __pyx_t_1);
19424     __Pyx_GIVEREF(__pyx_t_2);
19425     PyTuple_SET_ITEM(__pyx_t_3, 1, __pyx_t_2);
19426     __pyx_t_1 = 0;
19427     __pyx_t_2 = 0;
19428     __pyx_t_2 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_3, NULL); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19429     __Pyx_GOTREF(__pyx_t_2);
19430     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19431     __pyx_t_3 = PySet_New(__pyx_t_2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,
__pyx_L1_error)
19432     __Pyx_GOTREF(__pyx_t_3);
19433     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19434     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_index_set),
__pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19435     __Pyx_GOTREF(__pyx_t_2);
19436     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19437     __pyx_r = __pyx_t_2;
19438     __pyx_t_2 = 0;
19439     goto __pyx_L0;
19440
19441     /* "PyClical.pyx":1949
19442     *     return clifford(index_set(obj))
19443     *
19444     * def istpq(p, q):          # ««««««««
19445     *     """
19446     *     Abbreviation for index_set({-q,...p}).
19447     */
19448
19449     /* function exit code */
19450     __pyx_L1_error;;
19451     __Pyx_XDECREF(__pyx_t_1);
19452     __Pyx_XDECREF(__pyx_t_2);
19453     __Pyx_XDECREF(__pyx_t_3);
19454     __Pyx_AddTraceback("PyClical.istpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19455     __pyx_r = NULL;
19456     __pyx_L0;;
19457     __Pyx_XGIVEREF(__pyx_r);
19458     __Pyx_RefNannyFinishContext();
19459     return __pyx_r;
19460 }
19461

```



```

19462         /* "PyClical.pyx":1962
19463     *
19464     * # Doctest interface.
19465     * def _test():
19466     *     import PyClical, doctest
19467     *     return doctest.testmod(PyClical)
19468 */
19469
19470     /* Python wrapper */
19471     static PyObject *__pyx_pw_8PyClical_93_test(PyObject *__pyx_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
19472     static PyMethodDef __pyx_mdef_8PyClical_93_test = {"_test",
(PyCFunction)__pyx_pw_8PyClical_93_test, METH_NOARGS, 0};
19473     static PyObject *__pyx_pw_8PyClical_93_test(PyObject *__pyx_self, CYTHON_UNUSED
PyObject *unused) {
19474         PyObject *__pyx_r = 0;
19475         __Pyx_RefNannyDeclarations
19476         __Pyx_RefNannySetupContext("_test (wrapper)", 0);
19477         __pyx_r = __pyx_pf_8PyClical_92_test(__pyx_self);
19478
19479         /* function exit code */
19480         __Pyx_RefNannyFinishContext();
19481         return __pyx_r;
19482     }
19483
19484     static PyObject *__pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject *__pyx_self) {
19485         PyObject *__pyx_v_PyClical = NULL;
19486         PyObject *__pyx_v_doctest = NULL;
19487         PyObject *__pyx_r = NULL;
19488         __Pyx_RefNannyDeclarations
19489         PyObject *__pyx_t_1 = NULL;
19490         PyObject *__pyx_t_2 = NULL;
19491         PyObject *__pyx_t_3 = NULL;
19492         int __pyx_lineno = 0;
19493         const char *__pyx_filename = NULL;
19494         int __pyx_clineno = 0;
19495         __Pyx_RefNannySetupContext("_test", 0);
19496
19497         /* "PyClical.pyx":1963
19498     * # Doctest interface.
19499     * def _test():
19500     *     import PyClical, doctest
19501     *     return doctest.testmod(PyClical)
19502     *
19503     */
19504         __pyx_t_1 = __Pyx_Import(__pyx_n_s_PyClical, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19505         __Pyx_GOTREF(__pyx_t_1);
19506         __pyx_v_PyClical = __pyx_t_1;
19507         __pyx_t_1 = 0;
19508         __pyx_t_1 = __Pyx_Import(__pyx_n_s_doctest, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19509         __Pyx_GOTREF(__pyx_t_1);
19510         __pyx_v_doctest = __pyx_t_1;
19511         __pyx_t_1 = 0;
19512
19513         /* "PyClical.pyx":1964
19514     * def _test():
19515     *     import PyClical, doctest
19516     *     return doctest.testmod(PyClical)
19517     *
19518     * if __name__ == "__main__":
19519     */
19520         __Pyx_XDECREF(__pyx_r);
19521         __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_v_doctest, __pyx_n_s_testmod); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1964, __pyx_L1_error)
19522         __Pyx_GOTREF(__pyx_t_2);
19523         __pyx_t_3 = NULL;
19524         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
19525             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
19526             if (likely(__pyx_t_3)) {
19527                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
19528                 __Pyx_INCREF(__pyx_t_3);
19529                 __Pyx_INCREF(function);
19530                 __Pyx_DECREF_SET(__pyx_t_2, function);
19531             }
19532         }
19533         __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_2, __pyx_t_3,
__pyx_v_PyClical) : __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_v_PyClical);
19534         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
19535         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1964, __pyx_L1_error)
19536         __Pyx_GOTREF(__pyx_t_1);
19537         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19538         __pyx_r = __pyx_t_1;
19539         __pyx_t_1 = 0;
19540         goto __pyx_L0;
19541

```

```

19542         /* "PyCliclcal.pyx":1962
19543         *
19544         * # Doctest interface.
19545         * def _test():
19546             import PyCliclcal, doctest
19547             return doctest.testmod(PyCliclcal)
19548         */
19549
19550         /* function exit code */
19551         __pyx_L1_error++;
19552         __Pyx_XDECREF(__pyx_t_1);
19553         __Pyx_XDECREF(__pyx_t_2);
19554         __Pyx_XDECREF(__pyx_t_3);
19555         __Pyx_AddTraceback("PyCliclcal._test", __pyx_clineno, __pyx_lineno, __pyx_filename);
19556         __pyx_r = NULL;
19557         __pyx_L0++;
19558         __Pyx_XDECREF(__pyx_v_PyCliclcal);
19559         __Pyx_XDECREF(__pyx_v_doctest);
19560         __Pyx_XGIVEREF(__pyx_r);
19561         __Pyx_RefNannyFinishContext();
19562         return __pyx_r;
19563     }
19564
19565     /* "string.to_py":31
19566     *
19567     * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19568     * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19569     *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19570     * cdef extern from *:
19571     */
19572
19573     static CYTHON_INLINE PyObject
19574     __pyx_convert_PyObject_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19575         PyObject *__pyx_r = NULL;
19576         __Pyx_RefNannyDeclarations
19577         PyObject *__pyx_t_1 = NULL;
19578         int __pyx_lineno = 0;
19579         const char *__pyx_filename = NULL;
19580         int __pyx_clineno = 0;
19581         __Pyx_RefNannySetupContext("__pyx_convert_PyObject_string_to_py_std__in_string", 0);
19582
19583         /* "string.to_py":32
19584         * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19585         * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19586         *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19587         * cdef extern from *:
19588         *     cdef object __Pyx_PyUnicode_FromStringAndSize(const char*, size_t)
19589         */
19590         __Pyx_XDECREF(__pyx_r);
19591         __pyx_t_1 = __Pyx_PyObject_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19592         (unlikely(!__pyx_t_1)) __PYX_ERR(1, 32, __pyx_L1_error)
19593         __Pyx_GOTREF(__pyx_t_1);
19594         __pyx_r = __pyx_t_1;
19595         __pyx_t_1 = 0;
19596         goto __pyx_L0;
19597
19598         /* "string.to_py":31
19599         *
19600         * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19601         * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19602         *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19603         * cdef extern from *:
19604         */
19605
19606         /* function exit code */
19607         __pyx_L1_error++;
19608         __Pyx_XDECREF(__pyx_t_1);
19609
19610         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyObject_string_to_py_std__in_string", __pyx_clineno,
19611         __pyx_lineno, __pyx_filename);
19612         __pyx_r = 0;
19613         __pyx_L0++;
19614         __Pyx_XGIVEREF(__pyx_r);
19615         __Pyx_RefNannyFinishContext();
19616         return __pyx_r;
19617     }
19618
19619     /* "string.to_py":37
19620     *
19621     * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19622     * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19623     *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19624     * cdef extern from *:
19625     */

```

```

19622
19623         static CYTHON_INLINE PyObject
19624         *__pyx_convert_PyUnicode_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19625             PyObject *__pyx_r = NULL;
19626             __Pyx_RefNannyDeclarations
19627             PyObject *__pyx_t_1 = NULL;
19628             int __pyx_lineno = 0;
19629             const char *__pyx_filename = NULL;
19630             int __pyx_clineno = 0;
19631             __Pyx_RefNannySetupContext("__pyx_convert_PyUnicode_string_to_py_std__in_string",
19632             0);
19633
19634             /* "string.to_py":38
19635             * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19636             * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19637             *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size()) # ««««««««
19638             * cdef extern from *:
19639             *     cdef object __Pyx_PyStr_FromStringAndSize(const char*, size_t)
19640             */
19641             __Pyx_XDECREF(__pyx_r);
19642             __pyx_t_1 = __Pyx_PyUnicode_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19643             if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 38, __pyx_L1_error)
19644             __Pyx_GOTREF(__pyx_t_1);
19645             __pyx_r = __pyx_t_1;
19646             __pyx_t_1 = 0;
19647             goto __pyx_L0;
19648
19649             /* "string.to_py":37
19650             * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19651             * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19652             *     # ««««««««
19653             *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19654             * cdef extern from *:
19655             */
19656             /* function exit code */
19657             __pyx_L1_error:;
19658             __Pyx_XDECREF(__pyx_t_1);
19659
19660             __Pyx_AddTraceback("string.to_py.__pyx_convert_PyUnicode_string_to_py_std__in_string", __pyx_clineno,
19661             __pyx_lineno, __pyx_filename);
19662             __pyx_r = 0;
19663             __pyx_L0:;
19664             __Pyx_XGIVEREF(__pyx_r);
19665             __Pyx_RefNannyFinishContext();
19666             return __pyx_r;
19667         }
19668
19669         /* "string.to_py":43
19670         * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19671         * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s): #
19672         *     # ««««««««
19673         *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19674         * cdef extern from *:
19675         */
19676         static CYTHON_INLINE PyObject
19677         *__pyx_convert_PyStr_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19678             PyObject *__pyx_r = NULL;
19679             __Pyx_RefNannyDeclarations
19680             PyObject *__pyx_t_1 = NULL;
19681             int __pyx_lineno = 0;
19682             const char *__pyx_filename = NULL;
19683             int __pyx_clineno = 0;
19684             __Pyx_RefNannySetupContext("__pyx_convert_PyStr_string_to_py_std__in_string", 0);
19685
19686             /* "string.to_py":44
19687             * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19688             * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s):
19689             *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size()) # ««««««««
19690             * cdef extern from *:
19691             *     cdef object __Pyx_PyBytes_FromStringAndSize(const char*, size_t)
19692             */
19693             __Pyx_XDECREF(__pyx_r);
19694             __pyx_t_1 = __Pyx_PyStr_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19695             (unlikely(!__pyx_t_1)) __PYX_ERR(1, 44, __pyx_L1_error)
19696             __Pyx_GOTREF(__pyx_t_1);
19697             __pyx_r = __pyx_t_1;
19698             __pyx_t_1 = 0;
19699             goto __pyx_L0;
19700
19701             /* "string.to_py":43
19702             * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19703             * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s): #

```

```

19700 *         return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19701 * cdef extern from *:
19702 */
19703
19704         /* function exit code */
19705         __pyx_L1_error++;
19706         __Pyx_XDECREF(__pyx_t_1);
19707         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyStr_string_to_py_std__in_string",
__pyx_clineno, __pyx_lineno, __pyx_filename);
19708         __pyx_r = 0;
19709         __pyx_L0;
19710         __Pyx_XGIVEREF(__pyx_r);
19711         __Pyx_RefNannyFinishContext();
19712         return __pyx_r;
19713     }
19714
19715     /* "string.to_py":49
19716 *
19717 * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19718 * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19719 # <<<<<<<<<
19719 *         return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19720 * cdef extern from *:
19721 */
19722
19723     static CYTHON_INLINE PyObject
19724 *__pyx_convert_PyBytes_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19725     PyObject *__pyx_r = NULL;
19726     __Pyx_RefNannyDeclarations
19727     PyObject *__pyx_t_1 = NULL;
19728     int __pyx_lineno = 0;
19729     const char *__pyx_filename = NULL;
19730     int __pyx_clineno = 0;
19731     __Pyx_RefNannySetupContext("__pyx_convert_PyBytes_string_to_py_std__in_string", 0);
19732
19733     /* "string.to_py":50
19733 * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19734 * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19735 *         return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size()) # <<<<<<<<<
19736 * cdef extern from *:
19737 *         cdef object __Pyx_PyByteArray_FromStringAndSize(const char*, size_t)
19738 */
19739         __Pyx_XDECREF(__pyx_r);
19740         __pyx_t_1 = __Pyx_PyBytes_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 50, __pyx_L1_error)
19741         __Pyx_GOTREF(__pyx_t_1);
19742         __pyx_r = __pyx_t_1;
19743         __pyx_t_1 = 0;
19744         goto __pyx_L0;
19745
19746     /* "string.to_py":49
19747 *
19748 * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19749 * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19750 # <<<<<<<<<
19750 *         return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19751 * cdef extern from *:
19752 */
19753
19754         /* function exit code */
19755         __pyx_L1_error++;
19756         __Pyx_XDECREF(__pyx_t_1);
19757         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyBytes_string_to_py_std__in_string",
__pyx_clineno, __pyx_lineno, __pyx_filename);
19758         __pyx_r = 0;
19759         __pyx_L0;
19760         __Pyx_XGIVEREF(__pyx_r);
19761         __Pyx_RefNannyFinishContext();
19762         return __pyx_r;
19763     }
19764
19765     /* "string.to_py":55
19766 *
19767 * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19768 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19769 # <<<<<<<<<
19769 *         return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19770 *
19771 */
19772
19773     static CYTHON_INLINE PyObject
19774 *__pyx_convert_PyByteArray_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19775     PyObject *__pyx_r = NULL;
19776     __Pyx_RefNannyDeclarations
19777     PyObject *__pyx_t_1 = NULL;
19778     int __pyx_lineno = 0;

```

```

19778         const char *__pyx_filename = NULL;
19779         int __pyx_clineno = 0;
19780         __Pyx_RefNannySetupContext("__pyx_convert_PyByteArray_string_to_py_std__in_string",
0);
19781
19782         /* "string.to_py":56
19783  * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19784  * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19785  *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size()) # ««««««««
19786  *
19787  */
19788         __Pyx_XDECREF(__pyx_r);
19789         __pyx_t_1 = __Pyx_PyByteArray_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19790         if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 56, __pyx_L1_error)
19791         __Pyx_GOTREF(__pyx_t_1);
19792         __pyx_r = __pyx_t_1;
19793         __pyx_t_1 = 0;
19794         goto __pyx_L0;
19795
19796         /* "string.to_py":55
19797  * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19798  * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19799  *     # ««««««««
19800  *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19801  *
19802  */
19803         /* function exit code */
19804         __pyx_L1_error:;
19805         __Pyx_XDECREF(__pyx_t_1);
19806
19807         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyByteArray_string_to_py_std__in_string",
__pyx_clineno, __pyx_lineno, __pyx_filename);
19808         __pyx_r = 0;
19809         __pyx_L0:;
19810         __Pyx_XGIVEREF(__pyx_r);
19811         __Pyx_RefNannyFinishContext();
19812         return __pyx_r;
19813     }
19814     static struct __pyx_vtabstruct_8PyClical_index_set __pyx_vtable_8PyClical_index_set;
19815     static PyObject *__pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a,
PyObject *k) {
19816         struct __pyx_obj_8PyClical_index_set *p;
19817         PyObject *o;
19818         if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
19819             o = (*t->tp_alloc)(t, 0);
19820         } else {
19821             o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
19822         }
19823         if (unlikely(!o)) return 0;
19824         p = ((struct __pyx_obj_8PyClical_index_set *)o);
19825         p->__pyx_vtab = __pyx_vtabptr_8PyClical_index_set;
19826         if (unlikely(__pyx_pw_8PyClical_9index_set_3__cinit__(o, a, k) < 0)) goto bad;
19827         return o;
19828     bad:
19829         Py_DECREF(o); o = 0;
19830         return NULL;
19831     }
19832
19833     static void __pyx_tp_dealloc_8PyClical_index_set(PyObject *o) {
19834         #if CYTHON_USE_TP_FINALIZE
19835         if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
Py_TYPE(o)->tp_finalize) && (!PyType_IS_GC(Py_TYPE(o)) || !PyGC_FINALIZED(o))) {
19836             if (PyObject_CallFinalizerFromDealloc(o)) return;
19837         }
19838         #endif
19839         {
19840             PyObject *etype, *eval, *etb;
19841             PyErr_Fetch(&etype, &eval, &etb);
19842             __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
19843             __pyx_pw_8PyClical_9index_set_5__dealloc__(o);
19844             __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
19845             PyErr_Restore(etype, eval, etb);
19846         }
19847         (*Py_TYPE(o)->tp_free)(o);
19848     }
19849     static PyObject *__pyx_sq_item_8PyClical_index_set(PyObject *o, Py_ssize_t i) {
19850         PyObject *r;
19851         PyObject *x = PyInt_FromSsize_t(i); if (!x) return 0;
19852         r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
19853         Py_DECREF(x);
19854         return r;
19855     }
19856
19857     static int __pyx_mp_ass_subscript_8PyClical_index_set(PyObject *o, PyObject *i,

```

```

PyObject *v) {
19858     if (v) {
19859         return __pyx_pw_8PyClical_9index_set_9__setitem__(o, i, v);
19860     }
19861     else {
19862         PyErr_Format(PyExc_NotImplementedError,
19863             "Subscript deletion not supported by %.200s", Py_TYPE(o)->tp_name);
19864         return -1;
19865     }
19866 }
19867
19868 static PyMethodDef __pyx_methods_8PyClical_index_set[] = {
19869     {"copy", (PyCFunction)__pyx_pw_8PyClical_9index_set_1copy, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_copy},
19870     {"count", (PyCFunction)__pyx_pw_8PyClical_9index_set_32count, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_31count},
19871     {"count_neg", (PyCFunction)__pyx_pw_8PyClical_9index_set_34count_neg, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_33count_neg},
19872     {"count_pos", (PyCFunction)__pyx_pw_8PyClical_9index_set_36count_pos, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_35count_pos},
19873     {"min", (PyCFunction)__pyx_pw_8PyClical_9index_set_38min, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_37min},
19874     {"max", (PyCFunction)__pyx_pw_8PyClical_9index_set_40max, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_39max},
19875     {"hash_fn", (PyCFunction)__pyx_pw_8PyClical_9index_set_42hash_fn, METH_NOARGS,
__pyx_doc_8PyClical_9index_set_41hash_fn},
19876     {"sign_of_mult", (PyCFunction)__pyx_pw_8PyClical_9index_set_44sign_of_mult, METH_O,
__pyx_doc_8PyClical_9index_set_43sign_of_mult},
19877     {"sign_of_square", (PyCFunction)__pyx_pw_8PyClical_9index_set_46sign_of_square,
METH_NOARGS, __pyx_doc_8PyClical_9index_set_45sign_of_square},
19878     {"__reduce_cython__",
(PyCFunction)__pyx_pw_8PyClical_9index_set_52__reduce_cython__, METH_NOARGS, 0},
19879     {"__setstate_cython__",
(PyCFunction)__pyx_pw_8PyClical_9index_set_54__setstate_cython__, METH_O, 0},
19880     {0, 0, 0, 0}
19881 };
19882
19883 static PyNumberMethods __pyx_tp_as_number_index_set = {
19884     0, /*nb_add*/
19885     0, /*nb_subtract*/
19886     0, /*nb_multiply*/
19887     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19888     0, /*nb_divide*/
19889     #endif
19890     0, /*nb_remainder*/
19891     0, /*nb_divmod*/
19892     0, /*nb_power*/
19893     0, /*nb_negative*/
19894     0, /*nb_positive*/
19895     0, /*nb_absolute*/
19896     0, /*nb_nonzero*/
19897     __pyx_pw_8PyClical_9index_set_18__invert__, /*nb_invert*/
19898     0, /*nb_lshift*/
19899     0, /*nb_rshift*/
19900     __pyx_pw_8PyClical_9index_set_24__and__, /*nb_and*/
19901     __pyx_pw_8PyClical_9index_set_20__xor__, /*nb_xor*/
19902     __pyx_pw_8PyClical_9index_set_28__or__, /*nb_or*/
19903     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19904     0, /*nb_coerce*/
19905     #endif
19906     0, /*nb_int*/
19907     #if PY_MAJOR_VERSION < 3
19908     0, /*nb_long*/
19909     #else
19910     0, /*reserved*/
19911     #endif
19912     0, /*nb_float*/
19913     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19914     0, /*nb_oct*/
19915     #endif
19916     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19917     0, /*nb_hex*/
19918     #endif
19919     0, /*nb_inplace_add*/
19920     0, /*nb_inplace_subtract*/
19921     0, /*nb_inplace_multiply*/
19922     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19923     0, /*nb_inplace_divide*/
19924     #endif
19925     0, /*nb_inplace_remainder*/
19926     0, /*nb_inplace_power*/
19927     0, /*nb_inplace_lshift*/

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19928         0, /*nb_inplace_rshift*/
19929         __pyx_pw_8PyClical_9index_set_26__iand__, /*nb_inplace_and*/
19930         __pyx_pw_8PyClical_9index_set_22__ixor__, /*nb_inplace_xor*/
19931         __pyx_pw_8PyClical_9index_set_30__ior__, /*nb_inplace_or*/
19932         0, /*nb_floor_divide*/
19933         0, /*nb_true_divide*/
19934         0, /*nb_inplace_floor_divide*/
19935         0, /*nb_inplace_true_divide*/
19936         0, /*nb_index*/
19937         #if PY_VERSION_HEX >= 0x03050000
19938         0, /*nb_matrix_multiply*/
19939         #endif
19940         #if PY_VERSION_HEX >= 0x03050000
19941         0, /*nb_inplace_matrix_multiply*/
19942         #endif
19943     };
19944
19945     static PySequenceMethods __pyx_tp_as_sequence_index_set = {
19946         0, /*sq_length*/
19947         0, /*sq_concat*/
19948         0, /*sq_repeat*/
19949         __pyx_sq_item_8PyClical_index_set, /*sq_item*/
19950         0, /*sq_slice*/
19951         0, /*sq_ass_item*/
19952         0, /*sq_ass_slice*/
19953         __pyx_pw_8PyClical_9index_set_13__contains__, /*sq_contains*/
19954         0, /*sq_inplace_concat*/
19955         0, /*sq_inplace_repeat*/
19956     };
19957
19958     static PyMappingMethods __pyx_tp_as_mapping_index_set = {
19959         0, /*mp_length*/
19960         __pyx_pw_8PyClical_9index_set_11__getitem__, /*mp_subscript*/
19961         __pyx_mp_ass_subscript_8PyClical_index_set, /*mp_ass_subscript*/
19962     };
19963
19964     static PyTypeObject __pyx_type_8PyClical_index_set = {
19965         PyVarObject_HEAD_INIT(0, 0)
19966         "PyClical.index_set", /*tp_name*/
19967         sizeof(struct __pyx_obj_8PyClical_index_set), /*tp_basicsize*/
19968         0, /*tp_itemsize*/
19969         __pyx_tp_dealloc_8PyClical_index_set, /*tp_dealloc*/
19970         #if PY_VERSION_HEX < 0x030800b4
19971         0, /*tp_print*/
19972         #endif
19973         #if PY_VERSION_HEX >= 0x030800b4
19974         0, /*tp_vectorcall_offset*/
19975         #endif
19976         0, /*tp_getattr*/
19977         0, /*tp_setattr*/
19978         #if PY_MAJOR_VERSION < 3
19979         0, /*tp_compare*/
19980         #endif
19981         #if PY_MAJOR_VERSION >= 3
19982         0, /*tp_as_async*/
19983         #endif
19984         __pyx_pw_8PyClical_9index_set_48__repr__, /*tp_repr*/
19985         &__pyx_tp_as_number_index_set, /*tp_as_number*/
19986         &__pyx_tp_as_sequence_index_set, /*tp_as_sequence*/
19987         &__pyx_tp_as_mapping_index_set, /*tp_as_mapping*/
19988         0, /*tp_hash*/
19989         0, /*tp_call*/
19990         __pyx_pw_8PyClical_9index_set_50__str__, /*tp_str*/
19991         0, /*tp_getattro*/
19992         0, /*tp_setattro*/
19993         0, /*tp_as_buffer*/
19994
19995         Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
19996         /*tp_flags*/
19997
19998         "\n    Python class index_set wraps C++ class IndexSet.\n    ", /*tp_doc*/
19999         0, /*tp_traverse*/
20000         0, /*tp_clear*/
20001         __pyx_pw_8PyClical_9index_set_7__richcmp__, /*tp_richcompare*/
20002         0, /*tp_weaklistoffset*/
20003         __pyx_pw_8PyClical_9index_set_15__iter__, /*tp_iter*/
20004         0, /*tp_iternext*/
20005         __pyx_methods_8PyClical_index_set, /*tp_methods*/
20006         0, /*tp_members*/
20007         0, /*tp_getset*/
20008         0, /*tp_base*/
20009         0, /*tp_dict*/
20010         0, /*tp_descr_get*/
20011         0, /*tp_descr_set*/
20012         0, /*tp_dictoffset*/
20013         0, /*tp_init*/
20014         0, /*tp_alloc*/
20015         __pyx_tp_new_8PyClical_index_set, /*tp_new*/

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20013         0, /*tp_free*/
20014         0, /*tp_is_gc*/
20015         0, /*tp_bases*/
20016         0, /*tp_mro*/
20017         0, /*tp_cache*/
20018         0, /*tp_subclasses*/
20019         0, /*tp_weaklist*/
20020         0, /*tp_del*/
20021         0, /*tp_version_tag*/
20022         #if PY_VERSION_HEX >= 0x030400a1
20023         0, /*tp_finalize*/
20024         #endif
20025         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
>= 0x07030800)
20026         0, /*tp_vectorcall*/
20027         #endif
20028         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20029         0, /*tp_print*/
20030         #endif
20031         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20032         0, /*tp_pypy_flags*/
20033         #endif
20034     };
20035     static struct __pyx_vtabstruct_8PyClical_clifford __pyx_vtable_8PyClical_clifford;
20036
20037     PyObject *k) {
20038         struct __pyx_obj_8PyClical_clifford *p;
20039         PyObject *o;
20040         if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
20041             o = (*t->tp_alloc)(t, 0);
20042         } else {
20043             o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
20044         }
20045         if (unlikely(!o)) return 0;
20046         p = ((struct __pyx_obj_8PyClical_clifford *)o);
20047         p->__pyx_vtab = __pyx_vtabptr_8PyClical_clifford;
20048         if (unlikely(__pyx_pw_8PyClical_8clifford_3__cinit__(o, a, k) < 0)) goto bad;
20049         return o;
20050     bad:
20051         Py_DECREF(o); o = 0;
20052         return NULL;
20053     }
20054
20055     static void __pyx_tp_dealloc_8PyClical_clifford(PyObject *o) {
20056         #if CYTHON_USE_TP_FINALIZE
20057         if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
Py_TYPE(o)->tp_finalize) && (!PyType_IS_GC(Py_TYPE(o)) || !_PyGC_FINALIZED(o))) {
20058             if (PyObject_CallFinalizerFromDealloc(o)) return;
20059         }
20060         #endif
20061         {
20062             PyObject *etype, *eval, *etb;
20063             PyErr_Fetch(&etype, &eval, &etb);
20064             __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
20065             __pyx_pw_8PyClical_8clifford_5__dealloc__(o);
20066             __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
20067             PyErr_Restore(etype, eval, etb);
20068         }
20069         (*Py_TYPE(o)->tp_free)(o);
20070     }
20071     static PyObject *__pyx_sq_item_8PyClical_clifford(PyObject *o, Py_ssize_t i) {
20072         PyObject *r;
20073         PyObject *x = PyInt_FromSsize_t(i); if(!x) return 0;
20074         r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
20075         Py_DECREF(x);
20076         return r;
20077     }
20078
20079     static PyMethodDef __pyx_methods_8PyClical_clifford[] = {
20080         {"copy", (PyCFunction)__pyx_pw_8PyClical_8clifford_1copy, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_1copy},
20081         {"reframe", (PyCFunction)__pyx_pw_8PyClical_8clifford_11reframe, METH_O,
__pyx_doc_8PyClical_8clifford_10reframe},
20082         {"inv", (PyCFunction)__pyx_pw_8PyClical_8clifford_49inv, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_48inv},
20083         {"pow", (PyCFunction)__pyx_pw_8PyClical_8clifford_57pow, METH_O,
__pyx_doc_8PyClical_8clifford_56pow},
20084         {"outer_pow", (PyCFunction)__pyx_pw_8PyClical_8clifford_59outer_pow, METH_O,
__pyx_doc_8PyClical_8clifford_58outer_pow},
20085         {"scalar", (PyCFunction)__pyx_pw_8PyClical_8clifford_63scalar, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_62scalar},
20086         {"pure", (PyCFunction)__pyx_pw_8PyClical_8clifford_65pure, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_64pure},
20087         {"even", (PyCFunction)__pyx_pw_8PyClical_8clifford_67even, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_66even},
20088         {"odd", (PyCFunction)__pyx_pw_8PyClical_8clifford_69odd, METH_NOARGS,

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__pyx_doc_8PyClical_8clifford_68odd},
20089     {"vector_part",
(PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_8clifford_71vector_part,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_8clifford_70vector_part},
20090     {"involute", (PyCFunction)__pyx_pw_8PyClical_8clifford_73involute, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_72involute},
20091     {"reverse", (PyCFunction)__pyx_pw_8PyClical_8clifford_75reverse, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_74reverse},
20092     {"conj", (PyCFunction)__pyx_pw_8PyClical_8clifford_77conj, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_76conj},
20093     {"quad", (PyCFunction)__pyx_pw_8PyClical_8clifford_79quad, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_78quad},
20094     {"norm", (PyCFunction)__pyx_pw_8PyClical_8clifford_81norm, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_80norm},
20095     {"abs", (PyCFunction)__pyx_pw_8PyClical_8clifford_83abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_82abs},
20096     {"max_abs", (PyCFunction)__pyx_pw_8PyClical_8clifford_85max_abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_84max_abs},
20097     {"truncated", (PyCFunction)__pyx_pw_8PyClical_8clifford_87truncated, METH_O,
__pyx_doc_8PyClical_8clifford_86truncated},
20098     {"isinf", (PyCFunction)__pyx_pw_8PyClical_8clifford_89isinf, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_88isinf},
20099     {"isnan", (PyCFunction)__pyx_pw_8PyClical_8clifford_91isnan, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_90isnan},
20100     {"frame", (PyCFunction)__pyx_pw_8PyClical_8clifford_93frame, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_92frame},
20101     {"__reduce_cython__", (PyCFunction)__pyx_pw_8PyClical_8clifford_99__reduce_cython__,
METH_NOARGS, 0},
20102     {"__setstate_cython__",
(PyCFunction)__pyx_pw_8PyClical_8clifford_101__setstate_cython__, METH_O, 0),
20103     {0, 0, 0, 0}
20104 };
20105
20106 static PyNumberMethods __pyx_tp_as_number_clifford = {
20107     __pyx_pw_8PyClical_8clifford_21__add__, /*nb_add*/
20108     __pyx_pw_8PyClical_8clifford_25__sub__, /*nb_subtract*/
20109     __pyx_pw_8PyClical_8clifford_29__mul__, /*nb_multiply*/
20110     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20111         0, /*nb_divide*/
20112     #endif
20113     __pyx_pw_8PyClical_8clifford_33__mod__, /*nb_remainder*/
20114     0, /*nb_divmod*/
20115     __pyx_pw_8PyClical_8clifford_55__pow__, /*nb_power*/
20116     __pyx_pw_8PyClical_8clifford_17__neg__, /*nb_negative*/
20117     __pyx_pw_8PyClical_8clifford_19__pos__, /*nb_positive*/
20118     0, /*nb_absolute*/
20119     0, /*nb_nonzero*/
20120     0, /*nb_invert*/
20121     0, /*nb_lshift*/
20122     0, /*nb_rshift*/
20123     __pyx_pw_8PyClical_8clifford_37__and__, /*nb_and*/
20124     __pyx_pw_8PyClical_8clifford_41__xor__, /*nb_xor*/
20125     __pyx_pw_8PyClical_8clifford_51__or__, /*nb_or*/
20126     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20127         0, /*nb_coerce*/
20128     #endif
20129     0, /*nb_int*/
20130     #if PY_MAJOR_VERSION < 3
20131         0, /*nb_long*/
20132     #else
20133         0, /*reserved*/
20134     #endif
20135     0, /*nb_float*/
20136     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20137         0, /*nb_oct*/
20138     #endif
20139     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20140         0, /*nb_hex*/
20141     #endif
20142     __pyx_pw_8PyClical_8clifford_23__iadd__, /*nb_inplace_add*/
20143     __pyx_pw_8PyClical_8clifford_27__isub__, /*nb_inplace_subtract*/
20144     __pyx_pw_8PyClical_8clifford_31__imul__, /*nb_inplace_multiply*/
20145     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20146         __pyx_pw_8PyClical_8clifford_47__idiv__, /*nb_inplace_divide*/
20147     #endif
20148     __pyx_pw_8PyClical_8clifford_35__imod__, /*nb_inplace_remainder*/
20149     0, /*nb_inplace_power*/
20150     0, /*nb_inplace_lshift*/
20151     0, /*nb_inplace_rshift*/
20152     __pyx_pw_8PyClical_8clifford_39__iand__, /*nb_inplace_and*/
20153     __pyx_pw_8PyClical_8clifford_43__ixor__, /*nb_inplace_xor*/
20154     __pyx_pw_8PyClical_8clifford_53__ior__, /*nb_inplace_or*/

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20155         0, /*nb_floor_divide*/
20156         __pyx_pw_8PyClical_8clifford_45_truediv__, /*nb_true_divide*/
20157         0, /*nb_inplace_floor_divide*/
20158         0, /*nb_inplace_true_divide*/
20159         0, /*nb_index*/
20160         #if PY_VERSION_HEX >= 0x03050000
20161         0, /*nb_matrix_multiply*/
20162         #endif
20163         #if PY_VERSION_HEX >= 0x03050000
20164         0, /*nb_inplace_matrix_multiply*/
20165         #endif
20166     };
20167
20168     static PySequenceMethods __pyx_tp_as_sequence_clifford = {
20169         0, /*sq_length*/
20170         0, /*sq_concat*/
20171         0, /*sq_repeat*/
20172         __pyx_sq_item_8PyClical_clifford, /*sq_item*/
20173         0, /*sq_slice*/
20174         0, /*sq_ass_item*/
20175         0, /*sq_ass_slice*/
20176         __pyx_pw_8PyClical_8clifford_7_contains__, /*sq_contains*/
20177         0, /*sq_inplace_concat*/
20178         0, /*sq_inplace_repeat*/
20179     };
20180
20181     static PyMappingMethods __pyx_tp_as_mapping_clifford = {
20182         0, /*mp_length*/
20183         __pyx_pw_8PyClical_8clifford_15_getitem__, /*mp_subscript*/
20184         0, /*mp_ass_subscript*/
20185     };
20186
20187     static PyTypeObject __pyx_type_8PyClical_clifford = {
20188         PyVarObject_HEAD_INIT(0, 0)
20189         "PyClical.clifford", /*tp_name*/
20190         sizeof(struct __pyx_obj_8PyClical_clifford), /*tp_basicsize*/
20191         0, /*tp_itemsize*/
20192         __pyx_tp_dealloc_8PyClical_clifford, /*tp_dealloc*/
20193         #if PY_VERSION_HEX < 0x030800b4
20194         0, /*tp_print*/
20195         #endif
20196         #if PY_VERSION_HEX >= 0x030800b4
20197         0, /*tp_vectorcall_offset*/
20198         #endif
20199         0, /*tp_getattr*/
20200         0, /*tp_setattr*/
20201         #if PY_MAJOR_VERSION < 3
20202         0, /*tp_compare*/
20203         #endif
20204         #if PY_MAJOR_VERSION >= 3
20205         0, /*tp_as_async*/
20206         #endif
20207         __pyx_pw_8PyClical_8clifford_95_repr__, /*tp_repr*/
20208         &__pyx_tp_as_number_clifford, /*tp_as_number*/
20209         &__pyx_tp_as_sequence_clifford, /*tp_as_sequence*/
20210         &__pyx_tp_as_mapping_clifford, /*tp_as_mapping*/
20211         0, /*tp_hash*/
20212         __pyx_pw_8PyClical_8clifford_61_call__, /*tp_call*/
20213         __pyx_pw_8PyClical_8clifford_97_str__, /*tp_str*/
20214         0, /*tp_getattro*/
20215         0, /*tp_setattro*/
20216         0, /*tp_as_buffer*/
20217
20218         Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
20219         /*tp_flags*/
20220         "\n    Python class clifford wraps C++ class Clifford.\n    ", /*tp_doc*/
20221         0, /*tp_traverse*/
20222         0, /*tp_clear*/
20223         __pyx_pw_8PyClical_8clifford_13_richcmp__, /*tp_richcompare*/
20224         0, /*tp_weaklistoffset*/
20225         __pyx_pw_8PyClical_8clifford_9_iter__, /*tp_iter*/
20226         0, /*tp_iternext*/
20227         __pyx_methods_8PyClical_clifford, /*tp_methods*/
20228         0, /*tp_members*/
20229         0, /*tp_getset*/
20230         0, /*tp_base*/
20231         0, /*tp_dict*/
20232         0, /*tp_descr_get*/
20233         0, /*tp_descr_set*/
20234         0, /*tp_dictoffset*/
20235         0, /*tp_init*/
20236         0, /*tp_alloc*/
20237         __pyx_tp_new_8PyClical_clifford, /*tp_new*/
20238         0, /*tp_free*/
20239         0, /*tp_is_gc*/
20240         0, /*tp_bases*/
20241         0, /*tp_mro*/

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20240         0, /*tp_cache*/
20241         0, /*tp_subclasses*/
20242         0, /*tp_weaklist*/
20243         0, /*tp_del*/
20244         0, /*tp_version_tag*/
20245         #if PY_VERSION_HEX >= 0x030400a1
20246         0, /*tp_finalize*/
20247         #endif
20248         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
    >= 0x07030800)
20249         0, /*tp_vectorcall*/
20250         #endif
20251         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20252         0, /*tp_print*/
20253         #endif
20254         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20255         0, /*tp_pypy_flags*/
20256         #endif
20257     };
20258
20259     static struct __pyx_obj_8PyClical__pyx_scope_struct____iter__
    *__pyx_freelist_8PyClical__pyx_scope_struct____iter__[8];
20260     static int __pyx_freecount_8PyClical__pyx_scope_struct____iter__ = 0;
20261
20262     static PyObject * __pyx_tp_new_8PyClical__pyx_scope_struct____iter__(PyTypeObject *t,
    CYTHON_UNUSED PyObject *a, CYTHON_UNUSED PyObject *k) {
20263         PyObject *o;
20264         if (CYTHON_COMPILING_IN_CPYTHON &&
    likely((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ > 0) & (t->tp_basicsize ==
    sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20265             o =
    (PyObject*)__pyx_freelist_8PyClical__pyx_scope_struct____iter__[--__pyx_freecount_8PyClical__pyx_scope_struct____iter__];
20266             memset(o, 0, sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__));
20267             (void) PyObject_INIT(o, t);
20268             PyObject_GC_Track(o);
20269         } else {
20270             o = (*t->tp_alloc)(t, 0);
20271             if (unlikely(!o)) return 0;
20272         }
20273         return o;
20274     }
20275
20276     static void __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__(PyObject *o) {
20277         struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
    __pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20278         PyObject_GC_UnTrack(o);
20279         Py_CLEAR(p->__pyx_v_idx);
20280         Py_CLEAR(p->__pyx_v_self);
20281         Py_CLEAR(p->__pyx_t_0);
20282         if (CYTHON_COMPILING_IN_CPYTHON &&
    ((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ < 8) & (Py_TYPE(o)->tp_basicsize ==
    sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20283             __pyx_freelist_8PyClical__pyx_scope_struct____iter__[__pyx_freecount_8PyClical__pyx_scope_struct____iter__++]
    = ((struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o);
20284         } else {
20285             (*Py_TYPE(o)->tp_free)(o);
20286         }
20287     }
20288
20289     static int __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__(PyObject *o,
    visitproc v, void *a) {
20290         int e;
20291         struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
    __pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20292         if (p->__pyx_v_idx) {
20293             e = (*v)(p->__pyx_v_idx, a); if (e) return e;
20294         }
20295         if (p->__pyx_v_self) {
20296             e = (*v)((PyObject *)p->__pyx_v_self, a); if (e) return e;
20297         }
20298         if (p->__pyx_t_0) {
20299             e = (*v)(p->__pyx_t_0, a); if (e) return e;
20300         }
20301         return 0;
20302     }
20303
20304     static PyTypeObject __pyx_type_8PyClical__pyx_scope_struct____iter__ = {
20305         PyVarObject_HEAD_INIT(0, 0)
20306         "PyClical.__pyx_scope_struct____iter__", /*tp_name*/
20307         sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__), /*tp_basicsize*/
20308         0, /*tp_itemsize*/
20309         __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__, /*tp_dealloc*/
20310         #if PY_VERSION_HEX < 0x030800b4
20311         0, /*tp_print*/
20312         #endif
20313         #if PY_VERSION_HEX >= 0x030800b4

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20314         0, /*tp_vectorcall_offset*/
20315         #endif
20316         0, /*tp_getattr*/
20317         0, /*tp_setattr*/
20318         #if PY_MAJOR_VERSION < 3
20319         0, /*tp_compare*/
20320         #endif
20321         #if PY_MAJOR_VERSION >= 3
20322         0, /*tp_as_async*/
20323         #endif
20324         0, /*tp_repr*/
20325         0, /*tp_as_number*/
20326         0, /*tp_as_sequence*/
20327         0, /*tp_as_mapping*/
20328         0, /*tp_hash*/
20329         0, /*tp_call*/
20330         0, /*tp_str*/
20331         0, /*tp_getattro*/
20332         0, /*tp_setattro*/
20333         0, /*tp_as_buffer*/
20334
Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_HAVE_GC,
/*tp_flags*/

