glucat

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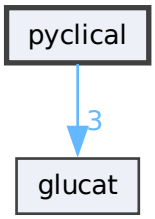
- file framed_multi_imp.h
- file generation.h
- file generation_imp.h
- file global.h
- file glucat.h
- file glucat_config.h
- file glucat_imp.h
- file index_set.h
- file index_set_imp.h
- file long_double.h
- file matrix.h
- file matrix_imp.h
- file matrix_multi.h
- file matrix_multi_imp.h
- file portability.h
- file promotion.h
- file qd.h
- file random.h
- file scalar.h
- file scalar_imp.h
- file tuning.h

6.2 pyclical Directory Reference

Directory dependency graph for pyclical:

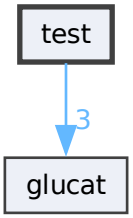


Files

- file glucat.pxd
- file PyClical.h
- file PyClical.pxd
- file PyClical.pyx

6.3 test Directory Reference

Directory dependency graph for test:



Files

- file control.h
- file driver.h
- file timing.h
- file try_catch.h
- file tuning.h
- file undefine.h

Chapter 7

Namespace Documentation

7.1 cga3 Namespace Reference

Definitions for 3D Conformal Geometric Algebra [DL].

Functions

- template<typename Multivector_T>
  Multivector_T cga3 (const Multivector_T &x)

  Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].
- template<typename Multivector_T>
  Multivector_T cga3std (const Multivector_T &X)

  Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].
- template<typename Multivector_T>
  Multivector_T agc3 (const Multivector_T &X)

  Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

7.1.1 Detailed Description

Definitions for 3D Conformal Geometric Algebra [DL].

7.1.2 Function Documentation

7.1.2.1 agc3()

```
template<typename Multivector_T>
Multivector_T cga3::agc3 (
            const Multivector_T & X)  [inline]
```

Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Definition at line 126 of file PyClical.h.

References agc3(), and cga3std().

Referenced by agc3().

7.1.2.2 cga3()

```
template<typename Multivector_T>
Multivector_T cga3::cga3 (
            const Multivector_T & x)  [inline]
```

Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].

Definition at line 103 of file PyClical.h.

7.1.2.3 cga3std()

```
template<typename Multivector_T>
Multivector_T cga3::cga3std (
            const Multivector_T & X)  [inline]
```

Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].

Definition at line 114 of file PyClical.h.

References cga3std().

Referenced by agc3(), and cga3std().

7.2 glucat Namespace Reference

Namespaces

- namespace gen
- namespace matrix
- namespace timing

Classes

- class basis_table

  Table of basis elements used as a cache by basis_element().
- class bool_to_type

  Bool to type.
- class clifford_algebra

  clifford_algebra<> declares the operations of a Clifford algebra
- class compare_types

  Type comparison.
- class compare_types< T, T >
- class control_t

  Parameters to control tests.
- struct CTAssertion

  Compile time assertion.
- struct CTAssertion< true >
- class error

  Specific exception class.

- class framed_multi

  A framed_multi<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector.
- class glucat_error

  Abstract exception class.
- class index_set

  Index set class based on std::bitset<> in Gnu standard C++ library.
- class index_set_hash
- class matrix_multi

  A matrix_multi<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector.
- class numeric_traits

  Extra traits which extend numeric limits.
- class random_generator

  Random number generator with single instance per Scalar_T.
- class sorted_range

  Sorted range for use with output.
- class sorted_range< Sorted_Map_T, Sorted_Map_T >

Typedefs

- using index_t = int

  Size of index_t should be enough to represent LO, HI.
- using set_value_t = unsigned long

  Size of set_value_t should be enough to contain index_set<LO,HI>.
- typedef int(∗) intfn()

  For exception catching: pointer to function returning int.
- typedef int(∗) intintfn(int)

  For exception catching: pointer to function of int returning int.
- using tuning_slow
- using tuning_naive
- using tuning_fast

Functions

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for inequality of multivectors.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool

  Test for inequality of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for inequality of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto error_squared_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Quadratic norm error tolerance relative to a specific multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto error_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T

  Relative or absolute error using the quadratic norm.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool

  Test for approximate equality of multivectors.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for approximate equality of multivectors.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inner product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric multiplicative inverse.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Integer power of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Multivector power of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto outer_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Outer product power of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Real part: synonym for scalar part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Imaginary part: deprecated (always 0).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Pure part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Even part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Odd part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto vector_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >

  Vector part of multivector, as a vector_t with respect to frame().
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Main involution, each {i} is replaced by -{i} in each term, eg. {1}{2} -> (-{2})*(-{1}).*
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Reversion, eg. {1}{2} -> {2}*{1}.*
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Conjugation, rev o invo == invo o rev.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  *Scalar_T quadratic form == (rev(x)*x)(0).*
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar_T norm == sum of norm of coordinates.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Absolute value == sqrt(norm).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto max_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Maximum of absolute values of components of multivector: multivector infinity norm.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of -1 which commutes with all members of the frame of the given multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto clifford_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune↩_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse sine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse sine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Hyperbolic sine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune↩ _P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune↩ _P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static void check_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar↩ _T, LO, HI, Tune_P > &i, const bool prechecked=false)

  Check that i is a valid complexifier for val.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term) -> std::ostream &

  Write term to output.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  static auto crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T

  Coordinate of product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >

  Product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

Natural logarithm of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI>
  static auto crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T

  Coordinate of product of terms.
- _GLUCAT_CTAssert (std::numeric_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const index_t BITS_PER_CHAR

  If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.
- _GLUCAT_CTAssert (_GLUCAT_BITS_PER_ULONG==BITS_PER_SET_VALUE, BitsPerULongDoesNot↩ MatchSetValueT) const index_t DEFAULT_LO

  Default lowest index in an index set.
- template<typename LHS_T, typename RHS_T>
  auto pos_mod (LHS_T lhs, RHS_T rhs) -> LHS_T

  Modulo function which works reliably for lhs < 0.
- template<const index_t LO, const index_t HI>
  auto operator^ (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Symmetric set difference: exclusive or.
- template<const index_t LO, const index_t HI>
  auto operator& (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Set intersection: and.
- template<const index_t LO, const index_t HI>
  auto operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Set union: or.
- template<const index_t LO, const index_t HI>
  auto compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int

  "lexicographic compare" eg. {3,4,5} is less than {3,7,8}
- _GLUCAT_CTAssert (sizeof(set_value_t) >=sizeof(std::bitset< DEFAULT_HI-DEFAULT_LO >), Default_↩ index_set_too_big_for_value) template< const index_t LO

  Size of set_value_t should be enough to contain bitset<DEFAULT_HI-DEFAULT_LO>.
- const index_t HI auto operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &
- template<const index_t LO, const index_t HI>
  auto operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &

  Read in index set.
- auto sign_of_square (index_t j) -> int

  Square of generator {j}.
- template<const index_t LO, const index_t HI>
  auto min_neg (const index_set< LO, HI > &ist) -> index_t

  Minimum negative index, or 0 if none.
- template<const index_t LO, const index_t HI>
  auto max_pos (const index_set< LO, HI > &ist) -> index_t

  Maximum positive index, or 0 if none.
- template<const index_t LO, const index_t HI>
  auto operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &

  Write out index set.
- static auto inverse_reversed_gray (unsigned long x) -> unsigned long

  Inverse reversed Gray code.
- static auto inverse_gray (unsigned long x) -> unsigned long

  Inverse Gray code.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator* (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

Geometric product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator^ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator& (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator% (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto star (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator/ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator| (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs_reframed) -> const index_set< LO, HI >

  Find a common frame for operands of a binary operator.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- auto offset_level (const index_t p, const index_t q) -> index_t

  Determine the log2 dim corresponding to signature p, q.
- template<typename Matrix_Index_T, const index_t LO, const index_t HI>
  static auto folded_dim (const index_set< LO, HI > &sub) -> Matrix_Index_T

  Determine the matrix dimension of the fold of a subalegbra.
- template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
  static auto fast (const Matrix_T &X, index_t level) -> Multivector_T

  Inverse generalized Fast Fourier Transform.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size>
  static auto pade_approx (const std::array< Scalar_T, Size > &numer, const std::array< Scalar_T, Size > &denom, const matrix_multi< Scalar_T, LO, HI, Tune_P > &X) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Pade' approximation.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static void db_step (matrix_multi< Scalar_T, LO, HI, Tune_P > &M, matrix_multi< Scalar_T, LO, HI, Tune_P > &Y)

  Single step of product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto db_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 4)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto cr_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_Y_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 1)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Cyclic reduction square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto pade_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Pade' approximation of log.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto cascade_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Incomplete square root cascade and Pade' approximation of log.
- template<typename Scalar_T>
  auto log2 (const Scalar_T &x) -> Scalar_T

  Log base 2 of scalar.
- template<typename Scalar_T>
  auto to_promote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::promoted::type

  Cast to promote.
- template<typename Scalar_T>
  auto to_demote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::demoted::type

  Cast to demote.
- int try_catch (intfn f)

  Exception catching for functions returning int.
- int try_catch (intintfn f, int arg)

  Exception catching for functions of int returning int.

Variables

- template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
  const Scalar_T clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation = std::↩
  numeric_limits<Scalar_T>::epsilon()

  Default for truncation.
- const double MS_PER_S = 1000.0

  Timing constant: deprecated here - moved to test/timing.h.
- const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits

  Number of bits in set_value_t.
- const index_t DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)

  Default highest index in an index set.
- static const long double l_pi = 3.14159265358979323846264338327950 29L
- static const long double l_ln2 = 0.69314718055994530941723212145817 66L
- const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits
- const unsigned int Tuning_Max_Threshold = 1 << Tuning_Int_Digits
- const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Slow_Basis_Max_Count = 0
- const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0
- const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
- const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
- const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0
- const unsigned int Tuning_Fast_Div_Max_Steps = 0
- const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256
- const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256
- const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16
- const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8
- const unsigned int Tuning_Fast_Basis_Max_Count = 1
- const unsigned int Tuning_Fast_Fast_Size_Threshold = 0
- const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0
- const unsigned int Tuning_Fast_Products_Size_Threshold = 0

7.2.1 Typedef Documentation

7.2.1.1 index_t

```
using glucat::index_t = int
```

Size of index_t should be enough to represent LO, HI.

Definition at line 77 of file global.h.

7.2.1.2 intfn

```
typedef int(*) glucat::intfn()
```

For exception catching: pointer to function returning int.

Definition at line 37 of file try_catch.h.

7.2.1.3 intintfn

```
typedef int(*) glucat::intintfn(int)
```

For exception catching: pointer to function of int returning int.

Definition at line 40 of file try_catch.h.

7.2.1.4 set_value_t

```
using glucat::set_value_t = unsigned long
```

Size of set_value_t should be enough to contain index_set<LO,HI>.

Definition at line 79 of file global.h.

7.2.1.5 tuning_fast

```
using glucat::tuning_fast
```

Initial value:

```
tuning
   <
     Tuning_Fast_Mult_Matrix_Threshold,
     Tuning_Fast_Div_Max_Steps,
     Tuning_Fast_CR_Sqrt_Max_Steps,
     Tuning_Fast_DB_Sqrt_Max_Steps,
     Tuning_Fast_Log_Max_Outer_Steps,
     Tuning_Fast_Log_Max_Inner_Steps,
     Tuning_Fast_Basis_Max_Count,
     Tuning_Fast_Fast_Size_Threshold,
     Tuning_Fast_Inv_Fast_Dim_Threshold,
     Tuning_Fast_Products_Size_Threshold,
     Tuning_Default_Denom_Different_Bits,
     Tuning_Default_Extra_Different_Bits,
     Tuning_Default_Function_Precision
   >
```

Definition at line 97 of file tuning.h.

7.2.1.6 tuning_naive

```
using glucat::tuning_naive
```

Initial value:

```
tuning
   <
     Tuning_Naive_Mult_Matrix_Threshold,
     Tuning_Default_Div_Max_Steps,
     Tuning_Default_CR_Sqrt_Max_Steps,
     Tuning_Default_DB_Sqrt_Max_Steps,
     Tuning_Default_Log_Max_Outer_Steps,
     Tuning_Default_Log_Max_Inner_Steps,
     Tuning_Naive_Basis_Max_Count,
     Tuning_Naive_Fast_Size_Threshold,
     Tuning_Naive_Inv_Fast_Dim_Threshold,
     Tuning_Default_Products_Size_Threshold,
     Tuning_Default_Denom_Different_Bits,
     Tuning_Default_Extra_Different_Bits,
     Tuning_Default_Function_Precision
   >
```

Definition at line 69 of file tuning.h.

7.2.1.7 tuning_slow

```
using glucat::tuning_slow
```

Initial value:

```
tuning
   <
     Tuning_Slow_Mult_Matrix_Threshold,
     Tuning_Default_Div_Max_Steps,
     Tuning_Default_CR_Sqrt_Max_Steps,
     Tuning_Default_DB_Sqrt_Max_Steps,
     Tuning_Default_Log_Max_Outer_Steps,
     Tuning_Default_Log_Max_Inner_Steps,
     Tuning_Slow_Basis_Max_Count,
     Tuning_Slow_Fast_Size_Threshold,
     Tuning_Slow_Inv_Fast_Dim_Threshold,
     Tuning_Slow_Products_Size_Threshold,
     Tuning_Default_Denom_Different_Bits,
     Tuning_Default_Extra_Different_Bits,
     Tuning_Default_Function_Precision
   >
```

Definition at line 47 of file tuning.h.

7.2.2 Function Documentation

7.2.2.1 _GLUCAT_CTAssert() [1/3]

```
glucat::_GLUCAT_CTAssert (
             _GLUCAT_BITS_PER_ULONG  = =BITS_PER_SET_VALUE,
             BitsPerULongDoesNotMatchSetValueT ) const
```

Default lowest index in an index set.

References _GLUCAT_CTAssert(), and BITS_PER_SET_VALUE.

7.2.2.2 _GLUCAT_CTAssert() [2/3]

```
glucat::_GLUCAT_CTAssert (
             sizeof(set_value_t) >=sizeof(std::bitset< DEFAULT_HI-DEFAULT_LO >) ,
             Default_index_set_too_big_for_value ) const
```

Size of set_value_t should be enough to contain bitset<DEFAULT_HI-DEFAULT_LO>.

Write out index set

References _GLUCAT_CTAssert(), and DEFAULT_HI.

7.2.2.3 _GLUCAT_CTAssert() [3/3]

```
glucat::_GLUCAT_CTAssert (
             std::numeric_limits< unsigned char >::radix  = =2,
             CannotDetermineBitsPerChar ) const
```

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

Number of bits per char is used to determine number of bits in set_value_t

References _GLUCAT_CTAssert().

Referenced by _GLUCAT_CTAssert(), _GLUCAT_CTAssert(), and _GLUCAT_CTAssert().

7.2.2.4 abs()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::abs (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Absolute value == sqrt(norm).

Definition at line 577 of file clifford_algebra_imp.h.

References abs(), and glucat::numeric_traits< Scalar_T >::sqrt().

Referenced by abs(), PyClical.clifford::abs(), clifford_to_str(), matrix_log(), and matrix_sqrt().

7.2.2.5 acos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acos (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse cosine of multivector.

Definition at line 902 of file clifford_algebra_imp.h.

References acos(), and complexifier().

7.2.2.6 acos() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acos (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse cosine of multivector with specified complexifier.

Definition at line 882 of file clifford_algebra_imp.h.

References acos(), acosh(), and check_complex().

Referenced by acos(), and acos().