20335         0, /*tp_doc*/
20336         __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__, /*tp_traverse*/
20337         0, /*tp_clear*/
20338         0, /*tp_richcompare*/
20339         0, /*tp_weaklistoffset*/
20340         0, /*tp_iter*/
20341         0, /*tp_iternext*/
20342         0, /*tp_methods*/
20343         0, /*tp_members*/
20344         0, /*tp_getset*/
20345         0, /*tp_base*/
20346         0, /*tp_dict*/
20347         0, /*tp_descr_get*/
20348         0, /*tp_descr_set*/
20349         0, /*tp_dictoffset*/
20350         0, /*tp_init*/
20351         0, /*tp_alloc*/
20352         __pyx_tp_new_8PyClical__pyx_scope_struct____iter__, /*tp_new*/
20353         0, /*tp_free*/
20354         0, /*tp_is_gc*/
20355         0, /*tp_bases*/
20356         0, /*tp_mro*/
20357         0, /*tp_cache*/
20358         0, /*tp_subclasses*/
20359         0, /*tp_weaklist*/
20360         0, /*tp_del*/
20361         0, /*tp_version_tag*/
20362         #if PY_VERSION_HEX >= 0x030400a1
20363         0, /*tp_finalize*/
20364         #endif
20365         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
>= 0x07030800)
20366         0, /*tp_vectorcall*/
20367         #endif
20368         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20369         0, /*tp_print*/
20370         #endif
20371         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20372         0, /*tp_pypy_flags*/
20373         #endif
20374     };
20375
20376     static PyMethodDef __pyx_methods[] = {
20377         {"compare",
(PyCFunction)(void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_3compare, METH_VARARGS|METH_KEYWORDS,
__pyx_doc_8PyClical_2compare},
20378         {"min_neg", (PyCFunction) __pyx_pw_8PyClical_5min_neg, METH_O,
__pyx_doc_8PyClical_4min_neg},
20379         {"max_pos", (PyCFunction) __pyx_pw_8PyClical_7max_pos, METH_O,
__pyx_doc_8PyClical_6max_pos},
20380         {"error_squared_tol", (PyCFunction) __pyx_pw_8PyClical_11error_squared_tol, METH_O,
__pyx_doc_8PyClical_10error_squared_tol},
20381         {"error_squared",
(PyCFunction)(void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_13error_squared,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_12error_squared},
20382         {"approx_equal",
(PyCFunction)(void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_15approx_equal,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_14approx_equal},
20383         {"inv", (PyCFunction) __pyx_pw_8PyClical_17inv, METH_O, __pyx_doc_8PyClical_16inv},
20384         {"scalar", (PyCFunction) __pyx_pw_8PyClical_19scalar, METH_O,
__pyx_doc_8PyClical_18scalar},
20385         {"real", (PyCFunction) __pyx_pw_8PyClical_21real, METH_O,
__pyx_doc_8PyClical_20real},
20386         {"imag", (PyCFunction) __pyx_pw_8PyClical_23imag, METH_O,

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__pyx_doc_8PyClical_22imag},
20387     {"pure", (PyCFunction)__pyx_pw_8PyClical_25pure, METH_O,
__pyx_doc_8PyClical_24pure},
20388     {"even", (PyCFunction)__pyx_pw_8PyClical_27even, METH_O,
__pyx_doc_8PyClical_26even},
20389     {"odd", (PyCFunction)__pyx_pw_8PyClical_29odd, METH_O, __pyx_doc_8PyClical_28odd},
20390     {"involute", (PyCFunction)__pyx_pw_8PyClical_31involute, METH_O,
__pyx_doc_8PyClical_30involute},
20391     {"reverse", (PyCFunction)__pyx_pw_8PyClical_33reverse, METH_O,
__pyx_doc_8PyClical_32reverse},
20392     {"conj", (PyCFunction)__pyx_pw_8PyClical_35conj, METH_O,
__pyx_doc_8PyClical_34conj},
20393     {"quad", (PyCFunction)__pyx_pw_8PyClical_37quad, METH_O,
__pyx_doc_8PyClical_36quad},
20394     {"norm", (PyCFunction)__pyx_pw_8PyClical_39norm, METH_O,
__pyx_doc_8PyClical_38norm},
20395     {"abs", (PyCFunction)__pyx_pw_8PyClical_41abs, METH_O, __pyx_doc_8PyClical_40abs},
20396     {"max_abs", (PyCFunction)__pyx_pw_8PyClical_43max_abs, METH_O,
__pyx_doc_8PyClical_42max_abs},
20397     {"pow", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_45pow,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_44pow},
20398     {"outer_pow",
(PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_47outer_pow,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_46outer_pow},
20399     {"complexifier", (PyCFunction)__pyx_pw_8PyClical_49complexifier, METH_O,
__pyx_doc_8PyClical_48complexifier},
20400     {"sqrt", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_51sqrt,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_50sqrt},
20401     {"exp", (PyCFunction)__pyx_pw_8PyClical_53exp, METH_O, __pyx_doc_8PyClical_52exp},
20402     {"log", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_55log,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_54log},
20403     {"cos", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_57cos,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_56cos},
20404     {"acos", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_59acos,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_58acos},
20405     {"cosh", (PyCFunction)__pyx_pw_8PyClical_61cosh, METH_O,
__pyx_doc_8PyClical_60cosh},
20406     {"acosh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_63acosh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_62acosh},
20407     {"sin", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_65sin,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_64sin},
20408     {"asin", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_67asin,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_66asin},
20409     {"sinh", (PyCFunction)__pyx_pw_8PyClical_69sinh, METH_O,
__pyx_doc_8PyClical_68sinh},
20410     {"asinh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_71asinh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_70asinh},
20411     {"tan", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_73tan,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_72tan},
20412     {"atan", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_75atan,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_74atan},
20413     {"tanh", (PyCFunction)__pyx_pw_8PyClical_77tanh, METH_O,
__pyx_doc_8PyClical_76tanh},
20414     {"atanh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_79atanh,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_78atanh},
20415     {"random_clifford",
(PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_81random_clifford,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_80random_clifford},
20416     {"cga3", (PyCFunction)__pyx_pw_8PyClical_83cga3, METH_O,
__pyx_doc_8PyClical_82cga3},
20417     {"cga3std", (PyCFunction)__pyx_pw_8PyClical_85cga3std, METH_O,
__pyx_doc_8PyClical_84cga3std},
20418     {"agc3", (PyCFunction)__pyx_pw_8PyClical_87agc3, METH_O,
__pyx_doc_8PyClical_86agc3},
20419     {0, 0, 0, 0}
20420 };
20421
20422     #if PY_MAJOR_VERSION >= 3
20423     #if CYTHON_PEP489_MULTI_PHASE_INIT
20424     static PyObject* __pyx_pymod_create(PyObject *spec, PyModuleDef *def); /*proto*/
20425     static int __pyx_pymod_exec_PyClical(PyObject* module); /*proto*/
20426     static PyModuleDef_Slot __pyx_moduledef_slots[] = {
20427         {Py_mod_create, (void*)__pyx_pymod_create},
20428         {Py_mod_exec, (void*)__pyx_pymod_exec_PyClical},
20429         {0, NULL}
20430     };
20431     #endif
20432
20433     static struct PyModuleDef __pyx_moduledef = {
20434         PyModuleDef_HEAD_INIT,
20435         "PyClical",
20436         0, /* m_doc */
20437         #if CYTHON_PEP489_MULTI_PHASE_INIT
20438         0, /* m_size */
20439         #else
20440         -1, /* m_size */
20441         #endif

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20442         __pyx_methods /* m_methods */,
20443         #if CYTHON_PEP489_MULTI_PHASE_INIT
20444         __pyx_moduledef_slots, /* m_slots */
20445         #else
20446         NULL, /* m_reload */
20447         #endif
20448         NULL, /* m_traverse */
20449         NULL, /* m_clear */
20450         NULL /* m_free */
20451     };
20452     #endif
20453     #ifndef CYTHON_SMALL_CODE
20454     #if defined(__clang__)
20455         #define CYTHON_SMALL_CODE
20456     #elif defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 3))
20457         #define CYTHON_SMALL_CODE __attribute__((cold))
20458     #else
20459         #define CYTHON_SMALL_CODE
20460     #endif
20461     #endif
20462
20463     static __Pyx_StringTabEntry __pyx_string_tab[] = {
20464         {&__pyx_kp_u_, __pyx_k_, sizeof(__pyx_k_), 0, 1, 0, 0},
20465         {&__pyx_kp_u_Abbreviation_for_clifford_index, __pyx_k_Abbreviation_for_clifford_index,
20466         sizeof(__pyx_k_Abbreviation_for_clifford_index), 0, 1, 0, 0},
20467         {&__pyx_kp_u_Abbreviation_for_index_set_q_p, __pyx_k_Abbreviation_for_index_set_q_p,
20468         sizeof(__pyx_k_Abbreviation_for_index_set_q_p), 0, 1, 0, 0},
20469         {&__pyx_kp_u_Absolute_value_of_multivector_m, __pyx_k_Absolute_value_of_multivector_m,
20470         sizeof(__pyx_k_Absolute_value_of_multivector_m), 0, 1, 0, 0},
20471         {&__pyx_kp_u_Absolute_value_square_root_of_n, __pyx_k_Absolute_value_square_root_of_n,
20472         sizeof(__pyx_k_Absolute_value_square_root_of_n), 0, 1, 0, 0},
20473         {&__pyx_kp_u_Cannot_initialize_clifford_objec, __pyx_k_Cannot_initialize_clifford_objec,
20474         sizeof(__pyx_k_Cannot_initialize_clifford_objec), 0, 1, 0, 0},
20475         {&__pyx_kp_u_Cannot_initialize_index_set_obje, __pyx_k_Cannot_initialize_index_set_obje,
20476         sizeof(__pyx_k_Cannot_initialize_index_set_obje), 0, 1, 0, 0},
20477         {&__pyx_kp_u_Cannot_reframe, __pyx_k_Cannot_reframe, sizeof(__pyx_k_Cannot_reframe), 0, 1, 0, 0},
20478         {&__pyx_kp_u_Cannot_take_vector_part_of, __pyx_k_Cannot_take_vector_part_of,
20479         sizeof(__pyx_k_Cannot_take_vector_part_of), 0, 1, 0, 0},
20480         {&__pyx_kp_u_Cardinality_Number_of_indices_i, __pyx_k_Cardinality_Number_of_indices_i,
20481         sizeof(__pyx_k_Cardinality_Number_of_indices_i), 0, 1, 0, 0},
20482         {&__pyx_kp_u_Check_if_a_multivector_contains, __pyx_k_Check_if_a_multivector_contains,
20483         sizeof(__pyx_k_Check_if_a_multivector_contains), 0, 1, 0, 0},
20484         {&__pyx_kp_u_Check_if_a_multivector_contains_2, __pyx_k_Check_if_a_multivector_contains_2,
20485         sizeof(__pyx_k_Check_if_a_multivector_contains_2), 0, 1, 0, 0},
20486         {&__pyx_kp_u_Conjugation_reverse_o_involute, __pyx_k_Conjugation_reverse_o_involute,
20487         sizeof(__pyx_k_Conjugation_reverse_o_involute), 0, 1, 0, 0},
20488         {&__pyx_kp_u_Conjugation_reverse_o_involute_2, __pyx_k_Conjugation_reverse_o_involute_2,
20489         sizeof(__pyx_k_Conjugation_reverse_o_involute_2), 0, 1, 0, 0},
20490         {&__pyx_kp_u_Contraction_print_clifford_l_cl, __pyx_k_Contraction_print_clifford_l_cl,
20491         sizeof(__pyx_k_Contraction_print_clifford_l_cl), 0, 1, 0, 0},
20492         {&__pyx_kp_u_Contraction_x_clifford_l_x_clif, __pyx_k_Contraction_x_clifford_l_x_clif,
20493         sizeof(__pyx_k_Contraction_x_clifford_l_x_clif), 0, 1, 0, 0},
20494         {&__pyx_kp_u_Convert_CGA3_null_vector_to_Euc, __pyx_k_Convert_CGA3_null_vector_to_Euc,
20495         sizeof(__pyx_k_Convert_CGA3_null_vector_to_Euc), 0, 1, 0, 0},
20496         {&__pyx_kp_u_Convert_CGA3_null_vector_to_sta, __pyx_k_Convert_CGA3_null_vector_to_sta,
20497         sizeof(__pyx_k_Convert_CGA3_null_vector_to_sta), 0, 1, 0, 0},
20498         {&__pyx_kp_u_Convert_Euclidean_3D_multivecto, __pyx_k_Convert_Euclidean_3D_multivecto,
20499         sizeof(__pyx_k_Convert_Euclidean_3D_multivecto), 0, 1, 0, 0},
20500         {&__pyx_kp_u_Copy_this_clifford_object_x_cli, __pyx_k_Copy_this_clifford_object_x_cli,
20501         sizeof(__pyx_k_Copy_this_clifford_object_x_cli), 0, 1, 0, 0},
20502         {&__pyx_kp_u_Copy_this_index_set_object_s_in, __pyx_k_Copy_this_index_set_object_s_in,
20503         sizeof(__pyx_k_Copy_this_index_set_object_s_in), 0, 1, 0, 0},
20504         {&__pyx_kp_u_Cosine_of_multivector_with_opti, __pyx_k_Cosine_of_multivector_with_opti,
20505         sizeof(__pyx_k_Cosine_of_multivector_with_opti), 0, 1, 0, 0},
20506         {&__pyx_kp_u_Even_part_of_multivector_sum_of, __pyx_k_Even_part_of_multivector_sum_of,
20507         sizeof(__pyx_k_Even_part_of_multivector_sum_of), 0, 1, 0, 0},
20508         {&__pyx_kp_u_Even_part_of_multivector_sum_of_2, __pyx_k_Even_part_of_multivector_sum_of_2,
20509         sizeof(__pyx_k_Even_part_of_multivector_sum_of_2), 0, 1, 0, 0},
20510         {&__pyx_kp_u_Exponential_of_multivector_x_cl, __pyx_k_Exponential_of_multivector_x_cl,
20511         sizeof(__pyx_k_Exponential_of_multivector_x_cl), 0, 1, 0, 0},
20512         {&__pyx_kp_u_Geometric_difference_print_clif, __pyx_k_Geometric_difference_print_clif,
20513         sizeof(__pyx_k_Geometric_difference_print_clif), 0, 1, 0, 0},
20514         {&__pyx_kp_u_Geometric_difference_x_clifford, __pyx_k_Geometric_difference_x_clifford,
20515         sizeof(__pyx_k_Geometric_difference_x_clifford), 0, 1, 0, 0},
20516         {&__pyx_kp_u_Geometric_multiplicative_invers, __pyx_k_Geometric_multiplicative_invers,
20517         sizeof(__pyx_k_Geometric_multiplicative_invers), 0, 1, 0, 0},
20518         {&__pyx_kp_u_Geometric_multiplicative_invers_2, __pyx_k_Geometric_multiplicative_invers_2,
20519         sizeof(__pyx_k_Geometric_multiplicative_invers_2), 0, 1, 0, 0},
20520         {&__pyx_kp_u_Geometric_product_print_cliffor, __pyx_k_Geometric_product_print_cliffor,
20521         sizeof(__pyx_k_Geometric_product_print_cliffor), 0, 1, 0, 0},
20522         {&__pyx_kp_u_Geometric_product_x_clifford_2, __pyx_k_Geometric_product_x_clifford_2,
20523         sizeof(__pyx_k_Geometric_product_x_clifford_2), 0, 1, 0, 0},
20524         {&__pyx_kp_u_Geometric_quotient_print_cliffo, __pyx_k_Geometric_quotient_print_cliffo,
20525         sizeof(__pyx_k_Geometric_quotient_print_cliffo), 0, 1, 0, 0},
20526         {&__pyx_kp_u_Geometric_quotient_x_clifford_l, __pyx_k_Geometric_quotient_x_clifford_l,
20527         sizeof(__pyx_k_Geometric_quotient_x_clifford_l), 0, 1, 0, 0},
20528         {&__pyx_kp_u_Geometric_sum_print_clifford_l, __pyx_k_Geometric_sum_print_clifford_l,

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sizeof(__pyx_k_Geometric_sum_print_clifford_l), 0, 1, 0, 0),
20498 {&__pyx_kp_u_Geometric_sum_x_clifford_l_x_cl, __pyx_k_Geometric_sum_x_clifford_l_x_cl,
sizeof(__pyx_k_Geometric_sum_x_clifford_l_x_cl), 0, 1, 0, 0),
20499 {&__pyx_kp_u_Get_the_value_of_an_index_set_o, __pyx_k_Get_the_value_of_an_index_set_o,
sizeof(__pyx_k_Get_the_value_of_an_index_set_o), 0, 1, 0, 0),
20500 {&__pyx_kp_u_Hyperbolic_cosine_of_multivecto, __pyx_k_Hyperbolic_cosine_of_multivecto,
sizeof(__pyx_k_Hyperbolic_cosine_of_multivecto), 0, 1, 0, 0),
20501 {&__pyx_kp_u_Hyperbolic_sine_of_multivector, __pyx_k_Hyperbolic_sine_of_multivector,
sizeof(__pyx_k_Hyperbolic_sine_of_multivector), 0, 1, 0, 0),
20502 {&__pyx_kp_u_Hyperbolic_tangent_of_multivect, __pyx_k_Hyperbolic_tangent_of_multivect,
sizeof(__pyx_k_Hyperbolic_tangent_of_multivect), 0, 1, 0, 0),
20503 {&__pyx_kp_u_Imaginary_part_deprecated_alway, __pyx_k_Imaginary_part_deprecated_alway,
sizeof(__pyx_k_Imaginary_part_deprecated_alway), 0, 1, 0, 0),
20504 {&__pyx_n_s_IndexError, __pyx_k_IndexError, sizeof(__pyx_k_IndexError), 0, 0, 1, 1),
20505 {&__pyx_kp_u_Inner_product_print_clifford_l, __pyx_k_Inner_product_print_clifford_l,
sizeof(__pyx_k_Inner_product_print_clifford_l), 0, 1, 0, 0),
20506 {&__pyx_kp_u_Inner_product_x_clifford_l_x_cl, __pyx_k_Inner_product_x_clifford_l_x_cl,
sizeof(__pyx_k_Inner_product_x_clifford_l_x_cl), 0, 1, 0, 0),
20507 {&__pyx_kp_u_Integer_power_of_multivector_ob, __pyx_k_Integer_power_of_multivector_ob,
sizeof(__pyx_k_Integer_power_of_multivector_ob), 0, 1, 0, 0),
20508 {&__pyx_n_s_Integral, __pyx_k_Integral, sizeof(__pyx_k_Integral), 0, 0, 1, 1),
20509 {&__pyx_kp_u_Inverse_cosine_of_multivector_w, __pyx_k_Inverse_cosine_of_multivector_w,
sizeof(__pyx_k_Inverse_cosine_of_multivector_w), 0, 1, 0, 0),
20510 {&__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu, __pyx_k_Inverse_hyperbolic_cosine_of_mu,
sizeof(__pyx_k_Inverse_hyperbolic_cosine_of_mu), 0, 1, 0, 0),
20511 {&__pyx_kp_u_Inverse_hyperbolic_sine_of_mult, __pyx_k_Inverse_hyperbolic_sine_of_mult,
sizeof(__pyx_k_Inverse_hyperbolic_sine_of_mult), 0, 1, 0, 0),
20512 {&__pyx_kp_u_Inverse_hyperbolic_tangent_of_m, __pyx_k_Inverse_hyperbolic_tangent_of_m,
sizeof(__pyx_k_Inverse_hyperbolic_tangent_of_m), 0, 1, 0, 0),
20513 {&__pyx_kp_u_Inverse_sine_of_multivector_wit, __pyx_k_Inverse_sine_of_multivector_wit,
sizeof(__pyx_k_Inverse_sine_of_multivector_wit), 0, 1, 0, 0),
20514 {&__pyx_kp_u_Inverse_tangent_of_multivector, __pyx_k_Inverse_tangent_of_multivector,
sizeof(__pyx_k_Inverse_tangent_of_multivector), 0, 1, 0, 0),
20515 {&__pyx_kp_u_Iterate_over_the_indices_of_an, __pyx_k_Iterate_over_the_indices_of_an,
sizeof(__pyx_k_Iterate_over_the_indices_of_an), 0, 1, 0, 0),
20516 {&__pyx_kp_u_Main_involution_each_i_is_repla, __pyx_k_Main_involution_each_i_is_repla,
sizeof(__pyx_k_Main_involution_each_i_is_repla), 0, 1, 0, 0),
20517 {&__pyx_kp_u_Main_involution_each_i_is_repla_2, __pyx_k_Main_involution_each_i_is_repla_2,
sizeof(__pyx_k_Main_involution_each_i_is_repla_2), 0, 1, 0, 0),
20518 {&__pyx_kp_u_Maximum_absolute_value_of_coord, __pyx_k_Maximum_absolute_value_of_coord,
sizeof(__pyx_k_Maximum_absolute_value_of_coord), 0, 1, 0, 0),
20519 {&__pyx_kp_u_Maximum_member_index_set_l_l_2, __pyx_k_Maximum_member_index_set_l_l_2,
sizeof(__pyx_k_Maximum_member_index_set_l_l_2), 0, 1, 0, 0),
20520 {&__pyx_kp_u_Maximum_of_absolute_values_of_c, __pyx_k_Maximum_of_absolute_values_of_c,
sizeof(__pyx_k_Maximum_of_absolute_values_of_c), 0, 1, 0, 0),
20521 {&__pyx_kp_u_Maximum_positive_index_or_0_if, __pyx_k_Maximum_positive_index_or_0_if,
sizeof(__pyx_k_Maximum_positive_index_or_0_if), 0, 1, 0, 0),
20522 {&__pyx_kp_u_Minimum_member_index_set_l_l_2, __pyx_k_Minimum_member_index_set_l_l_2,
sizeof(__pyx_k_Minimum_member_index_set_l_l_2), 0, 1, 0, 0),
20523 {&__pyx_kp_u_Minimum_negative_index_or_0_if, __pyx_k_Minimum_negative_index_or_0_if,
sizeof(__pyx_k_Minimum_negative_index_or_0_if), 0, 1, 0, 0),
20524 {&__pyx_kp_u_Natural_logarithm_of_multivecto, __pyx_k_Natural_logarithm_of_multivecto,
sizeof(__pyx_k_Natural_logarithm_of_multivecto), 0, 1, 0, 0),
20525 {&__pyx_kp_u_Norm_sum_of_squares_of_coordina, __pyx_k_Norm_sum_of_squares_of_coordina,
sizeof(__pyx_k_Norm_sum_of_squares_of_coordina), 0, 1, 0, 0),
20526 {&__pyx_n_s_NotImplemented, __pyx_k_NotImplemented, sizeof(__pyx_k_NotImplemented), 0, 0, 1, 1),
20527 {&__pyx_kp_u_Not_applicable, __pyx_k_Not_applicable, sizeof(__pyx_k_Not_applicable), 0, 1, 0, 0),
20528 {&__pyx_kp_u_Not_applicable_for_a_in_cliffor, __pyx_k_Not_applicable_for_a_in_cliffor,
sizeof(__pyx_k_Not_applicable_for_a_in_cliffor), 0, 1, 0, 0),
20529 {&__pyx_kp_u_Number_of_negative_indices_incl, __pyx_k_Number_of_negative_indices_incl,
sizeof(__pyx_k_Number_of_negative_indices_incl), 0, 1, 0, 0),
20530 {&__pyx_kp_u_Number_of_positive_indices_incl, __pyx_k_Number_of_positive_indices_incl,
sizeof(__pyx_k_Number_of_positive_indices_incl), 0, 1, 0, 0),
20531 {&__pyx_kp_u_Odd_part_of_multivector_sum_of, __pyx_k_Odd_part_of_multivector_sum_of,
sizeof(__pyx_k_Odd_part_of_multivector_sum_of), 0, 1, 0, 0),
20532 {&__pyx_kp_u_Odd_part_of_multivector_sum_of_2, __pyx_k_Odd_part_of_multivector_sum_of_2,
sizeof(__pyx_k_Odd_part_of_multivector_sum_of_2), 0, 1, 0, 0),
20533 {&__pyx_kp_u_Outer_product_power_of_multivec, __pyx_k_Outer_product_power_of_multivec,
sizeof(__pyx_k_Outer_product_power_of_multivec), 0, 1, 0, 0),
20534 {&__pyx_kp_u_Outer_product_power_x_clifford, __pyx_k_Outer_product_power_x_clifford,
sizeof(__pyx_k_Outer_product_power_x_clifford), 0, 1, 0, 0),
20535 {&__pyx_kp_u_Outer_product_print_clifford_l, __pyx_k_Outer_product_print_clifford_l,
sizeof(__pyx_k_Outer_product_print_clifford_l), 0, 1, 0, 0),
20536 {&__pyx_kp_u_Outer_product_x_clifford_l_x_cl, __pyx_k_Outer_product_x_clifford_l_x_cl,
sizeof(__pyx_k_Outer_product_x_clifford_l_x_cl), 0, 1, 0, 0),
20537 {&__pyx_kp_u_Power_self_to_the_m_x_clifford, __pyx_k_Power_self_to_the_m_x_clifford,
sizeof(__pyx_k_Power_self_to_the_m_x_clifford), 0, 1, 0, 0),
20538 {&__pyx_kp_u_Power_self_to_the_m_x_clifford_2, __pyx_k_Power_self_to_the_m_x_clifford_2,
sizeof(__pyx_k_Power_self_to_the_m_x_clifford_2), 0, 1, 0, 0),
20539 {&__pyx_kp_u_Pure_grade_vector_part_print_cl, __pyx_k_Pure_grade_vector_part_print_cl,
sizeof(__pyx_k_Pure_grade_vector_part_print_cl), 0, 1, 0, 0),
20540 {&__pyx_kp_u_Pure_part_print_clifford_l_l_1, __pyx_k_Pure_part_print_clifford_l_l_1,
sizeof(__pyx_k_Pure_part_print_clifford_l_l_1), 0, 1, 0, 0),
20541 {&__pyx_kp_u_Pure_part_print_pure_clifford_l, __pyx_k_Pure_part_print_pure_clifford_l,
sizeof(__pyx_k_Pure_part_print_pure_clifford_l), 0, 1, 0, 0),
20542 {&__pyx_kp_u_Put_self_into_a_larger_frame_co, __pyx_k_Put_self_into_a_larger_frame_co,
sizeof(__pyx_k_Put_self_into_a_larger_frame_co), 0, 1, 0, 0),

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20543     {&__pyx_n_s_PyClical, __pyx_k_PyClical, sizeof(__pyx_k_PyClical), 0, 0, 1, 1},
20544     {&__pyx_kp_s_PyClical_pyx, __pyx_k_PyClical_pyx, sizeof(__pyx_k_PyClical_pyx), 0, 0, 1, 0},
20545     {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print, __pyx_k_Quadratic_form_rev_x_x_0_print,
sizeof(__pyx_k_Quadratic_form_rev_x_x_0_print), 0, 1, 0, 0},
20546     {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2, __pyx_k_Quadratic_form_rev_x_x_0_print_2,
sizeof(__pyx_k_Quadratic_form_rev_x_x_0_print_2), 0, 1, 0, 0},
20547     {&__pyx_kp_u_Quadratic_norm_error_tolerance, __pyx_k_Quadratic_norm_error_tolerance,
sizeof(__pyx_k_Quadratic_norm_error_tolerance), 0, 1, 0, 0},
20548     {&__pyx_kp_u_Random_multivector_within_a_fra, __pyx_k_Random_multivector_within_a_fra,
sizeof(__pyx_k_Random_multivector_within_a_fra), 0, 1, 0, 0},
20549     {&__pyx_n_s_Real, __pyx_k_Real, sizeof(__pyx_k_Real), 0, 0, 1, 1},
20550     {&__pyx_kp_u_Real_part_synonym_for_scalar_pa, __pyx_k_Real_part_synonym_for_scalar_pa,
sizeof(__pyx_k_Real_part_synonym_for_scalar_pa), 0, 1, 0, 0},
20551     {&__pyx_kp_u_Relative_or_absolute_error_usin, __pyx_k_Relative_or_absolute_error_usin,
sizeof(__pyx_k_Relative_or_absolute_error_usin), 0, 1, 0, 0},
20552     {&__pyx_kp_u_Remove_all_terms_of_self_with_r, __pyx_k_Remove_all_terms_of_self_with_r,
sizeof(__pyx_k_Remove_all_terms_of_self_with_r), 0, 1, 0, 0},
20553     {&__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve, __pyx_k_Reversion_eg_1_2_2_1_print_reve,
sizeof(__pyx_k_Reversion_eg_1_2_2_1_print_reve), 0, 1, 0, 0},
20554     {&__pyx_kp_u_Reversion_eg_clifford_1_cliffor, __pyx_k_Reversion_eg_clifford_1_cliffor,
sizeof(__pyx_k_Reversion_eg_clifford_1_cliffor), 0, 1, 0, 0},
20555     {&__pyx_n_s_RuntimeError, __pyx_k_RuntimeError, sizeof(__pyx_k_RuntimeError), 0, 0, 1, 1},
20556     {&__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc, __pyx_k_Scalar_part_clifford_1_1_1_2_sc,
sizeof(__pyx_k_Scalar_part_clifford_1_1_1_2_sc), 0, 1, 0, 0},
20557     {&__pyx_kp_u_Scalar_part_scalar_clifford_1_1, __pyx_k_Scalar_part_scalar_clifford_1_1,
sizeof(__pyx_k_Scalar_part_scalar_clifford_1_1), 0, 1, 0, 0},
20558     {&__pyx_n_s_Sequence, __pyx_k_Sequence, sizeof(__pyx_k_Sequence), 0, 0, 1, 1},
20559     {&__pyx_kp_u_Set_complement_not_print_index, __pyx_k_Set_complement_not_print_index,
sizeof(__pyx_k_Set_complement_not_print_index), 0, 1, 0, 0},
20560     {&__pyx_kp_u_Set_intersection_and_print_inde, __pyx_k_Set_intersection_and_print_inde,
sizeof(__pyx_k_Set_intersection_and_print_inde), 0, 1, 0, 0},
20561     {&__pyx_kp_u_Set_intersection_and_x_index_se, __pyx_k_Set_intersection_and_x_index_se,
sizeof(__pyx_k_Set_intersection_and_x_index_se), 0, 1, 0, 0},
20562     {&__pyx_kp_u_Set_the_value_of_an_index_set_o, __pyx_k_Set_the_value_of_an_index_set_o,
sizeof(__pyx_k_Set_the_value_of_an_index_set_o), 0, 1, 0, 0},
20563     {&__pyx_kp_u_Set_union_or_print_index_set_1, __pyx_k_Set_union_or_print_index_set_1,
sizeof(__pyx_k_Set_union_or_print_index_set_1), 0, 1, 0, 0},
20564     {&__pyx_kp_u_Set_union_or_x_index_set_1_x_in, __pyx_k_Set_union_or_x_index_set_1_x_in,
sizeof(__pyx_k_Set_union_or_x_index_set_1_x_in), 0, 1, 0, 0},
20565     {&__pyx_kp_u_Sign_of_geometric_product_of_tw, __pyx_k_Sign_of_geometric_product_of_tw,
sizeof(__pyx_k_Sign_of_geometric_product_of_tw), 0, 1, 0, 0},
20566     {&__pyx_kp_u_Sign_of_geometric_square_of_a_C, __pyx_k_Sign_of_geometric_square_of_a_C,
sizeof(__pyx_k_Sign_of_geometric_square_of_a_C), 0, 1, 0, 0},
20567     {&__pyx_kp_u_Sine_of_multivector_with_option, __pyx_k_Sine_of_multivector_with_option,
sizeof(__pyx_k_Sine_of_multivector_with_option), 0, 1, 0, 0},
20568     {&__pyx_kp_u_Square_root_of_1_which_commutates, __pyx_k_Square_root_of_1_which_commutates,
sizeof(__pyx_k_Square_root_of_1_which_commutates), 0, 1, 0, 0},
20569     {&__pyx_kp_u_Square_root_of_multivector_with, __pyx_k_Square_root_of_multivector_with,
sizeof(__pyx_k_Square_root_of_multivector_with), 0, 1, 0, 0},
20570     {&__pyx_kp_u_Subalgebra_generated_by_all_gen, __pyx_k_Subalgebra_generated_by_all_gen,
sizeof(__pyx_k_Subalgebra_generated_by_all_gen), 0, 1, 0, 0},
20571     {&__pyx_kp_u_Subscripting_map_from_index_set, __pyx_k_Subscripting_map_from_index_set,
sizeof(__pyx_k_Subscripting_map_from_index_set), 0, 1, 0, 0},
20572     {&__pyx_kp_u_Symmetric_set_difference_exclus, __pyx_k_Symmetric_set_difference_exclus,
sizeof(__pyx_k_Symmetric_set_difference_exclus), 0, 1, 0, 0},
20573     {&__pyx_kp_u_Symmetric_set_difference_exclus_2, __pyx_k_Symmetric_set_difference_exclus_2,
sizeof(__pyx_k_Symmetric_set_difference_exclus_2), 0, 1, 0, 0},
20574     {&__pyx_kp_u_Tangent_of_multivector_with_opt, __pyx_k_Tangent_of_multivector_with_opt,
sizeof(__pyx_k_Tangent_of_multivector_with_opt), 0, 1, 0, 0},
20575     {&__pyx_kp_u_Test_for_approximate_equality_o, __pyx_k_Test_for_approximate_equality_o,
sizeof(__pyx_k_Test_for_approximate_equality_o), 0, 1, 0, 0},
20576     {&__pyx_kp_u_Tests_for_functions_that_Doctes, __pyx_k_Tests_for_functions_that_Doctes,
sizeof(__pyx_k_Tests_for_functions_that_Doctes), 0, 1, 0, 0},
20577     {&__pyx_kp_u_Tests_for_functions_that_Doctes_2, __pyx_k_Tests_for_functions_that_Doctes_2,
sizeof(__pyx_k_Tests_for_functions_that_Doctes_2), 0, 1, 0, 0},
20578     {&__pyx_kp_u_The_informal_string_representat, __pyx_k_The_informal_string_representat,
sizeof(__pyx_k_The_informal_string_representat), 0, 1, 0, 0},
20579     {&__pyx_kp_u_The_informal_string_representat_2, __pyx_k_The_informal_string_representat_2,
sizeof(__pyx_k_The_informal_string_representat_2), 0, 1, 0, 0},
20580     {&__pyx_kp_u_The_official_string_representat, __pyx_k_The_official_string_representat,
sizeof(__pyx_k_The_official_string_representat), 0, 1, 0, 0},
20581     {&__pyx_kp_u_The_official_string_representat_2, __pyx_k_The_official_string_representat_2,
sizeof(__pyx_k_The_official_string_representat_2), 0, 1, 0, 0},
20582     {&__pyx_kp_u_This_comparison_operator_is_not, __pyx_k_This_comparison_operator_is_not,
sizeof(__pyx_k_This_comparison_operator_is_not), 0, 1, 0, 0},
20583     {&__pyx_kp_u_Transform_left_hand_side_using, __pyx_k_Transform_left_hand_side_using,
sizeof(__pyx_k_Transform_left_hand_side_using), 0, 1, 0, 0},
20584     {&__pyx_kp_u_Transform_left_hand_side_using_2, __pyx_k_Transform_left_hand_side_using_2,
sizeof(__pyx_k_Transform_left_hand_side_using_2), 0, 1, 0, 0},
20585     {&__pyx_n_s_TypeError, __pyx_k_TypeError, sizeof(__pyx_k_TypeError), 0, 0, 1, 1},
20586     {&__pyx_kp_u_UTF_8, __pyx_k_UTF_8, sizeof(__pyx_k_UTF_8), 0, 1, 0, 0},
20587     {&__pyx_kp_u_Unary_print_clifford_1_1, __pyx_k_Unary_print_clifford_1_1,
sizeof(__pyx_k_Unary_print_clifford_1_1), 0, 1, 0, 0},
20588     {&__pyx_kp_u_Unary_print_clifford_1_1_2, __pyx_k_Unary_print_clifford_1_1_2,
sizeof(__pyx_k_Unary_print_clifford_1_1_2), 0, 1, 0, 0},
20589     {&__pyx_n_s_ValueError, __pyx_k_ValueError, sizeof(__pyx_k_ValueError), 0, 0, 1, 1},
20590     {&__pyx_kp_u_Vector_part_of_multivector_as_a, __pyx_k_Vector_part_of_multivector_as_a,