7.2.2.7 acosh() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acosh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse hyperbolic cosine of multivector.

Definition at line 843 of file clifford_algebra_imp.h.

References acosh(), and complexifier().

7.2.2.8 acosh() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::acosh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse hyperbolic cosine of multivector with specified complexifier.

Definition at line 824 of file clifford_algebra_imp.h.

References acosh(), check_complex(), log(), norm(), and sqrt().

Referenced by acos(), acosh(), and acosh().

7.2.2.9 approx_equal() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::approx_equal (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool  [inline]
```

Test for approximate equality of multivectors.

Definition at line 169 of file clifford_algebra_imp.h.

References approx_equal(), and error_squared_tol().

7.2.2.10 approx_equal() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::approx_equal (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
            const Scalar_T threshold,
            const Scalar_T tolerance) -> bool  [inline]
```

Test for approximate equality of multivectors.

Definition at line 154 of file clifford_algebra_imp.h.

References approx_equal(), and error_squared().

Referenced by approx_equal(), approx_equal(), and matrix_sqrt().

7.2.2.11 asin() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asin (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse sine of multivector.

Definition at line 1007 of file clifford_algebra_imp.h.

References asin(), and complexifier().

7.2.2.12 asin() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asin (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse sine of multivector with specified complexifier.

Definition at line 987 of file clifford_algebra_imp.h.

References asin(), asinh(), and check_complex().

Referenced by asin(), and asin().

7.2.2.13 asinh() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asinh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse hyperbolic sine of multivector.

Definition at line 948 of file clifford_algebra_imp.h.

References asinh(), and complexifier().

7.2.2.14 asinh() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::asinh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse hyperbolic sine of multivector with specified complexifier.

Definition at line 929 of file clifford_algebra_imp.h.

References asinh(), check_complex(), log(), norm(), and sqrt().

Referenced by asin(), asinh(), and asinh().

7.2.2.15 atan() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::atan (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse tangent of multivector.

Definition at line 1107 of file clifford_algebra_imp.h.

References atan(), and complexifier().

7.2.2.16 atan() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::atan (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val,
             const Multivector< Scalar_T, LO, HI, Tune_P > & i,
             const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse tangent of multivector with specified complexifier.

Definition at line 1087 of file clifford_algebra_imp.h.

References atan(), atanh(), and check_complex().

Referenced by atan(), and atan().

7.2.2.17 atanh() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::atanh (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Inverse hyperbolic tangent of multivector.

Definition at line 1051 of file clifford_algebra_imp.h.

References atanh(), and complexifier().

7.2.2.18 atanh() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::atanh (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val,
             const Multivector< Scalar_T, LO, HI, Tune_P > & i,
             const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Inverse hyperbolic tangent of multivector with specified complexifier.

Definition at line 1034 of file clifford_algebra_imp.h.

References atanh(), check_complex(), log(), and norm().

Referenced by atan(), atanh(), and atanh().

7.2.2.19 cascade_log()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cascade_log (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [static]
```

Incomplete square root cascade and Pade' approximation of log.

Definition at line 1910 of file matrix_multi_imp.h.

References cascade_log(), db_step(), epsilon, glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), norm(), and pade_log().

Referenced by cascade_log(), and matrix_log().

7.2.2.20 check_complex()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
void glucat::check_complex (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false)  [inline], [static]
```

Check that i is a valid complexifier for val.

Definition at line 652 of file clifford_algebra_imp.h.

References check_complex(), and complexifier().

Referenced by acos(), acosh(), asin(), asinh(), atan(), atanh(), check_complex(), cos(), log(), log(), sin(), sqrt(), sqrt(), and tan().

7.2.2.21 clifford_exp()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::clifford_exp (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 689 of file clifford_algebra_imp.h.

References clifford_exp(), and log2().

Referenced by clifford_exp(), exp(), and exp().

7.2.2.22 compare()

```
template<const index_t LO, const index_t HI>
auto glucat::compare (
            const index_set< LO, HI > & a,
            const index_set< LO, HI > & b) -> int  [inline]
```

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

Lexicographic ordering of two sets: -1 if a<b, +1 if a>b, 0 if a==b.

Definition at line 574 of file index_set_imp.h.

References compare().

Referenced by compare().

7.2.2.23 complexifier()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::complexifier (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >
```

Square root of -1 which commutes with all members of the frame of the given multivector.

Definition at line 592 of file clifford_algebra_imp.h.

References complexifier(), and pos_mod().

Referenced by acos(), acosh(), asin(), asinh(), atan(), atanh(), check_complex(), complexifier(), cos(), elliptic(), log(), sin(), sqrt(), and tan().

7.2.2.24 conj()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::conj (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Conjugation, rev o invo == invo o rev.

Definition at line 553 of file clifford_algebra_imp.h.

References conj().

Referenced by conj().

7.2.2.25 cos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cos (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Cosine of multivector.

Definition at line 873 of file clifford_algebra_imp.h.

References complexifier(), and cos().

7.2.2.26 cos() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cos (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Cosine of multivector with specified complexifier.

Definition at line 850 of file clifford_algebra_imp.h.

References check_complex(), cos(), and exp().

Referenced by cos(), cos(), and tan().

7.2.2.27 cosh()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cosh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Hyperbolic cosine of multivector.

Definition at line 806 of file clifford_algebra_imp.h.

References cosh(), and exp().

Referenced by cosh(), and tanh().

7.2.2.28 cr_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::cr_sqrt (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
            Scalar_T norm_Y_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) ->
const matrix_multi< Scalar_T, LO, HI, Tune_P >  [static]
```

Cyclic reduction square root iteration.

Definition at line 1344 of file matrix_multi_imp.h.

References cr_sqrt(), glucat::numeric_traits< Scalar_T >::NaN(), and norm().

Referenced by cr_sqrt(), and matrix_sqrt().

7.2.2.29 crd_of_mult() [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::crd_of_mult (
            const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
            const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T  [inline],
[static]
```

Coordinate of product of terms.

References crd_of_mult(), exp(), log(), and sqrt().

Referenced by crd_of_mult(), operator%(), operator&(), operator*(), and operator^().

7.2.2.30 crd_of_mult() [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::crd_of_mult (
            const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
            const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> Scalar_T  [inline],
[static]
```

Coordinate of product of terms.

Definition at line 1704 of file framed_multi_imp.h.

References crd_of_mult().

7.2.2.31 db_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::db_sqrt (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
            Scalar_T norm_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) ->
const matrix_multi< Scalar_T, LO, HI, Tune_P >  [static]
```

Product form of Denman-Beavers square root iteration.

Definition at line 1317 of file matrix_multi_imp.h.

References db_sqrt(), db_step(), glucat::numeric_traits< Scalar_T >::NaN(), and norm().

Referenced by db_sqrt(), and matrix_sqrt().

7.2.2.32 db_step()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
void glucat::db_step (
            matrix_multi< Scalar_T, LO, HI, Tune_P > & M,
            matrix_multi< Scalar_T, LO, HI, Tune_P > & Y)  [inline], [static]
```

Single step of product form of Denman-Beavers square root iteration.

Definition at line 1305 of file matrix_multi_imp.h.

References db_step(), and inv().

Referenced by cascade_log(), db_sqrt(), and db_step().

7.2.2.33 elliptic()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::elliptic (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Square root of -1 which commutes with all members of the frame of the given multivector The name "elliptic" is now deprecated: use "complexifier" instead.

Definition at line 643 of file clifford_algebra_imp.h.

References complexifier(), and elliptic().

Referenced by elliptic().

7.2.2.34 error_squared()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::error_squared (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
            const Scalar_T threshold) -> Scalar_T  [inline]
```

Relative or absolute error using the quadratic norm.

Definition at line 134 of file clifford_algebra_imp.h.

References error_squared(), and norm().

Referenced by approx_equal(), and error_squared().

7.2.2.35 error_squared_tol()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::error_squared_tol (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T
```

Quadratic norm error tolerance relative to a specific multivector.

Definition at line 112 of file clifford_algebra_imp.h.

References error_squared_tol(), and glucat::numeric_traits< Scalar_T >::pow().

Referenced by approx_equal(), and error_squared_tol().

7.2.2.36 even()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::even (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Even part.

Definition at line 513 of file clifford_algebra_imp.h.

References even().

Referenced by even().

7.2.2.37 exp() [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::exp (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & val) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 1745 of file framed_multi_imp.h.

References clifford_exp(), exp(), and scalar().

Referenced by cos(), cosh(), crd_of_mult(), exp(), exp(), matrix_log(), matrix_sqrt(), pow(), sin(), and sinh().

7.2.2.38 exp() [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::exp (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >
```

Exponential of multivector.

Definition at line 2074 of file matrix_multi_imp.h.

References clifford_exp(), exp(), glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), and glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar().

7.2.2.39 fast()

```
template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
auto glucat::fast (
             const Matrix_T & X,
             index_t level) -> Multivector_T  [static]
```

Inverse generalized Fast Fourier Transform.

Definition at line 1024 of file matrix_multi_imp.h.

References fast(), glucat::matrix::signed_perm_nork(), and glucat::matrix::unit().

Referenced by fast(), and glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi().

7.2.2.40 folded_dim()

```
template<typename Matrix_Index_T, const index_t LO, const index_t HI>
auto glucat::folded_dim (
             const index_set< LO, HI > & sub) -> Matrix_Index_T  [inline], [static]
```

Determine the matrix dimension of the fold of a subalegbra.

Definition at line 101 of file matrix_multi_imp.h.

References folded_dim(), and offset_level().

Referenced by folded_dim(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::matrix_multi< Scalar_T, LO, H
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(),
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(),
and glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi().

7.2.2.41 imag()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::imag (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Imaginary part: deprecated (always 0).

Definition at line 497 of file clifford_algebra_imp.h.

References imag().

Referenced by imag().

7.2.2.42 inv()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::inv (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Geometric multiplicative inverse.

Definition at line 400 of file clifford_algebra_imp.h.

References inv().

Referenced by db_step(), inv(), and matrix_log().

7.2.2.43 inverse_gray()

```
auto glucat::inverse_gray (
            unsigned long x) -> unsigned long  [inline], [static]
```

Inverse Gray code.

Definition at line 863 of file index_set_imp.h.

References inverse_gray().

Referenced by inverse_gray(), and glucat::index_set< LO, HI >::sign_of_mult().

7.2.2.44 inverse_reversed_gray()

```
auto glucat::inverse_reversed_gray (
            unsigned long x) -> unsigned long  [inline], [static]
```

Inverse reversed Gray code.

Definition at line 846 of file index_set_imp.h.

References inverse_reversed_gray().

Referenced by inverse_reversed_gray(), and glucat::index_set< LO, HI >::sign_of_mult().

7.2.2.45 involute()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::involute (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1}∗{2} -> (-{2})∗(-{1}).

Main involution, each {i} is replaced by -{i} in each term, eg. {1} ->-{1}.

Definition at line 537 of file clifford_algebra_imp.h.

References involute().

Referenced by involute().

7.2.2.46 log() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
            bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1795 of file framed_multi_imp.h.

References check_complex(), and log().

7.2.2.47 log() [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
            bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 2033 of file matrix_multi_imp.h.

References check_complex(), glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), log(), and matrix_log().

7.2.2.48 log() [3/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Natural logarithm of multivector.

Definition at line 798 of file clifford_algebra_imp.h.

References complexifier(), and log().

7.2.2.49 log() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::log (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Natural logarithm of multivector with specified complexifier.

Definition at line 790 of file clifford_algebra_imp.h.

References log().

Referenced by acosh(), asinh(), atanh(), crd_of_mult(), log(), log(), log(), log(), and pow().

7.2.2.50 log2()

```
template<typename Scalar_T>
auto glucat::log2 (
            const Scalar_T & x) -> Scalar_T  [inline]
```

Log base 2 of scalar.

Definition at line 303 of file scalar.h.

References log2(), and glucat::numeric_traits< Scalar_T >::log2().

Referenced by clifford_exp(), and log2().

7.2.2.51 matrix_log()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_log (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
            const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1957 of file matrix_multi_imp.h.

References abs(), cascade_log(), glucat::matrix::classify_eigenvalues(), exp(), inv(), glucat::clifford_algebra< Scalar_T, Index_Set_T, matrix_log(), norm(), and glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar().

Referenced by log(), and matrix_log().

7.2.2.52 matrix_sqrt()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_sqrt (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
            const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1563 of file matrix_multi_imp.h.

References abs(), approx_equal(), glucat::matrix::classify_eigenvalues(), cr_sqrt(), db_sqrt(), pade::pade_sqrt_denom< Scalar_T >:: exp(), glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), matrix_sqrt(), norm(), pade::pade_sqrt_numer< Sca pade_approx(), pow(), and glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar().

Referenced by matrix_sqrt(), and sqrt().

7.2.2.53 max_abs()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::max_abs (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

Definition at line 585 of file clifford_algebra_imp.h.

References max_abs().

Referenced by max_abs().

7.2.2.54 max_pos()

```
template<const index_t LO, const index_t HI>
auto glucat::max_pos (
            const index_set< LO, HI > & ist) -> index_t  [inline]
```

Maximum positive index, or 0 if none.

Definition at line 977 of file index_set_imp.h.

References max_pos().

Referenced by max_pos(), and operator<<().

7.2.2.55 min_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::min_neg (
            const index_set< LO, HI > & ist) -> index_t  [inline]
```

Minimum negative index, or 0 if none.

Definition at line 970 of file index_set_imp.h.

References min_neg().

Referenced by min_neg(), and operator<<().

7.2.2.56 norm()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::norm (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Scalar_T norm == sum of norm of coordinates.

Definition at line 569 of file clifford_algebra_imp.h.

References norm().

Referenced by acosh(), asinh(), atanh(), cascade_log(), cr_sqrt(), db_sqrt(), error_squared(), matrix_log(), matrix_sqrt(), and norm().

7.2.2.57 odd()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::odd (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Odd part.

Definition at line 521 of file clifford_algebra_imp.h.

References odd().

Referenced by odd().

7.2.2.58 offset_level()

```
auto glucat::offset_level (
            const index_t p,
            const index_t q) -> index_t  [inline]
```

Determine the log2 dim corresponding to signature p, q.

Definition at line 86 of file matrix_multi_imp.h.

References offset_level(), and pos_mod().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element(), folded_dim(), and offset_level().

7.2.2.59 operator"!=() [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!= (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> bool  [inline]
```

Test for inequality of multivectors.

Definition at line 86 of file clifford_algebra_imp.h.

7.2.2.60 operator"!=() [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!= (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const Scalar_T & scr) -> bool  [inline]
```

Test for inequality of multivector and scalar.

Definition at line 94 of file clifford_algebra_imp.h.

7.2.2.61 operator"!=() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator!= (
            const Scalar_T & scr,
            const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> bool  [inline]
```

Test for inequality of scalar and multivector.

Definition at line 102 of file clifford_algebra_imp.h.

7.2.2.62 operator%() [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Left contraction.

Definition at line 597 of file framed_multi_imp.h.

References _GLUCAT_HASH_SIZE_T, and crd_of_mult().

7.2.2.63 operator%() [2/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Left contraction.

Definition at line 579 of file matrix_multi_imp.h.

7.2.2.64 operator%() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator% (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Left contraction.

Definition at line 322 of file clifford_algebra_imp.h.

7.2.2.65 operator&() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Inner product.

Definition at line 495 of file framed_multi_imp.h.

References _GLUCAT_HASH_SIZE_T, and crd_of_mult().

7.2.2.66 operator&() [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator& (
            const index_set< LO, HI > & lhs,
            const index_set< LO, HI > & rhs) -> const index_set< LO, HI >  [inline]
```

Set intersection: and.