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sizeof(_pyx_k_Vector_part_of_multivector_as_a), 0, 1, 0, 0},
20591 {&_pyx_kp_u_2, _pyx_k_2, sizeof(_pyx_k_2), 0, 1, 0, 0},
20592 {&_pyx_kp_u_5, _pyx_k_5, sizeof(_pyx_k_5), 0, 1, 0, 0},
20593 {&_pyx_kp_u_6, _pyx_k_6, sizeof(_pyx_k_6), 0, 1, 0, 0},
20594 {&_pyx_kp_u_7, _pyx_k_7, sizeof(_pyx_k_7), 0, 1, 0, 0},
20595 {&_pyx_kp_u_8, _pyx_k_8, sizeof(_pyx_k_8), 0, 1, 0, 0},
20596 {&_pyx_kp_u_9, _pyx_k_9, sizeof(_pyx_k_9), 0, 1, 0, 0},
20597 {&_pyx_n_s_abc, _pyx_k_abc, sizeof(_pyx_k_abc), 0, 0, 1, 1},
20598 {&_pyx_kp_u_abs_line_1522, _pyx_k_abs_line_1522, sizeof(_pyx_k_abs_line_1522), 0, 1, 0, 0},
20599 {&_pyx_n_s_acos, _pyx_k_acos, sizeof(_pyx_k_acos), 0, 0, 1, 1},
20600 {&_pyx_kp_u_acos_line_1668, _pyx_k_acos_line_1668, sizeof(_pyx_k_acos_line_1668), 0, 1, 0, 0},
20601 {&_pyx_n_s_acosh, _pyx_k_acosh, sizeof(_pyx_k_acosh), 0, 0, 1, 1},
20602 {&_pyx_kp_u_acosh_line_1705, _pyx_k_acosh_line_1705, sizeof(_pyx_k_acosh_line_1705), 0, 1, 0, 0},
20603 {&_pyx_kp_u_agc3_line_1893, _pyx_k_agc3_line_1893, sizeof(_pyx_k_agc3_line_1893), 0, 1, 0, 0},
20604 {&_pyx_kp_u_approx_equal_line_1359, _pyx_k_approx_equal_line_1359,
sizeof(_pyx_k_approx_equal_line_1359), 0, 1, 0, 0},
20605 {&_pyx_n_s_args, _pyx_k_args, sizeof(_pyx_k_args), 0, 0, 1, 1},
20606 {&_pyx_kp_u_as_frame, _pyx_k_as_frame, sizeof(_pyx_k_as_frame), 0, 1, 0, 0},
20607 {&_pyx_n_s_asin, _pyx_k_asin, sizeof(_pyx_k_asin), 0, 0, 1, 1},
20608 {&_pyx_kp_u_asin_line_1747, _pyx_k_asin_line_1747, sizeof(_pyx_k_asin_line_1747), 0, 1, 0, 0},
20609 {&_pyx_n_s_asinh, _pyx_k_asinh, sizeof(_pyx_k_asinh), 0, 0, 1, 1},
20610 {&_pyx_kp_u_asinh_line_1782, _pyx_k_asinh_line_1782, sizeof(_pyx_k_asinh_line_1782), 0, 1, 0, 0},
20611 {&_pyx_n_s_atan, _pyx_k_atan, sizeof(_pyx_k_atan), 0, 0, 1, 1},
20612 {&_pyx_kp_u_atan_line_1818, _pyx_k_atan_line_1818, sizeof(_pyx_k_atan_line_1818), 0, 1, 0, 0},
20613 {&_pyx_n_s_atanh, _pyx_k_atanh, sizeof(_pyx_k_atanh), 0, 0, 1, 1},
20614 {&_pyx_kp_u_atanh_line_1847, _pyx_k_atanh_line_1847, sizeof(_pyx_k_atanh_line_1847), 0, 1, 0, 0},
20615 {&_pyx_kp_u_cga3_line_1873, _pyx_k_cga3_line_1873, sizeof(_pyx_k_cga3_line_1873), 0, 1, 0, 0},
20616 {&_pyx_kp_u_cga3std_line_1882, _pyx_k_cga3std_line_1882, sizeof(_pyx_k_cga3std_line_1882), 0, 1,
0, 0},
20617 {&_pyx_n_s_cl, _pyx_k_cl, sizeof(_pyx_k_cl), 0, 0, 1, 1},
20618 {&_pyx_n_s_clifford, _pyx_k_clifford, sizeof(_pyx_k_clifford), 0, 0, 1, 1},
20619 {&_pyx_kp_u_clifford_add_line_740, _pyx_k_clifford_add_line_740,
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20620 {&_pyx_kp_u_clifford_and_line_836, _pyx_k_clifford_and_line_836,
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20621 {&_pyx_kp_u_clifford_call_line_1020, _pyx_k_clifford_call_line_1020,
sizeof(_pyx_k_clifford_call_line_1020), 0, 1, 0, 0},
20622 {&_pyx_kp_u_clifford_getitem_line_707, _pyx_k_clifford_getitem_line_707,
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20623 {&_pyx_kp_u_clifford_iadd_line_751, _pyx_k_clifford_iadd_line_751,
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20624 {&_pyx_kp_u_clifford_iand_line_851, _pyx_k_clifford_iand_line_851,
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20625 {&_pyx_kp_u_clifford_idiv_line_911, _pyx_k_clifford_idiv_line_911,
sizeof(_pyx_k_clifford_idiv_line_911), 0, 1, 0, 0},
20626 {&_pyx_kp_u_clifford_imod_line_821, _pyx_k_clifford_imod_line_821,
sizeof(_pyx_k_clifford_imod_line_821), 0, 1, 0, 0},
20627 {&_pyx_kp_u_clifford_imul_line_793, _pyx_k_clifford_imul_line_793,
sizeof(_pyx_k_clifford_imul_line_793), 0, 1, 0, 0},
20628 {&_pyx_kp_u_clifford_ior_line_950, _pyx_k_clifford_ior_line_950,
sizeof(_pyx_k_clifford_ior_line_950), 0, 1, 0, 0},
20629 {&_pyx_kp_u_clifford_isub_line_771, _pyx_k_clifford_isub_line_771,
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20630 {&_pyx_kp_u_clifford_iter_line_638, _pyx_k_clifford_iter_line_638,
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20631 {&_pyx_kp_u_clifford_ixor_line_881, _pyx_k_clifford_ixor_line_881,
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20632 {&_pyx_kp_u_clifford_mod_line_806, _pyx_k_clifford_mod_line_806,
sizeof(_pyx_k_clifford_mod_line_806), 0, 1, 0, 0},
20633 {&_pyx_kp_u_clifford_mul_line_780, _pyx_k_clifford_mul_line_780,
sizeof(_pyx_k_clifford_mul_line_780), 0, 1, 0, 0},
20634 {&_pyx_kp_u_clifford_neg_line_722, _pyx_k_clifford_neg_line_722,
sizeof(_pyx_k_clifford_neg_line_722), 0, 1, 0, 0},
20635 {&_pyx_kp_u_clifford_or_line_939, _pyx_k_clifford_or_line_939,
sizeof(_pyx_k_clifford_or_line_939), 0, 1, 0, 0},
20636 {&_pyx_kp_u_clifford_pos_line_731, _pyx_k_clifford_pos_line_731,
sizeof(_pyx_k_clifford_pos_line_731), 0, 1, 0, 0},
20637 {&_pyx_kp_u_clifford_pow_line_961, _pyx_k_clifford_pow_line_961,
sizeof(_pyx_k_clifford_pow_line_961), 0, 1, 0, 0},
20638 {&_pyx_kp_u_clifford_repr_line_1235, _pyx_k_clifford_repr_line_1235,
sizeof(_pyx_k_clifford_repr_line_1235), 0, 1, 0, 0},
20639 {&_pyx_kp_u_clifford_str_line_1244, _pyx_k_clifford_str_line_1244,
sizeof(_pyx_k_clifford_str_line_1244), 0, 1, 0, 0},
20640 {&_pyx_kp_u_clifford_sub_line_760, _pyx_k_clifford_sub_line_760,
sizeof(_pyx_k_clifford_sub_line_760), 0, 1, 0, 0},
20641 {&_pyx_kp_u_clifford_truediv_line_896, _pyx_k_clifford_truediv_line_896,
sizeof(_pyx_k_clifford_truediv_line_896), 0, 1, 0, 0},
20642 {&_pyx_kp_u_clifford_xor_line_866, _pyx_k_clifford_xor_line_866,
sizeof(_pyx_k_clifford_xor_line_866), 0, 1, 0, 0},
20643 {&_pyx_kp_u_clifford_abs_line_1175, _pyx_k_clifford_abs_line_1175,
sizeof(_pyx_k_clifford_abs_line_1175), 0, 1, 0, 0},
20644 {&_pyx_kp_u_clifford_conj_line_1138, _pyx_k_clifford_conj_line_1138,
sizeof(_pyx_k_clifford_conj_line_1138), 0, 1, 0, 0},
20645 {&_pyx_kp_u_clifford_copy_line_556, _pyx_k_clifford_copy_line_556,
sizeof(_pyx_k_clifford_copy_line_556), 0, 1, 0, 0},
20646 {&_pyx_kp_u_clifford_even_line_1061, _pyx_k_clifford_even_line_1061,
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20647     {&__pyx_kp_u_clifford_frame_line_1224, __pyx_k_clifford_frame_line_1224,
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20648     {&__pyx_n_s_clifford_hidden_doctests, __pyx_k_clifford_hidden_doctests,
sizeof(__pyx_k_clifford_hidden_doctests), 0, 0, 1, 1},
20649     {&__pyx_kp_u_clifford_hidden_doctests_line_12, __pyx_k_clifford_hidden_doctests_line_12,
sizeof(__pyx_k_clifford_hidden_doctests_line_12), 0, 1, 0, 0},
20650     {&__pyx_kp_u_clifford_inv_line_926, __pyx_k_clifford_inv_line_926,
sizeof(__pyx_k_clifford_inv_line_926), 0, 1, 0, 0},
20651     {&__pyx_kp_u_clifford_involute_line_1107, __pyx_k_clifford_involute_line_1107,
sizeof(__pyx_k_clifford_involute_line_1107), 0, 1, 0, 0},
20652     {&__pyx_kp_u_clifford_isinf_line_1206, __pyx_k_clifford_isinf_line_1206,
sizeof(__pyx_k_clifford_isinf_line_1206), 0, 1, 0, 0},
20653     {&__pyx_kp_u_clifford_isnan_line_1215, __pyx_k_clifford_isnan_line_1215,
sizeof(__pyx_k_clifford_isnan_line_1215), 0, 1, 0, 0},
20654     {&__pyx_kp_u_clifford_max_abs_line_1184, __pyx_k_clifford_max_abs_line_1184,
sizeof(__pyx_k_clifford_max_abs_line_1184), 0, 1, 0, 0},
20655     {&__pyx_kp_u_clifford_norm_line_1164, __pyx_k_clifford_norm_line_1164,
sizeof(__pyx_k_clifford_norm_line_1164), 0, 1, 0, 0},
20656     {&__pyx_kp_u_clifford_odd_line_1070, __pyx_k_clifford_odd_line_1070,
sizeof(__pyx_k_clifford_odd_line_1070), 0, 1, 0, 0},
20657     {&__pyx_kp_u_clifford_outer_pow_line_1004, __pyx_k_clifford_outer_pow_line_1004,
sizeof(__pyx_k_clifford_outer_pow_line_1004), 0, 1, 0, 0},
20658     {&__pyx_kp_u_clifford_pow_line_980, __pyx_k_clifford_pow_line_980,
sizeof(__pyx_k_clifford_pow_line_980), 0, 1, 0, 0},
20659     {&__pyx_kp_u_clifford_pure_line_1050, __pyx_k_clifford_pure_line_1050,
sizeof(__pyx_k_clifford_pure_line_1050), 0, 1, 0, 0},
20660     {&__pyx_kp_u_clifford_quad_line_1153, __pyx_k_clifford_quad_line_1153,
sizeof(__pyx_k_clifford_quad_line_1153), 0, 1, 0, 0},
20661     {&__pyx_kp_u_clifford_reframe_line_649, __pyx_k_clifford_reframe_line_649,
sizeof(__pyx_k_clifford_reframe_line_649), 0, 1, 0, 0},
20662     {&__pyx_kp_u_clifford_reverse_line_1123, __pyx_k_clifford_reverse_line_1123,
sizeof(__pyx_k_clifford_reverse_line_1123), 0, 1, 0, 0},
20663     {&__pyx_kp_u_clifford_scalar_line_1039, __pyx_k_clifford_scalar_line_1039,
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20664     {&__pyx_kp_u_clifford_truncated_line_1195, __pyx_k_clifford_truncated_line_1195,
sizeof(__pyx_k_clifford_truncated_line_1195), 0, 1, 0, 0},
20665     {&__pyx_kp_u_clifford_vector_part_line_1079, __pyx_k_clifford_vector_part_line_1079,
sizeof(__pyx_k_clifford_vector_part_line_1079), 0, 1, 0, 0},
20666     {&__pyx_n_s_cline_in_traceback, __pyx_k_cline_in_traceback, sizeof(__pyx_k_cline_in_traceback), 0,
0, 1, 1},
20667     {&__pyx_n_s_close, __pyx_k_close, sizeof(__pyx_k_close), 0, 0, 1, 1},
20668     {&__pyx_n_s_collections, __pyx_k_collections, sizeof(__pyx_k_collections), 0, 0, 1, 1},
20669     {&__pyx_kp_u_compare_line_492, __pyx_k_compare_line_492, sizeof(__pyx_k_compare_line_492), 0, 1, 0,
0},
20670     {&__pyx_kp_u_complexifier_line_1576, __pyx_k_complexifier_line_1576,
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20671     {&__pyx_n_s_conj, __pyx_k_conj, sizeof(__pyx_k_conj), 0, 0, 1, 1},
20672     {&__pyx_kp_u_conj_line_1485, __pyx_k_conj_line_1485, sizeof(__pyx_k_conj_line_1485), 0, 1, 0, 0},
20673     {&__pyx_n_s_copy, __pyx_k_copy, sizeof(__pyx_k_copy), 0, 0, 1, 1},
20674     {&__pyx_n_s_cos, __pyx_k_cos, sizeof(__pyx_k_cos), 0, 0, 1, 1},
20675     {&__pyx_kp_u_cos_line_1651, __pyx_k_cos_line_1651, sizeof(__pyx_k_cos_line_1651), 0, 1, 0, 0},
20676     {&__pyx_n_s_cosh, __pyx_k_cosh, sizeof(__pyx_k_cosh), 0, 0, 1, 1},
20677     {&__pyx_kp_u_cosh_line_1689, __pyx_k_cosh_line_1689, sizeof(__pyx_k_cosh_line_1689), 0, 1, 0, 0},
20678     {&__pyx_n_s_doctest, __pyx_k_doctest, sizeof(__pyx_k_doctest), 0, 0, 1, 1},
20679     {&__pyx_n_s_e, __pyx_k_e, sizeof(__pyx_k_e), 0, 0, 1, 1},
20680     {&__pyx_kp_u_e_line_1936, __pyx_k_e_line_1936, sizeof(__pyx_k_e_line_1936), 0, 1, 0, 0},
20681     {&__pyx_n_s_encode, __pyx_k_encode, sizeof(__pyx_k_encode), 0, 0, 1, 1},
20682     {&__pyx_kp_u_error_squared_line_1346, __pyx_k_error_squared_line_1346,
sizeof(__pyx_k_error_squared_line_1346), 0, 1, 0, 0},
20683     {&__pyx_kp_u_error_squared_tol_line_1337, __pyx_k_error_squared_tol_line_1337,
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20684     {&__pyx_n_s_even, __pyx_k_even, sizeof(__pyx_k_even), 0, 0, 1, 1},
20685     {&__pyx_kp_u_even_line_1437, __pyx_k_even_line_1437, sizeof(__pyx_k_even_line_1437), 0, 1, 0, 0},
20686     {&__pyx_n_s_exp, __pyx_k_exp, sizeof(__pyx_k_exp), 0, 0, 1, 1},
20687     {&__pyx_kp_u_exp_line_1614, __pyx_k_exp_line_1614, sizeof(__pyx_k_exp_line_1614), 0, 1, 0, 0},
20688     {&__pyx_n_s_fill, __pyx_k_fill, sizeof(__pyx_k_fill), 0, 0, 1, 1},
20689     {&__pyx_n_s_frm, __pyx_k_frm, sizeof(__pyx_k_frm), 0, 0, 1, 1},
20690     {&__pyx_kp_u_from, __pyx_k_from, sizeof(__pyx_k_from), 0, 1, 0, 0},
20691     {&__pyx_n_s_getstate, __pyx_k_getstate, sizeof(__pyx_k_getstate), 0, 0, 1, 1},
20692     {&__pyx_n_s_grade, __pyx_k_grade, sizeof(__pyx_k_grade), 0, 0, 1, 1},
20693     {&__pyx_n_s_i, __pyx_k_i, sizeof(__pyx_k_i), 0, 0, 1, 1},
20694     {&__pyx_kp_u_imag_line_1415, __pyx_k_imag_line_1415, sizeof(__pyx_k_imag_line_1415), 0, 1, 0, 0},
20695     {&__pyx_n_s_import, __pyx_k_import, sizeof(__pyx_k_import), 0, 0, 1, 1},
20696     {&__pyx_n_s_index_set, __pyx_k_index_set, sizeof(__pyx_k_index_set), 0, 0, 1, 1},
20697     {&__pyx_kp_u_index_set_and_line_271, __pyx_k_index_set_and_line_271,
sizeof(__pyx_k_index_set_and_line_271), 0, 1, 0, 0},
20698     {&__pyx_kp_u_index_set_getitem_line_191, __pyx_k_index_set_getitem_line_191,
sizeof(__pyx_k_index_set_getitem_line_191), 0, 1, 0, 0},
20699     {&__pyx_kp_u_index_set_iand_line_282, __pyx_k_index_set_iand_line_282,
sizeof(__pyx_k_index_set_iand_line_282), 0, 1, 0, 0},
20700     {&__pyx_kp_u_index_set_invert_line_240, __pyx_k_index_set_invert_line_240,
sizeof(__pyx_k_index_set_invert_line_240), 0, 1, 0, 0},
20701     {&__pyx_kp_u_index_set_ior_line_304, __pyx_k_index_set_ior_line_304,
sizeof(__pyx_k_index_set_ior_line_304), 0, 1, 0, 0},
20702     {&__pyx_n_s_index_set_iter, __pyx_k_index_set_iter, sizeof(__pyx_k_index_set_iter), 0, 0, 1,
1},
20703     {&__pyx_kp_u_index_set_iter_line_229, __pyx_k_index_set_iter_line_229,

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sizeof(__pyx_k_index_set__iter__line_229), 0, 1, 0, 0},
20704 {&__pyx_kp_u_index_set__ixor__line_260, __pyx_k_index_set__ixor__line_260,
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20705 {&__pyx_kp_u_index_set__or__line_293, __pyx_k_index_set__or__line_293,
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20706 {&__pyx_kp_u_index_set__repr__line_384, __pyx_k_index_set__repr__line_384,
sizeof(__pyx_k_index_set__repr__line_384), 0, 1, 0, 0},
20707 {&__pyx_kp_u_index_set__setitem__line_179, __pyx_k_index_set__setitem__line_179,
sizeof(__pyx_k_index_set__setitem__line_179), 0, 1, 0, 0},
20708 {&__pyx_kp_u_index_set__str__line_395, __pyx_k_index_set__str__line_395,
sizeof(__pyx_k_index_set__str__line_395), 0, 1, 0, 0},
20709 {&__pyx_kp_u_index_set__xor__line_249, __pyx_k_index_set__xor__line_249,
sizeof(__pyx_k_index_set__xor__line_249), 0, 1, 0, 0},
20710 {&__pyx_kp_u_index_set__copy_line_65, __pyx_k_index_set__copy_line_65,
sizeof(__pyx_k_index_set__copy_line_65), 0, 1, 0, 0},
20711 {&__pyx_kp_u_index_set__count_line_315, __pyx_k_index_set__count_line_315,
sizeof(__pyx_k_index_set__count_line_315), 0, 1, 0, 0},
20712 {&__pyx_kp_u_index_set__count_neg_line_324, __pyx_k_index_set__count_neg_line_324,
sizeof(__pyx_k_index_set__count_neg_line_324), 0, 1, 0, 0},
20713 {&__pyx_kp_u_index_set__count_pos_line_333, __pyx_k_index_set__count_pos_line_333,
sizeof(__pyx_k_index_set__count_pos_line_333), 0, 1, 0, 0},
20714 {&__pyx_n_s_index_set_hidden_doctests, __pyx_k_index_set_hidden_doctests,
sizeof(__pyx_k_index_set_hidden_doctests), 0, 0, 1, 1},
20715 {&__pyx_kp_u_index_set_hidden_doctests_line_4, __pyx_k_index_set_hidden_doctests_line_4,
sizeof(__pyx_k_index_set_hidden_doctests_line_4), 0, 1, 0, 0},
20716 {&__pyx_kp_u_index_set_max_line_351, __pyx_k_index_set_max_line_351,
sizeof(__pyx_k_index_set_max_line_351), 0, 1, 0, 0},
20717 {&__pyx_kp_u_index_set_min_line_342, __pyx_k_index_set_min_line_342,
sizeof(__pyx_k_index_set_min_line_342), 0, 1, 0, 0},
20718 {&__pyx_kp_u_index_set_sign_of_mult_line_366, __pyx_k_index_set_sign_of_mult_line_366,
sizeof(__pyx_k_index_set_sign_of_mult_line_366), 0, 1, 0, 0},
20719 {&__pyx_kp_u_index_set_sign_of_square_line_37, __pyx_k_index_set_sign_of_square_line_37,
sizeof(__pyx_k_index_set_sign_of_square_line_37), 0, 1, 0, 0},
20720 {&__pyx_n_s_inv, __pyx_k_inv, sizeof(__pyx_k_inv), 0, 0, 1, 1},
20721 {&__pyx_kp_u_inv_line_1378, __pyx_k_inv_line_1378, sizeof(__pyx_k_inv_line_1378), 0, 1, 0, 0},
20722 {&__pyx_kp_u_invalid, __pyx_k_invalid, sizeof(__pyx_k_invalid), 0, 1, 0, 0},
20723 {&__pyx_kp_u_invalid_string, __pyx_k_invalid_string, sizeof(__pyx_k_invalid_string), 0, 1, 0, 0},
20724 {&__pyx_n_s_involute, __pyx_k_involute, sizeof(__pyx_k_involute), 0, 0, 1, 1},
20725 {&__pyx_kp_u_involute_line_1455, __pyx_k_involute_line_1455, sizeof(__pyx_k_involute_line_1455), 0,
1, 0, 0},
20726 {&__pyx_n_s_ist, __pyx_k_ist, sizeof(__pyx_k_ist), 0, 0, 1, 1},
20727 {&__pyx_n_s_istpq, __pyx_k_istpq, sizeof(__pyx_k_istpq), 0, 0, 1, 1},
20728 {&__pyx_kp_u_istpq_line_1949, __pyx_k_istpq_line_1949, sizeof(__pyx_k_istpq_line_1949), 0, 1, 0, 0},
20729 {&__pyx_n_s_iter, __pyx_k_iter, sizeof(__pyx_k_iter), 0, 0, 1, 1},
20730 {&__pyx_n_s_ixt, __pyx_k_ixt, sizeof(__pyx_k_ixt), 0, 0, 1, 1},
20731 {&__pyx_kp_u_lexicographic_compare_eg_3_4_5, __pyx_k_lexicographic_compare_eg_3_4_5,
sizeof(__pyx_k_lexicographic_compare_eg_3_4_5), 0, 1, 0, 0},
20732 {&__pyx_n_s_lhs, __pyx_k_lhs, sizeof(__pyx_k_lhs), 0, 0, 1, 1},
20733 {&__pyx_n_s_log, __pyx_k_log, sizeof(__pyx_k_log), 0, 0, 1, 1},
20734 {&__pyx_kp_u_log_line_1628, __pyx_k_log_line_1628, sizeof(__pyx_k_log_line_1628), 0, 1, 0, 0},
20735 {&__pyx_n_s_m, __pyx_k_m, sizeof(__pyx_k_m), 0, 0, 1, 1},
20736 {&__pyx_n_s_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 0, 1, 1},
20737 {&__pyx_n_u_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 1, 0, 1},
20738 {&__pyx_n_s_math, __pyx_k_math, sizeof(__pyx_k_math), 0, 0, 1, 1},
20739 {&__pyx_n_s_max, __pyx_k_max, sizeof(__pyx_k_max), 0, 0, 1, 1},
20740 {&__pyx_kp_u_max_abs_line_1531, __pyx_k_max_abs_line_1531, sizeof(__pyx_k_max_abs_line_1531), 0, 1,
0, 0},
20741 {&__pyx_kp_u_max_pos_line_513, __pyx_k_max_pos_line_513, sizeof(__pyx_k_max_pos_line_513), 0, 1, 0,
0},
20742 {&__pyx_n_s_min, __pyx_k_min, sizeof(__pyx_k_min), 0, 0, 1, 1},
20743 {&__pyx_kp_u_min_neg_line_504, __pyx_k_min_neg_line_504, sizeof(__pyx_k_min_neg_line_504), 0, 1, 0,
0},
20744 {&__pyx_n_s_name, __pyx_k_name, sizeof(__pyx_k_name), 0, 0, 1, 1},
20745 {&__pyx_n_s_nbar3, __pyx_k_nbar3, sizeof(__pyx_k_nbar3), 0, 0, 1, 1},
20746 {&__pyx_n_s_ninf3, __pyx_k_ninf3, sizeof(__pyx_k_ninf3), 0, 0, 1, 1},
20747 {&__pyx_kp_u_no_default_reduce_due_to_non, __pyx_k_no_default_reduce_due_to_non,
sizeof(__pyx_k_no_default_reduce_due_to_non), 0, 0, 1, 0},
20748 {&__pyx_n_s_norm, __pyx_k_norm, sizeof(__pyx_k_norm), 0, 0, 1, 1},
20749 {&__pyx_kp_u_norm_line_1511, __pyx_k_norm_line_1511, sizeof(__pyx_k_norm_line_1511), 0, 1, 0, 0},
20750 {&__pyx_kp_u_norm_sum_of_squares_of_coordina, __pyx_k_norm_sum_of_squares_of_coordina,
sizeof(__pyx_k_norm_sum_of_squares_of_coordina), 0, 1, 0, 0},
20751 {&__pyx_n_s_numbers, __pyx_k_numbers, sizeof(__pyx_k_numbers), 0, 0, 1, 1},
20752 {&__pyx_n_s_obj, __pyx_k_obj, sizeof(__pyx_k_obj), 0, 0, 1, 1},
20753 {&__pyx_n_s_odd, __pyx_k_odd, sizeof(__pyx_k_odd), 0, 0, 1, 1},
20754 {&__pyx_kp_u_odd_line_1446, __pyx_k_odd_line_1446, sizeof(__pyx_k_odd_line_1446), 0, 1, 0, 0},
20755 {&__pyx_n_s_other, __pyx_k_other, sizeof(__pyx_k_other), 0, 0, 1, 1},
20756 {&__pyx_n_s_outer_pow, __pyx_k_outer_pow, sizeof(__pyx_k_outer_pow), 0, 0, 1, 1},
20757 {&__pyx_kp_u_outer_pow_line_1567, __pyx_k_outer_pow_line_1567, sizeof(__pyx_k_outer_pow_line_1567),
0, 1, 0, 0},
20758 {&__pyx_n_s_p, __pyx_k_p, sizeof(__pyx_k_p), 0, 0, 1, 1},
20759 {&__pyx_n_s_pi, __pyx_k_pi, sizeof(__pyx_k_pi), 0, 0, 1, 1},
20760 {&__pyx_n_s_pow, __pyx_k_pow, sizeof(__pyx_k_pow), 0, 0, 1, 1},
20761 {&__pyx_kp_u_pow_line_1543, __pyx_k_pow_line_1543, sizeof(__pyx_k_pow_line_1543), 0, 1, 0, 0},
20762 {&__pyx_n_s_pure, __pyx_k_pure, sizeof(__pyx_k_pure), 0, 0, 1, 1},
20763 {&__pyx_kp_u_pure_line_1426, __pyx_k_pure_line_1426, sizeof(__pyx_k_pure_line_1426), 0, 1, 0, 0},
20764 {&__pyx_n_s_pyx_vtable, __pyx_k_pyx_vtable, sizeof(__pyx_k_pyx_vtable), 0, 0, 1, 1},
20765 {&__pyx_n_s_q, __pyx_k_q, sizeof(__pyx_k_q), 0, 0, 1, 1},

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20766     {&__pyx_n_s_quad, __pyx_k_quad, sizeof(__pyx_k_quad), 0, 0, 1, 1},
20767     {&__pyx_kp_u_quad_line_1500, __pyx_k_quad_line_1500, sizeof(__pyx_k_quad_line_1500), 0, 1, 0, 0},
20768     {&__pyx_kp_u_random_clifford_line_1864, __pyx_k_random_clifford_line_1864,
sizeof(__pyx_k_random_clifford_line_1864), 0, 1, 0, 0},
20769     {&__pyx_n_s_range, __pyx_k_range, sizeof(__pyx_k_range), 0, 0, 1, 1},
20770     {&__pyx_kp_u_real_line_1404, __pyx_k_real_line_1404, sizeof(__pyx_k_real_line_1404), 0, 1, 0, 0},
20771     {&__pyx_n_s_reduce, __pyx_k_reduce, sizeof(__pyx_k_reduce), 0, 0, 1, 1},
20772     {&__pyx_n_s_reduce_cython, __pyx_k_reduce_cython, sizeof(__pyx_k_reduce_cython), 0, 0, 1, 1},
20773     {&__pyx_n_s_reduce_ex, __pyx_k_reduce_ex, sizeof(__pyx_k_reduce_ex), 0, 0, 1, 1},
20774     {&__pyx_n_s_reverse, __pyx_k_reverse, sizeof(__pyx_k_reverse), 0, 0, 1, 1},
20775     {&__pyx_kp_u_reverse_line_1470, __pyx_k_reverse_line_1470, sizeof(__pyx_k_reverse_line_1470), 0, 1,
0, 0},
20776     {&__pyx_n_s_rhs, __pyx_k_rhs, sizeof(__pyx_k_rhs), 0, 0, 1, 1},
20777     {&__pyx_n_s_scalar, __pyx_k_scalar, sizeof(__pyx_k_scalar), 0, 0, 1, 1},
20778     {&__pyx_n_s_scalar_epsilon, __pyx_k_scalar_epsilon, sizeof(__pyx_k_scalar_epsilon), 0, 0, 1, 1},
20779     {&__pyx_kp_u_scalar_line_1393, __pyx_k_scalar_line_1393, sizeof(__pyx_k_scalar_line_1393), 0, 1, 0,
0},
20780     {&__pyx_n_s_send, __pyx_k_send, sizeof(__pyx_k_send), 0, 0, 1, 1},
20781     {&__pyx_n_s_setstate, __pyx_k_setstate, sizeof(__pyx_k_setstate), 0, 0, 1, 1},
20782     {&__pyx_n_s_setstate_cython, __pyx_k_setstate_cython, sizeof(__pyx_k_setstate_cython), 0, 0, 1, 1},
20783     {&__pyx_n_s_sin, __pyx_k_sin, sizeof(__pyx_k_sin), 0, 0, 1, 1},
20784     {&__pyx_kp_u_sin_line_1728, __pyx_k_sin_line_1728, sizeof(__pyx_k_sin_line_1728), 0, 1, 0, 0},
20785     {&__pyx_n_s_sinh, __pyx_k_sinh, sizeof(__pyx_k_sinh), 0, 0, 1, 1},
20786     {&__pyx_kp_u_sinh_line_1768, __pyx_k_sinh_line_1768, sizeof(__pyx_k_sinh_line_1768), 0, 1, 0, 0},
20787     {&__pyx_n_s_sqrt, __pyx_k_sqrt, sizeof(__pyx_k_sqrt), 0, 0, 1, 1},
20788     {&__pyx_kp_u_sqrt_line_1591, __pyx_k_sqrt_line_1591, sizeof(__pyx_k_sqrt_line_1591), 0, 1, 0, 0},
20789     {&__pyx_n_s_tan, __pyx_k_tan, sizeof(__pyx_k_tan), 0, 0, 1, 1},
20790     {&__pyx_kp_u_tan_line_1801, __pyx_k_tan_line_1801, sizeof(__pyx_k_tan_line_1801), 0, 1, 0, 0},
20791     {&__pyx_n_s_tanh, __pyx_k_tanh, sizeof(__pyx_k_tanh), 0, 0, 1, 1},
20792     {&__pyx_kp_u_tanh_line_1835, __pyx_k_tanh_line_1835, sizeof(__pyx_k_tanh_line_1835), 0, 1, 0, 0},
20793     {&__pyx_n_s_tau, __pyx_k_tau, sizeof(__pyx_k_tau), 0, 0, 1, 1},
20794     {&__pyx_n_s_test, __pyx_k_test, sizeof(__pyx_k_test), 0, 0, 1, 1},
20795     {&__pyx_n_s_test_2, __pyx_k_test_2, sizeof(__pyx_k_test_2), 0, 0, 1, 1},
20796     {&__pyx_n_s_testmod, __pyx_k_testmod, sizeof(__pyx_k_testmod), 0, 0, 1, 1},
20797     {&__pyx_n_s_threshold, __pyx_k_threshold, sizeof(__pyx_k_threshold), 0, 0, 1, 1},
20798     {&__pyx_n_s_throw, __pyx_k_throw, sizeof(__pyx_k_throw), 0, 0, 1, 1},
20799     {&__pyx_kp_u_to_frame, __pyx_k_to_frame, sizeof(__pyx_k_to_frame), 0, 1, 0, 0},
20800     {&__pyx_n_s_tol, __pyx_k_tol, sizeof(__pyx_k_tol), 0, 0, 1, 1},
20801     {&__pyx_kp_u_using, __pyx_k_using, sizeof(__pyx_k_using), 0, 1, 0, 0},
20802     {&__pyx_kp_u_using_invalid, __pyx_k_using_invalid, sizeof(__pyx_k_using_invalid), 0, 1, 0, 0},
20803     {&__pyx_kp_u_utf_8, __pyx_k_utf_8, sizeof(__pyx_k_utf_8), 0, 1, 0, 0},
20804     {&__pyx_kp_u_value, __pyx_k_value, sizeof(__pyx_k_value), 0, 1, 0, 0},
20805     {&__pyx_n_s_version, __pyx_k_version, sizeof(__pyx_k_version), 0, 0, 1, 1},
20806     {&__pyx_n_s_xrange, __pyx_k_xrange, sizeof(__pyx_k_xrange), 0, 0, 1, 1},
20807     {0, 0, 0, 0, 0, 0, 0},
20808 };
20809 static CYTHON_SMALL_CODE int __Pyx_InitCachedBuiltins(void) {
20810     __pyx_builtin_IndexError = __Pyx_GetBuiltinName(__pyx_n_s_IndexError); if
(!__pyx_builtin_IndexError) __PYX_ERR(0, 103, __pyx_L1_error)
20811     __pyx_builtin_RuntimeError = __Pyx_GetBuiltinName(__pyx_n_s_RuntimeError); if
(!__pyx_builtin_RuntimeError) __PYX_ERR(0, 105, __pyx_L1_error)
20812     __pyx_builtin_TypeError = __Pyx_GetBuiltinName(__pyx_n_s_TypeError); if (!__pyx_builtin_TypeError)
__PYX_ERR(0, 105, __pyx_L1_error)
20813     __pyx_builtin_ValueError = __Pyx_GetBuiltinName(__pyx_n_s_ValueError); if
(!__pyx_builtin_ValueError) __PYX_ERR(0, 106, __pyx_L1_error)
20814     __pyx_builtin_NotImplemented = __Pyx_GetBuiltinName(__pyx_n_s_NotImplemented); if
(!__pyx_builtin_NotImplemented) __PYX_ERR(0, 159, __pyx_L1_error)
20815     __pyx_builtin_range = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_range) __PYX_ERR(0,
236, __pyx_L1_error)
20816     #if PY_MAJOR_VERSION >= 3
20817     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_xrange)
__PYX_ERR(0, 1099, __pyx_L1_error)
20818     #else
20819     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_xrange); if (!__pyx_builtin_xrange)
__PYX_ERR(0, 1099, __pyx_L1_error)
20820     #endif
20821     return 0;
20822     __pyx_L1_error:;
20823     return -1;
20824 }
20825
20826 static CYTHON_SMALL_CODE int __Pyx_InitCachedConstants(void) {
20827     __Pyx_RefNannyDeclarations
20828     __Pyx_RefNannySetupContext("__Pyx_InitCachedConstants", 0);
20829
20830     /* "(tree fragment)":2
20831     * def __reduce_cython__(self):
20832     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20833     * def __setstate_cython__(self, __pyx_state):
20834     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20835     */
20836     __pyx_tuple__3 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__3)) __PYX_ERR(1, 2, __pyx_L1_error)
20837     __Pyx_GOTREF(__pyx_tuple__3);
20838     __Pyx_GIVEREF(__pyx_tuple__3);
20839
20840     /* "(tree fragment)":4

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20841 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20842 * def __setstate_cython__(self, __pyx_state):
20843 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20844 */
20845 __pyx_tuple__4 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__4)) __PYX_ERR(1, 4, __pyx_L1_error)
20846 __Pyx_GOTREF(__pyx_tuple__4);
20847 __Pyx_GIVEREF(__pyx_tuple__4);
20848
20849 /* "PyClical.pyx":636
20850 *     TypeError: Not applicable.
20851 *     """
20852 *     raise TypeError("Not applicable.") # ««««««««
20853 *
20854 *     def __iter__(self):
20855 */
20856 __pyx_tuple__10 = PyTuple_Pack(1, __pyx_kp_u_Not_applicable); if (unlikely(!__pyx_tuple__10))
__PYX_ERR(0, 636, __pyx_L1_error)
20857 __Pyx_GOTREF(__pyx_tuple__10);
20858 __Pyx_GIVEREF(__pyx_tuple__10);
20859
20860 /* "(tree fragment)":2
20861 * def __reduce_cython__(self):
20862 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20863 * def __setstate_cython__(self, __pyx_state):
20864 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20865 */
20866 __pyx_tuple__11 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__11)) __PYX_ERR(1, 2, __pyx_L1_error)
20867 __Pyx_GOTREF(__pyx_tuple__11);
20868 __Pyx_GIVEREF(__pyx_tuple__11);
20869
20870 /* "(tree fragment)":4
20871 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20872 * def __setstate_cython__(self, __pyx_state):
20873 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__") # ««««««««
20874 */
20875 __pyx_tuple__12 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
(unlikely(!__pyx_tuple__12)) __PYX_ERR(1, 4, __pyx_L1_error)
20876 __Pyx_GOTREF(__pyx_tuple__12);
20877 __Pyx_GIVEREF(__pyx_tuple__12);
20878
20879 /* "PyClical.pyx":406
20880 *     return index_set_to_str( self.unwrap() ).decode()
20881 *
20882 * def index_set_hidden_doctests(): # ««««««««
20883 *     """
20884 *     Tests for functions that Doctest cannot see.
20885 */
20886 __pyx_codeobj__13 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_index_set_hidden_doctests, 406,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__13)) __PYX_ERR(0, 406, __pyx_L1_error)
20887
20888 /* "PyClical.pyx":1253
20889 *     return clifford_to_str( self.unwrap() ).decode()
20890 *
20891 * def clifford_hidden_doctests(): # ««««««««
20892 *     """
20893 *     Tests for functions that Doctest cannot see.
20894 */
20895 __pyx_codeobj__14 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_clifford_hidden_doctests, 1253,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__14)) __PYX_ERR(0, 1253, __pyx_L1_error)
20896
20897 /* "PyClical.pyx":1907
20898 * scalar_epsilon = epsilon
20899 *
20900 * pi = atan(clifford(1.0)) * 4.0 # ««««««««
20901 * tau = atan(clifford(1.0)) * 8.0
20902 *
20903 */
20904 __pyx_tuple__15 = PyTuple_Pack(1, __pyx_float_1_0); if (unlikely(!__pyx_tuple__15)) __PYX_ERR(0,
1907, __pyx_L1_error)
20905 __Pyx_GOTREF(__pyx_tuple__15);
20906 __Pyx_GIVEREF(__pyx_tuple__15);
20907
20908 /* "PyClical.pyx":1936
20909 *     """
20910 *
20911 *     def e(obj): # ««««««««
20912 *         """
20913 *         Abbreviation for clifford(index_set(obj)).
20914 */
20915 __pyx_tuple__16 = PyTuple_Pack(1, __pyx_n_s_obj); if (unlikely(!__pyx_tuple__16)) __PYX_ERR(0, 1936,
__pyx_L1_error)

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20916 __Pyx_GOTREF(__pyx_tuple__16);
20917 __Pyx_GIVEREF(__pyx_tuple__16);
20918 __pyx_codeobj__17 = (PyObject *)__Pyx_PyCode_New(1, 0, 1, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__16, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClicl.pyx, __pyx_n_s_e, 1936, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__17)) __PYX_ERR(0, 1936, __pyx_L1_error)

20919
20920 /* "PyClicl.pyx":1949
20921 *     return clifford(index_set(obj))
20922 *
20923 * def istpq(p, q):                # ««««««««
20924 *     """
20925 *     Abbreviation for index_set({-q,...p}).
20926 */
20927 __pyx_tuple__18 = PyTuple_Pack(2, __pyx_n_s_p, __pyx_n_s_q); if (unlikely(!__pyx_tuple__18))
__PYX_ERR(0, 1949, __pyx_L1_error)
20928 __Pyx_GOTREF(__pyx_tuple__18);
20929 __Pyx_GIVEREF(__pyx_tuple__18);
20930 __pyx_codeobj__19 = (PyObject *)__Pyx_PyCode_New(2, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__18, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClicl.pyx, __pyx_n_s_istpq, 1949, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__19)) __PYX_ERR(0, 1949, __pyx_L1_error)

20931
20932 /* "PyClicl.pyx":1958
20933 *     return index_set(set(range(-q,p+1)))
20934 *
20935 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].          #
20936 * ««««««««
20937 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
20938 *
20939 */
20940 __pyx_tuple__20 = PyTuple_Pack(1, __pyx_int_4); if (unlikely(!__pyx_tuple__20)) __PYX_ERR(0, 1958,
__pyx_L1_error)
20941 __Pyx_GOTREF(__pyx_tuple__20);
20942 __Pyx_GIVEREF(__pyx_tuple__20);
20943 __pyx_tuple__21 = PyTuple_Pack(1, __pyx_int_neg_1); if (unlikely(!__pyx_tuple__21)) __PYX_ERR(0,
1958, __pyx_L1_error)
20944 __Pyx_GOTREF(__pyx_tuple__21);
20945 __Pyx_GIVEREF(__pyx_tuple__21);
20946
20947 /* "PyClicl.pyx":1962
20948 *
20949 * # Doctest interface.
20950 * def _test():                # ««««««««
20951 *     import PyClicl, doctest
20952 *     return doctest.testmod(PyClicl)
20953 */
20954 __pyx_tuple__22 = PyTuple_Pack(2, __pyx_n_s_PyClicl, __pyx_n_s_doctest); if
(unlikely(!__pyx_tuple__22)) __PYX_ERR(0, 1962, __pyx_L1_error)
20955 __Pyx_GOTREF(__pyx_tuple__22);
20956 __Pyx_GIVEREF(__pyx_tuple__22);
20957 __pyx_codeobj__23 = (PyObject *)__Pyx_PyCode_New(0, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__22, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClicl.pyx, __pyx_n_s_test, 1962, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__23)) __PYX_ERR(0, 1962, __pyx_L1_error)
20958 __Pyx_RefNannyFinishContext();
20959 return 0;
20960 __pyx_L1_error:;
20961 __Pyx_RefNannyFinishContext();
20962 return -1;
20963 }
20964 static CYTHON_SMALL_CODE int __Pyx_InitGlobals(void) {
20965     if (__Pyx_InitStrings(__pyx_string_tab) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
20966     __pyx_float_0_0 = PyFloat_FromDouble(0.0); if (unlikely(!__pyx_float_0_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20967     __pyx_float_1_0 = PyFloat_FromDouble(1.0); if (unlikely(!__pyx_float_1_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20968     __pyx_float_4_0 = PyFloat_FromDouble(4.0); if (unlikely(!__pyx_float_4_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20969     __pyx_float_8_0 = PyFloat_FromDouble(8.0); if (unlikely(!__pyx_float_8_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20970     __pyx_int_0 = PyInt_FromLong(0); if (unlikely(!__pyx_int_0)) __PYX_ERR(0, 1, __pyx_L1_error)
20971     __pyx_int_1 = PyInt_FromLong(1); if (unlikely(!__pyx_int_1)) __PYX_ERR(0, 1, __pyx_L1_error)
20972     __pyx_int_4 = PyInt_FromLong(4); if (unlikely(!__pyx_int_4)) __PYX_ERR(0, 1, __pyx_L1_error)
20973     __pyx_int_neg_1 = PyInt_FromLong(-1); if (unlikely(!__pyx_int_neg_1)) __PYX_ERR(0, 1,
__pyx_L1_error)
20974     return 0;
20975     __pyx_L1_error:;
20976     return -1;
20977 }
20978
20979 static CYTHON_SMALL_CODE int __Pyx_modinit_global_init_code(void); /*proto*/
20980 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_export_code(void); /*proto*/
20981 static CYTHON_SMALL_CODE int __Pyx_modinit_function_export_code(void); /*proto*/
20982 static CYTHON_SMALL_CODE int __Pyx_modinit_type_init_code(void); /*proto*/
20983 static CYTHON_SMALL_CODE int __Pyx_modinit_type_import_code(void); /*proto*/

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20984 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_import_code(void); /*proto*/
20985 static CYTHON_SMALL_CODE int __Pyx_modinit_function_import_code(void); /*proto*/
20986
20987 static int __Pyx_modinit_global_init_code(void) {
20988     __Pyx_RefNannyDeclarations
20989     __Pyx_RefNannySetupContext("__Pyx_modinit_global_init_code", 0);
20990     /*---- Global init code ----*/
20991     __Pyx_RefNannyFinishContext();
20992     return 0;
20993 }
20994
20995 static int __Pyx_modinit_variable_export_code(void) {
20996     __Pyx_RefNannyDeclarations
20997     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_export_code", 0);
20998     /*---- Variable export code ----*/
20999     __Pyx_RefNannyFinishContext();
21000     return 0;
21001 }
21002
21003 static int __Pyx_modinit_function_export_code(void) {
21004     __Pyx_RefNannyDeclarations
21005     __Pyx_RefNannySetupContext("__Pyx_modinit_function_export_code", 0);
21006     /*---- Function export code ----*/
21007     __Pyx_RefNannyFinishContext();
21008     return 0;
21009 }
21010
21011 static int __Pyx_modinit_type_init_code(void) {
21012     __Pyx_RefNannyDeclarations
21013     int __pyx_lineno = 0;
21014     const char *__pyx_filename = NULL;
21015     int __pyx_clineno = 0;
21016     __Pyx_RefNannySetupContext("__Pyx_modinit_type_init_code", 0);
21017     /*---- Type init code ----*/
21018     __pyx_vtabptr_8PyClical_index_set = &__pyx_vtable_8PyClical_index_set;
21019     __pyx_vtable_8PyClical_index_set.wrap = (PyObject *) (struct __pyx_obj_8PyClical_index_set *,
IndexSet)) __pyx_f_8PyClical_9index_set_wrap;
21020     __pyx_vtable_8PyClical_index_set.unwrap = (IndexSet *) (struct __pyx_obj_8PyClical_index_set
*) __pyx_f_8PyClical_9index_set_unwrap;
21021     __pyx_vtable_8PyClical_index_set.copy = (PyObject *) (struct __pyx_obj_8PyClical_index_set *, int
__pyx_skip_dispatch) __pyx_f_8PyClical_9index_set_copy;
21022     if (PyType_Ready(&__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46, __pyx_L1_error)
21023     #if PY_VERSION_HEX < 0x030800B1
21024     __pyx_type_8PyClical_index_set.tp_print = 0;
21025     #endif
21026     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical_index_set.tp_dictoffset && __pyx_type_8PyClical_index_set.tp_getattro ==
PyObject_GenericGetAttr)) {
21027         __pyx_type_8PyClical_index_set.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21028     }
21029     #if CYTHON_COMPILING_IN_CPYTHON
21030     {
21031         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__setitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21032         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21033             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21034             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__.doc =
__pyx_doc_8PyClical_9index_set_8__setitem__;
21035             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
21036         }
21037     }
21038     #endif
21039     #if CYTHON_COMPILING_IN_CPYTHON
21040     {
21041         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21042         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21043             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21044             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__.doc =
__pyx_doc_8PyClical_9index_set_10__getitem__;
21045             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
21046         }
21047     }
21048     #endif
21049     #if CYTHON_COMPILING_IN_CPYTHON
21050     {
21051         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
"__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21052         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21053             __pyx_wrapperbase_8PyClical_9index_set_12__contains__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21054             __pyx_wrapperbase_8PyClical_9index_set_12__contains__.doc =
__pyx_doc_8PyClical_9index_set_12__contains__;