Definition at line 186 of file index_set_imp.h.

7.2.2.67 operator&() [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Inner product.

Definition at line 560 of file matrix_multi_imp.h.

7.2.2.68 operator&() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator& (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Inner product.

Definition at line 307 of file clifford_algebra_imp.h.

7.2.2.69 operator∗() [1/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Geometric product.

Definition at line 374 of file framed_multi_imp.h.

References _GLUCAT_HASH_SIZE_T.

7.2.2.70 operator∗() [2/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Geometric product.

Definition at line 500 of file matrix_multi_imp.h.

References glucat::numeric_traits< Scalar_T >::NaN(), and reframe().

7.2.2.71 operator∗() [3/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Geometric product.

Definition at line 277 of file clifford_algebra_imp.h.

7.2.2.72 operator∗() [4/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >  [inline]
```

Product of multivector and scalar.

Definition at line 251 of file clifford_algebra_imp.h.

7.2.2.73 operator∗() [5/6]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator* (
            const Scalar_T & scr,
            const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Product of scalar and multivector.

Definition at line 262 of file clifford_algebra_imp.h.

7.2.2.74 operator∗() [6/6]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::operator* (
            const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
            const std::pair< const index_set< LO, HI >, Scalar_T > & rhs) -> const std::↩
pair< const index_set< LO, HI >, Scalar_T >  [inline]
```

Product of terms.

Definition at line 1712 of file framed_multi_imp.h.

References crd_of_mult().

7.2.2.75 operator+() [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Geometric sum.

Definition at line 206 of file clifford_algebra_imp.h.

7.2.2.76 operator+() [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >  [inline]
```

Geometric sum of multivector and scalar.

Definition at line 181 of file clifford_algebra_imp.h.

7.2.2.77 operator+() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator+ (
            const Scalar_T & scr,
            const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Geometric sum of scalar and multivector.

Definition at line 192 of file clifford_algebra_imp.h.

7.2.2.78 operator-() [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Geometric difference.

Definition at line 240 of file clifford_algebra_imp.h.

7.2.2.79 operator-() [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >  [inline]
```

Geometric difference of multivector and scalar.

Definition at line 217 of file clifford_algebra_imp.h.

7.2.2.80 operator-() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator- (
            const Scalar_T & scr,
            const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Geometric difference of scalar and multivector.

Definition at line 228 of file clifford_algebra_imp.h.

7.2.2.81 operator/() [1/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Geometric quotient.

Definition at line 731 of file framed_multi_imp.h.

7.2.2.82 operator/() [2/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >
```

Geometric quotient.

Definition at line 612 of file matrix_multi_imp.h.

References glucat::matrix::isnan(), and reframe().

7.2.2.83 operator/() [3/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Geometric quotient.

Definition at line 374 of file clifford_algebra_imp.h.

7.2.2.84 operator/() [4/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const Scalar_T & scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >  [inline]
```

Quotient of multivector and scalar.

Definition at line 348 of file clifford_algebra_imp.h.

7.2.2.85 operator/() [5/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator/ (
            const Scalar_T & scr,
            const Multivector< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Quotient of scalar and multivector.

Definition at line 359 of file clifford_algebra_imp.h.

7.2.2.86 operator<<() [1/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator<< (
            std::ostream & os,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream &
```

Write multivector to output.

Definition at line 1144 of file framed_multi_imp.h.

References scalar(), glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range(), and glucat::clifford_algebra< Scalar_T, Index_

7.2.2.87 operator<<() [2/5]

```
const index_t HI auto glucat::operator<< (
            std::ostream & os,
            const index_set< LO, HI > & ist) -> std::ostream &
```

References max_pos(), min_neg(), and sign_of_square().

7.2.2.88 operator<<() [3/5]

```
template<const index_t LO, const index_t HI>
auto glucat::operator<< (
            std::ostream & os,
            const index_set< LO, HI > & ist) -> std::ostream &
```

Write out index set.

Definition at line 611 of file index_set_imp.h.

7.2.2.89 operator<<() [4/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator<< (
            std::ostream & os,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::ostream &  [inline]
```

Write multivector to output.

Definition at line 952 of file matrix_multi_imp.h.

7.2.2.90 operator<<() [5/5]

```
template<typename Scalar_T, const index_t LO, const index_t HI>
auto glucat::operator<< (
             std::ostream & os,
             const std::pair< const index_set< LO, HI >, Scalar_T > & term) -> std::ostream &
```

Write term to output.

Definition at line 1204 of file framed_multi_imp.h.

References glucat::numeric_traits< Scalar_T >::to_double().

7.2.2.91 operator>>() [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator>> (
             std::istream & s,
             framed_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream &
```

Read multivector from input.

Definition at line 1243 of file framed_multi_imp.h.

References glucat::index_set< LO, HI >::index_set().

7.2.2.92 operator>>() [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::operator>> (
             std::istream & s,
             index_set< LO, HI > & ist) -> std::istream &
```

Read in index set.

Definition at line 634 of file index_set_imp.h.

7.2.2.93 operator>>() [3/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator>> (
             std::istream & s,
             matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> std::istream &  [inline]
```

Read multivector from input.

Definition at line 963 of file matrix_multi_imp.h.

7.2.2.94 operator^() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >
```

Outer product.

Definition at line 416 of file framed_multi_imp.h.

References _GLUCAT_HASH_SIZE_T, and crd_of_mult().

7.2.2.95 operator^() [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator^ (
            const index_set< LO, HI > & lhs,
            const index_set< LO, HI > & rhs) -> const index_set< LO, HI >  [inline]
```

Symmetric set difference: exclusive or.

Definition at line 161 of file index_set_imp.h.

7.2.2.96 operator^() [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Outer product.

Definition at line 541 of file matrix_multi_imp.h.

7.2.2.97 operator^() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator^ (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Outer product.

Definition at line 292 of file clifford_algebra_imp.h.

7.2.2.98 operator"|() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const framed_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Transformation via twisted adjoint action.

Definition at line 756 of file framed_multi_imp.h.

7.2.2.99 operator"|() [2/4]

```
template<const index_t LO, const index_t HI>
auto glucat::operator| (
            const index_set< LO, HI > & lhs,
            const index_set< LO, HI > & rhs) -> const index_set< LO, HI >  [inline]
```

Set union: or.

Definition at line 211 of file index_set_imp.h.

7.2.2.100 operator"|() [3/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [inline]
```

Transformation via twisted adjoint action.

Definition at line 714 of file matrix_multi_imp.h.

7.2.2.101 operator"|() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::operator| (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Transformation via twisted adjoint action.

Definition at line 389 of file clifford_algebra_imp.h.

7.2.2.102 outer_pow()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::outer_pow (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Outer product power of multivector.

Definition at line 470 of file clifford_algebra_imp.h.

References outer_pow().

Referenced by outer_pow().

7.2.2.103 pade_approx()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t
Size>
auto glucat::pade_approx (
            const std::array< Scalar_T, Size > & numer,
            const std::array< Scalar_T, Size > & denom,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & X) -> const matrix_multi< Scalar↩
_T, LO, HI, Tune_P >  [inline], [static]
```

Pade' approximation.

Definition at line 1242 of file matrix_multi_imp.h.

References pade_approx().

Referenced by matrix_sqrt(), pade_approx(), and pade_log().

7.2.2.104 pade_log()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pade_log (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & val) -> const matrix_multi<
Scalar_T, LO, HI, Tune_P >  [static]
```

Pade' approximation of log.

Definition at line 1890 of file matrix_multi_imp.h.

References pade::pade_log_denom< Scalar_T >::denom, glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), pade::pade_log_numer< Scalar_T >::numer, pade_approx(), and pade_log().

Referenced by cascade_log(), and pade_log().

7.2.2.105 pos_mod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::pos_mod (
             LHS_T lhs,
             RHS_T rhs) -> LHS_T  [inline]
```

Modulo function which works reliably for lhs < 0.

Definition at line 117 of file global.h.

References pos_mod().

Referenced by complexifier(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi(), glucat::framed_multi< Scalar_T
glucat::gen::generator_table< Matrix_T >::gen_vector(), offset_level(), glucat::gen::generator_table< Matrix_T >::operator()(),
and pos_mod().

7.2.2.106 pow() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
             const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
             const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> const Multivector< Scalar_T, LO,
HI, Tune_P >  [inline]
```

Multivector power of multivector.

Definition at line 446 of file clifford_algebra_imp.h.

References exp(), log(), and pow().

7.2.2.107 pow() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pow (
             const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
             int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Integer power of multivector.

Definition at line 407 of file clifford_algebra_imp.h.

References pow().

Referenced by matrix_sqrt(), pow(), and pow().

7.2.2.108 pure()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::pure (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Pure part.

Definition at line 505 of file clifford_algebra_imp.h.

References pure().

Referenced by pure().

7.2.2.109 quad()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::quad (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Scalar_T quadratic form == (rev(x)∗x)(0).

Definition at line 561 of file clifford_algebra_imp.h.

References quad().

Referenced by quad().

7.2.2.110 real()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::real (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Real part: synonym for scalar part.

Definition at line 486 of file clifford_algebra_imp.h.

References real().

Referenced by real().

7.2.2.111 reframe()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::reframe (
             const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
             const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs,
             matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs_reframed,
             matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs_reframed) -> const index_set< LO,
HI >  [inline]
```

Find a common frame for operands of a binary operator.

Definition at line 343 of file matrix_multi_imp.h.

References reframe().

Referenced by operator∗(), operator/(), and reframe().

7.2.2.112 reverse()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::reverse (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Reversion, eg. {1}∗{2} -> {2}∗{1}.

Definition at line 545 of file clifford_algebra_imp.h.

References reverse().

Referenced by reverse().

7.2.2.113 scalar()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::scalar (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> Scalar_T  [inline]
```

Scalar part.

Definition at line 478 of file clifford_algebra_imp.h.

References scalar().

Referenced by exp(), operator<<(), and scalar().

7.2.2.114 sign_of_square()

```
auto glucat::sign_of_square (
            index_t j) -> int  [inline]
```

Square of generator {j}.

Square of generator index j.

Definition at line 963 of file index_set_imp.h.

References sign_of_square().

Referenced by operator<<(), and sign_of_square().

7.2.2.115 sin() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Sine of multivector.

Definition at line 978 of file clifford_algebra_imp.h.

References complexifier(), and sin().

7.2.2.116 sin() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sin (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val,
            const Multivector< Scalar_T, LO, HI, Tune_P > & i,
            const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
```

Sine of multivector with specified complexifier.

Definition at line 955 of file clifford_algebra_imp.h.

References check_complex(), exp(), and sin().

Referenced by sin(), sin(), and tan().

7.2.2.117 sinh()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sinh (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Hyperbolic sine of multivector.

Definition at line 910 of file clifford_algebra_imp.h.

References exp(), and sinh().

Referenced by sinh(), and tanh().

7.2.2.118 sqrt() [1/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
             const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
             const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
             bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1722 of file framed_multi_imp.h.

References check_complex(), and sqrt().

7.2.2.119 sqrt() [2/4]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
             const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
             const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
             bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >
```

Square root of multivector with specified complexifier.

Definition at line 1657 of file matrix_multi_imp.h.

References check_complex(), glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan(), matrix_sqrt(), and sqrt().

7.2.2.120 sqrt() [3/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Square root of multivector.

Definition at line 682 of file clifford_algebra_imp.h.

References complexifier(), and sqrt().

7.2.2.121 sqrt() [4/4]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::sqrt (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val,
             const Multivector< Scalar_T, LO, HI, Tune_P > & i,
             const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Square root of multivector with specified complexifier.

Definition at line 674 of file clifford_algebra_imp.h.

References sqrt().

Referenced by acosh(), asinh(), crd_of_mult(), sqrt(), sqrt(), sqrt(), and sqrt().

7.2.2.122 star() [1/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
            const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T
```

Hestenes scalar product.

Definition at line 683 of file framed_multi_imp.h.

References star().

7.2.2.123 star() [2/3]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & lhs,
            const matrix_multi< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T  [inline]
```

Hestenes scalar product.

Definition at line 598 of file matrix_multi_imp.h.

References star().

7.2.2.124 star() [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T,
const index_t LO, const index_t HI, typename Tune_P>
auto glucat::star (
            const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
            const RHS< Scalar_T, LO, HI, Tune_P > & rhs) -> Scalar_T  [inline]
```

Hestenes scalar product.

Definition at line 337 of file clifford_algebra_imp.h.

References star().

Referenced by star(), star(), and star().

7.2.2.125 tan() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::tan (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Tangent of multivector.

Definition at line 1078 of file clifford_algebra_imp.h.

References complexifier(), and tan().

7.2.2.126 tan() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::tan (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val,
             const Multivector< Scalar_T, LO, HI, Tune_P > & i,
             const bool prechecked = false) -> const Multivector< Scalar_T, LO, HI, Tune_P >
[inline]
```

Tangent of multivector with specified complexifier.

Definition at line 1059 of file clifford_algebra_imp.h.

References check_complex(), cos(), sin(), and tan().

Referenced by tan(), and tan().

7.2.2.127 tanh()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::tanh (
             const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const Multivector< Scalar↩
_T, LO, HI, Tune_P >  [inline]
```

Hyperbolic tangent of multivector.

Definition at line 1015 of file clifford_algebra_imp.h.

References cosh(), sinh(), and tanh().

Referenced by tanh().

7.2.2.128 to_demote()

```
template<typename Scalar_T>
auto glucat::to_demote (
             const Scalar_T & val) -> typename numeric_traits< Scalar_T >::demoted::type
[inline]
```

Cast to demote.

Definition at line 135 of file scalar_imp.h.

References to_demote(), and glucat::numeric_traits< Scalar_T >::to_scalar_t().

Referenced by to_demote().

7.2.2.129 to_promote()

```
template<typename Scalar_T>
auto glucat::to_promote (
            const Scalar_T & val) -> typename numeric_traits< Scalar_T >::promoted::type
[inline]
```

Cast to promote.

Definition at line 125 of file scalar_imp.h.

References to_promote(), and glucat::numeric_traits< Scalar_T >::to_scalar_t().

Referenced by to_promote().

7.2.2.130 try_catch() [1/2]

```
int glucat::try_catch (
            intfn f)
```

Exception catching for functions returning int.

Definition at line 49 of file try_catch.h.

References try_catch().

Referenced by glucat::control_t::call(), glucat::control_t::call(), try_catch(), and try_catch().

7.2.2.131 try_catch() [2/2]

```
int glucat::try_catch (
            intintfn f,
            int arg)
```

Exception catching for functions of int returning int.

Definition at line 64 of file try_catch.h.

References try_catch().

7.2.2.132 vector_part()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::vector_part (
            const Multivector< Scalar_T, LO, HI, Tune_P > & val) -> const std::vector< Scalar↩
_T >  [inline]
```

Vector part of multivector, as a vector_t with respect to frame().

Definition at line 529 of file clifford_algebra_imp.h.

References vector_part().

Referenced by vector_part().

7.2.3 Variable Documentation

7.2.3.1 BITS_PER_SET_VALUE

```
const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits
```

Number of bits in set_value_t.

Definition at line 103 of file global.h.

Referenced by _GLUCAT_CTAssert().