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21055     ((PyWrapperDescrObject *)wrapper)->d_base =
21056     &__pyx_wrapperbase_8PyClical_9index_set_12__contains__;
21057 }
21058 #endif
21059 #if CYTHON_COMPILING_IN_CPYTHON
21060 {
21061     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21062     "__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21063     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21064         __pyx_wrapperbase_8PyClical_9index_set_14__iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21065         __pyx_wrapperbase_8PyClical_9index_set_14__iter__.doc = __pyx_doc_8PyClical_9index_set_14__iter__;
21066         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_14__iter__;
21067     }
21068 #endif
21069 #if CYTHON_COMPILING_IN_CPYTHON
21070 {
21071     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21072     "__invert__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21073     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21074         __pyx_wrapperbase_8PyClical_9index_set_17__invert__ = *((PyWrapperDescrObject
21075 *)wrapper)->d_base;
21076         __pyx_wrapperbase_8PyClical_9index_set_17__invert__.doc =
21077         __pyx_doc_8PyClical_9index_set_17__invert__;
21078         ((PyWrapperDescrObject *)wrapper)->d_base =
21079         &__pyx_wrapperbase_8PyClical_9index_set_17__invert__;
21080     }
21081 #endif
21082 #if CYTHON_COMPILING_IN_CPYTHON
21083 {
21084     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21085     "__xor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21086     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21087         __pyx_wrapperbase_8PyClical_9index_set_19__xor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21088         __pyx_wrapperbase_8PyClical_9index_set_19__xor__.doc = __pyx_doc_8PyClical_9index_set_19__xor__;
21089         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_19__xor__;
21090     }
21091 #endif
21092 #if CYTHON_COMPILING_IN_CPYTHON
21093 {
21094     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21095     "__ixor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21096     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21097         __pyx_wrapperbase_8PyClical_9index_set_21__ixor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21098         __pyx_wrapperbase_8PyClical_9index_set_21__ixor__.doc =
21099         __pyx_doc_8PyClical_9index_set_21__ixor__;
21100         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_21__ixor__;
21101     }
21102 #endif
21103 #if CYTHON_COMPILING_IN_CPYTHON
21104 {
21105     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21106     "__and__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21107     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21108         __pyx_wrapperbase_8PyClical_9index_set_23__and__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21109         __pyx_wrapperbase_8PyClical_9index_set_23__and__.doc = __pyx_doc_8PyClical_9index_set_23__and__;
21110         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_23__and__;
21111     }
21112 #endif
21113 #if CYTHON_COMPILING_IN_CPYTHON
21114 {
21115     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set,
21116     "__iand__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21117     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21118         __pyx_wrapperbase_8PyClical_9index_set_25__iand__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21119         __pyx_wrapperbase_8PyClical_9index_set_25__iand__.doc =
21120         __pyx_doc_8PyClical_9index_set_25__iand__;
21121         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_25__iand__;
21122     }
21123 #endif
21124 #if CYTHON_COMPILING_IN_CPYTHON
21125 {
21126     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_index_set, "__or__");
21127     if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21128     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21129         __pyx_wrapperbase_8PyClical_9index_set_27__or__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21130         __pyx_wrapperbase_8PyClical_9index_set_27__or__.doc = __pyx_doc_8PyClical_9index_set_27__or__;
21131         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_27__or__;
21132     }
21133 #endif

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21128     #endif
21129     #if CYTHON_COMPILING_IN_CPYTHON
21130     {
21131         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21132         "__ior__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21133         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21134             __pyx_wrapperbase_8PyClical_9index_set_29__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21135             __pyx_wrapperbase_8PyClical_9index_set_29__ior__.doc = __pyx_doc_8PyClical_9index_set_29__ior__;
21136             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_29__ior__;
21137         }
21138     #endif
21139     #if CYTHON_COMPILING_IN_CPYTHON
21140     {
21141         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21142         "__repr__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21143         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21144             __pyx_wrapperbase_8PyClical_9index_set_47__repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21145             __pyx_wrapperbase_8PyClical_9index_set_47__repr__.doc = __pyx_doc_8PyClical_9index_set_47__repr__;
21146             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_47__repr__;
21147         }
21148     #endif
21149     #if CYTHON_COMPILING_IN_CPYTHON
21150     {
21151         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21152         "__str__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21153         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21154             __pyx_wrapperbase_8PyClical_9index_set_49__str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21155             __pyx_wrapperbase_8PyClical_9index_set_49__str__.doc = __pyx_doc_8PyClical_9index_set_49__str__;
21156             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_49__str__;
21157         }
21158     #endif
21159     if (__Pyx_SetVtable(__pyx_type_8PyClical_index_set.tp_dict, __pyx_vtabptr_8PyClical_index_set) < 0)
21160     __PYX_ERR(0, 46, __pyx_L1_error)
21161     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_index_set, (PyObject *) &__pyx_type_8PyClical_index_set) < 0)
21162     __PYX_ERR(0, 46, __pyx_L1_error)
21163     if (__Pyx_setup_reduce((PyObject *) &__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46,
21164     __pyx_L1_error)
21165     __pyx_ptype_8PyClical_index_set = &__pyx_type_8PyClical_index_set;
21166     __pyx_vtabptr_8PyClical_clifford = &__pyx_vtable_8PyClical_clifford;
21167     __pyx_vtable_8PyClical_clifford.wrap = (PyObject *)(&struct __pyx_obj_8PyClical_clifford *,
21168     Clifford))__pyx_f_8PyClical_8clifford_wrap;
21169     __pyx_vtable_8PyClical_clifford.unwrap = (Clifford *)(&struct __pyx_obj_8PyClical_clifford
21170     *)__pyx_f_8PyClical_8clifford_unwrap;
21171     __pyx_vtable_8PyClical_clifford.copy = (PyObject *)(&struct __pyx_obj_8PyClical_clifford *, int
21172     __pyx_skip_dispatch))__pyx_f_8PyClical_8clifford_copy;
21173     if (PyType_Ready(&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537, __pyx_L1_error)
21174     #if PY_VERSION_HEX < 0x030800B1
21175     __pyx_type_8PyClical_clifford.tp_print = 0;
21176     #endif
21177     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
21178     likely(!__pyx_type_8PyClical_clifford.tp_dictoffset && __pyx_type_8PyClical_clifford.tp_getattro ==
21179     PyObject_GenericGetAttr)) {
21180         __pyx_type_8PyClical_clifford.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21181     }
21182     #if CYTHON_COMPILING_IN_CPYTHON
21183     {
21184         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21185         "__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21186         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21187             __pyx_wrapperbase_8PyClical_8clifford_6__contains__ = *((PyWrapperDescrObject
21188             *)wrapper)->d_base;
21189             __pyx_wrapperbase_8PyClical_8clifford_6__contains__.doc =
21190             __pyx_doc_8PyClical_8clifford_6__contains__;
21191             ((PyWrapperDescrObject *)wrapper)->d_base =
21192             &__pyx_wrapperbase_8PyClical_8clifford_6__contains__;
21193         }
21194     #endif
21195     #if CYTHON_COMPILING_IN_CPYTHON
21196     {
21197         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21198         "__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21199         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21200             __pyx_wrapperbase_8PyClical_8clifford_8__iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21201             __pyx_wrapperbase_8PyClical_8clifford_8__iter__.doc = __pyx_doc_8PyClical_8clifford_8__iter__;
21202             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_8__iter__;
21203         }
21204     #endif
21205     #if CYTHON_COMPILING_IN_CPYTHON
21206     {
21207         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21208         "__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)

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21197     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21198         __pyx_wrapperbase_8PyClical_8clifford_14__getitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21199         __pyx_wrapperbase_8PyClical_8clifford_14__getitem__.doc =
__pyx_doc_8PyClical_8clifford_14__getitem__;
21200         ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_8clifford_14__getitem__;
21201     }
21202 }
21203 #endif
21204 #if CYTHON_COMPILING_IN_CPYTHON
21205 {
21206     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__neg__");
21207     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21208     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21209         __pyx_wrapperbase_8PyClical_8clifford_16__neg__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21210         __pyx_wrapperbase_8PyClical_8clifford_16__neg__.doc = __pyx_doc_8PyClical_8clifford_16__neg__;
21211         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_16__neg__;
21212     }
21213 #endif
21214 #if CYTHON_COMPILING_IN_CPYTHON
21215 {
21216     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__pos__");
21217     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21218     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21219         __pyx_wrapperbase_8PyClical_8clifford_18__pos__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21220         __pyx_wrapperbase_8PyClical_8clifford_18__pos__.doc = __pyx_doc_8PyClical_8clifford_18__pos__;
21221         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_18__pos__;
21222     }
21223 #endif
21224 #if CYTHON_COMPILING_IN_CPYTHON
21225 {
21226     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__add__");
21227     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21228     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21229         __pyx_wrapperbase_8PyClical_8clifford_20__add__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21230         __pyx_wrapperbase_8PyClical_8clifford_20__add__.doc = __pyx_doc_8PyClical_8clifford_20__add__;
21231         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_20__add__;
21232     }
21233 #endif
21234 #if CYTHON_COMPILING_IN_CPYTHON
21235 {
21236     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
"__iadd__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21237     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21238         __pyx_wrapperbase_8PyClical_8clifford_22__iadd__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21239         __pyx_wrapperbase_8PyClical_8clifford_22__iadd__.doc = __pyx_doc_8PyClical_8clifford_22__iadd__;
21240         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_22__iadd__;
21241     }
21242 #endif
21243 #if CYTHON_COMPILING_IN_CPYTHON
21244 {
21245     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__sub__");
21246     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21247     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21248         __pyx_wrapperbase_8PyClical_8clifford_24__sub__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21249         __pyx_wrapperbase_8PyClical_8clifford_24__sub__.doc = __pyx_doc_8PyClical_8clifford_24__sub__;
21250         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_24__sub__;
21251     }
21252 #endif
21253 #if CYTHON_COMPILING_IN_CPYTHON
21254 {
21255     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
"__isub__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21256     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21257         __pyx_wrapperbase_8PyClical_8clifford_26__isub__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21258         __pyx_wrapperbase_8PyClical_8clifford_26__isub__.doc = __pyx_doc_8PyClical_8clifford_26__isub__;
21259         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_26__isub__;
21260     }
21261 #endif
21262 #if CYTHON_COMPILING_IN_CPYTHON
21263 {
21264     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__mul__");
21265     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21266     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21267         __pyx_wrapperbase_8PyClical_8clifford_28__mul__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21268         __pyx_wrapperbase_8PyClical_8clifford_28__mul__.doc = __pyx_doc_8PyClical_8clifford_28__mul__;
21269         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_28__mul__;
21270     }
21271 #endif
21272 }
21273 #endif

```

```

21274 #if CYTHON_COMPILING_IN_CPYTHON
21275 {
21276     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21277         "__imul__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21278     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21279         __pyx_wrapperbase_8PyClical_8clifford_30__imul__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21280         __pyx_wrapperbase_8PyClical_8clifford_30__imul__.doc = __pyx_doc_8PyClical_8clifford_30__imul__;
21281         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_30__imul__;
21282     }
21283 #endif
21284 #if CYTHON_COMPILING_IN_CPYTHON
21285 {
21286     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__mod__");
21287     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21288     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21289         __pyx_wrapperbase_8PyClical_8clifford_32__mod__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21290         __pyx_wrapperbase_8PyClical_8clifford_32__mod__.doc = __pyx_doc_8PyClical_8clifford_32__mod__;
21291         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_32__mod__;
21292     }
21293 #endif
21294 #if CYTHON_COMPILING_IN_CPYTHON
21295 {
21296     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21297         "__imod__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21298     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21299         __pyx_wrapperbase_8PyClical_8clifford_34__imod__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21300         __pyx_wrapperbase_8PyClical_8clifford_34__imod__.doc = __pyx_doc_8PyClical_8clifford_34__imod__;
21301         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_34__imod__;
21302     }
21303 #endif
21304 #if CYTHON_COMPILING_IN_CPYTHON
21305 {
21306     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__and__");
21307     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21308     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21309         __pyx_wrapperbase_8PyClical_8clifford_36__and__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21310         __pyx_wrapperbase_8PyClical_8clifford_36__and__.doc = __pyx_doc_8PyClical_8clifford_36__and__;
21311         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_36__and__;
21312     }
21313 #endif
21314 #if CYTHON_COMPILING_IN_CPYTHON
21315 {
21316     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21317         "__iand__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21318     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21319         __pyx_wrapperbase_8PyClical_8clifford_38__iand__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21320         __pyx_wrapperbase_8PyClical_8clifford_38__iand__.doc = __pyx_doc_8PyClical_8clifford_38__iand__;
21321         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_38__iand__;
21322     }
21323 #endif
21324 #if CYTHON_COMPILING_IN_CPYTHON
21325 {
21326     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__xor__");
21327     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21328     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21329         __pyx_wrapperbase_8PyClical_8clifford_40__xor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21330         __pyx_wrapperbase_8PyClical_8clifford_40__xor__.doc = __pyx_doc_8PyClical_8clifford_40__xor__;
21331         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_40__xor__;
21332     }
21333 #endif
21334 #if CYTHON_COMPILING_IN_CPYTHON
21335 {
21336     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21337         "__ixor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21338     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21339         __pyx_wrapperbase_8PyClical_8clifford_42__ixor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21340         __pyx_wrapperbase_8PyClical_8clifford_42__ixor__.doc = __pyx_doc_8PyClical_8clifford_42__ixor__;
21341         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_42__ixor__;
21342     }
21343 #endif
21344 #if CYTHON_COMPILING_IN_CPYTHON
21345 {
21346     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21347         "__truediv__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21348     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21349         __pyx_wrapperbase_8PyClical_8clifford_44__truediv__ = *((PyWrapperDescrObject
21350 *)wrapper)->d_base;
21351         __pyx_wrapperbase_8PyClical_8clifford_44__truediv__.doc =
21352         __pyx_doc_8PyClical_8clifford_44__truediv__;
21353         ((PyWrapperDescrObject *)wrapper)->d_base =

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        &__pyx_wrapperbase_8PyClical_8clifford_44__truediv__;
21351     }
21352 }
21353 #endif
21354 #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
21355 #if CYTHON_COMPILING_IN_CPYTHON
21356 {
21357     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21358 "    __idiv__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21359     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21360         __pyx_wrapperbase_8PyClical_8clifford_46__idiv__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21361         __pyx_wrapperbase_8PyClical_8clifford_46__idiv__.doc = __pyx_doc_8PyClical_8clifford_46__idiv__;
21362         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_46__idiv__;
21363     }
21364 #endif
21365 #endif
21366 #if CYTHON_COMPILING_IN_CPYTHON
21367 {
21368     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__or__");
21369     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21370     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21371         __pyx_wrapperbase_8PyClical_8clifford_50__or__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21372         __pyx_wrapperbase_8PyClical_8clifford_50__or__.doc = __pyx_doc_8PyClical_8clifford_50__or__;
21373         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_50__or__;
21374     }
21375 #endif
21376 #if CYTHON_COMPILING_IN_CPYTHON
21377 {
21378     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__ior__");
21379     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21380     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21381         __pyx_wrapperbase_8PyClical_8clifford_52__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21382         __pyx_wrapperbase_8PyClical_8clifford_52__ior__.doc = __pyx_doc_8PyClical_8clifford_52__ior__;
21383         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_52__ior__;
21384     }
21385 #endif
21386 #if CYTHON_COMPILING_IN_CPYTHON
21387 {
21388     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__pow__");
21389     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21390     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21391         __pyx_wrapperbase_8PyClical_8clifford_54__pow__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21392         __pyx_wrapperbase_8PyClical_8clifford_54__pow__.doc = __pyx_doc_8PyClical_8clifford_54__pow__;
21393         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_54__pow__;
21394     }
21395 #endif
21396 #if CYTHON_COMPILING_IN_CPYTHON
21397 {
21398     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21399 "    __call__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21400     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21401         __pyx_wrapperbase_8PyClical_8clifford_60__call__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21402         __pyx_wrapperbase_8PyClical_8clifford_60__call__.doc = __pyx_doc_8PyClical_8clifford_60__call__;
21403         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_60__call__;
21404     }
21405 #endif
21406 #if CYTHON_COMPILING_IN_CPYTHON
21407 {
21408     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21409 "    __repr__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21410     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21411         __pyx_wrapperbase_8PyClical_8clifford_94__repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21412         __pyx_wrapperbase_8PyClical_8clifford_94__repr__.doc = __pyx_doc_8PyClical_8clifford_94__repr__;
21413         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_94__repr__;
21414     }
21415 #endif
21416 #if CYTHON_COMPILING_IN_CPYTHON
21417 {
21418     PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__str__");
21419     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21420     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21421         __pyx_wrapperbase_8PyClical_8clifford_96__str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21422         __pyx_wrapperbase_8PyClical_8clifford_96__str__.doc = __pyx_doc_8PyClical_8clifford_96__str__;
21423         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_96__str__;
21424     }
21425 #endif
21426 if (__Pyx_SetVtable(__pyx_type_8PyClical_clifford.tp_dict, __pyx_vtabptr_8PyClical_clifford) < 0)
21427     __PYX_ERR(0, 537, __pyx_L1_error)
21427 if (PyObject_SetAttr(__pyx_m, __pyx_n_s_clifford, (PyObject *) &__pyx_type_8PyClical_clifford) < 0)
21427     __PYX_ERR(0, 537, __pyx_L1_error)

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21428     if (__Pyx_setup_reduce((PyObject*)&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537,
__pyx_L1_error)
21429     __pyx_ptype_8PyClical_clifford = &__pyx_type_8PyClical_clifford;
21430     if (PyType_Ready(&__pyx_type_8PyClical__pyx_scope_struct__iter__) < 0) __PYX_ERR(0, 229,
__pyx_L1_error)
21431     #if PY_VERSION_HEX < 0x030800B1
21432     __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_print = 0;
21433     #endif
21434     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical__pyx_scope_struct__iter__.tp_dictoffset &&
__pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro == PyObject_GenericGetAttr)) {
21435         __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro =
__Pyx_PyObject_GenericGetAttrNoDict;
21436     }
21437     __pyx_ptype_8PyClical__pyx_scope_struct__iter__ =
&__pyx_type_8PyClical__pyx_scope_struct__iter__;
21438     __Pyx_RefNannyFinishContext();
21439     return 0;
21440     __pyx_L1_error:;
21441     __Pyx_RefNannyFinishContext();
21442     return -1;
21443 }
21444
21445 static int __Pyx_modinit_type_import_code(void) {
21446     __Pyx_RefNannyDeclarations
21447     __Pyx_RefNannySetupContext("__Pyx_modinit_type_import_code", 0);
21448     /*--- Type import code ---*/
21449     __Pyx_RefNannyFinishContext();
21450     return 0;
21451 }
21452
21453 static int __Pyx_modinit_variable_import_code(void) {
21454     __Pyx_RefNannyDeclarations
21455     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_import_code", 0);
21456     /*--- Variable import code ---*/
21457     __Pyx_RefNannyFinishContext();
21458     return 0;
21459 }
21460
21461 static int __Pyx_modinit_function_import_code(void) {
21462     __Pyx_RefNannyDeclarations
21463     __Pyx_RefNannySetupContext("__Pyx_modinit_function_import_code", 0);
21464     /*--- Function import code ---*/
21465     __Pyx_RefNannyFinishContext();
21466     return 0;
21467 }
21468
21469
21470 #ifndef CYTHON_NO_PYINIT_EXPORT
21471 #define __Pyx_PyMODINIT_FUNC PyMODINIT_FUNC
21472 #elif PY_MAJOR_VERSION < 3
21473 #ifdef __cplusplus
21474 #define __Pyx_PyMODINIT_FUNC extern "C" void
21475 #else
21476 #define __Pyx_PyMODINIT_FUNC void
21477 #endif
21478 #else
21479 #ifdef __cplusplus
21480 #define __Pyx_PyMODINIT_FUNC extern "C" PyObject *
21481 #else
21482 #define __Pyx_PyMODINIT_FUNC PyObject *
21483 #endif
21484 #endif
21485
21486
21487 #if PY_MAJOR_VERSION < 3
21488     __Pyx_PyMODINIT_FUNC initPyClical(void) CYTHON_SMALL_CODE; /*proto*/
21489     __Pyx_PyMODINIT_FUNC initPyClical(void)
21490     #else
21491     __Pyx_PyMODINIT_FUNC PyInit_PyClical(void) CYTHON_SMALL_CODE; /*proto*/
21492     __Pyx_PyMODINIT_FUNC PyInit_PyClical(void)
21493     #if CYTHON_PEP489_MULTI_PHASE_INIT
21494     {
21495         return PyModuleDef_Init(&__pyx_moduledef);
21496     }
21497     static CYTHON_SMALL_CODE int __Pyx_check_single_interpreter(void) {
21498         #if PY_VERSION_HEX >= 0x030700A1
21499         static PY_INT64_T main_interpreter_id = -1;
21500         PY_INT64_T current_id = PyInterpreterState_GetID(PyThreadState_Get()->interp);
21501         if (main_interpreter_id == -1) {
21502             main_interpreter_id = current_id;
21503             return (unlikely(current_id == -1)) ? -1 : 0;
21504         } else if (unlikely(main_interpreter_id != current_id))
21505             #else
21506             static PyInterpreterState *main_interpreter = NULL;
21507             PyInterpreterState *current_interpreter = PyThreadState_Get()->interp;
21508             if (!main_interpreter) {

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21509     main_interpreter = current_interpreter;
21510 } else if (unlikely(main_interpreter != current_interpreter))
21511 #endif
21512 {
21513     PyErr_SetString(
21514         PyExc_ImportError,
21515         "Interpreter change detected - this module can only be loaded into one interpreter per
process.");
21516     return -1;
21517 }
21518     return 0;
21519 }
21520 static CYTHON_SMALL_CODE int __Pyx_copy_spec_to_module(PyObject *spec, PyObject *moddict, const char*
from_name, const char* to_name, int allow_none) {
21521     PyObject *value = PyObject_GetAttrString(spec, from_name);
21522     int result = 0;
21523     if (likely(value)) {
21524         if (allow_none || value != Py_None) {
21525             result = PyDict_SetItemString(moddict, to_name, value);
21526         }
21527         Py_DECREF(value);
21528     } else if (PyErr_ExceptionMatches(PyExc_AttributeError)) {
21529         PyErr_Clear();
21530     } else {
21531         result = -1;
21532     }
21533     return result;
21534 }
21535 static CYTHON_SMALL_CODE PyObject* __pyx_pymod_create(PyObject *spec, CYTHON_UNUSED PyModuleDef *def)
{
21536     PyObject *module = NULL, *moddict, *modname;
21537     if (__Pyx_check_single_interpreter())
21538         return NULL;
21539     if (__pyx_m)
21540         return __Pyx_NewRef(__pyx_m);
21541     modname = PyObject_GetAttrString(spec, "name");
21542     if (unlikely(!modname)) goto bad;
21543     module = PyModule_NewObject(modname);
21544     Py_DECREF(modname);
21545     if (unlikely(!module)) goto bad;
21546     moddict = PyModule_GetDict(module);
21547     if (unlikely(!moddict)) goto bad;
21548     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "loader", "__loader__", 1) < 0)) goto bad;
21549     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "origin", "__file__", 1) < 0)) goto bad;
21550     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "parent", "__package__", 1) < 0)) goto bad;
21551     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "submodule_search_locations", "__path__", 0)
< 0)) goto bad;
21552     return module;
21553 bad:
21554     Py_XDECREF(module);
21555     return NULL;
21556 }
21557
21558
21559 static CYTHON_SMALL_CODE int __pyx_pymod_exec_PyClical(PyObject *__pyx_pyinit_module)
21560 #endif
21561 #endif
21562 {
21563     PyObject *__pyx_t_1 = NULL;
21564     PyObject *__pyx_t_2 = NULL;
21565     PyObject *__pyx_t_3 = NULL;
21566     int __pyx_t_4;
21567     int __pyx_lineno = 0;
21568     const char *__pyx_filename = NULL;
21569     int __pyx_clineno = 0;
21570     __Pyx_RefNannyDeclarations
21571     #if CYTHON_PEP489_MULTI_PHASE_INIT
21572     if (__pyx_m) {
21573         if (__pyx_m == __pyx_pyinit_module) return 0;
21574         PyErr_SetString(PyExc_RuntimeError, "Module 'PyClical' has already been imported.
Re-initialisation is not supported.");
21575         return -1;
21576     }
21577     #elif PY_MAJOR_VERSION >= 3
21578     if (__pyx_m) return __Pyx_NewRef(__pyx_m);
21579     #endif
21580     #if CYTHON_REFNANNY
21581     __Pyx_RefNanny = __Pyx_RefNannyImportAPI("refnanny");
21582     if (!__Pyx_RefNanny) {
21583         PyErr_Clear();
21584         __Pyx_RefNanny = __Pyx_RefNannyImportAPI("Cython.Runtime.refnanny");
21585     } if (!__Pyx_RefNanny)
21586         Py_FatalError("failed to import 'refnanny' module");
21587     }
21588 #endif
21589     __Pyx_RefNannySetupContext("__Pyx_PyMODINIT_FUNC PyInit_PyClical(void)", 0);
21590     if (__Pyx_check_binary_version() < 0) __PYX_ERR(0, 1, __pyx_L1_error)

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21591 #ifdef __Pxy_PyFrame_Initialize_Offsets
21592 __Pxy_PyFrame_Initialize_Offsets();
21593 #endif
21594 __pyx_empty_tuple = PyTuple_New(0); if (unlikely(!__pyx_empty_tuple)) __PYX_ERR(0, 1,
__pyx_L1_error)
21595 __pyx_empty_bytes = PyBytes_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_bytes)) __PYX_ERR(0,
1, __pyx_L1_error)
21596 __pyx_empty_unicode = PyUnicode_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_unicode))
__PYX_ERR(0, 1, __pyx_L1_error)
21597 #ifdef __Pyx_CyFunction_USED
21598 if (__pyx_CyFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21599 #endif
21600 #ifdef __Pyx_FusedFunction_USED
21601 if (__pyx_FusedFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21602 #endif
21603 #ifdef __Pyx_Coroutine_USED
21604 if (__pyx_Coroutine_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21605 #endif
21606 #ifdef __Pyx_Generator_USED
21607 if (__pyx_Generator_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21608 #endif
21609 #ifdef __Pyx_AsyncGen_USED
21610 if (__pyx_AsyncGen_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21611 #endif
21612 #ifdef __Pyx_StopAsyncIteration_USED
21613 if (__pyx_StopAsyncIteration_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21614 #endif
21615 /----- Library function declarations -----*/
21616 /----- Threads initialization code -----*/
21617 #if defined(WITH_THREAD) && PY_VERSION_HEX < 0x030700F0 && defined(__PYX_FORCE_INIT_THREADS) &&
__PYX_FORCE_INIT_THREADS
21618 PyEval_InitThreads();
21619 #endif
21620 /----- Module creation code -----*/
21621 #if CYTHON_PEP489_MULTI_PHASE_INIT
21622 __pyx_m = __pyx_pyinit_module;
21623 Py_INCREF(__pyx_m);
21624 #else
21625 #if PY_MAJOR_VERSION < 3
21626 __pyx_m = Py_InitModule4("PyClical", __pyx_methods, 0, 0, PYTHON_API_VERSION); Py_XINCREF(__pyx_m);
21627 #else
21628 __pyx_m = PyModule_Create(&__pyx_moduledef);
21629 #endif
21630 if (unlikely(!__pyx_m)) __PYX_ERR(0, 1, __pyx_L1_error)
21631 #endif
21632 __pyx_d = PyModule_GetDict(__pyx_m); if (unlikely(!__pyx_d)) __PYX_ERR(0, 1, __pyx_L1_error)
21633 Py_INCREF(__pyx_d);
21634 __pyx_b = PyImport_AddModule(__Pyx_BUILTIN_MODULE_NAME); if (unlikely(!__pyx_b)) __PYX_ERR(0, 1,
__pyx_L1_error)
21635 Py_INCREF(__pyx_b);
21636 __pyx_cython_runtime = PyImport_AddModule((char *) "cython_runtime"); if
(unlikely(!__pyx_cython_runtime)) __PYX_ERR(0, 1, __pyx_L1_error)
21637 Py_INCREF(__pyx_cython_runtime);
21638 if (PyObject_SetAttrString(__pyx_m, "__builtins__", __pyx_b) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
21639 /----- Initialize various global constants etc. -----*/
21640 if (__Pyx_InitGlobals() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21641 #if PY_MAJOR_VERSION < 3 && (__PYX_DEFAULT_STRING_ENCODING_IS_ASCII ||
__PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT)
21642 if (__Pyx_init_sys_getdefaultencoding_params() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21643 #endif
21644 if (__pyx_module_is_main_PyClical) {
21645     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_name, __pyx_n_s_main) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21646 }
21647 #if PY_MAJOR_VERSION >= 3
21648 {
21649     PyObject *modules = PyImport_GetModuleDict(); if (unlikely(!modules)) __PYX_ERR(0, 1,
__pyx_L1_error)
21650     if (!PyDict_GetItemString(modules, "PyClical")) {
21651         if (unlikely(PyDict_SetItemString(modules, "PyClical", __pyx_m) < 0)) __PYX_ERR(0, 1,
__pyx_L1_error)
21652     }
21653 }
21654 #endif
21655 /----- Builtin init code -----*/
21656 if (__Pyx_InitCachedBuiltins() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21657 /----- Constants init code -----*/
21658 if (__Pyx_InitCachedConstants() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21659 /----- Global type/function init code -----*/
21660 (void)__Pyx_modinit_global_init_code();
21661 (void)__Pyx_modinit_variable_export_code();
21662 (void)__Pyx_modinit_function_export_code();
21663 if (unlikely(__Pyx_modinit_type_init_code() < 0)) __PYX_ERR(0, 1, __pyx_L1_error)
21664 (void)__Pyx_modinit_type_import_code();
21665 (void)__Pyx_modinit_variable_import_code();
21666 (void)__Pyx_modinit_function_import_code();
21667 /----- Execution code -----*/
21668 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)

```



```

21669     if (__Pyx_patch_abc() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21670     #endif
21671
21672     /* "PyClical.pyx":29
21673     * # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
21674     *
21675     * import math                # ««««««««
21676     * import numbers
21677     * import collections
21678     */
21679     __pyx_t_1 = __Pyx_Import(__pyx_n_s_math, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 29,
__pyx_L1_error)
21680     __Pyx_GOTREF(__pyx_t_1);
21681     if (PyDict_SetItem(__pyx_d, __pyx_n_s_math, __pyx_t_1) < 0) __PYX_ERR(0, 29, __pyx_L1_error)
21682     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21683
21684     /* "PyClical.pyx":30
21685     *
21686     * import math
21687     * import numbers                # ««««««««
21688     * import collections
21689     *
21690     */
21691     __pyx_t_1 = __Pyx_Import(__pyx_n_s_numbers, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 30,
__pyx_L1_error)
21692     __Pyx_GOTREF(__pyx_t_1);
21693     if (PyDict_SetItem(__pyx_d, __pyx_n_s_numbers, __pyx_t_1) < 0) __PYX_ERR(0, 30, __pyx_L1_error)
21694     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21695
21696     /* "PyClical.pyx":31
21697     * import math
21698     * import numbers
21699     * import collections                # ««««««««
21700     *
21701     * from PyClical cimport *
21702     */
21703     __pyx_t_1 = __Pyx_Import(__pyx_n_s_collections, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 31,
__pyx_L1_error)
21704     __Pyx_GOTREF(__pyx_t_1);
21705     if (PyDict_SetItem(__pyx_d, __pyx_n_s_collections, __pyx_t_1) < 0) __PYX_ERR(0, 31, __pyx_L1_error)
21706     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21707
21708     /* "PyClical.pyx":35
21709     * from PyClical cimport *
21710     *
21711     * __version__ = str(glucat_package_version,'utf-8')                # ««««««««
21712     *
21713     * # Forward reference
21714     */
21715     __pyx_t_1 = __pyx_convert_PyBytes_string_to_py_std_in_string(glucat_package_version); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21716     __Pyx_GOTREF(__pyx_t_1);
21717     __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 35, __pyx_L1_error)
21718     __Pyx_GOTREF(__pyx_t_2);
21719     __Pyx_GIVEREF(__pyx_t_1);
21720     PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
21721     __Pyx_INCREF(__pyx_kp_u_utf_8);
21722     __Pyx_GIVEREF(__pyx_kp_u_utf_8);
21723     PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_kp_u_utf_8);
21724     __pyx_t_1 = 0;
21725     __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)(&PyUnicode_Type)), __pyx_t_2, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21726     __Pyx_GOTREF(__pyx_t_1);
21727     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21728     if (PyDict_SetItem(__pyx_d, __pyx_n_s_version, __pyx_t_1) < 0) __PYX_ERR(0, 35, __pyx_L1_error)
21729     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21730
21731     /* "PyClical.pyx":406
21732     *         return index_set_to_str( self.unwrap() ).decode()
21733     *
21734     * def index_set_hidden_doctests():                # ««««««««
21735     *     """
21736     *     Tests for functions that Doctest cannot see.
21737     */
21738     __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_1index_set_hidden_doctests, NULL,
__pyx_n_s_PyClical); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 406, __pyx_L1_error)
21739     __Pyx_GOTREF(__pyx_t_1);
21740     if (PyDict_SetItem(__pyx_d, __pyx_n_s_index_set_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 406,
__pyx_L1_error)
21741     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21742
21743     /* "PyClical.pyx":1253
21744     *         return clifford_to_str( self.unwrap() ).decode()
21745     *
21746     * def clifford_hidden_doctests():                # ««««««««
21747     *     """
21748     *     Tests for functions that Doctest cannot see.

```



```

21749 */
21750 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_9clifford_hidden_doctests, NULL,
__pyx_n_s_PyClical); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1253, __pyx_L1_error)
21751 __Pyx_GOTREF(__pyx_t_1);
21752 if (PyDict_SetItem(__pyx_d, __pyx_n_s_clifford_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 1253,
__pyx_L1_error)
21753 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21754
21755 /* "PyClical.pyx":1905
21756 *
21757 * # Some abbreviations.
21758 * scalar_epsilon = epsilon # ««««««««
21759 *
21760 * pi = atan(clifford(1.0)) * 4.0
21761 */
21762 __pyx_t_1 = PyFloat_FromDouble(epsilon); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1905,
__pyx_L1_error)
21763 __Pyx_GOTREF(__pyx_t_1);
21764 if (PyDict_SetItem(__pyx_d, __pyx_n_s_scalar_epsilon, __pyx_t_1) < 0) __PYX_ERR(0, 1905,
__pyx_L1_error)
21765 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21766
21767 /* "PyClical.pyx":1907
21768 * scalar_epsilon = epsilon
21769 *
21770 * pi = atan(clifford(1.0)) * 4.0 # ««««««««
21771 * tau = atan(clifford(1.0)) * 8.0
21772 *
21773 */
21774 __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1907, __pyx_L1_error)
21775 __Pyx_GOTREF(__pyx_t_1);
21776 __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1907,
__pyx_L1_error)
21777 __Pyx_GOTREF(__pyx_t_2);
21778 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21779 __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_4_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1907, __pyx_L1_error)
21780 __Pyx_GOTREF(__pyx_t_1);
21781 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21782 if (PyDict_SetItem(__pyx_d, __pyx_n_s_pi, __pyx_t_1) < 0) __PYX_ERR(0, 1907, __pyx_L1_error)
21783 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21784
21785 /* "PyClical.pyx":1908
21786 *
21787 * pi = atan(clifford(1.0)) * 4.0
21788 * tau = atan(clifford(1.0)) * 8.0 # ««««««««
21789 *
21790 * cl = clifford
21791 */
21792 __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1908, __pyx_L1_error)
21793 __Pyx_GOTREF(__pyx_t_1);
21794 __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1908,
__pyx_L1_error)
21795 __Pyx_GOTREF(__pyx_t_2);
21796 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21797 __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_8_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1908, __pyx_L1_error)
21798 __Pyx_GOTREF(__pyx_t_1);
21799 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21800 if (PyDict_SetItem(__pyx_d, __pyx_n_s_tau, __pyx_t_1) < 0) __PYX_ERR(0, 1908, __pyx_L1_error)
21801 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21802
21803 /* "PyClical.pyx":1910
21804 * tau = atan(clifford(1.0)) * 8.0
21805 *
21806 * cl = clifford # ««««««««
21807 * ""
21808 * Abbreviation for clifford.
21809 */
21810 if (PyDict_SetItem(__pyx_d, __pyx_n_s_cl, ((PyObject *)__pyx_ptype_8PyClical_clifford)) < 0)
__PYX_ERR(0, 1910, __pyx_L1_error)
21811
21812 /* "PyClical.pyx":1928
21813 * ""
21814 *
21815 * ist = index_set # ««««««««
21816 * ""
21817 * Abbreviation for index_set.
21818 */
21819 if (PyDict_SetItem(__pyx_d, __pyx_n_s_ist, ((PyObject *)__pyx_ptype_8PyClical_index_set)) < 0)
__PYX_ERR(0, 1928, __pyx_L1_error)
21820
21821 /* "PyClical.pyx":1936
21822 * ""
21823 *

```

```

21824 * def e(obj):          # ««««««««
21825 *     """
21826 *     Abbreviation for clifford(index_set(obj)).
21827 */
21828 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_89e, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1936, __pyx_L1_error)
21829 __Pyx_GOTREF(__pyx_t_1);
21830 if (PyDict_SetItem(__pyx_d, __pyx_n_s_e, __pyx_t_1) < 0) __PYX_ERR(0, 1936, __pyx_L1_error)
21831 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21832
21833 /* "PyClical.pyx":1949
21834 *     return clifford(index_set(obj))
21835 *
21836 * def istpq(p, q):      # ««««««««
21837 *     """
21838 *     Abbreviation for index_set({-q,...p}).
21839 */
21840 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_91istpq, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1949, __pyx_L1_error)
21841 __Pyx_GOTREF(__pyx_t_1);
21842 if (PyDict_SetItem(__pyx_d, __pyx_n_s_istpq, __pyx_t_1) < 0) __PYX_ERR(0, 1949, __pyx_L1_error)
21843 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21844
21845 /* "PyClical.pyx":1958
21846 *     return index_set(set(range(-q,p+1)))
21847 *
21848 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].          #
21849 * ««««««««
21850 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
21851 *
21852 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21853 __Pyx_GOTREF(__pyx_t_1);
21854 __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1958, __pyx_L1_error)
21855 __Pyx_GOTREF(__pyx_t_2);
21856 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21857 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21858 __Pyx_GOTREF(__pyx_t_1);
21859 __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 1958, __pyx_L1_error)
21860 __Pyx_GOTREF(__pyx_t_3);
21861 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21862 __pyx_t_1 = PyNumber_Add(__pyx_t_2, __pyx_t_3); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21863 __Pyx_GOTREF(__pyx_t_1);
21864 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21865 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21866 if (PyDict_SetItem(__pyx_d, __pyx_n_s_ninf3, __pyx_t_1) < 0) __PYX_ERR(0, 1958, __pyx_L1_error)
21867 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21868
21869 /* "PyClical.pyx":1959
21870 *
21871 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
21872 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].          # ««««««««
21873 *
21874 * # Doctest interface.
21875 */
21876 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21877 __Pyx_GOTREF(__pyx_t_1);
21878 __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 1959, __pyx_L1_error)
21879 __Pyx_GOTREF(__pyx_t_3);
21880 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21881 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21882 __Pyx_GOTREF(__pyx_t_1);
21883 __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1959, __pyx_L1_error)
21884 __Pyx_GOTREF(__pyx_t_2);
21885 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21886 __pyx_t_1 = PyNumber_Subtract(__pyx_t_3, __pyx_t_2); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_L1_error)
21887 __Pyx_GOTREF(__pyx_t_1);
21888 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21889 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21890 if (PyDict_SetItem(__pyx_d, __pyx_n_s_nbar3, __pyx_t_1) < 0) __PYX_ERR(0, 1959, __pyx_L1_error)
21891 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21892
21893 /* "PyClical.pyx":1962
21894 *
21895 * # Doctest interface.
21896 * def _test():          # ««««««««
21897 *     import PyClical, doctest

```