7.2.3.2 clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↩
truncation = std::numeric_limits<Scalar_T>::epsilon()
```

Default for truncation.

Definition at line 74 of file clifford_algebra_imp.h.

7.2.3.3 DEFAULT_HI

```
const index_t glucat::DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)
```

Default highest index in an index set.

Definition at line 111 of file global.h.

Referenced by _GLUCAT_CTAssert().

7.2.3.4 l_ln2

```
const long double glucat::l_ln2 = 0.693147180559945309417232121458176 6L  [static]
```

Definition at line 44 of file long_double.h.

Referenced by glucat::numeric_traits< Scalar_T >::ln_2().

7.2.3.5 l_pi

```
const long double glucat::l_pi = 3.14159265358979323846264338327950 29L  [static]
```

Definition at line 43 of file long_double.h.

Referenced by glucat::numeric_traits< Scalar_T >::pi().

7.2.3.6 MS_PER_S

```
const double glucat::MS_PER_S = 1000.0
```

Timing constant: deprecated here - moved to test/timing.h.

Definition at line 83 of file global.h.

7.2.3.7 Tuning_Fast_Basis_Max_Count

```
const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
```

Definition at line 92 of file tuning.h.

7.2.3.8 Tuning_Fast_CR_Sqrt_Max_Steps

```
const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
```

Definition at line 88 of file tuning.h.

7.2.3.9 Tuning_Fast_DB_Sqrt_Max_Steps

```
const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
```

Definition at line 89 of file tuning.h.

7.2.3.10 Tuning_Fast_Div_Max_Steps

```
const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
```

Definition at line 87 of file tuning.h.

7.2.3.11 Tuning_Fast_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
```

Definition at line 93 of file tuning.h.

7.2.3.12 Tuning_Fast_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
```

Definition at line 94 of file tuning.h.

7.2.3.13 Tuning_Fast_Log_Max_Inner_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
```

Definition at line 91 of file tuning.h.

7.2.3.14 Tuning_Fast_Log_Max_Outer_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
```

Definition at line 90 of file tuning.h.

7.2.3.15 Tuning_Fast_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
```

Definition at line 86 of file tuning.h.

7.2.3.16 Tuning_Fast_Products_Size_Threshold

```
const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0
```

Definition at line 95 of file tuning.h.

7.2.3.17 Tuning_Int_Digits

```
const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
```

Definition at line 36 of file tuning.h.

7.2.3.18 Tuning_Max_Threshold

```
const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning_Int_Digits
```

Definition at line 37 of file tuning.h.

7.2.3.19 Tuning_Naive_Basis_Max_Count

```
const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
```

Definition at line 65 of file tuning.h.

7.2.3.20 Tuning_Naive_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 66 of file tuning.h.

7.2.3.21 Tuning_Naive_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 67 of file tuning.h.

7.2.3.22 Tuning_Naive_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
```

Definition at line 64 of file tuning.h.

7.2.3.23 Tuning_Slow_Basis_Max_Count

```
const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
```

Definition at line 42 of file tuning.h.

7.2.3.24 Tuning_Slow_Fast_Size_Threshold

```
const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 43 of file tuning.h.

7.2.3.25 Tuning_Slow_Inv_Fast_Dim_Threshold

```
const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 44 of file tuning.h.

7.2.3.26 Tuning_Slow_Mult_Matrix_Threshold

```
const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
```

Definition at line 41 of file tuning.h.

7.2.3.27 Tuning_Slow_Products_Size_Threshold

```
const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 45 of file tuning.h.

7.3 glucat::gen Namespace Reference

Classes

- class generator_table

  Table of generators for specific signatures.

Typedefs

- using signature_t = std::pair<index_t, index_t>

  A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min().

Variables

- static const std::array< index_t, 8 > offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}

  Offsets between the current signature and that of the real superalgebra.

7.3.1 Typedef Documentation

7.3.1.1 signature_t

```
using glucat::gen::signature_t = std::pair<index_t, index_t>
```

A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min().

Definition at line 48 of file generation.h.

7.3.2 Variable Documentation

7.3.2.1 offset_to_super

```
const std::array<index_t, 8> glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}  [static]
```

Offsets between the current signature and that of the real superalgebra.

Definition at line 86 of file generation.h.

7.4 glucat::matrix Namespace Reference

Classes

- struct eig_genus

  Structure containing classification of eigenvalues.

Typedefs

- using eig_case_t

  Classification of eigenvalues of a matrix.

Functions

- template<typename LHS_T, typename RHS_T>
  auto kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Kronecker tensor product of matrices - as per Matlab kron.
- template<typename LHS_T, typename RHS_T>
  auto mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Sparse Kronecker tensor product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  auto nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T

  Left inverse of Kronecker product.
- template<typename LHS_T, typename RHS_T>
  auto signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Left inverse of Kronecker product where lhs is a signed permutation matrix.
- template<typename Matrix_T>
  auto nnz (const Matrix_T &m) -> typename Matrix_T::size_type

  Number of non-zeros.
- template<typename Matrix_T>
  auto isinf (const Matrix_T &m) -> bool

  Infinite.
- template<typename Matrix_T>
  auto isnan (const Matrix_T &m) -> bool

  Not a Number.
- template<typename Matrix_T>
  auto unit (const typename Matrix_T::size_type n) -> const Matrix_T

  Unit matrix - as per Matlab eye.
- template<typename LHS_T, typename RHS_T>
  auto mono_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  auto sparse_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of sparse matrices.
- template<typename LHS_T, typename RHS_T>
  auto prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of matrices.
- template<typename Scalar_T, typename LHS_T, typename RHS_T>
  auto inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T

  *Inner product: sum(x(i,j)*y(i,j))/x.nrows().*
- template<typename Matrix_T>
  auto norm_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type

  Square of Frobenius norm.
- template<typename Matrix_T>
  auto trace (const Matrix_T &val) -> typename Matrix_T::value_type

  Matrix trace.

- template<typename Matrix_T>
  auto eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >

  Eigenvalues of a matrix.
- template<typename Matrix_T>
  auto classify_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >

  Classify the eigenvalues of a matrix.
- template<typename LHS_T, typename RHS_T>
  void nork_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)

  Utility routine for nork: calculate result for a range of indices.
- template<typename Matrix_T>
  static auto to_blaze (const Matrix_T &val) -> blaze::DynamicMatrix< double, blaze::rowMajor >

  Convert matrix to Blaze format.

7.4.1 Typedef Documentation

7.4.1.1 eig_case_t

```
using glucat::matrix::eig_case_t
```

Initial value:

```
enum {
     safe_eigs,
     neg_real_eigs,
     both_eigs}
```

Classification of eigenvalues of a matrix.

Definition at line 133 of file matrix.h.

7.4.2 Function Documentation

7.4.2.1 classify_eigenvalues()

```
template<typename Matrix_T>
auto glucat::matrix::classify_eigenvalues (
             const Matrix_T & val) -> eig_genus< Matrix_T >
```

Classify the eigenvalues of a matrix.

Definition at line 492 of file matrix_imp.h.

References classify_eigenvalues(), eigenvalues(), epsilon, glucat::matrix::eig_genus< Matrix_T >::m_eig_case, glucat::matrix::eig_genus< Matrix_T >::m_is_singular, glucat::matrix::eig_genus< Matrix_T >::m_safe_arg, glucat::numeric_traits< Scalar_T >::pi(), and glucat::numeric_traits< Scalar_T >::to_double().

Referenced by classify_eigenvalues(), glucat::matrix_log(), and glucat::matrix_sqrt().

7.4.2.2 eigenvalues()

```
template<typename Matrix_T>
auto glucat::matrix::eigenvalues (
            const Matrix_T & val) -> std::vector< std::complex< double > >
```

Eigenvalues of a matrix.

Definition at line 464 of file matrix_imp.h.

References eigenvalues(), and to_blaze().

Referenced by classify_eigenvalues(), and eigenvalues().

7.4.2.3 inner()

```
template<typename Scalar_T, typename LHS_T, typename RHS_T>
auto glucat::matrix::inner (
            const LHS_T & lhs,
            const RHS_T & rhs) -> Scalar_T
```

Inner product: sum(x(i,j)∗y(i,j))/x.nrows().