```

21898 *      return doctest.testmod(PyClical)
21899 */
21900 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_93_test, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1962, __pyx_L1_error)
21901 __Pyx_GOTREF(__pyx_t_1);
21902 if (PyDict_SetItem(__pyx_d, __pyx_n_s_test, __pyx_t_1) < 0) __PYX_ERR(0, 1962, __pyx_L1_error)
21903 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21904
21905 /* "PyClical.pyx":1966
21906 *      return doctest.testmod(PyClical)
21907 *
21908 * if __name__ == "__main__":          # ««««««««
21909 *     _test()
21910 */
21911 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_name); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1966,
__pyx_L1_error)
21912 __Pyx_GOTREF(__pyx_t_1);
21913 __pyx_t_4 = (__Pyx_PyUnicode_Equals(__pyx_t_1, __pyx_n_u_main, Py_EQ)); if (unlikely(__pyx_t_4 < 0))
__PYX_ERR(0, 1966, __pyx_L1_error)
21914 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21915 if (__pyx_t_4) {
21916
21917     /* "PyClical.pyx":1967
21918     *
21919     * if __name__ == "__main__":
21920     *     _test()          # ««««««««
21921     */
21922     __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_test); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1967,
__pyx_L1_error)
21923     __Pyx_GOTREF(__pyx_t_1);
21924     __pyx_t_2 = __Pyx_PyObject_CallNoArg(__pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1967,
__pyx_L1_error)
21925     __Pyx_GOTREF(__pyx_t_2);
21926     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21927     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21928
21929     /* "PyClical.pyx":1966
21930     *      return doctest.testmod(PyClical)
21931     *
21932     * if __name__ == "__main__":          # ««««««««
21933     *     _test()
21934     */
21935     }
21936
21937     /* "PyClical.pyx":1
21938     * # -*- coding: utf-8 -*-          # ««««««««
21939     * # cython: language_level=3
21940     * # distutils: language = c++
21941     */
21942     __pyx_t_2 = __Pyx_PyDict_NewPresized(111); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1, __pyx_L1_error)
21943     __Pyx_GOTREF(__pyx_t_2);
21944     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_copy_line_65,
__pyx_kp_u_Copy_this_index_set_object_s_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21945     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_setitem_line_179,
__pyx_kp_u_Set_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21946     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_getitem_line_191,
__pyx_kp_u_Get_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21947     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_iter_line_229,
__pyx_kp_u_Iterate_over_the_indices_of_an) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21948     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_invert_line_240,
__pyx_kp_u_Set_complement_not_print_index) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21949     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_xor_line_249,
__pyx_kp_u_Symmetric_set_difference_exclus) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21950     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ixor_line_260,
__pyx_kp_u_Symmetric_set_difference_exclus_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21951     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_and_line_271,
__pyx_kp_u_Set_intersection_and_print_inde) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21952     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_iand_line_282,
__pyx_kp_u_Set_intersection_and_x_index_se) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21953     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_or_line_293,
__pyx_kp_u_Set_union_or_print_index_set_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21954     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ior_line_304,
__pyx_kp_u_Set_union_or_x_index_set_1_x_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21955     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_line_315,
__pyx_kp_u_Cardinality_Number_of_indices_i) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21956     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_neg_line_324,
__pyx_kp_u_Number_of_negative_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21957     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_pos_line_333,
__pyx_kp_u_Number_of_positive_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21958     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_min_line_342,
__pyx_kp_u_Minimum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21959     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_max_line_351,
__pyx_kp_u_Maximum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21960     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_mult_line_366,
__pyx_kp_u_Sign_of_geometric_product_of_tw) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21961     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_square_line_37,
__pyx_kp_u_Sign_of_geometric_square_of_a_C) < 0) __PYX_ERR(0, 1, __pyx_L1_error)

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21962 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set__repr__line_384,
21963 __pyx_kp_u_The_official_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21963 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set__str__line_395,
__pyx_kp_u_The_informal_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21964 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_hidden_doctests_line_4,
__pyx_kp_u_Tests_for_functions_that_Doctes) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21965 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_compare_line_492,
__pyx_kp_u_lexicographic_compare_eg_3_4_5) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21966 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_min_neg_line_504,
__pyx_kp_u_Minimum_negative_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21967 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_pos_line_513,
__pyx_kp_u_Maximum_positive_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21968 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_copy_line_556,
__pyx_kp_u_Copy_this_clifford_object_x_cli) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21969 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iter__line_638,
__pyx_kp_u_Not_applicable_for_a_in_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21970 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_reframe_line_649,
__pyx_kp_u_Put_self_into_a_larger_frame_co) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21971 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_getitem__line_707,
__pyx_kp_u_Subscripting_map_from_index_set) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21972 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_neg__line_722,
__pyx_kp_u_Unary_print_clifford_l_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21973 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pos__line_731,
__pyx_kp_u_Unary_print_clifford_l_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21974 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_add__line_740,
__pyx_kp_u_Geometric_sum_print_clifford_l) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21975 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iadd__line_751,
__pyx_kp_u_Geometric_sum_x_clifford_l_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21976 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_sub__line_760,
__pyx_kp_u_Geometric_difference_print_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21977 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isub__line_771,
__pyx_kp_u_Geometric_difference_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21978 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_mul__line_780,
__pyx_kp_u_Geometric_product_print_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21979 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_imul__line_793,
__pyx_kp_u_Geometric_product_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21980 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_mod__line_806,
__pyx_kp_u_Contraction_print_clifford_l_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21981 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_imod__line_821,
__pyx_kp_u_Contraction_x_clifford_l_x_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21982 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_and__line_836,
__pyx_kp_u_Inner_product_print_clifford_l) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21983 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iand__line_851,
__pyx_kp_u_Inner_product_x_clifford_l_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21984 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_xor__line_866,
__pyx_kp_u_Outer_product_print_clifford_l) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21985 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_ixor__line_881,
__pyx_kp_u_Outer_product_x_clifford_l_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21986 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_truediv__line_896,
__pyx_kp_u_Geometric_quotient_print_cliffo) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21987 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_idiv__line_911,
__pyx_kp_u_Geometric_quotient_x_clifford_l) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21988 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_inv_line_926,
__pyx_kp_u_Geometric_multiplicative_invers) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21989 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_or__line_939,
__pyx_kp_u_Transform_left_hand_side_using) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21990 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_ior__line_950,
__pyx_kp_u_Transform_left_hand_side_using_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21991 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pow__line_961,
__pyx_kp_u_Power_self_to_the_m_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21992 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pow_line_980,
__pyx_kp_u_Power_self_to_the_m_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21993 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_outer_pow_line_1004,
__pyx_kp_u_Outer_product_power_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21994 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_call_line_1020,
__pyx_kp_u_Pure_grade_vector_part_print_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21995 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_scalar_line_1039,
__pyx_kp_u_Scalar_part_clifford_l_l_1_2_sc) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21996 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pure_line_1050,
__pyx_kp_u_Pure_part_print_clifford_l_l_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21997 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_even_line_1061,
__pyx_kp_u_Even_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21998 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_odd_line_1070,
__pyx_kp_u_Odd_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21999 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_vector_part_line_1079,
__pyx_kp_u_Vector_part_of_multivector_as_a) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22000 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_involute_line_1107,
__pyx_kp_u_Main_involution_each_i_is_repla) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22001 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_reverse_line_1123,
__pyx_kp_u_Reversion_eg_clifford_l_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22002 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_conj_line_1138,
__pyx_kp_u_Conjugation_reverse_o_involute) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22003 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_quad_line_1153,
__pyx_kp_u_Quadratic_form_rev_x_x_0_print) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22004 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_norm_line_1164,
__pyx_kp_u_Norm_sum_of_squares_of_coordina) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22005 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_abs_line_1175,

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__pyx_kp_u_Absolute_value_square_root_of_n) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22006 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_max_abs_line_1184,
__pyx_kp_u_Maximum_of_absolute_values_of_c) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22007 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_truncated_line_1195,
__pyx_kp_u_Remove_all_terms_of_self_with_r) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22008 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isinf_line_1206,
__pyx_kp_u_Check_if_a_multivector_contains) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22009 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isnan_line_1215,
__pyx_kp_u_Check_if_a_multivector_contains_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22010 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_frame_line_1224,
__pyx_kp_u_Subalgebra_generated_by_all_gen) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22011 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_repr_line_1235,
__pyx_kp_u_The_official_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22012 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_str_line_1244,
__pyx_kp_u_The_informal_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22013 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_hidden_doctests_line_12,
__pyx_kp_u_Tests_for_functions_that_Doctes_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22014 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_error_squared_tol_line_1337,
__pyx_kp_u_Quadratic_norm_error_tolerance) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22015 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_error_squared_line_1346,
__pyx_kp_u_Relative_or_absolute_error_usin) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22016 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_approx_equal_line_1359,
__pyx_kp_u_Test_for_approximate_equality_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22017 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_inv_line_1378,
__pyx_kp_u_Geometric_multiplicative_invers_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22018 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_scalar_line_1393,
__pyx_kp_u_Scalar_part_scalar_clifford_l_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22019 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_real_line_1404, __pyx_kp_u_Real_part_synonym_for_scalar_pa)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22020 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_imag_line_1415, __pyx_kp_u_Imaginary_part_deprecated_alway)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22021 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pure_line_1426, __pyx_kp_u_Pure_part_print_pure_clifford_l)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22022 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_even_line_1437,
__pyx_kp_u_Even_part_of_multivector_sum_of_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22023 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_odd_line_1446, __pyx_kp_u_Odd_part_of_multivector_sum_of_2)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22024 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_involute_line_1455,
__pyx_kp_u_Main_involution_each_i_is_repla_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22025 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_reverse_line_1470,
__pyx_kp_u_Reversion_eg_l_2_2_l_print_reve) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22026 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_conj_line_1485,
__pyx_kp_u_Conjugation_reverse_o_involute_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22027 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_quad_line_1500,
__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22028 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_norm_line_1511, __pyx_kp_u_norm_sum_of_squares_of_coordina)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22029 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_abs_line_1522, __pyx_kp_u_Absolute_value_of_multivector_m)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22030 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_abs_line_1531,
__pyx_kp_u_Maximum_absolute_value_of_coord) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22031 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pow_line_1543, __pyx_kp_u_Integer_power_of_multivector_ob)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22032 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_outer_pow_line_1567,
__pyx_kp_u_Outer_product_power_of_multivec) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22033 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_complexifier_line_1576,
__pyx_kp_u_Square_root_of_l_which_commutes) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22034 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sqrt_line_1591, __pyx_kp_u_Square_root_of_multivector_with)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22035 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_exp_line_1614, __pyx_kp_u_Exponential_of_multivector_x_cl)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22036 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_log_line_1628, __pyx_kp_u_Natural_logarithm_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22037 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cos_line_1651, __pyx_kp_u_Cosine_of_multivector_with_opti)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22038 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acos_line_1668, __pyx_kp_u_Inverse_cosine_of_multivector_w)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22039 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cosh_line_1689, __pyx_kp_u_Hyperbolic_cosine_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22040 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acosh_line_1705,
__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22041 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sin_line_1728, __pyx_kp_u_Sine_of_multivector_with_option)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22042 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asin_line_1747, __pyx_kp_u_Inverse_sine_of_multivector_wit)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22043 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sinh_line_1768, __pyx_kp_u_Hyperbolic_sine_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22044 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asinh_line_1782,
__pyx_kp_u_Inverse_hyperbolic_sine_of_mult) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22045 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tan_line_1801, __pyx_kp_u_Tangent_of_multivector_with_opt)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22046 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atan_line_1818, __pyx_kp_u_Inverse_tangent_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22047 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tanh_line_1835, __pyx_kp_u_Hyperbolic_tangent_of_multivect)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22048 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atanh_line_1847,
__pyx_kp_u_Inverse_hyperbolic_tangent_of_m) < 0) __PYX_ERR(0, 1, __pyx_L1_error)

```

```

22049     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_random_clifford_line_1864,
22050         __pyx_kp_u_Random_multivector_within_a_fra) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22051     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3_line_1873, __pyx_kp_u_Convert_Euclidean_3D_multivecto)
22052         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22053     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3std_line_1882,
22054         __pyx_kp_u_Convert_CGA3_null_vector_to_sta) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22055     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_agc3_line_1893, __pyx_kp_u_Convert_CGA3_null_vector_to_Euc)
22056         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22057     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_e_line_1936, __pyx_kp_u_Abbreviation_for_clifford_index) <
22058         0) __PYX_ERR(0, 1, __pyx_L1_error)
22059     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_istpq_line_1949, __pyx_kp_u_Abbreviation_for_index_set_q_p)
22060         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22061     if (PyDict_SetItem(__pyx_d, __pyx_n_s_test_2, __pyx_t_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22062     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
22063
22064     /* "string.to_py":55
22065     *
22066     * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
22067     * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
22068     *     # ««««««««
22069     *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
22070     *
22071     */
22072
22073     /**** Wrapped vars code ****/
22074
22075     goto __pyx_L0;
22076     __pyx_L1_error:;
22077     __Pyx_XDECREF(__pyx_t_1);
22078     __Pyx_XDECREF(__pyx_t_2);
22079     __Pyx_XDECREF(__pyx_t_3);
22080     if (__pyx_m) {
22081         if (__pyx_d) {
22082             __Pyx_AddTraceback("init PyClical", __pyx_clineno, __pyx_lineno, __pyx_filename);
22083         }
22084         Py_CLEAR(__pyx_m);
22085     } else if (!PyErr_Occurred()) {
22086         PyErr_SetString(PyExc_ImportError, "init PyClical");
22087     }
22088     __pyx_L0:;
22089     __Pyx_RefNannyFinishContext();
22090     #if CYTHON_PEP489_MULTI_PHASE_INIT
22091     return (__pyx_m != NULL) ? 0 : -1;
22092     #elif PY_MAJOR_VERSION >= 3
22093     return __pyx_m;
22094     #else
22095     return;
22096     #endif
22097 }
22098
22099 /* --- Runtime support code --- */
22100 /* Refnanny */
22101 #if CYTHON_REFNANNY
22102 static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname) {
22103     PyObject *m = NULL, *p = NULL;
22104     void *r = NULL;
22105     m = PyImport_ImportModule(modname);
22106     if (!m) goto end;
22107     p = PyObject_GetAttrString(m, "RefNannyAPI");
22108     if (!p) goto end;
22109     r = PyLong_AsVoidPtr(p);
22110 end:
22111     Py_XDECREF(p);
22112     Py_XDECREF(m);
22113     return (__Pyx_RefNannyAPIStruct *)r;
22114 }
22115 #endif
22116
22117 /* PyObjectGetAttrStr */
22118 #if CYTHON_USE_TYPE_SLOTS
22119 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name) {
22120     PyTypeObject* tp = Py_TYPE(obj);
22121     if (likely(tp->tp_getattro))
22122         return tp->tp_getattro(obj, attr_name);
22123 #if PY_MAJOR_VERSION < 3
22124     if (likely(tp->tp_getattr))
22125         return tp->tp_getattr(obj, PyString_AS_STRING(attr_name));
22126 #endif
22127     return PyObject_GetAttr(obj, attr_name);
22128 }
22129 #endif
22130
22131 /* GetBuiltinName */
22132 static PyObject* __Pyx_GetBuiltinName(PyObject *name) {
22133     PyObject* result = __Pyx_PyObject_GetAttrStr(__pyx_b, name);
22134     if (unlikely(!result)) {
22135         PyErr_Format(PyExc_NameError,

```

```

22129 #if PY_MAJOR_VERSION >= 3
22130     "name '%U' is not defined", name);
22131 #else
22132     "name '%.200s' is not defined", PyString_AS_STRING(name));
22133 #endif
22134 }
22135 return result;
22136 }
22137
22138 /* PyCFunctionFastCall */
22139 #if CYTHON_FAST_PYCALL
22140 static CYTHON_INLINE PyObject * __Pyx_PyCFunction_FastCall(PyObject *func_obj, PyObject **args,
22141     Py_ssize_t nargs) {
22142     PyCFunctionObject *func = (PyCFunctionObject*)func_obj;
22143     PyCFunction meth = PyCFunction_GET_FUNCTION(func);
22144     PyObject *self = PyCFunction_GET_SELF(func);
22145     int flags = PyCFunction_GET_FLAGS(func);
22146     assert(PyCFunction_Check(func));
22147     assert(METH_FASTCALL == (flags & ~(METH_CLASS | METH_STATIC | METH_COEXIST | METH_KEYWORDS |
22148     METH_STACKLESS)));
22149     assert(nargs >= 0);
22150     assert(nargs == 0 || args != NULL);
22151     /* _PyCFunction_FastCallDict() must not be called with an exception set,
22152     because it may clear it (directly or indirectly) and so the
22153     caller loses its exception */
22154     assert(!PyErr_Occurred());
22155     if ((PY_VERSION_HEX < 0x030700A0) || unlikely(flags & METH_KEYWORDS)) {
22156         return ((*(__Pyx_PyCFunctionFastWithKeywords)(void*)meth)) (self, args, nargs, NULL);
22157     } else {
22158         return ((*(__Pyx_PyCFunctionFast)(void*)meth)) (self, args, nargs);
22159     }
22160 }
22161 #endif
22162 /* PyFunctionFastCall */
22163 #if CYTHON_FAST_PYCALL
22164 static PyObject* __Pyx_PyFunction_FastCallNoKw(PyCodeObject *co, PyObject **args, Py_ssize_t na,
22165     PyObject *globals) {
22166     PyFrameObject *f;
22167     PyThreadState *tstate = __Pyx_PyThreadState_Current;
22168     PyObject **fastlocals;
22169     Py_ssize_t i;
22170     PyObject *result;
22171     assert(globals != NULL);
22172     /* XXX Perhaps we should create a specialized
22173     PyFrame_New() that doesn't take locals, but does
22174     take builtins without sanity checking them.
22175     */
22176     assert(tstate != NULL);
22177     f = PyFrame_New(tstate, co, globals, NULL);
22178     if (f == NULL) {
22179         return NULL;
22180     }
22181     fastlocals = __Pyx_PyFrame_GetLocalsplus(f);
22182     for (i = 0; i < na; i++) {
22183         Py_INCREF(*args);
22184         fastlocals[i] = *args++;
22185     }
22186     result = PyEval_EvalFrameEx(f, 0);
22187     ++tstate->recursion_depth;
22188     Py_DECREF(f);
22189     --tstate->recursion_depth;
22190     return result;
22191 }
22192 #if 1 || PY_VERSION_HEX < 0x030600B1
22193 static PyObject * __Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs,
22194     PyObject *kwargs) {
22195     PyCodeObject *co = (PyCodeObject *)PyFunction_GET_CODE(func);
22196     PyObject *globals = PyFunction_GET_GLOBALS(func);
22197     PyObject *argdefs = PyFunction_GET_DEFAULTS(func);
22198     PyObject *closure;
22199     #if PY_MAJOR_VERSION >= 3
22200     PyObject *kwdefs;
22201     #endif
22202     PyObject *kwtuple, **k;
22203     PyObject **d;
22204     Py_ssize_t nd;
22205     Py_ssize_t nk;
22206     PyObject *result;
22207     assert(kwargs == NULL || PyDict_Check(kwargs));
22208     nk = kwargs ? PyDict_Size(kwargs) : 0;
22209     if (Py_EnterRecursiveCall((char*)" while calling a Python object")) {
22210         return NULL;
22211     }
22212     if (

```



```

22213 #endif
22214         likely(kwargs == NULL || nk == 0) &&
22215         co->co_flags == (CO_OPTIMIZED | CO_NEWLOCALS | CO_NOFREE)) {
22216     if (argdefs == NULL && co->co_argcount == nargs) {
22217         result = __Pyx_PyFunction_FastCallNoKw(co, args, nargs, globals);
22218         goto done;
22219     }
22220     else if (nargs == 0 && argdefs != NULL
22221             && co->co_argcount == Py_SIZE(argdefs)) {
22222         /* function called with no arguments, but all parameters have
22223            a default value: use default values as arguments .*/
22224         args = &PyTuple_GET_ITEM(argdefs, 0);
22225         result = __Pyx_PyFunction_FastCallNoKw(co, args, Py_SIZE(argdefs), globals);
22226         goto done;
22227     }
22228 }
22229 if (kwargs != NULL) {
22230     Py_ssize_t pos, i;
22231     kwtuple = PyTuple_New(2 * nk);
22232     if (kwtuple == NULL) {
22233         result = NULL;
22234         goto done;
22235     }
22236     k = &PyTuple_GET_ITEM(kwtuple, 0);
22237     pos = i = 0;
22238     while (PyDict_Next(kwargs, &pos, &k[i], &k[i+1])) {
22239         Py_INCREF(k[i]);
22240         Py_INCREF(k[i+1]);
22241         i += 2;
22242     }
22243     nk = i / 2;
22244 }
22245 else {
22246     kwtuple = NULL;
22247     k = NULL;
22248 }
22249 closure = PyFunction_GET_CLOSURE(func);
22250 #if PY_MAJOR_VERSION >= 3
22251 kwdefs = PyFunction_GET_KW_DEFAULTS(func);
22252 #endif
22253 if (argdefs != NULL) {
22254     d = &PyTuple_GET_ITEM(argdefs, 0);
22255     nd = Py_SIZE(argdefs);
22256 }
22257 else {
22258     d = NULL;
22259     nd = 0;
22260 }
22261 #if PY_MAJOR_VERSION >= 3
22262 result = PyEval_EvalCodeEx((PyObject*)co, globals, (PyObject *)NULL,
22263                             args, (int)nargs,
22264                             k, (int)nk,
22265                             d, (int)nd, kwdefs, closure);
22266 #else
22267 result = PyEval_EvalCodeEx(co, globals, (PyObject *)NULL,
22268                             args, (int)nargs,
22269                             k, (int)nk,
22270                             d, (int)nd, closure);
22271 #endif
22272 Py_XDECREF(kwtuple);
22273 done:
22274     Py_LeaveRecursiveCall();
22275     return result;
22276 }
22277 #endif
22278 #endif
22279
22280 /* PyObjectCall */
22281 #if CYTHON_COMPILING_IN_CPYTHON
22282 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw) {
22283     PyObject *result;
22284     ternaryfunc call = Py_TYPE(func)->tp_call;
22285     if (unlikely(!call))
22286         return PyObject_Call(func, arg, kw);
22287     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22288         return NULL;
22289     result = (*call)(func, arg, kw);
22290     Py_LeaveRecursiveCall();
22291     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22292         PyErr_SetString(
22293             PyExc_SystemError,
22294             "NULL result without error in PyObject_Call");
22295     }
22296     return result;
22297 }
22298 #endif
22299

```



```

22300 /* PyObjectCallMethO */
22301 #if CYTHON_COMPILING_IN_CPYTHON
22302 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMethO(PyObject *func, PyObject *arg) {
22303     PyObject *self, *result;
22304     PyCFunction cfunc;
22305     cfunc = PyCFunction_GET_FUNCTION(func);
22306     self = PyCFunction_GET_SELF(func);
22307     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22308         return NULL;
22309     result = cfunc(self, arg);
22310     Py_LeaveRecursiveCall();
22311     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22312         PyErr_SetString(
22313             PyExc_SystemError,
22314             "NULL result without error in PyObject_Call");
22315     }
22316     return result;
22317 }
22318 #endif
22319
22320 /* PyObjectCallOneArg */
22321 #if CYTHON_COMPILING_IN_CPYTHON
22322 static PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22323     PyObject *result;
22324     PyObject *args = PyTuple_New(1);
22325     if (unlikely(!args)) return NULL;
22326     Py_INCREF(arg);
22327     PyTuple_SET_ITEM(args, 0, arg);
22328     result = __Pyx_PyObject_Call(func, args, NULL);
22329     Py_DECREF(args);
22330     return result;
22331 }
22332 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22333 #if CYTHON_FAST_PYCALL
22334     if (PyFunction_Check(func)) {
22335         return __Pyx_PyFunction_FastCall(func, &arg, 1);
22336     }
22337 #endif
22338     if (likely(PyCFunction_Check(func))) {
22339         if (likely(PyCFunction_GET_FLAGS(func) & METH_O)) {
22340             return __Pyx_PyObject_CallMethO(func, arg);
22341         }
22342 #if CYTHON_FAST_PYCCALL
22343     } else if (__Pyx_PyFastCFunction_Check(func)) {
22344         return __Pyx_PyCFunction_FastCall(func, &arg, 1);
22345     }
22346 #endif
22347     return __Pyx_PyObject_CallOneArg(func, arg);
22348 }
22349 #else
22350 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22351     PyObject *result;
22352     PyObject *args = PyTuple_Pack(1, arg);
22353     if (unlikely(!args)) return NULL;
22354     result = __Pyx_PyObject_Call(func, args, NULL);
22355     Py_DECREF(args);
22356     return result;
22357 }
22358 #endif
22359
22360 /* PyErrFetchRestore */
22361 #if CYTHON_FAST_THREAD_STATE
22362 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
22363 *value, PyObject *tb) {
22364     PyObject *tmp_type, *tmp_value, *tmp_tb;
22365     tmp_type = tstate->curexc_type;
22366     tmp_value = tstate->curexc_value;
22367     tmp_tb = tstate->curexc_traceback;
22368     tstate->curexc_type = type;
22369     tstate->curexc_value = value;
22370     tstate->curexc_traceback = tb;
22371     Py_XDECREF(tmp_type);
22372     Py_XDECREF(tmp_value);
22373     Py_XDECREF(tmp_tb);
22374 }
22375 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
22376 **value, PyObject **tb) {
22377     *type = tstate->curexc_type;
22378     *value = tstate->curexc_value;
22379     *tb = tstate->curexc_traceback;
22380     tstate->curexc_type = 0;
22381     tstate->curexc_value = 0;
22382     tstate->curexc_traceback = 0;
22383 }
22384 #endif
22385
22386 /* WriteUnraisableException */

```

```

22385 static void __Pyx_WriteUnraisable(const char *name, CYTHON_UNUSED int clineno,
22386                                     CYTHON_UNUSED int lineno, CYTHON_UNUSED const char *filename,
22387                                     int full_traceback, CYTHON_UNUSED int nogil) {
22388     PyObject *old_exc, *old_val, *old_tb;
22389     PyObject *ctx;
22390     __Pyx_PyThreadState_declare
22391 #ifdef WITH_THREAD
22392     PyGILState_STATE state;
22393     if (nogil)
22394         state = PyGILState_Ensure();
22395 #ifdef _MSC_VER
22396     else state = (PyGILState_STATE)-1;
22397 #endif
22398 #endif
22399     __Pyx_PyThreadState_assign
22400     __Pyx_ErrFetch(&old_exc, &old_val, &old_tb);
22401     if (full_traceback) {
22402         Py_XINCREf(old_exc);
22403         Py_XINCREf(old_val);
22404         Py_XINCREf(old_tb);
22405         __Pyx_ErrRestore(old_exc, old_val, old_tb);
22406         PyErr_PrintEx(1);
22407     }
22408     #if PY_MAJOR_VERSION < 3
22409     ctx = PyString_FromString(name);
22410     #else
22411     ctx = PyUnicode_FromString(name);
22412     #endif
22413     __Pyx_ErrRestore(old_exc, old_val, old_tb);
22414     if (!ctx) {
22415         PyErr_WriteUnraisable(Py_None);
22416     } else {
22417         PyErr_WriteUnraisable(ctx);
22418         Py_DECREF(ctx);
22419     }
22420 #ifdef WITH_THREAD
22421     if (nogil)
22422         PyGILState_Release(state);
22423 #endif
22424 }
22425
22426 /* PyDictVersioning */
22427 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
22428 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj) {
22429     PyObject *dict = Py_TYPE(obj)->tp_dict;
22430     return likely(dict) ? __PYX_GET_DICT_VERSION(dict) : 0;
22431 }
22432 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj) {
22433     PyObject **dictptr = NULL;
22434     Py_ssize_t offset = Py_TYPE(obj)->tp_dictoffset;
22435     if (offset) {
22436 #if CYTHON_COMPILING_IN_CPYTHON
22437         dictptr = (likely(offset > 0)) ? (PyObject **) ((char *)obj + offset) :
22438         _PyObject_GetDictPtr(obj);
22439 #else
22439         dictptr = _PyObject_GetDictPtr(obj);
22440 #endif
22441     }
22442     return (dictptr && *dictptr) ? __PYX_GET_DICT_VERSION(*dictptr) : 0;
22443 }
22444 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
22445 PY_UINT64_T obj_dict_version) {
22446     PyObject *dict = Py_TYPE(obj)->tp_dict;
22447     if (unlikely(!dict) || unlikely(tp_dict_version != __PYX_GET_DICT_VERSION(dict)))
22448         return 0;
22449     return obj_dict_version == __Pyx_get_object_dict_version(obj);
22450 #endif
22451
22452 /* PyObjectCallNoArg */
22453 #if CYTHON_COMPILING_IN_CPYTHON
22454 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func) {
22455     #if CYTHON_FAST_PYCALL
22456     if (PyFunction_Check(func)) {
22457         return __Pyx_PyFunction_FastCall(func, NULL, 0);
22458     }
22459 #endif
22460 #ifdef __Pyx_CyFunction_USED
22461     if (likely(PyCFunction_Check(func) || __Pyx_CyFunction_Check(func)))
22462     #else
22463     if (likely(PyCFunction_Check(func)))
22464     #endif
22465     {
22466         if (likely(PyCFunction_GET_FLAGS(func) & METH_NOARGS)) {
22467             return __Pyx_PyObject_CallMethO(func, NULL);
22468         }
22469     }

```

```

22470     return __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL);
22471 }
22472 #endif
22473
22474 /* RaiseDoubleKeywords */
22475 static void __Pyx_RaiseDoubleKeywordsError(
22476     const char* func_name,
22477     PyObject* kw_name)
22478 {
22479     PyErr_Format(PyExc_TypeError,
22480         #if PY_MAJOR_VERSION >= 3
22481         "%s() got multiple values for keyword argument '%U'", func_name, kw_name);
22482         #else
22483         "%s() got multiple values for keyword argument '%s'", func_name,
22484         PyString_AsString(kw_name));
22485         #endif
22486 }
22487
22488 /* ParseKeywords */
22489 static int __Pyx_ParseOptionalKeywords(
22490     PyObject* kwds,
22491     PyObject** argnames[],
22492     PyObject* kwds2,
22493     PyObject* values[],
22494     Py_ssize_t num_pos_args,
22495     const char* function_name)
22496 {
22497     PyObject* key = 0, *value = 0;
22498     Py_ssize_t pos = 0;
22499     PyObject*** name;
22500     PyObject*** first_kw_arg = argnames + num_pos_args;
22501     while (PyDict_Next(kwds, &pos, &key, &value)) {
22502         name = first_kw_arg;
22503         while (*name && (**name != key)) name++;
22504         if (*name) {
22505             values[name-argnames] = value;
22506             continue;
22507         }
22508         name = first_kw_arg;
22509         #if PY_MAJOR_VERSION < 3
22510         if (likely(PyString_Check(key))) {
22511             while (*name) {
22512                 if ((CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**name) == PyString_GET_SIZE(key))
22513                     && _PyString_Eq(**name, key)) {
22514                     values[name-argnames] = value;
22515                     break;
22516                 }
22517                 name++;
22518             }
22519             if (*name) continue;
22520             else {
22521                 PyObject*** argname = argnames;
22522                 while (argname != first_kw_arg) {
22523                     if (**argname == key) || (
22524                         CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**argname) ==
22525                         PyString_GET_SIZE(key))
22526                         && _PyString_Eq(**argname, key)) {
22527                         goto arg_passed_twice;
22528                     }
22529                     argname++;
22530                 }
22531             } else
22532             #endif
22533             if (likely(PyUnicode_Check(key))) {
22534                 while (*name) {
22535                     int cmp = (**name == key) ? 0 :
22536                         #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22537                         (__Pyx_PyUnicode_GET_LENGTH(**name) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1 :
22538                         #endif
22539                         PyUnicode_Compare(**name, key);
22540                     if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22541                     if (cmp == 0) {
22542                         values[name-argnames] = value;
22543                         break;
22544                     }
22545                     name++;
22546                 }
22547                 if (*name) continue;
22548                 else {
22549                     PyObject*** argname = argnames;
22550                     while (argname != first_kw_arg) {
22551                         int cmp = (**argname == key) ? 0 :
22552                             #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22553                             (__Pyx_PyUnicode_GET_LENGTH(**argname) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1 :
22554                             :
22555                             #endif

```

```

22555         PyUnicode_Compare(**argname, key);
22556         if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22557         if (cmp == 0) goto arg_passed_twice;
22558         argname++;
22559     }
22560 }
22561 } else
22562     goto invalid_keyword_type;
22563 if (kwds2) {
22564     if (unlikely(PyDict_SetItem(kwds2, key, value))) goto bad;
22565 } else {
22566     goto invalid_keyword;
22567 }
22568 }
22569 return 0;
22570 arg_passed_twice:
22571     __Pyx_RaiseDoubleKeywordsError(function_name, key);
22572     goto bad;
22573 invalid_keyword_type:
22574     PyErr_Format(PyExc_TypeError,
22575                 "%.200s() keywords must be strings", function_name);
22576     goto bad;
22577 invalid_keyword:
22578     PyErr_Format(PyExc_TypeError,
22579                 #if PY_MAJOR_VERSION < 3
22580                 "%.200s() got an unexpected keyword argument '%.200s'",
22581                 function_name, PyString_AsString(key));
22582                 #else
22583                 "%s() got an unexpected keyword argument '%U'",
22584                 function_name, key);
22585                 #endif
22586 bad:
22587     return -1;
22588 }
22589
22590 /* RaiseArgTupleInvalid */
22591 static void __Pyx_RaiseArgtupleInvalid(
22592     const char* func_name,
22593     int exact,
22594     Py_ssize_t num_min,
22595     Py_ssize_t num_max,
22596     Py_ssize_t num_found)
22597 {
22598     Py_ssize_t num_expected;
22599     const char *more_or_less;
22600     if (num_found < num_min) {
22601         num_expected = num_min;
22602         more_or_less = "at least";
22603     } else {
22604         num_expected = num_max;
22605         more_or_less = "at most";
22606     }
22607     if (exact) {
22608         more_or_less = "exactly";
22609     }
22610     PyErr_Format(PyExc_TypeError,
22611                 "%.200s() takes %.8s %" CYTHON_FORMAT_SSIZE_T "d positional argument%.1s (%"
22612                 CYTHON_FORMAT_SSIZE_T "d given)",
22613                 func_name, more_or_less, num_expected,
22614                 (num_expected == 1) ? "" : "s", num_found);
22615 }
22616
22617 /* GetModuleGlobalName */
22618 #if CYTHON_USE_DICT_VERSIONS
22619 static PyObject *__Pyx_GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
22620 **dict_cached_value)
22621 #else
22622 static PyObject *__Pyx_GetModuleGlobalName(PyObject *name)
22623 #endif
22624 {
22625     PyObject *result;
22626     #if !CYTHON_AVOID_BORROWED_REFS
22627     #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1
22628     result = _PyDict_GetItem_KnownHash(__pyx_d, name, ((PyASCIIObject *) name)->hash);
22629     __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22630     if (likely(result)) {
22631         return __Pyx_NewRef(result);
22632     } else if (unlikely(PyErr_Occurred())) {
22633         return NULL;
22634     }
22635     #else
22636     result = PyDict_GetItem(__pyx_d, name);
22637     __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22638     if (likely(result)) {
22639         return __Pyx_NewRef(result);
22640     }
22641     #endif
22642 }
22643 #endif

```

```

22640 #else
22641     result = PyObject_GetItem(__pyx_d, name);
22642     __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22643     if (likely(result)) {
22644         return __Pyx_NewRef(result);
22645     }
22646     PyErr_Clear();
22647 #endif
22648     return __Pyx_GetBuiltinName(name);
22649 }
22650
22651 /* GetTopmostException */
22652 #if CYTHON_USE_EXC_INFO_STACK
22653 static _PyErr_StackItem *
22654 __Pyx_PyErr_GetTopmostException(PyThreadState *tstate)
22655 {
22656     _PyErr_StackItem *exc_info = tstate->exc_info;
22657     while ((exc_info->exc_type == NULL || exc_info->exc_type == Py_None) &&
22658           exc_info->previous_item != NULL)
22659     {
22660         exc_info = exc_info->previous_item;
22661     }
22662     return exc_info;
22663 }
22664 #endif
22665
22666 /* SaveResetException */
22667 #if CYTHON_FAST_THREAD_STATE
22668 static CYTHON_INLINE void __Pyx_ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
22669 **value, PyObject **tb) {
22670     #if CYTHON_USE_EXC_INFO_STACK
22671     _PyErr_StackItem *exc_info = __Pyx_PyErr_GetTopmostException(tstate);
22672     *type = exc_info->exc_type;
22673     *value = exc_info->exc_value;
22674     *tb = exc_info->exc_traceback;
22675     #else
22676     *type = tstate->exc_type;
22677     *value = tstate->exc_value;
22678     *tb = tstate->exc_traceback;
22679     #endif
22680     Py_XINCREF(*type);
22681     Py_XINCREF(*value);
22682     Py_XINCREF(*tb);
22683 }
22684 static CYTHON_INLINE void __Pyx_ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
22685 *value, PyObject *tb) {
22686     PyObject *tmp_type, *tmp_value, *tmp_tb;
22687     #if CYTHON_USE_EXC_INFO_STACK
22688     _PyErr_StackItem *exc_info = tstate->exc_info;
22689     tmp_type = exc_info->exc_type;
22690     tmp_value = exc_info->exc_value;
22691     tmp_tb = exc_info->exc_traceback;
22692     exc_info->exc_type = type;
22693     exc_info->exc_value = value;
22694     exc_info->exc_traceback = tb;
22695     #else
22696     tmp_type = tstate->exc_type;
22697     tmp_value = tstate->exc_value;
22698     tmp_tb = tstate->exc_traceback;
22699     tstate->exc_type = type;
22700     tstate->exc_value = value;
22701     tstate->exc_traceback = tb;
22702     #endif
22703     Py_XDECREF(tmp_type);
22704     Py_XDECREF(tmp_value);
22705     Py_XDECREF(tmp_tb);
22706 }
22707 #endif
22708 /* PyErrExceptionMatches */
22709 #if CYTHON_FAST_THREAD_STATE
22710 static int __Pyx_PyErr_ExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
22711     Py_ssize_t i, n;
22712     n = PyTuple_GET_SIZE(tuple);
22713     #if PY_MAJOR_VERSION >= 3
22714     for (i=0; i<n; i++) {
22715         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
22716     }
22717     #endif
22718     for (i=0; i<n; i++) {
22719         if (__Pyx_PyErr_GivenExceptionMatches(exc_type, PyTuple_GET_ITEM(tuple, i))) return 1;
22720     }
22721     return 0;
22722 }
22723 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatchesInState(PyThreadState* tstate, PyObject* err) {
22724     PyObject *exc_type = tstate->curexc_type;
22725     if (exc_type == err) return 1;

```

```

22725     if (unlikely(!exc_type)) return 0;
22726     if (unlikely(PyTuple_Check(err)))
22727         return __Pyx_PyErr_ExceptionMatchesTuple(exc_type, err);
22728     return __Pyx_PyErr_GivenExceptionMatches(exc_type, err);
22729 }
22730 #endif
22731
22732 /* GetException */
22733 #if CYTHON_FAST_THREAD_STATE
22734 static int __Pyx_GetException(PyThreadState *tstate, PyObject **type, PyObject **value, PyObject
    **tb)
22735 #else
22736 static int __Pyx_GetException(PyObject **type, PyObject **value, PyObject **tb)
22737 #endif
22738 {
22739     PyObject *local_type, *local_value, *local_tb;
22740     #if CYTHON_FAST_THREAD_STATE
22741     PyObject *tmp_type, *tmp_value, *tmp_tb;
22742     local_type = tstate->curexc_type;
22743     local_value = tstate->curexc_value;
22744     local_tb = tstate->curexc_traceback;
22745     tstate->curexc_type = 0;
22746     tstate->curexc_value = 0;
22747     tstate->curexc_traceback = 0;
22748     #else
22749     PyErr_Fetch(&local_type, &local_value, &local_tb);
22750     #endif
22751     PyErr_NormalizeException(&local_type, &local_value, &local_tb);
22752     #if CYTHON_FAST_THREAD_STATE
22753     if (unlikely(tstate->curexc_type))
22754     #else
22755     if (unlikely(PyErr_Occurred()))
22756     #endif
22757         goto bad;
22758     #if PY_MAJOR_VERSION >= 3
22759     if (local_tb) {
22760         if (unlikely(PyException_SetTraceback(local_value, local_tb) < 0))
22761             goto bad;
22762     }
22763     #endif
22764     Py_XINCREF(local_tb);
22765     Py_XINCREF(local_type);
22766     Py_XINCREF(local_value);
22767     *type = local_type;
22768     *value = local_value;
22769     *tb = local_tb;
22770     #if CYTHON_FAST_THREAD_STATE
22771     #if CYTHON_USE_EXC_INFO_STACK
22772     {
22773         _PyErr_StackItem *exc_info = tstate->exc_info;
22774         tmp_type = exc_info->exc_type;
22775         tmp_value = exc_info->exc_value;
22776         tmp_tb = exc_info->exc_traceback;
22777         exc_info->exc_type = local_type;
22778         exc_info->exc_value = local_value;
22779         exc_info->exc_traceback = local_tb;
22780     }
22781     #else
22782     tmp_type = tstate->exc_type;
22783     tmp_value = tstate->exc_value;
22784     tmp_tb = tstate->exc_traceback;
22785     tstate->exc_type = local_type;
22786     tstate->exc_value = local_value;
22787     tstate->exc_traceback = local_tb;
22788     #endif
22789     Py_XDECREF(tmp_type);
22790     Py_XDECREF(tmp_value);
22791     Py_XDECREF(tmp_tb);
22792     #else
22793     PyErr_SetExcInfo(local_type, local_value, local_tb);
22794     #endif
22795     return 0;
22796 bad:
22797     *type = 0;
22798     *value = 0;
22799     *tb = 0;
22800     Py_XDECREF(local_type);
22801     Py_XDECREF(local_value);
22802     Py_XDECREF(local_tb);
22803     return -1;
22804 }
22805
22806 /* RaiseException */
22807 #if PY_MAJOR_VERSION < 3
22808 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb,
22809     CYTHON_UNUSED PyObject *cause) {
22810     __Pyx_PyThreadState_declare

```

```

22811     Py_XINCREf(type);
22812     if (!value || value == Py_None)
22813         value = NULL;
22814     else
22815         Py_INCREF(value);
22816     if (!tb || tb == Py_None)
22817         tb = NULL;
22818     else {
22819         Py_INCREF(tb);
22820         if (!PyTraceBack_Check(tb)) {
22821             PyErr_SetString(PyExc_TypeError,
22822                 "raise: arg 3 must be a traceback or None");
22823             goto raise_error;
22824         }
22825     }
22826     if (PyType_Check(type)) {
22827 #if CYTHON_COMPILING_IN_PYPY
22828         if (!value) {
22829             Py_INCREF(Py_None);
22830             value = Py_None;
22831         }
22832 #endif
22833         PyErr_NormalizeException(&type, &value, &tb);
22834     } else {
22835         if (value) {
22836             PyErr_SetString(PyExc_TypeError,
22837                 "instance exception may not have a separate value");
22838             goto raise_error;
22839         }
22840         value = type;
22841         type = (PyObject*) Py_TYPE(type);
22842         Py_INCREF(type);
22843         if (!PyType_IsSubtype((PyTypeObject *)type, (PyTypeObject *)PyExc_BaseException)) {
22844             PyErr_SetString(PyExc_TypeError,
22845                 "raise: exception class must be a subclass of BaseException");
22846             goto raise_error;
22847         }
22848     }
22849     __Pyx_PyThreadState_assign
22850     __Pyx_ErrRestore(type, value, tb);
22851     return;
22852 raise_error:
22853     Py_XDECREF(value);
22854     Py_XDECREF(type);
22855     Py_XDECREF(tb);
22856     return;
22857 }
22858 #else
22859 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb, PyObject *cause) {
22860     PyObject* owned_instance = NULL;
22861     if (tb == Py_None) {
22862         tb = 0;
22863     } else if (tb && !PyTraceBack_Check(tb)) {
22864         PyErr_SetString(PyExc_TypeError,
22865             "raise: arg 3 must be a traceback or None");
22866         goto bad;
22867     }
22868     if (value == Py_None)
22869         value = 0;
22870     if (PyExceptionInstance_Check(type)) {
22871         if (value) {
22872             PyErr_SetString(PyExc_TypeError,
22873                 "instance exception may not have a separate value");
22874             goto bad;
22875         }
22876         value = type;
22877         type = (PyObject*) Py_TYPE(value);
22878     } else if (PyExceptionClass_Check(type)) {
22879         PyObject *instance_class = NULL;
22880         if (value && PyExceptionInstance_Check(value)) {
22881             instance_class = (PyObject*) Py_TYPE(value);
22882             if (instance_class != type) {
22883                 int is_subclass = PyObject_IsSubclass(instance_class, type);
22884                 if (!is_subclass) {
22885                     instance_class = NULL;
22886                 } else if (unlikely(is_subclass == -1)) {
22887                     goto bad;
22888                 } else {
22889                     type = instance_class;
22890                 }
22891             }
22892         }
22893         if (!instance_class) {
22894             PyObject *args;
22895             if (!value)
22896                 args = PyTuple_New(0);
22897             else if (PyTuple_Check(value)) {

```

```

22898         Py_INCREF(value);
22899         args = value;
22900     } else
22901     {
22902         args = PyTuple_Pack(1, value);
22903         if (!args)
22904             goto bad;
22905         owned_instance = PyObject_Call(type, args, NULL);
22906         Py_DECREF(args);
22907         if (!owned_instance)
22908             goto bad;
22909         value = owned_instance;
22910         if (!PyExceptionInstance_Check(value)) {
22911             PyErr_Format(PyExc_TypeError,
22912                          "calling %R should have returned an instance of "
22913                          "BaseException, not %R",
22914                          type, Py_TYPE(value));
22915             goto bad;
22916         }
22917     } else {
22918         PyErr_SetString(PyExc_TypeError,
22919                        "raise: exception class must be a subclass of BaseException");
22920         goto bad;
22921     }
22922     if (cause) {
22923         PyObject *fixed_cause;
22924         if (cause == Py_None) {
22925             fixed_cause = NULL;
22926         } else if (PyExceptionClass_Check(cause)) {
22927             fixed_cause = PyObject_CallObject(cause, NULL);
22928             if (fixed_cause == NULL)
22929                 goto bad;
22930         } else if (PyExceptionInstance_Check(cause)) {
22931             fixed_cause = cause;
22932             Py_INCREF(fixed_cause);
22933         } else {
22934             PyErr_SetString(PyExc_TypeError,
22935                            "exception causes must derive from "
22936                            "BaseException");
22937             goto bad;
22938         }
22939         PyException_SetCause(value, fixed_cause);
22940     }
22941     PyErr_SetObject(type, value);
22942     if (tb) {
22943 #if CYTHON_COMPILING_IN_PYPY
22944         PyObject *tmp_type, *tmp_value, *tmp_tb;
22945         PyErr_Fetch(&tmp_type, &tmp_value, &tmp_tb);
22946         Py_INCREF(tb);
22947         PyErr_Restore(tmp_type, tmp_value, tb);
22948         Py_XDECREF(tmp_tb);
22949 #else
22950         PyThreadState *tstate = __Pyx_PyThreadState_Current;
22951         PyObject* tmp_tb = tstate->curexc_traceback;
22952         if (tb != tmp_tb) {
22953             Py_INCREF(tb);
22954             tstate->curexc_traceback = tb;
22955             Py_XDECREF(tmp_tb);
22956         }
22957 #endif
22958     }
22959     bad:
22960     Py_XDECREF(owned_instance);
22961     return;
22962 }
22963 #endif
22964
22965 /* PyObjectCall2Args */
22966 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject*
22967 arg2) {
22968     PyObject *args, *result = NULL;
22969     #if CYTHON_FAST_PYCALL
22970     if (PyFunction_Check(function)) {
22971         PyObject *args[2] = {arg1, arg2};
22972         return __Pyx_PyFunction_FastCall(function, args, 2);
22973     }
22974     #endif
22975     #if CYTHON_FAST_PYCCALL
22976     if (__Pyx_PyFastCFunction_Check(function)) {
22977         PyObject *args[2] = {arg1, arg2};
22978         return __Pyx_PyCFunction_FastCall(function, args, 2);
22979     }
22980     #endif
22981     args = PyTuple_New(2);
22982     if (unlikely(!args)) goto done;
22983     Py_INCREF(arg1);
22984     PyTuple_SET_ITEM(args, 0, arg1);

```



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```

23063         } else if (8 * sizeof(PY_LONG_LONG) - 1 > 3 * PyLong_SHIFT) {
23064             lla = (PY_LONG_LONG) (((((((unsigned PY_LONG_LONG) digits[2]) « PyLong_SHIFT) |
(unsigned PY_LONG_LONG) digits[1]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[0]));
23065             goto long_long;
23066 #endif
23067         }
23068         CYTHON_FALLTHROUGH;
23069         case -4:
23070             if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
23071                 a = -(long) (((((((unsigned long) digits[3]) « PyLong_SHIFT) | (unsigned
long) digits[2]) « PyLong_SHIFT) | (unsigned long) digits[1]) « PyLong_SHIFT) | (unsigned
long) digits[0]));
23072                 break;
23073 #ifndef HAVE_LONG_LONG
23074             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23075                 lla = -(PY_LONG_LONG) (((((((unsigned PY_LONG_LONG) digits[3]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[1])
« PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[0]));
23076                 goto long_long;
23077 #endif
23078             }
23079             CYTHON_FALLTHROUGH;
23080             case 4:
23081                 if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
23082                     a = (long) (((((((unsigned long) digits[3]) « PyLong_SHIFT) | (unsigned
long) digits[2]) « PyLong_SHIFT) | (unsigned long) digits[1]) « PyLong_SHIFT) | (unsigned
long) digits[0]));
23083                     break;
23084 #ifndef HAVE_LONG_LONG
23085                 } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23086                     lla = (PY_LONG_LONG) (((((((unsigned PY_LONG_LONG) digits[3]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG) digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[1]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG) digits[0]));
23087                     goto long_long;
23088 #endif
23089                 }
23090                 CYTHON_FALLTHROUGH;
23091                 default: return PyLong_Type.tp_as_number->nb_add(op1, op2);
23092             }
23093         }
23094         x = a + b;
23095         return PyLong_FromLong(x);
23096 #ifndef HAVE_LONG_LONG
23097         long_long:
23098             llx = lla + llb;
23099             return PyLong_FromLongLong(llx);
23100 #endif
23101     }
23102 }
23103 #endif
23104 if (PyFloat_CheckExact(op1)) {
23105     const long b = intval;
23106     double a = PyFloat_AS_DOUBLE(op1);
23107     double result;
23108     PyFPE_START_PROTECT("add", return NULL)
23109     result = ((double)a) + (double)b;
23110     PyFPE_END_PROTECT(result)
23111     return PyFloat_FromDouble(result);
23112 }
23113 return (inplace ? PyNumber_InPlaceAdd : PyNumber_Add)(op1, op2);
23114 #endif
23115 }
23116 #endif
23117
23118 /* decode_c_bytes */
23119 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
23120     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
23121     const char* encoding, const char* errors,
23122     PyObject* (*decode_func)(const char*s, Py_ssize_t size, const char*errors)) {
23123     if (unlikely((start < 0) | (stop < 0))) {
23124         if (start < 0) {
23125             start += length;
23126             if (start < 0)
23127                 start = 0;
23128         }
23129         if (stop < 0)
23130             stop += length;
23131     }
23132     if (stop > length)
23133         stop = length;
23134     if (unlikely(stop <= start))
23135         return __Pyx_NewRef(__pyx_empty_unicode);
23136     length = stop - start;
23137     cstring += start;
23138     if (decode_func) {
23139         return decode_func(cstring, length, errors);
23140     } else {

```

```

23141         return PyUnicode_Decode(cstring, length, encoding, errors);
23142     }
23143 }
23144
23145 /* SwapException */
23146 #if CYTHON_FAST_THREAD_STATE
23147 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyThreadState *tstate, PyObject **type, PyObject
    **value, PyObject **tb) {
23148     PyObject *tmp_type, *tmp_value, *tmp_tb;
23149     #if CYTHON_USE_EXC_INFO_STACK
23150     _PyErr_StackItem *exc_info = tstate->exc_info;
23151     tmp_type = exc_info->exc_type;
23152     tmp_value = exc_info->exc_value;
23153     tmp_tb = exc_info->exc_traceback;
23154     exc_info->exc_type = *type;
23155     exc_info->exc_value = *value;
23156     exc_info->exc_traceback = *tb;
23157     #else
23158     tmp_type = tstate->exc_type;
23159     tmp_value = tstate->exc_value;
23160     tmp_tb = tstate->exc_traceback;
23161     tstate->exc_type = *type;
23162     tstate->exc_value = *value;
23163     tstate->exc_traceback = *tb;
23164     #endif
23165     *type = tmp_type;
23166     *value = tmp_value;
23167     *tb = tmp_tb;
23168 }
23169 #else
23170 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject **type, PyObject **value, PyObject **tb) {
23171     PyObject *tmp_type, *tmp_value, *tmp_tb;
23172     PyErr_GetExcInfo(&tmp_type, &tmp_value, &tmp_tb);
23173     PyErr_SetExcInfo(*type, *value, *tb);
23174     *type = tmp_type;
23175     *value = tmp_value;
23176     *tb = tmp_tb;
23177 }
23178 #endif
23179
23180 /* SetItemInt */
23181 static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v) {
23182     int r;
23183     if (!j) return -1;
23184     r = PyObject_SetItem(o, j, v);
23185     Py_DECREF(j);
23186     return r;
23187 }
23188 static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v, int is_list,
    CYTHON_NCP_UNUSED int wraparound, CYTHON_NCP_UNUSED int
23189     boundscheck) {
23190 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS && CYTHON_USE_TYPE_SLOTS
23191     if (is_list || PyList_CheckExact(o)) {
23192         Py_ssize_t n = (!wraparound) ? i : ((likely(i >= 0)) ? i : i + PyList_GET_SIZE(o));
23193         if (!(!boundscheck) || likely(__Pyx_is_valid_index(n, PyList_GET_SIZE(o)))) {
23194             PyObject* old = PyList_GET_ITEM(o, n);
23195             Py_INCREF(v);
23196             PyList_SET_ITEM(o, n, v);
23197             Py_DECREF(old);
23198             return 1;
23199         }
23200     } else {
23201         PySequenceMethods *m = Py_TYPE(o)->tp_as_sequence;
23202         if (likely(m && m->sq_ass_item)) {
23203             if (wraparound && unlikely(i < 0) && likely(m->sq_length)) {
23204                 Py_ssize_t l = m->sq_length(o);
23205                 if (likely(l >= 0)) {
23206                     i += l;
23207                 } else {
23208                     if (!PyErr_ExceptionMatches(PyExc_OverflowError))
23209                         return -1;
23210                     PyErr_Clear();
23211                 }
23212             }
23213             return m->sq_ass_item(o, i, v);
23214         }
23215     }
23216 #else
23217 #if CYTHON_COMPILING_IN_PYPY
23218     if (is_list || (PySequence_Check(o) && !PyDict_Check(o)))
23219 #else
23220     if (is_list || PySequence_Check(o))
23221 #endif
23222     {
23223         return PySequence_SetItem(o, i, v);
23224     }
23225 #endif

```

```

23226     return __Pyx_SetItemInt_Generic(o, PyInt_FromSsize_t(i), v);
23227 }
23228
23229 /* ArgTypeTest */
23230 static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact)
23231 {
23232     if (unlikely(!type)) {
23233         PyErr_SetString(PyExc_SystemError, "Missing type object");
23234         return 0;
23235     }
23236     else if (exact) {
23237         #if PY_MAJOR_VERSION == 2
23238             if ((type == &PyBaseString_Type) && likely(__Pyx_PyBaseString_CheckExact(obj))) return 1;
23239             #endif
23240     }
23241     else {
23242         if (likely(__Pyx_TypeCheck(obj, type))) return 1;
23243     }
23244     PyErr_Format(PyExc_TypeError,
23245         "Argument '%.200s' has incorrect type (expected %.200s, got %.200s)",
23246         name, type->tp_name, Py_TYPE(obj)->tp_name);
23247     return 0;
23248 }
23249
23250 /* Import */
23251 static PyObject *__Pyx_Import(PyObject *name, PyObject *from_list, int level) {
23252     PyObject *empty_list = 0;
23253     PyObject *module = 0;
23254     PyObject *global_dict = 0;
23255     PyObject *empty_dict = 0;
23256     PyObject *list;
23257     #if PY_MAJOR_VERSION < 3
23258     PyObject *py_import;
23259     py_import = __Pyx_PyObject_GetAttrStr(__pyx_b, __pyx_n_s_import);
23260     if (!py_import)
23261         goto bad;
23262     #endif
23263     if (from_list)
23264         list = from_list;
23265     else {
23266         empty_list = PyList_New(0);
23267         if (!empty_list)
23268             goto bad;
23269         list = empty_list;
23270     }
23271     global_dict = PyModule_GetDict(__pyx_m);
23272     if (!global_dict)
23273         goto bad;
23274     empty_dict = PyDict_New();
23275     if (!empty_dict)
23276         goto bad;
23277     {
23278         #if PY_MAJOR_VERSION >= 3
23279         if (level == -1) {
23280             if ((1) && (strchr(__Pyx_MODULE_NAME, '.'))) {
23281                 module = PyImport_ImportModuleLevelObject(
23282                     name, global_dict, empty_dict, list, 1);
23283                 if (!module) {
23284                     if (!PyErr_ExceptionMatches(PyExc_ImportError))
23285                         goto bad;
23286                     PyErr_Clear();
23287                 }
23288             }
23289             level = 0;
23290         }
23291         #endif
23292         if (!module) {
23293             #if PY_MAJOR_VERSION < 3
23294             PyObject *py_level = PyInt_FromLong(level);
23295             if (!py_level)
23296                 goto bad;
23297             module = PyObject_CallFunctionObjArgs(py_import,
23298                 name, global_dict, empty_dict, list, py_level, (PyObject *)NULL);
23299             Py_DECREF(py_level);
23300             #else
23301             module = PyImport_ImportModuleLevelObject(
23302                 name, global_dict, empty_dict, list, level);
23303             #endif
23304         }
23305     }
23306     bad:
23307     #if PY_MAJOR_VERSION < 3
23308     Py_XDECREF(py_import);
23309     #endif
23310     Py_XDECREF(empty_list);
23311     Py_XDECREF(empty_dict);
23312     return module;

```

```

23313 }
23314
23315 /* PyObject_GenericGetAttrNoDict */
23316 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23317 static PyObject* __Pyx_RaiseGenericGetAttributeError(PyTypeObject *tp, PyObject *attr_name) {
23318     PyErr_Format(PyExc_AttributeError,
23319 #if PY_MAJOR_VERSION >= 3
23320         "%0.50s' object has no attribute '%U'",
23321         tp->tp_name, attr_name);
23322 #else
23323         "%0.50s' object has no attribute '%.400s'",
23324         tp->tp_name, PyString_AS_STRING(attr_name));
23325 #endif
23326     return NULL;
23327 }
23328 static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject* attr_name)
23329 {
23330     PyObject *descr;
23331     PyTypeObject *tp = Py_TYPE(obj);
23332     if (unlikely(!PyString_Check(attr_name))) {
23333         return PyObject_GenericGetAttr(obj, attr_name);
23334     }
23335     assert(!tp->tp_dictoffset);
23336     descr = _PyType_Lookup(tp, attr_name);
23337     if (unlikely(!descr)) {
23338         return __Pyx_RaiseGenericGetAttributeError(tp, attr_name);
23339     }
23340     Py_INCREF(descr);
23341     #if PY_MAJOR_VERSION < 3
23342     if (likely(PyType_HasFeature(Py_TYPE(descr), Py_TPFLAGS_HAVE_CLASS)))
23343     #endif
23344     {
23345         descrgetfunc f = Py_TYPE(descr)->tp_descr_get;
23346         if (unlikely(!f)) {
23347             PyObject *res = f(descr, obj, (PyObject *)tp);
23348             Py_DECREF(descr);
23349             return res;
23350         }
23351         return descr;
23352     }
23353 #endif
23354
23355 /* PyObject_GenericGetAttr */
23356 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23357 static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name) {
23358     if (unlikely(Py_TYPE(obj)->tp_dictoffset)) {
23359         return PyObject_GenericGetAttr(obj, attr_name);
23360     }
23361     return __Pyx_PyObject_GenericGetAttrNoDict(obj, attr_name);
23362 }
23363 #endif
23364
23365 /* SetVTable */
23366 static int __Pyx_SetVtable(PyObject *dict, void *vtable) {
23367     #if PY_VERSION_HEX >= 0x02070000
23368     PyObject *ob = PyCapsule_New(vtable, 0, 0);
23369     #else
23370     PyObject *ob = PyCObject_FromVoidPtr(vtable, 0);
23371     #endif
23372     if (!ob)
23373         goto bad;
23374     if (PyDict_SetItem(dict, __pyx_n_s_pyx_vtable, ob) < 0)
23375         goto bad;
23376     Py_DECREF(ob);
23377     return 0;
23378 bad:
23379     Py_XDECREF(ob);
23380     return -1;
23381 }
23382
23383 /* PyObject_GetAttrStrNoError */
23384 static void __Pyx_PyObject_GetAttrStr_ClearAttributeError(void) {
23385     __Pyx_PyThreadState_declare
23386     __Pyx_PyThreadState_assign
23387     if (likely(__Pyx_PyErr_ExceptionMatches(PyExc_AttributeError)))
23388         __Pyx_PyErr_Clear();
23389 }
23390 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name) {
23391     PyObject *result;
23392     #if CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_TYPE_SLOTS && PY_VERSION_HEX >= 0x030700B1
23393     PyTypeObject* tp = Py_TYPE(obj);
23394     if (likely(tp->tp_getattro == PyObject_GenericGetAttr)) {
23395         return _PyObject_GenericGetAttrWithDict(obj, attr_name, NULL, 1);
23396     }
23397 #endif
23398     result = __Pyx_PyObject_GetAttrStr(obj, attr_name);

```

```

23399     if (unlikely(!result)) {
23400         __Pyx_PyObject_GetAttrStr_ClearAttributeError();
23401     }
23402     return result;
23403 }
23404
23405 /* SetupReduce */
23406 static int __Pyx_setup_reduce_is_named(PyObject* meth, PyObject* name) {
23407     int ret;
23408     PyObject *name_attr;
23409     name_attr = __Pyx_PyObject_GetAttrStr(meth, __pyx_n_s_name);
23410     if (likely(name_attr)) {
23411         ret = PyObject_RichCompareBool(name_attr, name, Py_EQ);
23412     } else {
23413         ret = -1;
23414     }
23415     if (unlikely(ret < 0)) {
23416         PyErr_Clear();
23417         ret = 0;
23418     }
23419     Py_XDECREF(name_attr);
23420     return ret;
23421 }
23422 static int __Pyx_setup_reduce(PyObject* type_obj) {
23423     int ret = 0;
23424     PyObject *object_reduce = NULL;
23425     PyObject *object_reduce_ex = NULL;
23426     PyObject *reduce = NULL;
23427     PyObject *reduce_ex = NULL;
23428     PyObject *reduce_cython = NULL;
23429     PyObject *setstate = NULL;
23430     PyObject *setstate_cython = NULL;
23431 #if CYTHON_USE_PYTYPE_LOOKUP
23432     if (_PyType_Lookup((PyTypeObject*)type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23433 #else
23434     if (PyObject_HasAttr(type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23435 #endif
23436 #if CYTHON_USE_PYTYPE_LOOKUP
23437     object_reduce_ex = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce_ex); if (!object_reduce_ex)
23438         goto __PYX_BAD;
23439 #else
23440     object_reduce_ex = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce_ex);
23441     if (!object_reduce_ex) goto __PYX_BAD;
23442 #endif
23443     reduce_ex = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce_ex); if (unlikely(!reduce_ex))
23444         goto __PYX_BAD;
23445     if (reduce_ex == object_reduce_ex) {
23446 #if CYTHON_USE_PYTYPE_LOOKUP
23447         object_reduce = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce); if (!object_reduce) goto
23448         __PYX_BAD;
23449 #else
23450         object_reduce = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce); if
23451         (!object_reduce) goto __PYX_BAD;
23452 #endif
23453     }
23454     reduce = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce); if (unlikely(!reduce)) goto
23455     __PYX_BAD;
23456     if (reduce == object_reduce || __Pyx_setup_reduce_is_named(reduce, __pyx_n_s_reduce_cython)) {
23457         reduce_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj, __pyx_n_s_reduce_cython);
23458         if (likely(reduce_cython)) {
23459             ret = PyDict_SetItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_reduce,
23460             reduce_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23461             ret = PyDict_DelItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_reduce_cython); if
23462             (unlikely(ret < 0)) goto __PYX_BAD;
23463         } else if (reduce == object_reduce || PyErr_Occurred()) {
23464             goto __PYX_BAD;
23465         }
23466         setstate = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_setstate);
23467         if (!setstate) PyErr_Clear();
23468         if (!setstate || __Pyx_setup_reduce_is_named(setstate, __pyx_n_s_setstate_cython)) {
23469             setstate_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj,
23470             __pyx_n_s_setstate_cython);
23471             if (likely(setstate_cython)) {
23472                 ret = PyDict_SetItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_setstate,
23473                 setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23474                 ret = PyDict_DelItem(((PyTypeObject*)type_obj)->tp_dict,
23475                 __pyx_n_s_setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23476             } else if (!setstate || PyErr_Occurred()) {
23477                 goto __PYX_BAD;
23478             }
23479         }
23480         PyType_Modified((PyTypeObject*)type_obj);
23481     }
23482     goto __PYX_GOOD;
23483 }
23484 __PYX_BAD:
23485     if (!PyErr_Occurred())
23486         PyErr_Format(PyExc_RuntimeError, "Unable to initialize pickling for %s",

```

```

        ((PyTypeObject*)type_obj)->tp_name);
23475     ret = -1;
23476 __PYX_GOOD:
23477 #if !CYTHON_USE_PYTYPE_LOOKUP
23478     Py_XDECREF(object_reduce);
23479     Py_XDECREF(object_reduce_ex);
23480 #endif
23481     Py_XDECREF(reduce);
23482     Py_XDECREF(reduce_ex);
23483     Py_XDECREF(reduce_cython);
23484     Py_XDECREF(setstate);
23485     Py_XDECREF(setstate_cython);
23486     return ret;
23487 }
23488
23489 /* BytesEquals */
23490 static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals) {
23491 #if CYTHON_COMPILING_IN_PYPY
23492     return PyObject_RichCompareBool(s1, s2, equals);
23493 #else
23494     if (s1 == s2) {
23495         return (equals == Py_EQ);
23496     } else if (PyBytes_CheckExact(s1) & PyBytes_CheckExact(s2)) {
23497         const char *ps1, *ps2;
23498         Py_ssize_t length = PyBytes_GET_SIZE(s1);
23499         if (length != PyBytes_GET_SIZE(s2))
23500             return (equals == Py_NE);
23501         ps1 = PyBytes_AS_STRING(s1);
23502         ps2 = PyBytes_AS_STRING(s2);
23503         if (ps1[0] != ps2[0]) {
23504             return (equals == Py_NE);
23505         } else if (length == 1) {
23506             return (equals == Py_EQ);
23507         } else {
23508             int result;
23509 #if CYTHON_USE_UNICODE_INTERNALS
23510             Py_hash_t hash1, hash2;
23511             hash1 = ((PyBytesObject*)s1)->ob_shash;
23512             hash2 = ((PyBytesObject*)s2)->ob_shash;
23513             if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23514                 return (equals == Py_NE);
23515             }
23516 #endif
23517             result = memcmp(ps1, ps2, (size_t)length);
23518             return (equals == Py_EQ) ? (result == 0) : (result != 0);
23519         }
23520     } else if ((s1 == Py_None) & PyBytes_CheckExact(s2)) {
23521         return (equals == Py_NE);
23522     } else if ((s2 == Py_None) & PyBytes_CheckExact(s1)) {
23523         return (equals == Py_NE);
23524     } else {
23525         int result;
23526         PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23527         if (!py_result)
23528             return -1;
23529         result = __Pyx_PyObject_IsTrue(py_result);
23530         Py_DECREF(py_result);
23531         return result;
23532     }
23533 #endif
23534 }
23535
23536 /* UnicodeEquals */
23537 static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals) {
23538 #if CYTHON_COMPILING_IN_PYPY
23539     return PyObject_RichCompareBool(s1, s2, equals);
23540 #else
23541 #if PY_MAJOR_VERSION < 3
23542     PyObject* owned_ref = NULL;
23543 #endif
23544     int s1_is_unicode, s2_is_unicode;
23545     if (s1 == s2) {
23546         goto return_eq;
23547     }
23548     s1_is_unicode = PyUnicode_CheckExact(s1);
23549     s2_is_unicode = PyUnicode_CheckExact(s2);
23550 #if PY_MAJOR_VERSION < 3
23551     if ((s1_is_unicode & (!s2_is_unicode)) && PyString_CheckExact(s2)) {
23552         owned_ref = PyUnicode_FromObject(s2);
23553         if (unlikely(!owned_ref))
23554             return -1;
23555         s2 = owned_ref;
23556         s2_is_unicode = 1;
23557     } else if ((s2_is_unicode & (!s1_is_unicode)) && PyString_CheckExact(s1)) {
23558         owned_ref = PyUnicode_FromObject(s1);
23559         if (unlikely(!owned_ref))
23560             return -1;

```

```

23561         s1 = owned_ref;
23562         s1_is_unicode = 1;
23563     } else if (((!s2_is_unicode) & (!s1_is_unicode))) {
23564         return __Pyx_PyBytes_Equals(s1, s2, equals);
23565     }
23566 #endif
23567     if (s1_is_unicode & s2_is_unicode) {
23568         Py_ssize_t length;
23569         int kind;
23570         void *data1, *data2;
23571         if (unlikely(__Pyx_PyUnicode_READY(s1) < 0) || unlikely(__Pyx_PyUnicode_READY(s2) < 0))
23572             return -1;
23573         length = __Pyx_PyUnicode_GET_LENGTH(s1);
23574         if (length != __Pyx_PyUnicode_GET_LENGTH(s2)) {
23575             goto return_ne;
23576         }
23577 #if CYTHON_USE_UNICODE_INTERNALS
23578     {
23579         Py_hash_t hash1, hash2;
23580         #if CYTHON_PEP393_ENABLED
23581             hash1 = ((PyASCIIObject*)s1)->hash;
23582             hash2 = ((PyASCIIObject*)s2)->hash;
23583         #else
23584             hash1 = ((PyUnicodeObject*)s1)->hash;
23585             hash2 = ((PyUnicodeObject*)s2)->hash;
23586         #endif
23587         if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23588             goto return_ne;
23589         }
23590     }
23591 #endif
23592     kind = __Pyx_PyUnicode_KIND(s1);
23593     if (kind != __Pyx_PyUnicode_KIND(s2)) {
23594         goto return_ne;
23595     }
23596     data1 = __Pyx_PyUnicode_DATA(s1);
23597     data2 = __Pyx_PyUnicode_DATA(s2);
23598     if (__Pyx_PyUnicode_READ(kind, data1, 0) != __Pyx_PyUnicode_READ(kind, data2, 0)) {
23599         goto return_ne;
23600     } else if (length == 1) {
23601         goto return_eq;
23602     } else {
23603         int result = memcmp(data1, data2, (size_t)(length * kind));
23604         #if PY_MAJOR_VERSION < 3
23605             Py_XDECREF(owned_ref);
23606         #endif
23607         return (equals == Py_EQ) ? (result == 0) : (result != 0);
23608     }
23609 } else if ((s1 == Py_None) & s2_is_unicode) {
23610     goto return_ne;
23611 } else if ((s2 == Py_None) & s1_is_unicode) {
23612     goto return_ne;
23613 } else {
23614     int result;
23615     PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23616     #if PY_MAJOR_VERSION < 3
23617     Py_XDECREF(owned_ref);
23618     #endif
23619     if (!py_result)
23620         return -1;
23621     result = __Pyx_PyObject_IsTrue(py_result);
23622     Py_DECREF(py_result);
23623     return result;
23624 }
23625 return_eq:
23626     #if PY_MAJOR_VERSION < 3
23627     Py_XDECREF(owned_ref);
23628     #endif
23629     return (equals == Py_EQ);
23630 return_ne:
23631     #if PY_MAJOR_VERSION < 3
23632     Py_XDECREF(owned_ref);
23633     #endif
23634     return (equals == Py_NE);
23635 #endif
23636 }
23637
23638 /* CLineInTraceback */
23639 #ifndef CYTHON_CLINE_IN_TRACEBACK
23640 static int __Pyx_CLineForTraceback(CYTHON_NCP_UNUSED PyThreadState *tstate, int c_line) {
23641     PyObject *use_cline;
23642     PyObject *ptype, *pvalue, *ptraceback;
23643     #if CYTHON_COMPILING_IN_CPYTHON
23644     PyObject **cython_runtime_dict;
23645     #endif
23646     if (unlikely(!__pyx_cython_runtime)) {
23647         return c_line;

```



```

23648     }
23649     __Pyx_ErrFetchInState(tstate, &ptype, &pvalue, &ptraceback);
23650 #if CYTHON_COMPILING_IN_CPYTHON
23651     cython_runtime_dict = _PyObject_GetDictPtr(__pyx_cython_runtime);
23652     if (likely(cython_runtime_dict)) {
23653         __PYX_PY_DICT_LOOKUP_IF_MODIFIED(
23654             use_cline, *cython_runtime_dict,
23655             __Pyx_PyDict_GetItemStr(*cython_runtime_dict, __pyx_n_s_cline_in_traceback))
23656     } else
23657 #endif
23658     {
23659         PyObject *use_cline_obj = __Pyx_PyObject_GetAttrStr(__pyx_cython_runtime,
23660             __pyx_n_s_cline_in_traceback);
23661         if (use_cline_obj) {
23662             use_cline = PyObject_Not(use_cline_obj) ? Py_False : Py_True;
23663             Py_DECREF(use_cline_obj);
23664         } else {
23665             PyErr_Clear();
23666             use_cline = NULL;
23667         }
23668         if (!use_cline) {
23669             c_line = 0;
23670             (void) PyObject_SetAttr(__pyx_cython_runtime, __pyx_n_s_cline_in_traceback, Py_False);
23671         }
23672         else if (use_cline == Py_False || (use_cline != Py_True && PyObject_Not(use_cline) != 0)) {
23673             c_line = 0;
23674         }
23675         __Pyx_ErrRestoreInState(tstate, ptype, pvalue, ptraceback);
23676         return c_line;
23677     }
23678 #endif
23679
23680 /* CodeObjectCache */
23681 static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line) {
23682     int start = 0, mid = 0, end = count - 1;
23683     if (end >= 0 && code_line > entries[end].code_line) {
23684         return count;
23685     }
23686     while (start < end) {
23687         mid = start + (end - start) / 2;
23688         if (code_line < entries[mid].code_line) {
23689             end = mid;
23690         } else if (code_line > entries[mid].code_line) {
23691             start = mid + 1;
23692         } else {
23693             return mid;
23694         }
23695     }
23696     if (code_line <= entries[mid].code_line) {
23697         return mid;
23698     } else {
23699         return mid + 1;
23700     }
23701 }
23702 static PyCodeObject* __pyx_find_code_object(int code_line) {
23703     PyCodeObject* code_object;
23704     int pos;
23705     if (unlikely(!code_line) || unlikely(!__pyx_code_cache.entries)) {
23706         return NULL;
23707     }
23708     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23709     if (unlikely(pos >= __pyx_code_cache.count) || unlikely(__pyx_code_cache.entries[pos].code_line !=
code_line)) {
23710         return NULL;
23711     }
23712     code_object = __pyx_code_cache.entries[pos].code_object;
23713     Py_INCREF(code_object);
23714     return code_object;
23715 }
23716 static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object) {
23717     int pos, i;
23718     __Pyx_CodeObjectCacheEntry* entries = __pyx_code_cache.entries;
23719     if (unlikely(!code_line)) {
23720         return;
23721     }
23722     if (unlikely(!entries)) {
23723         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Malloc(64*sizeof(__Pyx_CodeObjectCacheEntry));
23724         if (likely(entries)) {
23725             __pyx_code_cache.entries = entries;
23726             __pyx_code_cache.max_count = 64;
23727             __pyx_code_cache.count = 1;
23728             entries[0].code_line = code_line;
23729             entries[0].code_object = code_object;
23730             Py_INCREF(code_object);
23731         }
23732         return;

```

```

23733     }
23734     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23735     if ((pos < __pyx_code_cache.count) && unlikely(__pyx_code_cache.entries[pos].code_line ==
code_line)) {
23736         PyCodeObject* tmp = entries[pos].code_object;
23737         entries[pos].code_object = code_object;
23738         Py_DECREF(tmp);
23739         return;
23740     }
23741     if (__pyx_code_cache.count == __pyx_code_cache.max_count) {
23742         int new_max = __pyx_code_cache.max_count + 64;
23743         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Realloc(
23744             __pyx_code_cache.entries, ((size_t)new_max) * sizeof(__Pyx_CodeObjectCacheEntry));
23745         if (unlikely(!entries)) {
23746             return;
23747         }
23748         __pyx_code_cache.entries = entries;
23749         __pyx_code_cache.max_count = new_max;
23750     }
23751     for (i=__pyx_code_cache.count; i>pos; i--) {
23752         entries[i] = entries[i-1];
23753     }
23754     entries[pos].code_line = code_line;
23755     entries[pos].code_object = code_object;
23756     __pyx_code_cache.count++;
23757     Py_INCREF(code_object);
23758 }
23759
23760 /* AddTraceback */
23761 #include "compile.h"
23762 #include "frameobject.h"
23763 #include "traceback.h"
23764 static PyCodeObject* __Pyx_CreateCodeObjectForTraceback(
23765     const char *funcname, int c_line,
23766     int py_line, const char *filename) {
23767     PyCodeObject *py_code = NULL;
23768     PyObject *py_funcname = NULL;
23769     #if PY_MAJOR_VERSION < 3
23770     PyObject *py_srcfile = NULL;
23771     py_srcfile = PyString_FromString(filename);
23772     if (!py_srcfile) goto bad;
23773     #endif
23774     if (c_line) {
23775         #if PY_MAJOR_VERSION < 3
23776         py_funcname = PyString_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23777         if (!py_funcname) goto bad;
23778         #else
23779         py_funcname = PyUnicode_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23780         if (!py_funcname) goto bad;
23781         funcname = PyUnicode_AsUTF8(py_funcname);
23782         if (!funcname) goto bad;
23783         #endif
23784     }
23785     else {
23786         #if PY_MAJOR_VERSION < 3
23787         py_funcname = PyString_FromString(funcname);
23788         if (!py_funcname) goto bad;
23789         #endif
23790     }
23791     #if PY_MAJOR_VERSION < 3
23792     py_code = __Pyx_PyCode_New(
23793         0,
23794         0,
23795         0,
23796         0,
23797         0,
23798         __pyx_empty_bytes, /*PyObject *code,*/
23799         __pyx_empty_tuple, /*PyObject *consts,*/
23800         __pyx_empty_tuple, /*PyObject *names,*/
23801         __pyx_empty_tuple, /*PyObject *varnames,*/
23802         __pyx_empty_tuple, /*PyObject *freevars,*/
23803         __pyx_empty_tuple, /*PyObject *cellvars,*/
23804         py_srcfile, /*PyObject *filename,*/
23805         py_funcname, /*PyObject *name,*/
23806         py_line,
23807         __pyx_empty_bytes /*PyObject *notab*/
23808     );
23809     Py_DECREF(py_srcfile);
23810     #else
23811     py_code = PyCode_NewEmpty(filename, funcname, py_line);
23812     #endif
23813     Py_XDECREF(py_funcname); // XDECREF since it's only set on Py3 if cline
23814     return py_code;
23815 bad:
23816     Py_XDECREF(py_funcname);
23817     #if PY_MAJOR_VERSION < 3
23818     Py_XDECREF(py_srcfile);

```

```

23819     #endif
23820     return NULL;
23821 }
23822 static void __Pyx_AddTraceback(const char *funcname, int c_line,
23823                               int py_line, const char *filename) {
23824     PyCodeObject *py_code = 0;
23825     PyFrameObject *py_frame = 0;
23826     PyThreadState *tstate = __Pyx_PyThreadState_Current;
23827     if (c_line) {
23828         c_line = __Pyx_CLineForTraceback(tstate, c_line);
23829     }
23830     py_code = __pyx_find_code_object(c_line ? -c_line : py_line);
23831     if (!py_code) {
23832         py_code = __Pyx_CreateCodeObjectForTraceback(
23833             funcname, c_line, py_line, filename);
23834         if (!py_code) goto bad;
23835         __pyx_insert_code_object(c_line ? -c_line : py_line, py_code);
23836     }
23837     py_frame = PyFrame_New(
23838         tstate, /*PyThreadState *tstate,*/
23839         py_code, /*PyCodeObject *code,*/
23840         __pyx_d, /*PyObject *globals,*/
23841         0 /*PyObject *locals*/
23842     );
23843     if (!py_frame) goto bad;
23844     __Pyx_PyFrame_SetLineNumber(py_frame, py_line);
23845     PyTraceBack_Here(py_frame);
23846 bad:
23847     Py_XDECREF(py_code);
23848     Py_XDECREF(py_frame);
23849 }
23850
23851 /* CIntFromPyVerify */
23852 #define __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value)\
23853     __PYX__VERIFY_RETURN_INT(target_type, func_type, func_value, 0)
23854 #define __PYX_VERIFY_RETURN_INT_EXC(target_type, func_type, func_value)\
23855     __PYX__VERIFY_RETURN_INT(target_type, func_type, func_value, 1)
23856 #define __PYX__VERIFY_RETURN_INT(target_type, func_type, func_value, exc)\
23857     {\
23858         func_type value = func_value;\
23859         if (sizeof(target_type) < sizeof(func_type)) {\
23860             if (unlikely(value != (func_type) (target_type) value)) {\
23861                 func_type zero = 0;\
23862                 if (exc && unlikely(value == (func_type)-1 && PyErr_Occurred()))\
23863                     return (target_type) -1;\
23864                 if (is_unsigned && unlikely(value < zero))\
23865                     goto raise_neg_overflow;\
23866                 else\
23867                     goto raise_overflow;\
23868             }\
23869         }\
23870         return (target_type) value;\
23871     }
23872
23873 /* CIntFromPy */
23874 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *x) {
23875     #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23876     #pragma GCC diagnostic push
23877     #pragma GCC diagnostic ignored "-Wconversion"
23878     #endif
23879     const int neg_one = (int) -1, const_zero = (int) 0;
23880     #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23881     #pragma GCC diagnostic pop
23882     #endif
23883     const int is_unsigned = neg_one > const_zero;
23884     #if PY_MAJOR_VERSION < 3
23885     if (likely(PyInt_Check(x))) {
23886         if (sizeof(int) < sizeof(long)) {
23887             __PYX_VERIFY_RETURN_INT(int, long, PyInt_AS_LONG(x))
23888         } else {
23889             long val = PyInt_AS_LONG(x);
23890             if (is_unsigned && unlikely(val < 0)) {
23891                 goto raise_neg_overflow;
23892             }
23893             return (int) val;
23894         }
23895     } else
23896     #endif
23897     if (likely(PyLong_Check(x))) {
23898         if (is_unsigned) {
23899             #if CYTHON_USE_PYLONG_INTERNALS
23900             const digit* digits = ((PyLongObject*)x)->ob_digit;
23901             switch (Py_SIZE(x)) {
23902                 case 0: return (int) 0;
23903                 case 1: __PYX_VERIFY_RETURN_INT(int, digit, digits[0])
23904                 case 2:
23905                     if (8 * sizeof(int) > 1 * PyLong_SHIFT) {

```

```

23906         if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23907             __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned long)digits[1])
« PyLong_SHIFT) | (unsigned long)digits[0])))
23908         } else if (8 * sizeof(int) >= 2 * PyLong_SHIFT) {
23909             return (int) (((((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23910         }
23911     }
23912     break;
23913     case 3:
23914         if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23915             if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23916                 __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
23917             } else if (8 * sizeof(int) >= 3 * PyLong_SHIFT) {
23918                 return (int) (((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
PyLong_SHIFT) | (int)digits[0]));
23919             }
23920         }
23921     }
23922     break;
23923     case 4:
23924         if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
23925             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
                __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
23926             } else if (8 * sizeof(int) >= 4 * PyLong_SHIFT) {
23927                 return (int) (((((((((int)digits[3]) « PyLong_SHIFT) | (int)digits[2]) «
PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23928             }
23929         }
23930     }
23931     break;
23932 }
23933 #endif
23934 #if CYTHON_COMPILING_IN_CPYTHON
23935     if (unlikely(Py_SIZE(x) < 0)) {
23936         goto raise_neg_overflow;
23937     }
23938 #else
23939     {
23940         int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
23941         if (unlikely(result < 0))
23942             return (int) -1;
23943         if (unlikely(result == 1))
23944             goto raise_neg_overflow;
23945     }
23946 #endif
23947     if (sizeof(int) <= sizeof(unsigned long)) {
23948         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned long, PyLong_AsUnsignedLong(x))
23949     } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
23950         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
23951     }
23952 #endif
23953 } else {
23954     #if CYTHON_USE_PYLONG_INTERNALS
23955         const digit* digits = ((PyLongObject*)x)->ob_digit;
23956         switch (Py_SIZE(x)) {
23957             case 0: return (int) 0;
23958             case -1: __PYX_VERIFY_RETURN_INT(int, sdigit, (sdigit) ~((sdigit)digits[0]))
23959             case 1: __PYX_VERIFY_RETURN_INT(int, digit, +digits[0])
23960             case -2:
23961                 if (8 * sizeof(int) - 1 > 1 * PyLong_SHIFT) {
23962                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23963                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((unsigned long)digits[1]) «
PyLong_SHIFT) | (unsigned long)digits[0])))
23964                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23965                         return (int) (((int)-1)*((((int)digits[1]) « PyLong_SHIFT) |
(int)digits[0]));
23966                     }
23967                 }
23968                 break;
23969             case 2:
23970                 if (8 * sizeof(int) > 1 * PyLong_SHIFT) {
23971                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23972                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned long)digits[1])
« PyLong_SHIFT) | (unsigned long)digits[0])))
23973                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23974                         return (int) (((((((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23975                     }
23976                 }
23977                 break;
23978             case -3:
23979                 if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23980                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23981                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((((unsigned long)digits[2])
« PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))

```

```

23982         } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23983             return (int) (((int)-1)*((((int)digits[2]) « PyLong_SHIFT) |
(int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23984         }
23985     }
23986     break;
23987     case 3:
23988         if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23989             if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23990                 __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
23991             } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23992                 return (int) (((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
PyLong_SHIFT) | (int)digits[0]));
23993             }
23994         }
23995     break;
23996     case -4:
23997         if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23998             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
23999                 __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24000             } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
24001                 return (int) (((((((int)digits[3]) « PyLong_SHIFT) |
(int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
24002             }
24003         }
24004     break;
24005     case 4:
24006         if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
24007             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24008                 __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24009             } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
24010                 return (int) (((((((int)digits[3]) « PyLong_SHIFT) | (int)digits[2]) «
PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
24011             }
24012         }
24013     break;
24014 }
24015 #endif
24016     if (sizeof(int) <= sizeof(long)) {
24017         __PYX_VERIFY_RETURN_INT_EXC(int, long, PyLong_AsLong(x))
24018     #ifdef HAVE_LONG_LONG
24019     } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24020         __PYX_VERIFY_RETURN_INT_EXC(int, PY_LONG_LONG, PyLong_AsLongLong(x))
24021     #endif
24022 }
24023 }
24024 {
24025 #if CYTHON_COMPILING_IN_PYPY && !defined(_PyLong_AsByteArray)
24026     PyErr_SetString(PyExc_RuntimeError,
24027         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
numbers");
24028 #else
24029     int val;
24030     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24031     #if PY_MAJOR_VERSION < 3
24032         if (likely(v) && !PyLong_Check(v)) {
24033             PyObject *tmp = v;
24034             v = PyNumber_Long(tmp);
24035             Py_DECREF(tmp);
24036         }
24037     #endif
24038     if (likely(v)) {
24039         int one = 1; int is_little = (int)*(unsigned char *)&one;
24040         unsigned char *bytes = (unsigned char *)&val;
24041         int ret = _PyLong_AsByteArray((PyLongObject *)v,
bytes, sizeof(val),
is_little, !is_unsigned);
24042         Py_DECREF(v);
24043         if (likely(!ret))
24044             return val;
24045     }
24046     return (int) -1;
24047 }
24048 #endif
24049 }
24050 }
24051 else {
24052     int val;
24053     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24054     if (!tmp) return (int) -1;
24055     val = __Pyx_PyInt_As_int(tmp);
24056     Py_DECREF(tmp);
24057     return val;

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24058     }
24059 raise_overflow:
24060     PyErr_SetString(PyExc_OverflowError,
24061         "value too large to convert to int");
24062     return (int) -1;
24063 raise_neg_overflow:
24064     PyErr_SetString(PyExc_OverflowError,
24065         "can't convert negative value to int");
24066     return (int) -1;
24067 }
24068
24069 /* CIntToPy */
24070 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value) {
24071 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24072 #pragma GCC diagnostic push
24073 #pragma GCC diagnostic ignored "-Wconversion"
24074 #endif
24075     const int neg_one = (int) -1, const_zero = (int) 0;
24076 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24077 #pragma GCC diagnostic pop
24078 #endif
24079     const int is_unsigned = neg_one > const_zero;
24080     if (is_unsigned) {
24081         if (sizeof(int) < sizeof(long)) {
24082             return PyInt_FromLong((long) value);
24083         } else if (sizeof(int) <= sizeof(unsigned long)) {
24084             return PyLong_FromUnsignedLong((unsigned long) value);
24085 #ifdef HAVE_LONG_LONG
24086         } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
24087             return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24088 #endif
24089         }
24090     } else {
24091         if (sizeof(int) <= sizeof(long)) {
24092             return PyInt_FromLong((long) value);
24093 #ifdef HAVE_LONG_LONG
24094         } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24095             return PyLong_FromLongLong((PY_LONG_LONG) value);
24096 #endif
24097         }
24098     }
24099     {
24100         int one = 1; int little = (int)*(unsigned char *)&one;
24101         unsigned char *bytes = (unsigned char *)&value;
24102         return _PyLong_FromByteArray(bytes, sizeof(int),
24103             little, !is_unsigned);
24104     }
24105 }
24106
24107 /* CIntToPy */
24108 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value) {
24109 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24110 #pragma GCC diagnostic push
24111 #pragma GCC diagnostic ignored "-Wconversion"
24112 #endif
24113     const long neg_one = (long) -1, const_zero = (long) 0;
24114 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24115 #pragma GCC diagnostic pop
24116 #endif
24117     const int is_unsigned = neg_one > const_zero;
24118     if (is_unsigned) {
24119         if (sizeof(long) < sizeof(long)) {
24120             return PyInt_FromLong((long) value);
24121         } else if (sizeof(long) <= sizeof(unsigned long)) {
24122             return PyLong_FromUnsignedLong((unsigned long) value);
24123 #ifdef HAVE_LONG_LONG
24124         } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24125             return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24126 #endif
24127         }
24128     } else {
24129         if (sizeof(long) <= sizeof(long)) {
24130             return PyInt_FromLong((long) value);
24131 #ifdef HAVE_LONG_LONG
24132         } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24133             return PyLong_FromLongLong((PY_LONG_LONG) value);
24134 #endif
24135         }
24136     }
24137     {
24138         int one = 1; int little = (int)*(unsigned char *)&one;
24139         unsigned char *bytes = (unsigned char *)&value;
24140         return _PyLong_FromByteArray(bytes, sizeof(long),
24141             little, !is_unsigned);
24142     }
24143 }
24144

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```

24145 /* CIntFromPy */
24146 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *x) {
24147 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24148 #pragma GCC diagnostic push
24149 #pragma GCC diagnostic ignored "-Wconversion"
24150 #endif
24151     const long neg_one = (long) -1, const_zero = (long) 0;
24152 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24153 #pragma GCC diagnostic pop
24154 #endif
24155     const int is_unsigned = neg_one > const_zero;
24156 #if PY_MAJOR_VERSION < 3
24157     if (likely(PyInt_Check(x))) {
24158         if (sizeof(long) < sizeof(long)) {
24159             __PYX_VERIFY_RETURN_INT(long, long, PyInt_AS_LONG(x))
24160         } else {
24161             long val = PyInt_AS_LONG(x);
24162             if (is_unsigned && unlikely(val < 0)) {
24163                 goto raise_neg_overflow;
24164             }
24165             return (long) val;
24166         }
24167     } else
24168 #endif
24169     if (likely(PyLong_Check(x))) {
24170         if (is_unsigned) {
24171             #if CYTHON_USE_PYLONG_INTERNALS
24172                 const digit* digits = ((PyLongObject*)x)->ob_digit;
24173                 switch (Py_SIZE(x)) {
24174                     case 0: return (long) 0;
24175                     case 1: __PYX_VERIFY_RETURN_INT(long, digit, digits[0])
24176                     case 2:
24177                         if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24178                             if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24179                                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((unsigned long)digits[1])
24180                                     « PyLong_SHIFT) | (unsigned long)digits[0]))
24181                             } else if (8 * sizeof(long) >= 2 * PyLong_SHIFT) {
24182                                 return (long) (((((long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24183                             }
24184                             break;
24185                         case 3:
24186                             if (8 * sizeof(long) > 2 * PyLong_SHIFT) {
24187                                 if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24188                                     __PYX_VERIFY_RETURN_INT(long, unsigned long, ((((((unsigned
24189                                     long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
24190                                     long)digits[0])))
24191                                 } else if (8 * sizeof(long) >= 3 * PyLong_SHIFT) {
24192                                     return (long) (((((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
24193                                     PyLong_SHIFT) | (long)digits[0]));
24194                                 }
24195                                 break;
24196                             case 4:
24197                                 if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24198                                     if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24199                                         __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((((unsigned
24200                                         long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
24201                                         long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24202                                     } else if (8 * sizeof(long) >= 4 * PyLong_SHIFT) {
24203                                         return (long) ((((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
24204                                         « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24205                                     }
24206                                 }
24207                                 break;
24208                             }
24209                         #endif
24210                     #if CYTHON_COMPILING_IN_CPYTHON
24211                         if (unlikely(Py_SIZE(x) < 0)) {
24212                             goto raise_neg_overflow;
24213                         }
24214                     #else
24215                         {
24216                             int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
24217                             if (unlikely(result < 0))
24218                                 return (long) -1;
24219                             if (unlikely(result == 1))
24220                                 goto raise_neg_overflow;
24221                         }
24222                     #endif
24223                 if (sizeof(long) <= sizeof(unsigned long)) {
24224                     __PYX_VERIFY_RETURN_INT_EXC(long, unsigned long, PyLong_AsUnsignedLong(x))
24225                 } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24226                     __PYX_VERIFY_RETURN_INT_EXC(long, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
24227                 }
24228             }
24229         }
24230     }

```

```

24225     } else {
24226 #if CYTHON_USE_PYLONG_INTERNALS
24227     const digit* digits = ((PyLongObject*)x)->ob_digit;
24228     switch (Py_SIZE(x)) {
24229     case 0: return (long) 0;
24230     case -1: __PYX_VERIFY_RETURN_INT(long, sdigit, (sdigit) -(sdigit)digits[0]))
24231     case 1: __PYX_VERIFY_RETURN_INT(long, digit, +digits[0])
24232     case -2:
24233         if (8 * sizeof(long) - 1 > 1 * PyLong_SHIFT) {
24234             if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24235                 __PYX_VERIFY_RETURN_INT(long, long, -(long) (((unsigned long)digits[1])
24236 « PyLong_SHIFT) | (unsigned long)digits[0])))
24237             } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24238                 return (long) (((long)-1)*((((long)digits[1]) « PyLong_SHIFT) |
24239 (long)digits[0]]));
24240             }
24241         }
24242         break;
24243     case 2:
24244         if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24245             if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24246                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((unsigned long)digits[1])
24247 « PyLong_SHIFT) | (unsigned long)digits[0])))
24248             } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24249                 return (long) (((((long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24250             }
24251         }
24252         break;
24253     case -3:
24254         if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24255             if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24256                 __PYX_VERIFY_RETURN_INT(long, long, -(long) ((((((unsigned
24257 long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
24258 long)digits[0]))))
24259             } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24260                 return (long) (((long)-1)*((((((long)digits[2]) « PyLong_SHIFT) |
24261 (long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24262             }
24263         }
24264         break;
24265     case 3:
24266         if (8 * sizeof(long) > 2 * PyLong_SHIFT) {
24267             if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24268                 __PYX_VERIFY_RETURN_INT(long, unsigned long, ((((((unsigned
24269 long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
24270 long)digits[0]))))
24271             } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24272                 return (long) (((((((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
24273 PyLong_SHIFT) | (long)digits[0])));
24274             }
24275         }
24276         break;
24277     case -4:
24278         if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24279             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24280                 __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((((((unsigned
24281 long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
24282 long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))))
24283             } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24284                 return (long) (((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
24285 « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24286             }
24287         }
24288         break;
24289     case 4:
24290         if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24291             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24292                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((((unsigned
24293 long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
24294 long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))))
24295             } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24296                 return (long) (((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
24297 « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24298             }
24299         }
24300         break;
24301     }
24302 }
24303 #endif
24304 if (sizeof(long) <= sizeof(long)) {
24305     __PYX_VERIFY_RETURN_INT_EXC(long, long, PyLong_AsLong(x))
24306 #ifdef HAVE_LONG_LONG
24307 } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24308     __PYX_VERIFY_RETURN_INT_EXC(long, PY_LONG_LONG, PyLong_AsLongLong(x))
24309 #endif
24310 }
24311 }
24312 {

```



```

24297 #if CYTHON_COMPILING_IN_PYPY && !defined(_PyLong_AsByteArray)
24298     PyErr_SetString(PyExc_RuntimeError,
24299         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
        numbers");
24300 #else
24301     long val;
24302     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24303     #if PY_MAJOR_VERSION < 3
24304         if (likely(v) && !PyLong_Check(v)) {
24305             PyObject *tmp = v;
24306             v = PyNumber_Long(tmp);
24307             Py_DECREF(tmp);
24308         }
24309     #endif
24310     if (likely(v)) {
24311         int one = 1; int is_little = (int)*(unsigned char *)&one;
24312         unsigned char *bytes = (unsigned char *)&val;
24313         int ret = _PyLong_AsByteArray((PyLongObject *)v,
24314                                     bytes, sizeof(val),
24315                                     is_little, !is_unsigned);
24316         Py_DECREF(v);
24317         if (likely(!ret))
24318             return val;
24319     }
24320 #endif
24321     return (long) -1;
24322 }
24323 } else {
24324     long val;
24325     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24326     if (!tmp) return (long) -1;
24327     val = __Pyx_PyInt_As_long(tmp);
24328     Py_DECREF(tmp);
24329     return val;
24330 }
24331 raise_overflow:
24332     PyErr_SetString(PyExc_OverflowError,
24333         "value too large to convert to long");
24334     return (long) -1;
24335 raise_neg_overflow:
24336     PyErr_SetString(PyExc_OverflowError,
24337         "can't convert negative value to long");
24338     return (long) -1;
24339 }
24340
24341 /* FastTypeChecks */
24342 #if CYTHON_COMPILING_IN_CPYTHON
24343 static int __Pyx_InBases(PyTypeObject *a, PyTypeObject *b) {
24344     while (a) {
24345         a = a->tp_base;
24346         if (a == b)
24347             return 1;
24348     }
24349     return b == &PyBaseObject_Type;
24350 }
24351 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b) {
24352     PyObject *mro;
24353     if (a == b) return 1;
24354     mro = a->tp_mro;
24355     if (likely(mro)) {
24356         Py_ssize_t i, n;
24357         n = PyTuple_GET_SIZE(mro);
24358         for (i = 0; i < n; i++) {
24359             if (PyTuple_GET_ITEM(mro, i) == (PyObject *)b)
24360                 return 1;
24361         }
24362         return 0;
24363     }
24364     return __Pyx_InBases(a, b);
24365 }
24366 #if PY_MAJOR_VERSION == 2
24367 static int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1, PyObject*
exc_type2) {
24368     PyObject *exception, *value, *tb;
24369     int res;
24370     __Pyx_PyThreadState_declare
24371     __Pyx_PyThreadState_assign
24372     __Pyx_ErrFetch(&exception, &value, &tb);
24373     res = exc_type1 ? PyObject_IsSubclass(err, exc_type1) : 0;
24374     if (unlikely(res == -1)) {
24375         PyErr_WriteUnraisable(err);
24376         res = 0;
24377     }
24378     if (!res) {
24379         res = PyObject_IsSubclass(err, exc_type2);
24380         if (unlikely(res == -1)) {
24381             PyErr_WriteUnraisable(err);

```

```

24382         res = 0;
24383     }
24384 }
24385 __Pyx_ErrRestore(exception, value, tb);
24386 return res;
24387 }
24388 #else
24389 static CYTHON_INLINE int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1,
24390     PyObject *exc_type2) {
24391     int res = exc_type1 ? __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type1) : 0;
24392     if (!res) {
24393         res = __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type2);
24394     }
24395     return res;
24396 }
24397 #endif
24398 static int __Pyx_PyErr_GivenExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
24399     Py_ssize_t i, n;
24400     assert(PyExceptionClass_Check(exc_type));
24401     n = PyTuple_GET_SIZE(tuple);
24402     #if PY_MAJOR_VERSION >= 3
24403     for (i=0; i<n; i++) {
24404         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
24405     }
24406     #endif
24407     for (i=0; i<n; i++) {
24408         PyObject *t = PyTuple_GET_ITEM(tuple, i);
24409         #if PY_MAJOR_VERSION < 3
24410         if (likely(exc_type == t)) return 1;
24411         #endif
24412         if (likely(PyExceptionClass_Check(t))) {
24413             if (__Pyx_inner_PyErr_GivenExceptionMatches2(exc_type, NULL, t)) return 1;
24414         } else {
24415         }
24416     }
24417     return 0;
24418 }
24419 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject* exc_type) {
24420     if (likely(err == exc_type)) return 1;
24421     if (likely(PyExceptionClass_Check(err))) {
24422         if (likely(PyExceptionClass_Check(exc_type))) {
24423             return __Pyx_inner_PyErr_GivenExceptionMatches2(err, NULL, exc_type);
24424         } else if (likely(PyTuple_Check(exc_type))) {
24425             return __Pyx_PyErr_GivenExceptionMatchesTuple(err, exc_type);
24426         } else {
24427         }
24428     }
24429     return PyErr_GivenExceptionMatches(err, exc_type);
24430 }
24431 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *exc_type1,
24432     PyObject *exc_type2) {
24433     assert(PyExceptionClass_Check(exc_type1));
24434     assert(PyExceptionClass_Check(exc_type2));
24435     if (likely(err == exc_type1 || err == exc_type2)) return 1;
24436     if (likely(PyExceptionClass_Check(err))) {
24437         return __Pyx_inner_PyErr_GivenExceptionMatches2(err, exc_type1, exc_type2);
24438     }
24439     return (PyErr_GivenExceptionMatches(err, exc_type1) || PyErr_GivenExceptionMatches(err,
24440         exc_type2));
24441 }
24442 #endif
24443 /* FetchCommonType */
24444 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type) {
24445     PyObject* fake_module;
24446     PyTypeObject* cached_type = NULL;
24447     fake_module = PyImport_AddModule((char*) "_cython_" CYTHON_ABI);
24448     if (!fake_module) return NULL;
24449     Py_INCREF(fake_module);
24450     cached_type = (PyTypeObject*) PyObject_GetAttrString(fake_module, type->tp_name);
24451     if (cached_type) {
24452         if (!PyType_Check((PyObject*)cached_type)) {
24453             PyErr_Format(PyExc_TypeError,
24454                 "Shared Cython type %.200s is not a type object",
24455                 type->tp_name);
24456             goto bad;
24457         }
24458         if (cached_type->tp_basicsize != type->tp_basicsize) {
24459             PyErr_Format(PyExc_TypeError,
24460                 "Shared Cython type %.200s has the wrong size, try recompiling",
24461                 type->tp_name);
24462             goto bad;
24463         }
24464     } else {
24465         if (!PyErr_ExceptionMatches(PyExc_AttributeError)) goto bad;
24466         PyErr_Clear();
24467         if (PyType_Ready(type) < 0) goto bad;

```

```

24466         if (PyObject_SetAttrString(fake_module, type->tp_name, (PyObject*) type) < 0)
24467             goto bad;
24468         Py_INCREF(type);
24469         cached_type = type;
24470     }
24471 done:
24472     Py_DECREF(fake_module);
24473     return cached_type;
24474 bad:
24475     Py_XDECREF(cached_type);
24476     cached_type = NULL;
24477     goto done;
24478 }
24479
24480 /* PyObjectGetMethod */
24481 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method) {
24482     PyObject *attr;
24483     #if CYTHON_UNPACK_METHODS && CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_PYTYPE_LOOKUP
24484     PyTypeObject *tp = Py_TYPE(obj);
24485     PyObject *descr;
24486     descrgetfunc f = NULL;
24487     PyObject **dictptr, *dict;
24488     int meth_found = 0;
24489     assert (*method == NULL);
24490     if (unlikely(tp->tp_getattro != PyObject_GenericGetAttr)) {
24491         attr = __Pyx_PyObject_GetAttrStr(obj, name);
24492         goto try_unpack;
24493     }
24494     if (unlikely(tp->tp_dict == NULL) && unlikely(PyType_Ready(tp) < 0)) {
24495         return 0;
24496     }
24497     descr = _PyType_Lookup(tp, name);
24498     if (likely(descr != NULL)) {
24499         Py_INCREF(descr);
24500     #if PY_MAJOR_VERSION >= 3
24501         #ifdef __Pyx_CyFunction_USED
24502         if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type) ||
24503             __Pyx_CyFunction_Check(descr)))
24504             #else
24505             if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type)))
24506             #endif
24507         #else
24508         #ifdef __Pyx_CyFunction_USED
24509         if (likely(PyFunction_Check(descr) || __Pyx_CyFunction_Check(descr)))
24510         #else
24511         if (likely(PyFunction_Check(descr)))
24512         #endif
24513         {
24514             meth_found = 1;
24515         } else {
24516             f = Py_TYPE(descr)->tp_descr_get;
24517             if (f != NULL && PyDescr_IsData(descr)) {
24518                 attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24519                 Py_DECREF(descr);
24520                 goto try_unpack;
24521             }
24522         }
24523     }
24524     dictptr = _PyObject_GetDictPtr(obj);
24525     if (dictptr != NULL && (dict = *dictptr) != NULL) {
24526         Py_INCREF(dict);
24527         attr = __Pyx_PyDict_GetItemStr(dict, name);
24528         if (attr != NULL) {
24529             Py_INCREF(attr);
24530             Py_DECREF(dict);
24531             Py_XDECREF(descr);
24532             goto try_unpack;
24533         }
24534         Py_DECREF(dict);
24535     }
24536     if (meth_found) {
24537         *method = descr;
24538         return 1;
24539     }
24540     if (f != NULL) {
24541         attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24542         Py_DECREF(descr);
24543         goto try_unpack;
24544     }
24545     if (descr != NULL) {
24546         *method = descr;
24547         return 0;
24548     }
24549     PyErr_Format(PyExc_AttributeError,
24550     #if PY_MAJOR_VERSION >= 3
24551         "'%.50s' object has no attribute '%U'",

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```

24552         tp->tp_name, name);
24553 #else
24554         "%50s' object has no attribute '%.400s'",
24555         tp->tp_name, PyString_AS_STRING(name));
24556 #endif
24557     return 0;
24558 #else
24559     attr = __Pyx_PyObject_GetAttrStr(obj, name);
24560     goto try_unpack;
24561 #endif
24562 try_unpack:
24563 #if CYTHON_UNPACK_METHODS
24564     if (likely(attr) && PyMethod_Check(attr) && likely(PyMethod_GET_SELF(attr) == obj)) {
24565         PyObject *function = PyMethod_GET_FUNCTION(attr);
24566         Py_INCREF(function);
24567         Py_DECREF(attr);
24568         *method = function;
24569         return 1;
24570     }
24571 #endif
24572     *method = attr;
24573     return 0;
24574 }
24575
24576 /* PyObjectCallMethod1 */
24577 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* method, PyObject* arg) {
24578     PyObject *result = __Pyx_PyObject_CallOneArg(method, arg);
24579     Py_DECREF(method);
24580     return result;
24581 }
24582 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg) {
24583     PyObject *method = NULL, *result;
24584     int is_method = __Pyx_PyObject_GetMethod(obj, method_name, &method);
24585     if (likely(is_method)) {
24586         result = __Pyx_PyObject_Call2Args(method, obj, arg);
24587         Py_DECREF(method);
24588         return result;
24589     }
24590     if (unlikely(!method)) return NULL;
24591     return __Pyx_PyObject_CallMethod1(method, arg);
24592 }
24593
24594 /* CoroutineBase */
24595 #include <structmember.h>
24596 #include <frameobject.h>
24597 #define __Pyx_Coroutine_Undelegate(gen) Py_CLEAR((gen)->yieldfrom)
24598 static int __Pyx_PyGen_FetchStopIterationValue(CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject
**pvalue) {
24599     PyObject *et, *ev, *tb;
24600     PyObject *value = NULL;
24601     __Pyx_ErrFetch(&et, &ev, &tb);
24602     if (!et) {
24603         Py_XDECREF(tb);
24604         Py_XDECREF(ev);
24605         Py_INCREF(Py_None);
24606         *pvalue = Py_None;
24607         return 0;
24608     }
24609     if (likely(et == PyExc_StopIteration)) {
24610         if (!ev) {
24611             Py_INCREF(Py_None);
24612             value = Py_None;
24613         }
24614 #if PY_VERSION_HEX >= 0x030300A0
24615         else if (Py_TYPE(ev) == (PyTypeObject*)PyExc_StopIteration) {
24616             value = ((PyStopIterationObject *)ev)->value;
24617             Py_INCREF(value);
24618             Py_DECREF(ev);
24619         }
24620 #endif
24621         else if (unlikely(PyTuple_Check(ev))) {
24622             if (PyTuple_GET_SIZE(ev) >= 1) {
24623 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
24624                 value = PyTuple_GET_ITEM(ev, 0);
24625                 Py_INCREF(value);
24626 #else
24627                 value = PySequence_ITEM(ev, 0);
24628 #endif
24629             } else {
24630                 Py_INCREF(Py_None);
24631                 value = Py_None;
24632             }
24633             Py_DECREF(ev);
24634         }
24635         else if (!__Pyx_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration)) {
24636             value = ev;
24637         }

```

```

24638         if (likely(value)) {
24639             Py_XDECREF(tb);
24640             Py_DECREF(et);
24641             *pvalue = value;
24642             return 0;
24643         }
24644     } else if (!__Pyx_PyErr_GivenExceptionMatches(et, PyExc_StopIteration)) {
24645         __Pyx_ErrRestore(et, ev, tb);
24646         return -1;
24647     }
24648     PyErr_NormalizeException(&et, &ev, &tb);
24649     if (unlikely(!PyObject_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration))) {
24650         __Pyx_ErrRestore(et, ev, tb);
24651         return -1;
24652     }
24653     Py_XDECREF(tb);
24654     Py_DECREF(et);
24655     #if PY_VERSION_HEX >= 0x030300A0
24656     value = ((PyStopIterationObject *)ev)->value;
24657     Py_INCREF(value);
24658     Py_DECREF(ev);
24659     #else
24660     {
24661         PyObject* args = __Pyx_PyObject_GetAttrStr(ev, __pyx_n_s_args);
24662         Py_DECREF(ev);
24663         if (likely(args)) {
24664             value = PySequence_GetItem(args, 0);
24665             Py_DECREF(args);
24666         }
24667         if (unlikely(!value)) {
24668             __Pyx_ErrRestore(NULL, NULL, NULL);
24669             Py_INCREF(Py_None);
24670             value = Py_None;
24671         }
24672     }
24673     #endif
24674     *pvalue = value;
24675     return 0;
24676 }
24677 static CYTHON_INLINE
24678 void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *exc_state) {
24679     PyObject *t, *v, *tb;
24680     t = exc_state->exc_type;
24681     v = exc_state->exc_value;
24682     tb = exc_state->exc_traceback;
24683     exc_state->exc_type = NULL;
24684     exc_state->exc_value = NULL;
24685     exc_state->exc_traceback = NULL;
24686     Py_XDECREF(t);
24687     Py_XDECREF(v);
24688     Py_XDECREF(tb);
24689 }
24690 #define __Pyx_Coroutine_AlreadyRunningError(gen)  (__Pyx__Coroutine_AlreadyRunningError(gen),
(PyObject*)NULL)
24691 static void __Pyx__Coroutine_AlreadyRunningError(CYTHON_UNUSED __pyx_CoroutineObject *gen) {
24692     const char *msg;
24693     if ((0)) {
24694         #ifdef __Pyx_Coroutine_USED
24695     } else if (__Pyx_Coroutine_Check((PyObject*)gen)) {
24696         msg = "coroutine already executing";
24697         #endif
24698         #ifdef __Pyx_AsyncGen_USED
24699     } else if (__Pyx_AsyncGen_CheckExact((PyObject*)gen)) {
24700         msg = "async generator already executing";
24701         #endif
24702     } else {
24703         msg = "generator already executing";
24704     }
24705     PyErr_SetString(PyExc_ValueError, msg);
24706 }
24707 #define __Pyx_Coroutine_NotStartedError(gen)  (__Pyx__Coroutine_NotStartedError(gen), (PyObject*)NULL)
24708 static void __Pyx__Coroutine_NotStartedError(CYTHON_UNUSED PyObject *gen) {
24709     const char *msg;
24710     if ((0)) {
24711         #ifdef __Pyx_Coroutine_USED
24712     } else if (__Pyx_Coroutine_Check(gen)) {
24713         msg = "can't send non-None value to a just-started coroutine";
24714         #endif
24715         #ifdef __Pyx_AsyncGen_USED
24716     } else if (__Pyx_AsyncGen_CheckExact(gen)) {
24717         msg = "can't send non-None value to a just-started async generator";
24718         #endif
24719     } else {
24720         msg = "can't send non-None value to a just-started generator";
24721     }
24722     PyErr_SetString(PyExc_TypeError, msg);
24723 }

```

```

24724 #define __Pyx_Coroutine_AlreadyTerminatedError(gen, value, closing)
24725     (__Pyx_Coroutine_AlreadyTerminatedError(gen, value, closing), (PyObject*)NULL)
24726 static void __Pyx_Coroutine_AlreadyTerminatedError(CYTHON_UNUSED PyObject *gen, PyObject *value,
24727     CYTHON_UNUSED int closing) {
24728     #ifdef __Pyx_Coroutine_USED
24729     if (!closing && __Pyx_Coroutine_Check(gen)) {
24730         PyErr_SetString(PyExc_RuntimeError, "cannot reuse already awaited coroutine");
24731     } else
24732     #endif
24733     if (value) {
24734         #ifdef __Pyx_AsyncGen_USED
24735         if (__Pyx_AsyncGen_CheckExact(gen))
24736             PyErr_SetNone(__Pyx_PyExc_StopAsyncIteration);
24737         else
24738         #endif
24739         PyErr_SetNone(PyExc_StopIteration);
24740     }
24741 }
24742 static
24743 PyObject *__Pyx_Coroutine_SendEx(__pyx_CoroutineObject *self, PyObject *value, int closing) {
24744     __Pyx_PyThreadState_declare
24745     PyThreadState *tstate;
24746     __Pyx_ExcInfoStruct *exc_state;
24747     PyObject *retval;
24748     assert(!self->is_running);
24749     if (unlikely(self->resume_label == 0)) {
24750         if (unlikely(value && value != Py_None)) {
24751             return __Pyx_Coroutine_NotStartedError((PyObject*)self);
24752         }
24753     }
24754     if (unlikely(self->resume_label == -1)) {
24755         return __Pyx_Coroutine_AlreadyTerminatedError((PyObject*)self, value, closing);
24756     }
24757     #if CYTHON_FAST_THREAD_STATE
24758     __Pyx_PyThreadState_assign
24759     tstate = __pyx_tstate;
24760     #else
24761     tstate = __Pyx_PyThreadState_Current;
24762     #endif
24763     exc_state = &self->gi_exc_state;
24764     if (exc_state->exc_type) {
24765         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON
24766         #else
24767         if (exc_state->exc_traceback) {
24768             PyTracebackObject *tb = (PyTracebackObject *) exc_state->exc_traceback;
24769             PyFrameObject *f = tb->tb_frame;
24770             assert(f->f_back == NULL);
24771             #if PY_VERSION_HEX >= 0x030B00A1
24772             f->f_back = PyThreadState_GetFrame(tstate);
24773             #else
24774             Py_XINCREF(tstate->frame);
24775             f->f_back = tstate->frame;
24776             #endif
24777         }
24778         #endif
24779     }
24780     #if CYTHON_USE_EXC_INFO_STACK
24781     exc_state->previous_item = tstate->exc_info;
24782     tstate->exc_info = exc_state;
24783     #else
24784     if (exc_state->exc_type) {
24785         __Pyx_ExceptionSwap(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24786     } else {
24787         __Pyx_Coroutine_ExceptionClear(exc_state);
24788         __Pyx_ExceptionSave(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24789     }
24790     #endif
24791     self->is_running = 1;
24792     retval = self->body((PyObject *) self, tstate, value);
24793     self->is_running = 0;
24794     #if CYTHON_USE_EXC_INFO_STACK
24795     exc_state = &self->gi_exc_state;
24796     tstate->exc_info = exc_state->previous_item;
24797     exc_state->previous_item = NULL;
24798     __Pyx_Coroutine_ResetFrameBackpointer(exc_state);
24799     #endif
24800     return retval;
24801 }
24802 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state) {
24803     PyObject *exc_tb = exc_state->exc_traceback;
24804     if (likely(exc_tb)) {
24805         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON
24806         #else
24807         PyTracebackObject *tb = (PyTracebackObject *) exc_tb;
24808         PyFrameObject *f = tb->tb_frame;
24809         Py_CLEAR(f->f_back);
24810         #endif
24811     }
24812 }

```

```

24809     }
24810 }
24811 static CYTHON_INLINE
24812 PyObject *__Pyx_Coroutine_MethodReturn(CYTHON_UNUSED PyObject* gen, PyObject *retval) {
24813     if (unlikely(!retval)) {
24814         __Pyx_PyThreadState_declare
24815         __Pyx_PyThreadState_assign
24816         if (!__Pyx_PyErr_Occurred()) {
24817             PyObject *exc = PyExc_StopIteration;
24818             #ifdef __Pyx_AsyncGen_USED
24819             if (__Pyx_AsyncGen_CheckExact(gen))
24820                 exc = __Pyx_PyExc_StopAsyncIteration;
24821             #endif
24822             __Pyx_PyErr_SetNone(exc);
24823         }
24824     }
24825     return retval;
24826 }
24827 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
    PY_VERSION_HEX >= 0x030600B3)
24828 static CYTHON_INLINE
24829 PyObject *__Pyx_PyGen_Send(PyGenObject *gen, PyObject *arg) {
24830     #if PY_VERSION_HEX <= 0x030A00A1
24831         return _PyGen_Send(gen, arg);
24832     #else
24833         PyObject *result;
24834         if (PyIter_Send((PyObject*)gen, arg ? arg : Py_None, &result) == PYGEN_RETURN) {
24835             if (PyAsyncGen_CheckExact(gen)) {
24836                 assert(result == Py_None);
24837                 PyErr_SetNone(PyExc_StopAsyncIteration);
24838             }
24839             else if (result == Py_None) {
24840                 PyErr_SetNone(PyExc_StopIteration);
24841             }
24842             else {
24843                 _PyGen_SetStopIterationValue(result);
24844             }
24845             Py_CLEAR(result);
24846         }
24847         return result;
24848     #endif
24849 }
24850 #endif
24851 static CYTHON_INLINE
24852 PyObject *__Pyx_Coroutine_FinishDelegation(__pyx_CoroutineObject *gen) {
24853     PyObject *ret;
24854     PyObject *val = NULL;
24855     __Pyx_Coroutine_Undelegate(gen);
24856     __Pyx_PyGen__FetchStopIterationValue(__Pyx_PyThreadState_Current, &val);
24857     ret = __Pyx_Coroutine_SendEx(gen, val, 0);
24858     Py_XDECREF(val);
24859     return ret;
24860 }
24861 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value) {
24862     PyObject *retval;
24863     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24864     PyObject *yf = gen->yieldfrom;
24865     if (unlikely(gen->is_running))
24866         return __Pyx_Coroutine_AlreadyRunningError(gen);
24867     if (yf) {
24868         PyObject *ret;
24869         gen->is_running = 1;
24870         #ifdef __Pyx_Generator_USED
24871         if (__Pyx_Generator_CheckExact(yf)) {
24872             ret = __Pyx_Coroutine_Send(yf, value);
24873         } else
24874         #endif
24875         #ifdef __Pyx_Coroutine_USED
24876         if (__Pyx_Coroutine_Check(yf)) {
24877             ret = __Pyx_Coroutine_Send(yf, value);
24878         } else
24879         #endif
24880         #ifdef __Pyx_AsyncGen_USED
24881         if (__pyx_PyAsyncGenASend_CheckExact(yf)) {
24882             ret = __Pyx_async_gen_asend_send(yf, value);
24883         } else
24884         #endif
24885         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
    PY_VERSION_HEX >= 0x030600B3)
24886         if (PyGen_CheckExact(yf)) {
24887             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);
24888         } else
24889         #endif
24890         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03050000 && defined(PyCoro_CheckExact)
    && (defined(__linux__) || PY_VERSION_HEX >= 0x030600B3)
24891         if (PyCoro_CheckExact(yf)) {
24892             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);

```

```

24893         } else
24894         #endif
24895         {
24896             if (value == Py_None)
24897                 ret = Py_TYPE(yf)->tp_iternext(yf);
24898             else
24899                 ret = __Pyx_PyObject_CallMethod1(yf, __pyx_n_s_send, value);
24900         }
24901         gen->is_running = 0;
24902         if (likely(ret)) {
24903             return ret;
24904         }
24905         retval = __Pyx_Coroutine_FinishDelegation(gen);
24906     } else {
24907         retval = __Pyx_Coroutine_SendEx(gen, value, 0);
24908     }
24909     return __Pyx_Coroutine_MethodReturn(self, retval);
24910 }
24911 static int __Pyx_Coroutine_CloseIter(__pyx_CoroutineObject *gen, PyObject *yf) {
24912     PyObject *retval = NULL;
24913     int err = 0;
24914     #ifdef __Pyx_Generator_USED
24915     if (__Pyx_Generator_CheckExact(yf)) {
24916         retval = __Pyx_Coroutine_Close(yf);
24917         if (!retval)
24918             return -1;
24919     } else
24920     #endif
24921     #ifdef __Pyx_Coroutine_USED
24922     if (__Pyx_Coroutine_Check(yf)) {
24923         retval = __Pyx_Coroutine_Close(yf);
24924         if (!retval)
24925             return -1;
24926     } else
24927     if (__Pyx_CoroutineAwait_CheckExact(yf)) {
24928         retval = __Pyx_CoroutineAwait_Close((__pyx_CoroutineAwaitObject*)yf, NULL);
24929         if (!retval)
24930             return -1;
24931     } else
24932     #endif
24933     #ifdef __Pyx_AsyncGen_USED
24934     if (__pyx_PyAsyncGenASend_CheckExact(yf)) {
24935         retval = __Pyx_async_gen_asend_close(yf, NULL);
24936     } else
24937     if (__pyx_PyAsyncGenAThrow_CheckExact(yf)) {
24938         retval = __Pyx_async_gen_athrow_close(yf, NULL);
24939     } else
24940     #endif
24941     {
24942         PyObject *meth;
24943         gen->is_running = 1;
24944         meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_close);
24945         if (unlikely(!meth)) {
24946             if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
24947                 PyErr_WriteUnraisable(yf);
24948             }
24949             PyErr_Clear();
24950         } else {
24951             retval = PyObject_CallFunction(meth, NULL);
24952             Py_DECREF(meth);
24953             if (!retval)
24954                 err = -1;
24955         }
24956         gen->is_running = 0;
24957     }
24958     Py_XDECREF(retval);
24959     return err;
24960 }
24961 static PyObject *__Pyx_Generator_Next(PyObject *self) {
24962     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24963     PyObject *yf = gen->yieldfrom;
24964     if (unlikely(gen->is_running))
24965         return __Pyx_Coroutine_AlreadyRunningError(gen);
24966     if (yf) {
24967         PyObject *ret;
24968         gen->is_running = 1;
24969         #ifdef __Pyx_Generator_USED
24970         if (__Pyx_Generator_CheckExact(yf)) {
24971             ret = __Pyx_Generator_Next(yf);
24972         } else
24973         #endif
24974         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24975         if (PyGen_CheckExact(yf)) {
24976             ret = __Pyx_PyGen_Send((PyGenObject*)yf, NULL);
24977         } else
24978         #endif

```



```

24979     #ifdef __Pyx_Coroutine_USED
24980     if (__Pyx_Coroutine_Check(yf)) {
24981         ret = __Pyx_Coroutine_Send(yf, Py_None);
24982     } else
24983     #endif
24984         ret = Py_TYPE(yf)->tp_iternext(yf);
24985     gen->is_running = 0;
24986     if (likely(ret)) {
24987         return ret;
24988     }
24989     return __Pyx_Coroutine_FinishDelegation(gen);
24990 }
24991 return __Pyx_Coroutine_SendEx(gen, Py_None, 0);
24992 }
24993 static PyObject *__Pyx_Coroutine_Close_Method(PyObject *self, CYTHON_UNUSED PyObject *arg) {
24994     return __Pyx_Coroutine_Close(self);
24995 }
24996 static PyObject *__Pyx_Coroutine_Close(PyObject *self) {
24997     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
24998     PyObject *retval, *raised_exception;
24999     PyObject *yf = gen->yieldfrom;
25000     int err = 0;
25001     if (unlikely(gen->is_running))
25002         return __Pyx_Coroutine_AlreadyRunningError(gen);
25003     if (yf) {
25004         Py_INCREF(yf);
25005         err = __Pyx_Coroutine_CloseIter(gen, yf);
25006         __Pyx_Coroutine_Undelegate(gen);
25007         Py_DECREF(yf);
25008     }
25009     if (err == 0)
25010         PyErr_SetNone(PyExc_GeneratorExit);
25011     retval = __Pyx_Coroutine_SendEx(gen, NULL, 1);
25012     if (unlikely(retval)) {
25013         const char *msg;
25014         Py_DECREF(retval);
25015         if ((0)) {
25016             #ifdef __Pyx_Coroutine_USED
25017             } else if (__Pyx_Coroutine_Check(self)) {
25018                 msg = "coroutine ignored GeneratorExit";
25019             #endif
25020             #ifdef __Pyx_AsyncGen_USED
25021             } else if (__Pyx_AsyncGen_CheckExact(self)) {
25022                 #if PY_VERSION_HEX < 0x03060000
25023                 msg = "async generator ignored GeneratorExit - might require Python 3.6+ finalisation (PEP
25024                 525)";
25025             #else
25026                 msg = "async generator ignored GeneratorExit";
25027             #endif
25028             } else {
25029                 msg = "generator ignored GeneratorExit";
25030             }
25031             PyErr_SetString(PyExc_RuntimeError, msg);
25032             return NULL;
25033         }
25034         raised_exception = PyErr_Occurred();
25035         if (likely(!raised_exception || __Pyx_PyErr_GivenExceptionMatches2(raised_exception,
25036         PyExc_GeneratorExit, PyExc_StopIteration))) {
25037             if (raised_exception) PyErr_Clear();
25038             Py_INCREF(Py_None);
25039             return Py_None;
25040         }
25041         return NULL;
25042     }
25043     static PyObject *__Pyx__Coroutine_Throw(PyObject *self, PyObject *typ, PyObject *val, PyObject *tb,
25044     PyObject *args, int close_on_genexit) {
25045         __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25046         PyObject *yf = gen->yieldfrom;
25047         if (unlikely(gen->is_running))
25048             return __Pyx_Coroutine_AlreadyRunningError(gen);
25049         if (yf) {
25050             PyObject *ret;
25051             Py_INCREF(yf);
25052             if (__Pyx_PyErr_GivenExceptionMatches(typ, PyExc_GeneratorExit) && close_on_genexit) {
25053                 int err = __Pyx_Coroutine_CloseIter(gen, yf);
25054                 Py_DECREF(yf);
25055                 __Pyx_Coroutine_Undelegate(gen);
25056                 if (err < 0)
25057                     return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25058                 goto throw_here;
25059             }
25060             gen->is_running = 1;
25061             if (0)
25062                 #ifdef __Pyx_Generator_USED
25063                 || __Pyx_Generator_CheckExact(yf)
25064                 #endif

```

```

25064     #ifdef __Pyx_Coroutine_USED
25065     || __Pyx_Coroutine_Check(yf)
25066     #endif
25067     ) {
25068         ret = __Pyx_Coroutine_Throw(yf, typ, val, tb, args, close_on_genexit);
25069     #ifdef __Pyx_Coroutine_USED
25070     } else if (__Pyx_CoroutineAwait_CheckExact(yf)) {
25071         ret = __Pyx_Coroutine_Throw(((__pyx_CoroutineAwaitObject*)yf)->coroutine, typ, val, tb,
args, close_on_genexit);
25072     #endif
25073     } else {
25074         PyObject *meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_throw);
25075         if (unlikely(!meth)) {
25076             Py_DECREF(yf);
25077             if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
25078                 gen->is_running = 0;
25079                 return NULL;
25080             }
25081             PyErr_Clear();
25082             __Pyx_Coroutine_Undelegate(gen);
25083             gen->is_running = 0;
25084             goto throw_here;
25085         }
25086         if (likely(args)) {
25087             ret = PyObject_CallObject(meth, args);
25088         } else {
25089             ret = PyObject_CallFunctionObjArgs(meth, typ, val, tb, NULL);
25090         }
25091         Py_DECREF(meth);
25092     }
25093     gen->is_running = 0;
25094     Py_DECREF(yf);
25095     if (!ret) {
25096         ret = __Pyx_Coroutine_FinishDelegation(gen);
25097     }
25098     return __Pyx_Coroutine_MethodReturn(self, ret);
25099 }
25100 throw_here:
25101     __Pyx_Raise(typ, val, tb, NULL);
25102     return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25103 }
25104 static PyObject *__Pyx_Coroutine_Throw(PyObject *self, PyObject *args) {
25105     PyObject *typ;
25106     PyObject *val = NULL;
25107     PyObject *tb = NULL;
25108     if (!PyArg_UnpackTuple(args, (char *)"throw", 1, 3, &typ, &val, &tb))
25109         return NULL;
25110     return __Pyx_Coroutine_Throw(self, typ, val, tb, args, 1);
25111 }
25112 static CYTHON_INLINE int __Pyx_Coroutine_traverse_excstate(__Pyx_ExcInfoStruct *exc_state, visitproc
visit, void *arg) {
25113     Py_VISIT(exc_state->exc_type);
25114     Py_VISIT(exc_state->exc_value);
25115     Py_VISIT(exc_state->exc_traceback);
25116     return 0;
25117 }
25118 static int __Pyx_Coroutine_traverse(__pyx_CoroutineObject *gen, visitproc visit, void *arg) {
25119     Py_VISIT(gen->closure);
25120     Py_VISIT(gen->classobj);
25121     Py_VISIT(gen->yieldfrom);
25122     return __Pyx_Coroutine_traverse_excstate(&gen->gi_exc_state, visit, arg);
25123 }
25124 static int __Pyx_Coroutine_clear(PyObject *self) {
25125     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25126     Py_CLEAR(gen->closure);
25127     Py_CLEAR(gen->classobj);
25128     Py_CLEAR(gen->yieldfrom);
25129     __Pyx_Coroutine_ExceptionClear(&gen->gi_exc_state);
25130 #ifdef __Pyx_AsyncGen_USED
25131     if (__Pyx_AsyncGen_CheckExact(self)) {
25132         Py_CLEAR(((__pyx_PyAsyncGenObject*)gen)->ag_finalizer);
25133     }
25134 #endif
25135     Py_CLEAR(gen->gi_code);
25136     Py_CLEAR(gen->gi_frame);
25137     Py_CLEAR(gen->gi_name);
25138     Py_CLEAR(gen->gi_qualname);
25139     Py_CLEAR(gen->gi_modulename);
25140     return 0;
25141 }
25142 static void __Pyx_Coroutine_dealloc(PyObject *self) {
25143     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25144     PyObject_GC_UnTrack(gen);
25145     if (gen->gi_weakreflist != NULL)
25146         PyObject_ClearWeakRefs(self);
25147     if (gen->resume_label >= 0) {
25148         PyObject_GC_Track(self);

```

```

25149 #if PY_VERSION_HEX >= 0x030400a1 && CYTHON_USE_TP_FINALIZE
25150     if (PyObject_CallFinalizerFromDealloc(self))
25151 #else
25152     Py_TYPE(gen)->tp_del(self);
25153     if (Py_REFCNT(self) > 0)
25154 #endif
25155     {
25156         return;
25157     }
25158     PyObject_GC_UnTrack(self);
25159 }
25160 #ifdef __Pyx_AsyncGen_USED
25161     if (__Pyx_AsyncGen_CheckExact(self)) {
25162         /* We have to handle this case for asynchronous generators
25163            right here, because this code has to be between UNTRACK
25164            and GC_Del. */
25165         Py_CLEAR(((__pyx_PyAsyncGenObject*)self)->ag_finalizer);
25166     }
25167 #endif
25168     __Pyx_Coroutine_clear(self);
25169     PyObject_GC_Del(gen);
25170 }
25171 static void __Pyx_Coroutine_del(PyObject *self) {
25172     PyObject *error_type, *error_value, *error_traceback;
25173     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25174     __Pyx_PyThreadState_declare
25175     if (gen->resume_label < 0) {
25176         return;
25177     }
25178     #if !CYTHON_USE_TP_FINALIZE
25179     assert(self->ob_refcnt == 0);
25180     __Pyx_SET_REFCNT(self, 1);
25181 #endif
25182     __Pyx_PyThreadState_assign
25183     __Pyx_ErrFetch(&error_type, &error_value, &error_traceback);
25184     #ifdef __Pyx_AsyncGen_USED
25185     if (__Pyx_AsyncGen_CheckExact(self)) {
25186         __pyx_PyAsyncGenObject *agen = (__pyx_PyAsyncGenObject*)self;
25187         PyObject *finalizer = agen->ag_finalizer;
25188         if (finalizer && !agen->ag_closed) {
25189             PyObject *res = __Pyx_PyObject_CallOneArg(finalizer, self);
25190             if (unlikely(!res)) {
25191                 PyErr_WriteUnraisable(self);
25192             } else {
25193                 Py_DECREF(res);
25194             }
25195             __Pyx_ErrRestore(error_type, error_value, error_traceback);
25196             return;
25197         }
25198     }
25199 #endif
25200     if (unlikely(gen->resume_label == 0 && !error_value)) {
25201         #ifdef __Pyx_Coroutine_USED
25202         #ifdef __Pyx_Generator_USED
25203             if (!__Pyx_Generator_CheckExact(self))
25204         #endif
25205             {
25206                 PyObject_GC_UnTrack(self);
25207             }
25208             #if PY_MAJOR_VERSION >= 3 || defined(PyErr_WarnFormat)
25209             if (unlikely(PyErr_WarnFormat(PyExc_RuntimeWarning, 1, "coroutine '%.50s' was never awaited",
25210                 gen->gi_qualname) < 0))
25211                 PyErr_WriteUnraisable(self);
25212             #else
25213             {PyObject *msg;
25214             char *cmsg;
25215             #if CYTHON_COMPILING_IN_PYPY
25216             msg = NULL;
25217             cmsg = (char*) "coroutine was never awaited";
25218             #else
25219             char *cname;
25220             PyObject *qualname;
25221             qualname = gen->gi_qualname;
25222             cname = PyString_AS_STRING(qualname);
25223             msg = PyString_FromFormat("coroutine '%.50s' was never awaited", cname);
25224             if (unlikely(!msg)) {
25225                 PyErr_Clear();
25226                 cmsg = (char*) "coroutine was never awaited";
25227             } else {
25228                 cmsg = PyString_AS_STRING(msg);
25229             }
25230             #endif
25231             if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, cmsg, 1) < 0))
25232                 PyErr_WriteUnraisable(self);
25233             Py_XDECREF(msg);
25234         #endif
25235         PyObject_GC_Track(self);
25236     }

```

```

25235 #endif
25236     } else {
25237         PyObject *res = __Pyx_Coroutine_Close(self);
25238         if (unlikely(!res)) {
25239             if (PyErr_Occurred())
25240                 PyErr_WriteUnraisable(self);
25241         } else {
25242             Py_DECREF(res);
25243         }
25244     }
25245     __Pyx_ErrRestore(error_type, error_value, error_traceback);
25246 #if !CYTHON_USE_TP_FINALIZE
25247     assert(Py_REFCNT(self) > 0);
25248     if (--self->ob_refcnt == 0) {
25249         return;
25250     }
25251     {
25252         Py_ssize_t refcnt = Py_REFCNT(self);
25253         __Pyx_NewReference(self);
25254         __Pyx_SET_REFCNT(self, refcnt);
25255     }
25256 #if CYTHON_COMPILING_IN_CPYTHON
25257     assert(PyType_IS_GC(Py_TYPE(self)) &&
25258           _Py_AS_GC(self)->gc.gc_refs != _PyGC_REFS_UNTRACKED);
25259     _Py_DEC_REFTOTAL;
25260 #endif
25261 #ifdef COUNT_ALLOCS
25262     --Py_TYPE(self)->tp_frees;
25263     --Py_TYPE(self)->tp_allocs;
25264 #endif
25265 #endif
25266 }
25267 static PyObject *
25268 __Pyx_Coroutine_get_name(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25269 {
25270     PyObject *name = self->gi_name;
25271     if (unlikely(!name)) name = Py_None;
25272     Py_INCREF(name);
25273     return name;
25274 }
25275 static int
25276 __Pyx_Coroutine_set_name(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void *context)
25277 {
25278     PyObject *tmp;
25279 #if PY_MAJOR_VERSION >= 3
25280     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25281 #else
25282     if (unlikely(value == NULL || !PyString_Check(value)))
25283 #endif
25284     {
25285         PyErr_SetString(PyExc_TypeError,
25286             "__name__ must be set to a string object");
25287         return -1;
25288     }
25289     tmp = self->gi_name;
25290     Py_INCREF(value);
25291     self->gi_name = value;
25292     Py_XDECREF(tmp);
25293     return 0;
25294 }
25295 static PyObject *
25296 __Pyx_Coroutine_get_qualname(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25297 {
25298     PyObject *name = self->gi_qualname;
25299     if (unlikely(!name)) name = Py_None;
25300     Py_INCREF(name);
25301     return name;
25302 }
25303 static int
25304 __Pyx_Coroutine_set_qualname(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void
    *context)
25305 {
25306     PyObject *tmp;
25307 #if PY_MAJOR_VERSION >= 3
25308     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25309 #else
25310     if (unlikely(value == NULL || !PyString_Check(value)))
25311 #endif
25312     {
25313         PyErr_SetString(PyExc_TypeError,
25314             "__qualname__ must be set to a string object");
25315         return -1;
25316     }
25317     tmp = self->gi_qualname;
25318     Py_INCREF(value);
25319     self->gi_qualname = value;
25320     Py_XDECREF(tmp);

```

```

25321     return 0;
25322 }
25323 static PyObject *
25324 __Pyx_Coroutine_get_frame(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25325 {
25326     PyObject *frame = self->gi_frame;
25327     if (!frame) {
25328         if (unlikely(!self->gi_code)) {
25329             Py_RETURN_NONE;
25330         }
25331         frame = (PyObject *) PyFrame_New(
25332             PyThreadState_Get(), /*PyThreadState *tstate,*/
25333             (PyCodeObject*) self->gi_code, /*PyCodeObject *code,*/
25334             __pyx_d, /*PyObject *globals,*/
25335             0 /*PyObject *locals*/
25336         );
25337         if (unlikely(!frame))
25338             return NULL;
25339         self->gi_frame = frame;
25340     }
25341     Py_INCREF(frame);
25342     return frame;
25343 }
25344 static __pyx_CoroutineObject *__Pyx__Coroutine_New(
25345     PyTypeObject* type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
25346     PyObject *name, PyObject *qualname, PyObject *module_name) {
25347     __pyx_CoroutineObject *gen = PyObject_GC_New(__pyx_CoroutineObject, type);
25348     if (unlikely(!gen))
25349         return NULL;
25350     return __Pyx__Coroutine_NewInit(gen, body, code, closure, name, qualname, module_name);
25351 }
25352 static __pyx_CoroutineObject *__Pyx__Coroutine_NewInit(
25353     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
25354     *closure,
25355     PyObject *name, PyObject *qualname, PyObject *module_name) {
25356     gen->body = body;
25357     gen->closure = closure;
25358     Py_XINCREF(closure);
25359     gen->is_running = 0;
25360     gen->resume_label = 0;
25361     gen->classobj = NULL;
25362     gen->yieldfrom = NULL;
25363     gen->gi_exc_state.exc_type = NULL;
25364     gen->gi_exc_state.exc_value = NULL;
25365     gen->gi_exc_state.exc_traceback = NULL;
25366 #if CYTHON_USE_EXC_INFO_STACK
25367     gen->gi_exc_state.previous_item = NULL;
25368 #endif
25369     gen->gi_weakreflist = NULL;
25370     Py_XINCREF(qualname);
25371     gen->gi_qualname = qualname;
25372     Py_XINCREF(name);
25373     gen->gi_name = name;
25374     Py_XINCREF(module_name);
25375     gen->gi_module_name = module_name;
25376     Py_XINCREF(code);
25377     gen->gi_code = code;
25378     gen->gi_frame = NULL;
25379     PyObject_GC_Track(gen);
25380     return gen;
25381 }
25382 /* PatchModuleWithCoroutine */
25383 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code) {
25384     #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25385     int result;
25386     PyObject *globals, *result_obj;
25387     globals = PyDict_New(); if (unlikely(!globals)) goto ignore;
25388     result = PyDict_SetItemString(globals, "_cython_coroutine_type",
25389     #ifdef __Pyx_Coroutine_USED
25390         (PyObject*)__pyx_CoroutineType);
25391     #else
25392         Py_None);
25393     #endif
25394     if (unlikely(result < 0)) goto ignore;
25395     result = PyDict_SetItemString(globals, "_cython_generator_type",
25396     #ifdef __Pyx_Generator_USED
25397         (PyObject*)__pyx_GeneratorType);
25398     #else
25399         Py_None);
25400     #endif
25401     if (unlikely(result < 0)) goto ignore;
25402     if (unlikely(PyDict_SetItemString(globals, "_module", module) < 0)) goto ignore;
25403     if (unlikely(PyDict_SetItemString(globals, "__builtins__", __pyx_b) < 0)) goto ignore;
25404     result_obj = PyRun_String(py_code, Py_file_input, globals, globals);
25405     if (unlikely(!result_obj)) goto ignore;
25406     Py_DECREF(result_obj);

```

```

25407     Py_DECREF(globals);
25408     return module;
25409 ignore:
25410     Py_XDECREF(globals);
25411     PyErr_WriteUnraisable(module);
25412     if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, "Cython module failed to patch module with custom
type", 1) < 0)) {
25413         Py_DECREF(module);
25414         module = NULL;
25415     }
25416 #else
25417     py_code++;
25418 #endif
25419     return module;
25420 }
25421
25422 /* PatchGeneratorABC */
25423 #ifndef CYTHON_REGISTER_ABCS
25424 #define CYTHON_REGISTER_ABCS 1
25425 #endif
25426 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25427 static PyObject* __Pyx_patch_abc_module(PyObject *module);
25428 static PyObject* __Pyx_patch_abc_module(PyObject *module) {
25429     module = __Pyx_Coroutine_patch_module(
25430         module, ""
25431 "if _cython_generator_type is not None:\n"
25432 "    try: Generator = _module.Generator\n"
25433 "    except AttributeError: pass\n"
25434 "    else: Generator.register(_cython_generator_type)\n"
25435 "if _cython_coroutine_type is not None:\n"
25436 "    try: Coroutine = _module.Coroutine\n"
25437 "    except AttributeError: pass\n"
25438 "    else: Coroutine.register(_cython_coroutine_type)\n"
25439 );
25440     return module;
25441 }
25442 #endif
25443 static int __Pyx_patch_abc(void) {
25444     #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25445         static int abc_patched = 0;
25446         if (CYTHON_REGISTER_ABCS && !abc_patched) {
25447             PyObject *module;
25448             module = PyImport_ImportModule((PY_MAJOR_VERSION >= 3) ? "collections.abc" : "collections");
25449             if (!module) {
25450                 PyErr_WriteUnraisable(NULL);
25451                 if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning,
25452                     ((PY_MAJOR_VERSION >= 3) ?
25453                        "Cython module failed to register with collections.abc module" :
25454                        "Cython module failed to register with collections module"), 1) < 0)) {
25455                     return -1;
25456                 }
25457             } else {
25458                 module = __Pyx_patch_abc_module(module);
25459                 abc_patched = 1;
25460                 if (unlikely(!module))
25461                     return -1;
25462                 Py_DECREF(module);
25463             }
25464             module = PyImport_ImportModule("backports_abc");
25465             if (module) {
25466                 module = __Pyx_patch_abc_module(module);
25467                 Py_XDECREF(module);
25468             }
25469             if (!module) {
25470                 PyErr_Clear();
25471             }
25472         }
25473     #else
25474         if ((0)) __Pyx_Coroutine_patch_module(NULL, NULL);
25475     #endif
25476     return 0;
25477 }
25478
25479 /* Generator */
25480 static PyMethodDef __pyx_Generator_methods[] = {
25481     {"send", (PyCFunction) __Pyx_Coroutine_Send, METH_O,
25482      (char*) PyDoc_STR("send(arg) -> send 'arg' into generator,\nreturn next yielded value or raise
StopIteration.")},
25483     {"throw", (PyCFunction) __Pyx_Coroutine_Throw, METH_VARARGS,
25484      (char*) PyDoc_STR("throw(typ[,val[,tb]]) -> raise exception in generator,\nreturn next yielded
value or raise StopIteration.")},
25485     {"close", (PyCFunction) __Pyx_Coroutine_Close_Method, METH_NOARGS,
25486      (char*) PyDoc_STR("close() -> raise GeneratorExit inside generator.")},
25487     {0, 0, 0, 0}
25488 };
25489 static PyMemberDef __pyx_Generator_memberlist[] = {
25490     {(char *) "gi_running", T_BOOL, offsetof(__pyx_CoroutineObject, is_running), READONLY, NULL},

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```

25491     {(char*) "gi_yieldfrom", T_OBJECT, offsetof(__pyx_CoroutineObject, yieldfrom), READONLY,
25492      (char*) PyDoc_STR("object being iterated by 'yield from', or None")},
25493     {(char*) "gi_code", T_OBJECT, offsetof(__pyx_CoroutineObject, gi_code), READONLY, NULL},
25494     {0, 0, 0, 0, 0}
25495 };
25496 static PyGetSetDef __pyx_Generator_getsets[] = {
25497     {(char *) "__name__", (getter)__Pyx_Coroutine_get_name, (setter)__Pyx_Coroutine_set_name,
25498      (char*) PyDoc_STR("name of the generator"), 0},
25499     {(char *) "__qualname__", (getter)__Pyx_Coroutine_get_qualname,
25500      (setter)__Pyx_Coroutine_set_qualname,
25501      (char*) PyDoc_STR("qualified name of the generator"), 0},
25502     {(char *) "gi_frame", (getter)__Pyx_Coroutine_get_frame, NULL,
25503      (char*) PyDoc_STR("Frame of the generator"), 0},
25504     {0, 0, 0, 0, 0}
25505 };
25506 static PyTypeObject __pyx_GeneratorType_type = {
25507     PyVarObject_HEAD_INIT(0, 0)
25508     "generator",
25509     sizeof(__pyx_CoroutineObject),
25510     0,
25511     (destructor) __Pyx_Coroutine_dealloc,
25512     0,
25513     0,
25514     0,
25515     0,
25516     0,
25517     0,
25518     0,
25519     0,
25520     0,
25521     0,
25522     0,
25523     0,
25524     0,
25525     Py_TPFLAGS_DEFAULT | Py_TPFLAGS_HAVE_GC | Py_TPFLAGS_HAVE_FINALIZE,
25526     0,
25527     (traverseproc) __Pyx_Coroutine_traverse,
25528     0,
25529     0,
25530     offsetof(__pyx_CoroutineObject, gi_weakreflist),
25531     0,
25532     (iternextfunc) __Pyx_Generator_Next,
25533     __pyx_Generator_methods,
25534     __pyx_Generator_memberlist,
25535     __pyx_Generator_getsets,
25536     0,
25537     0,
25538     0,
25539     0,
25540     0,
25541     0,
25542     0,
25543     0,
25544     0,
25545     0,
25546     0,
25547     0,
25548     0,
25549     0,
25550     0,
25551     #if CYTHON_USE_TP_FINALIZE
25552     0,
25553     #else
25554     __Pyx_Coroutine_del,
25555     #endif
25556     0,
25557     #if CYTHON_USE_TP_FINALIZE
25558     __Pyx_Coroutine_del,
25559     #elif PY_VERSION_HEX >= 0x030400a1
25560     0,
25561     #endif
25562     #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM >= 0x07030800)
25563     0,
25564     #endif
25565     #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
25566     0,
25567     #endif
25568     #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
25569     0,
25570     #endif
25571 };
25572 static int __pyx_Generator_init(void) {
25573     __pyx_GeneratorType_type.tp_getattro = __Pyx_PyObject_GenericGetAttrNoDict;
25574     __pyx_GeneratorType_type.tp_iter = PyObject_SelfIter;
25575     __pyx_GeneratorType = __Pyx_FetchCommonType(&__pyx_GeneratorType_type);
25576     if (unlikely(!__pyx_GeneratorType)) {

```

```

25577         return -1;
25578     }
25579     return 0;
25580 }
25581
25582 /* CheckBinaryVersion */
25583 static int __Pyx_check_binary_version(void) {
25584     char ctversion[4], rtversion[4];
25585     PyOS_snprintf(ctversion, 4, "%d.%d", PY_MAJOR_VERSION, PY_MINOR_VERSION);
25586     PyOS_snprintf(rtversion, 4, "%s", Py_GetVersion());
25587     if (ctversion[0] != rtversion[0] || ctversion[2] != rtversion[2]) {
25588         char message[200];
25589         PyOS_snprintf(message, sizeof(message),
25590             "compiletime version %s of module '%.100s' "
25591             "does not match runtime version %s",
25592             ctversion, __Pyx_MODULE_NAME, rtversion);
25593         return PyErr_WarnEx(NULL, message, 1);
25594     }
25595     return 0;
25596 }
25597
25598 /* InitStrings */
25599 static int __Pyx_InitStrings(__Pyx_StringTabEntry *t) {
25600     while (t->p) {
25601         #if PY_MAJOR_VERSION < 3
25602         if (t->is_unicode) {
25603             *t->p = PyUnicode_DecodeUTF8(t->s, t->n - 1, NULL);
25604         } else if (t->intern) {
25605             *t->p = PyString_InternFromString(t->s);
25606         } else {
25607             *t->p = PyString_FromStringAndSize(t->s, t->n - 1);
25608         }
25609         #else
25610         if (t->is_unicode | t->is_str) {
25611             if (t->intern) {
25612                 *t->p = PyUnicode_InternFromString(t->s);
25613             } else if (t->encoding) {
25614                 *t->p = PyUnicode_Decode(t->s, t->n - 1, t->encoding, NULL);
25615             } else {
25616                 *t->p = PyUnicode_FromStringAndSize(t->s, t->n - 1);
25617             }
25618         } else {
25619             *t->p = PyBytes_FromStringAndSize(t->s, t->n - 1);
25620         }
25621         #endif
25622         if (!t->p)
25623             return -1;
25624         if (PyObject_Hash(*t->p) == -1)
25625             return -1;
25626         ++t;
25627     }
25628     return 0;
25629 }
25630
25631 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char* c_str) {
25632     return __Pyx_PyUnicode_FromStringAndSize(c_str, (Py_ssize_t)strlen(c_str));
25633 }
25634 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject* o) {
25635     Py_ssize_t ignore;
25636     return __Pyx_PyObject_AsStringAndSize(o, &ignore);
25637 }
25638 #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25639 #if !CYTHON_PEP393_ENABLED
25640 static const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25641     char* defenc_c;
25642     PyObject* defenc = _PyUnicode_AsDefaultEncodedString(o, NULL);
25643     if (!defenc) return NULL;
25644     defenc_c = PyBytes_AS_STRING(defenc);
25645     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25646     {
25647         char* end = defenc_c + PyBytes_GET_SIZE(defenc);
25648         char* c;
25649         for (c = defenc_c; c < end; c++) {
25650             if ((unsigned char) (*c) >= 128) {
25651                 PyUnicode_AsASCIIString(o);
25652                 return NULL;
25653             }
25654         }
25655     }
25656     #endif
25657     *length = PyBytes_GET_SIZE(defenc);
25658     return defenc_c;
25659 }
25660 #else
25661 static CYTHON_INLINE const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25662     if (unlikely(__Pyx_PyUnicode_READY(o) == -1)) return NULL;
25663     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII

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25664     if (likely(PyUnicode_IS_ASCII(o))) {
25665         *length = PyUnicode_GET_LENGTH(o);
25666         return PyUnicode_AsUTF8(o);
25667     } else {
25668         PyUnicode_AsASCIIString(o);
25669         return NULL;
25670     }
25671 #endif
25672     return PyUnicode_AsUTF8AndSize(o, length);
25673 #endif
25674 }
25675 #endif
25676 #endif
25677 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25678     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25679         if (
25680             #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25681                 __Pyx_sys_getdefaultencoding_not_ascii &&
25682             #endif
25683             PyUnicode_Check(o)) {
25684         return __Pyx_PyUnicode_AsStringAndSize(o, length);
25685     } else
25686 #endif
25687     #if (!CYTHON_COMPILING_IN_PYPY) || (defined(PyByteArray_AS_STRING) && defined(PyByteArray_GET_SIZE))
25688         if (PyByteArray_Check(o)) {
25689             *length = PyByteArray_GET_SIZE(o);
25690             return PyByteArray_AS_STRING(o);
25691         } else
25692 #endif
25693     {
25694         char* result;
25695         int r = PyBytes_AsStringAndSize(o, &result, length);
25696         if (unlikely(r < 0)) {
25697             return NULL;
25698         } else {
25699             return result;
25700         }
25701     }
25702 }
25703 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject* x) {
25704     int is_true = x == Py_True;
25705     if (is_true | (x == Py_False) | (x == Py_None)) return is_true;
25706     else return PyObject_IsTrue(x);
25707 }
25708 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject* x) {
25709     int retval;
25710     if (unlikely(!x)) return -1;
25711     retval = __Pyx_PyObject_IsTrue(x);
25712     Py_DECREF(x);
25713     return retval;
25714 }
25715 static PyObject* __Pyx_PyNumber_IntOrLongWrongResultType(PyObject* result, const char* type_name) {
25716     #if PY_MAJOR_VERSION >= 3
25717         if (PyLong_Check(result)) {
25718             if (PyErr_WarnFormat(PyExc_DeprecationWarning, 1,
25719                 "__int__ returned non-int (type %.200s). "
25720                 "The ability to return an instance of a strict subclass of int "
25721                 "is deprecated, and may be removed in a future version of Python.",
25722                 Py_TYPE(result)->tp_name)) {
25723                 Py_DECREF(result);
25724                 return NULL;
25725             }
25726             return result;
25727         }
25728     #endif
25729     PyErr_Format(PyExc_TypeError,
25730         "%.4s__ returned non-%.4s (type %.200s)",
25731         type_name, type_name, Py_TYPE(result)->tp_name);
25732     Py_DECREF(result);
25733     return NULL;
25734 }
25735 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x) {
25736     #if CYTHON_USE_TYPE_SLOTS
25737         PyNumberMethods *m;
25738     #endif
25739     const char *name = NULL;
25740     PyObject *res = NULL;
25741     #if PY_MAJOR_VERSION < 3
25742         if (likely(PyInt_Check(x) || PyLong_Check(x)))
25743             #else
25744             if (likely(PyLong_Check(x)))
25745             #endif
25746         return __Pyx_NewRef(x);
25747     #if CYTHON_USE_TYPE_SLOTS
25748         m = Py_TYPE(x)->tp_as_number;
25749         #if PY_MAJOR_VERSION < 3
25750             if (m && m->nb_int) {

```

```

25751     name = "int";
25752     res = m->nb_int(x);
25753 }
25754 else if (m && m->nb_long) {
25755     name = "long";
25756     res = m->nb_long(x);
25757 }
25758 #else
25759 if (likely(m && m->nb_int)) {
25760     name = "int";
25761     res = m->nb_int(x);
25762 }
25763 #endif
25764 #else
25765 if (!PyBytes_CheckExact(x) && !PyUnicode_CheckExact(x)) {
25766     res = PyNumber_Int(x);
25767 }
25768 #endif
25769 if (likely(res)) {
25770 #if PY_MAJOR_VERSION < 3
25771     if (unlikely(!PyInt_Check(res) && !PyLong_Check(res))) {
25772 #else
25773     if (unlikely(!PyLong_CheckExact(res))) {
25774 #endif
25775         return __Pyx_PyNumber_IntOrLongWrongResultType(res, name);
25776     }
25777 }
25778 else if (!PyErr_Occurred()) {
25779     PyErr_SetString(PyExc_TypeError,
25780         "an integer is required");
25781 }
25782 return res;
25783 }
25784 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject* b) {
25785     Py_ssize_t ival;
25786     PyObject *x;
25787 #if PY_MAJOR_VERSION < 3
25788     if (likely(PyInt_CheckExact(b))) {
25789         if (sizeof(Py_ssize_t) >= sizeof(long))
25790             return PyInt_AS_LONG(b);
25791         else
25792             return PyInt_AsSsize_t(b);
25793     }
25794 #endif
25795     if (likely(PyLong_CheckExact(b))) {
25796         #if CYTHON_USE_PYLONG_INTERNALS
25797         const digit* digits = ((PyLongObject*)b)->ob_digit;
25798         const Py_ssize_t size = Py_SIZE(b);
25799         if (likely(__Pyx_sst_abs(size) <= 1)) {
25800             ival = likely(size) ? digits[0] : 0;
25801             if (size == -1) ival = -ival;
25802             return ival;
25803         } else {
25804             switch (size) {
25805                 case 2:
25806                     if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25807                         return (Py_ssize_t) (((((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]));
25808                     }
25809                     break;
25810                 case -2:
25811                     if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25812                         return -(Py_ssize_t) (((((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]));
25813                     }
25814                     break;
25815                 case 3:
25816                     if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25817                         return (Py_ssize_t) ((((((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]));
25818                     }
25819                     break;
25820                 case -3:
25821                     if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25822                         return -(Py_ssize_t) ((((((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]));
25823                     }
25824                     break;
25825                 case 4:
25826                     if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25827                         return (Py_ssize_t) (((((((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]));
25828                     }
25829                     break;
25830                 case -4:
25831                     if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25832                         return -(Py_ssize_t) (((((((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]));
25833                     }

```

```

25834         break;
25835     }
25836 }
25837 #endif
25838 return PyLong_AsSsize_t(b);
25839 }
25840 x = PyNumber_Index(b);
25841 if (!x) return -1;
25842 ival = PyInt_AsSsize_t(x);
25843 Py_DECREF(x);
25844 return ival;
25845 }
25846 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject* o) {
25847     if (sizeof(Py_hash_t) == sizeof(Py_ssize_t)) {
25848         return (Py_hash_t) __Pyx_PyIndex_AsSsize_t(o);
25849 #if PY_MAJOR_VERSION < 3
25850     } else if (likely(PyInt_CheckExact(o))) {
25851         return PyInt_AS_LONG(o);
25852 #endif
25853     } else {
25854         Py_ssize_t ival;
25855         PyObject *x;
25856         x = PyNumber_Index(o);
25857         if (!x) return -1;
25858         ival = PyInt_AsLong(x);
25859         Py_DECREF(x);
25860         return ival;
25861     }
25862 }
25863 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b) {
25864     return b ? __Pyx_NewRef(Py_True) : __Pyx_NewRef(Py_False);
25865 }
25866 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t ival) {
25867     return PyInt_FromSize_t(ival);
25868 }
25869
25870
25871 #endif /* Py_PYTHON_H */

```

## 7.65 test/control.h File Reference

```
#include "glucat/glucat_config.h"
```

```
#include "test/try_catch.h"
```

Include dependency graph for control.h: This graph shows which files directly or indirectly include this file:

### Classes

- class [glucat::control\\_t](#)  
*Parameters to control tests.*

### Namespaces

- namespace [glucat](#)

## 7.66 control.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CONTROL_H
00002 #define _GLUCAT_CONTROL_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     control.h : Define and set parameters to control tests
00006     -----
00007     begin                : 2010-04-21
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi

```

```

00009 *****
00010
00011     This library is free software: you can redistribute it and/or modify
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00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031     See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033 #include "glucat/glucat_config.h"
00034 #include "test/try_catch.h"
00035
00036 namespace glucat
00037 {
00038     class control_t
00039     {
00040     private:
00041         bool m_valid;
00042         bool valid() const
00043         { return m_valid; }
00044
00045         bool m_catch_exceptions;
00046         bool catch_exceptions() const
00047         { return m_catch_exceptions; }
00048
00049         static bool m_verbose_output;
00050
00051         control_t(int argc, char ** argv);
00052         // Enforce singleton
00053         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00054         control_t() = default;
00055         ~control_t() = default;
00056         control_t(const control_t&) = delete;
00057         control_t& operator= (const control_t&) = delete;
00058
00059         friend class friend_for_private_destructor;
00060     public:
00061         static const control_t& control(int argc, char ** argv)
00062         { static const control_t c(argc, argv); return c; }
00063
00064         int call(intfn f) const;
00065         int call(intintfn f, int arg) const;
00066
00067         static bool verbose()
00068         { return m_verbose_output; }
00069     };
00070
00071 bool control_t::m_verbose_output = false;
00072
00073 control_t::
00074 control_t(int argc, char ** argv)
00075 : m_valid(true), m_catch_exceptions(true)
00076 {
00077     bool print_help = false;
00078     const std::string& arg_0_str = argv[0];
00079     const std::string& program_name = arg_0_str.substr(arg_0_str.find_last_of('/')+1);
00080     for (int arg_ndx = 1; arg_ndx < argc; ++arg_ndx)
00081     {
00082         const std::string& arg_str = argv[arg_ndx];
00083         bool valid = false;
00084         if (arg_str.substr(0,2) == "--")
00085         {
00086             valid = true;
00087             const std::string& arg_name = arg_str.substr(2);
00088             if (arg_name == "help")
00089             {
00090                 this->m_valid = false;
00091                 print_help = true;
00092             }
00093             else if (arg_name == "verbose")
00094                 this->m_verbose_output = true;
00095             else if (arg_name == "no-catch")

```

```

00111         this->m_catch_exceptions = false;
00112     else
00113         valid = false;
00114     }
00115     if (!valid)
00116     {
00117         std::cout << "Invalid argument: " << arg_str << std::endl;
00118         this->m_valid = false;
00119         print_help = true;
00120     }
00121     }
00122     if (print_help)
00123     {
00124         std::cout << program_name << " for " << GLUCAT_PACKAGE_NAME << " version " << GLUCAT_VERSION << ":" <<
std::endl;
00125         std::cout << "Usage: " << program_name << " [option ...]" << std::endl;
00126         std::cout << "Options:" << std::endl;
00127         std::cout << "  --help          : Print this summary." << std::endl;
00128         std::cout << "  --no-catch      : Do not catch exceptions." << std::endl;
00129         std::cout << "  --verbose       : Produce more detailed test output." << std::endl;
00130     }
00131 }
00132
00133 inline
00134 int
00135 control_t::
00136 call(intfn f) const
00137 {
00138     {
00139         if (valid())
00140             return (catch_exceptions())
? try_catch(f)
: (*f)();
00141         else
00142             return 1;
00143     }
00144 }
00145
00146 inline
00147 int
00148 control_t::
00149 call(intintfn f, int arg) const
00150 {
00151     {
00152         if (valid())
00153             return (catch_exceptions())
? try_catch(f, arg)
: (*f)(arg);
00154         else
00155             return 1;
00156     }
00157 }
00158 }
00159 }
00160 }
00161 #endif // _GLUCAT_CONTROL_H

```

## 7.67 test/driver.h File Reference

```

#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include "test/tuning.h"
#include "test/try_catch.h"
#include "test/control.h"
#include <cstdio>
Include dependency graph for driver.h:

```

## 7.68 driver.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_DRIVER_H
00002 #define GLUCAT_TEST_DRIVER_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     driver.h : Header for example and timing test driver
00006     -----
00007     begin          : Sun 2001-12-09
00008     copyright      : (C) 2001-2021 by Paul C. Leopardi

```

```

00009 *****
00010
00011     This library is free software: you can redistribute it and/or modify
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00016     This library is distributed in the hope that it will be useful,
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00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****
00031     See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/glucat.h"
00035 #include "glucat/glucat_imp.h"
00036 #include "test/tuning.h"
00037 #include "test/try_catch.h"
00038 #include "test/control.h"
00039 #include <cstdio>
00040
00041 #endif // GLUCAT_TEST_DRIVER_H

```

## 7.69 test/timing.h File Reference

### Namespaces

- namespace [glucat](#)
- namespace [glucat::timing](#)

### Functions

- static double [glucat::timing::elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

### Variables

- const double [glucat::timing::MS\\_PER\\_SEC](#) = 1000.0  
*Timing constant: milliseconds per second.*
- const double [glucat::timing::MS\\_PER\\_CLOCK](#) = MS\_PER\_SEC / double(CLOCKS\_PER\_SEC)  
*Timing constant: milliseconds per clock.*
- const int [glucat::timing::EXTRA\\_TRIALS](#) = 2  
*Timing constant: trial expansion factor.*

## 7.70 timing.h

Go to the documentation of this file.

```

00001 #ifndef GLUCAT_TEST_TIMING_H
00002 #define GLUCAT_TEST_TIMING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     timing.h : Common definitions for timing tests
00006     -----
00007     begin                : Tue 2012-03-27
00008     copyright             : (C) 2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
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00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 namespace glucat
00035 {
00036     namespace timing
00037     {
00038         const double MS_PER_SEC = 1000.0;
00039
00040         const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC);
00041
00042         const int EXTRA_TRIALS = 2;
00043
00044         inline
00045         static
00046         double
00047         elapsed(clock_t cpu_time)
00048         { return double(clock() - cpu_time) * MS_PER_CLOCK; }
00049     }
00050 }
00051 #endif // GLUCAT_TEST_TIMING_H

```

## 7.71 test/try\_catch.h File Reference

This graph shows which files directly or indirectly include this file:

### Namespaces

- namespace `glucat`

### Typedefs

- typedef int(\* `glucat::intfn`) ()  
For exception catching: pointer to function returning int.
- typedef int(\* `glucat::intintfn`) (int)  
For exception catching: pointer to function of int returning int.

## Functions

- `int glucat::try_catch (intfn f)`  
*Exception catching for functions returning int.*
- `int glucat::try_catch (intintfn f, int arg)`  
*Exception catching for functions of int returning int.*

## 7.72 try\_catch.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_TRY_CATCH_H
00002 #define _GLUCAT_TRY_CATCH_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   try_catch.h : Catch exceptions
00006   -----
00007   begin                : Sun 2001-12-20
00008   copyright            : (C) 2001-2010 by Paul C. Leopardi
00009   *****/
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00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   See also Arvind Raja's original header comments in glucat.h
00032   *****/
00033
00034 namespace glucat
00035 {
00037   typedef int (*intfn)();
00038
00040   typedef int (*intintfn)(int);
00041
00043   int try_catch(intfn f);
00044
00046   int try_catch(intintfn f, int arg);
00047
00049   int try_catch(intfn f)
00050   {
00051     int result = 0;
00052     try
00053     { result = (*f)(); }
00054     catch (const glucat_error& e)
00055     { e.print_error_msg(); }
00056     catch (const std::bad_alloc& e)
00057     { std::cerr << "bad_alloc" << std::endl; }
00058     catch (...)
00059     { std::cerr << "unexpected exception" << std::endl; }
00060     return result;
00061   }
00062
00064   int try_catch(intintfn f, int arg)
00065   {
00066     int result = 0;
00067     try
00068     { result = (*f)(arg); }
00069     catch (const glucat_error& e)
00070     { e.print_error_msg(); }
00071     catch (const std::bad_alloc& e)
00072     { std::cerr << "bad_alloc" << std::endl; }
00073     catch (...)
00074     { std::cerr << "unexpected exception" << std::endl; }

```



```
00075     return result;
00076 }
00077 }
00078 #endif // _GLUCAT_TRY_CATCH_H
```



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