Inner product: sum(lhs(i,j)∗rhs(i,j))/lhs.nrows().

Definition at line 368 of file matrix_imp.h.

References inner().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), and inner().

7.4.2.4 isinf()

```
template<typename Matrix_T>
auto glucat::matrix::isinf (
            const Matrix_T & m) -> bool
```

Infinite.

Definition at line 270 of file matrix_imp.h.

References glucat::numeric_traits< Scalar_T >::isInf(), and isinf().

Referenced by isinf().

7.4.2.5 isnan()

```
template<typename Matrix_T>
auto glucat::matrix::isnan (
            const Matrix_T & m) -> bool
```

Not a Number.

Definition at line 287 of file matrix_imp.h.

References glucat::numeric_traits< Scalar_T >::isNaN(), and isnan().

Referenced by isnan(), and glucat::operator/().

7.4.2.6 kron()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::kron (
             const LHS_T & lhs,
             const RHS_T & rhs) -> const RHS_T
```

Kronecker tensor product of matrices - as per Matlab kron.

Definition at line 78 of file matrix_imp.h.

References kron().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast(), and kron().

7.4.2.7 mono_kron()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_kron (
             const LHS_T & lhs,
             const RHS_T & rhs) -> const RHS_T
```

Sparse Kronecker tensor product of monomial matrices.

Definition at line 114 of file matrix_imp.h.

References mono_kron().

Referenced by glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1(), and mono_kron().

7.4.2.8 mono_prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::mono_prod (
             const ublas::matrix_expression< LHS_T > & lhs,
             const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↩
_type
```

Product of monomial matrices.

Definition at line 315 of file matrix_imp.h.

References mono_prod().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element(), glucat::gen::generator_table< Matrix_T >::gen_f
glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4(), glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1(),
and mono_prod().

7.4.2.9 nnz()

```
template<typename Matrix_T>
auto glucat::matrix::nnz (
             const Matrix_T & m) -> typename Matrix_T::size_type
```

Number of non-zeros.

Definition at line 253 of file matrix_imp.h.

References nnz().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), and nnz().

7.4.2.10 nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::nork (
             const LHS_T & lhs,
             const RHS_T & rhs,
             const bool mono = true) -> const RHS_T
```

Left inverse of Kronecker product.

Definition at line 177 of file matrix_imp.h.

References nork(), nork_range(), and norm_frob2().

Referenced by nork().

7.4.2.11 nork_range()

```
template<typename LHS_T, typename RHS_T>
void glucat::matrix::nork_range (
             RHS_T & result,
             const typename LHS_T::const_iterator2 lhs_it2,
             const RHS_T & rhs,
             const typename RHS_T::size_type res_s1,
             const typename RHS_T::size_type res_s2)
```

Utility routine for nork: calculate result for a range of indices.

Definition at line 147 of file matrix_imp.h.

References nork_range(), and glucat::numeric_traits< Scalar_T >::to_scalar_t().

Referenced by nork(), nork_range(), and signed_perm_nork().

7.4.2.12 norm_frob2()

```
template<typename Matrix_T>
auto glucat::matrix::norm_frob2 (
            const Matrix_T & val) -> typename Matrix_T::value_type
```

Square of Frobenius norm.

Definition at line 390 of file matrix_imp.h.

References glucat::numeric_traits< Scalar_T >::isNaN(), glucat::numeric_traits< Scalar_T >::NaN(), and norm_frob2().

Referenced by nork(), and norm_frob2().

7.4.2.13 prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::prod (
            const ublas::matrix_expression< LHS_T > & lhs,
            const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↩
_type  [inline]
```

Product of matrices.

Definition at line 356 of file matrix_imp.h.

References prod().

Referenced by prod().

7.4.2.14 signed_perm_nork()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::signed_perm_nork (
            const LHS_T & lhs,
            const RHS_T & rhs) -> const RHS_T
```

Left inverse of Kronecker product where lhs is a signed permutation matrix.

Definition at line 223 of file matrix_imp.h.

References nork_range(), and signed_perm_nork().

Referenced by glucat::fast(), and signed_perm_nork().

7.4.2.15 sparse_prod()

```
template<typename LHS_T, typename RHS_T>
auto glucat::matrix::sparse_prod (
            const ublas::matrix_expression< LHS_T > & lhs,
            const ublas::matrix_expression< RHS_T > & rhs) -> const typename RHS_T::expression↩
_type  [inline]
```

Product of sparse matrices.

Definition at line 345 of file matrix_imp.h.

References sparse_prod().

Referenced by sparse_prod().

7.4.2.16 to_blaze()

```
template<typename Matrix_T>
auto glucat::matrix::to_blaze (
            const Matrix_T & val) -> blaze::DynamicMatrix< double, blaze::rowMajor >  [static]
```

Convert matrix to Blaze format.

Definition at line 435 of file matrix_imp.h.

References to_blaze().

Referenced by eigenvalues(), and to_blaze().

7.4.2.17 trace()

```
template<typename Matrix_T>
auto glucat::matrix::trace (
            const Matrix_T & val) -> typename Matrix_T::value_type
```

Matrix trace.

Definition at line 411 of file matrix_imp.h.

References glucat::numeric_traits< Scalar_T >::isNaN(), glucat::numeric_traits< Scalar_T >::NaN(), and trace().

Referenced by trace().

7.4.2.18 unit()

```
template<typename Matrix_T>
auto glucat::matrix::unit (
            const typename Matrix_T::size_type n) -> const Matrix_T  [inline]
```

Unit matrix - as per Matlab eye.

Definition at line 305 of file matrix_imp.h.

References unit().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element(), glucat::fast(), glucat::framed_multi< Scalar_T, LO
glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1(), glucat::gen::generator_table< Matrix_T >::gen_vector(),
and unit().

7.5 glucat::timing Namespace Reference

Functions

- static double elapsed (clock_t cpu_time)

  Elapsed time in milliseconds.

Variables

- const double MS_PER_SEC = 1000.0

  Timing constant: milliseconds per second.
- const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)

  Timing constant: milliseconds per clock.
- const int EXTRA_TRIALS = 2

  Timing constant: trial expansion factor.

7.5.1 Function Documentation

7.5.1.1 elapsed()

```
double glucat::timing::elapsed (
            clock_t cpu_time)  [inline], [static]
```

Elapsed time in milliseconds.

Definition at line 51 of file timing.h.

References elapsed(), and MS_PER_CLOCK.

Referenced by elapsed().

7.5.2 Variable Documentation

7.5.2.1 EXTRA_TRIALS

```
const int glucat::timing::EXTRA_TRIALS = 2
```

Timing constant: trial expansion factor.

Definition at line 45 of file timing.h.

7.5.2.2 MS_PER_CLOCK

```
const double glucat::timing::MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)
```

Timing constant: milliseconds per clock.

Definition at line 42 of file timing.h.

Referenced by elapsed().

7.5.2.3 MS_PER_SEC

```
const double glucat::timing::MS_PER_SEC = 1000.0
```

Timing constant: milliseconds per second.

Definition at line 39 of file timing.h.

7.6 pade Namespace Reference

Classes

- struct pade_sqrt_numer

  Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct pade_sqrt_denom

  Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct pade_sqrt_numer< float >
- struct pade_sqrt_denom< float >
- struct pade_sqrt_numer< long double >
- struct pade_sqrt_denom< long double >
- struct pade_sqrt_numer< dd_real >
- struct pade_sqrt_denom< dd_real >
- struct pade_sqrt_numer< qd_real >
- struct pade_sqrt_denom< qd_real >
- struct pade_log_numer

  Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct pade_log_denom

  Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct pade_log_numer< float >
- struct pade_log_denom< float >
- struct pade_log_numer< long double >
- struct pade_log_denom< long double >
- struct pade_log_numer< dd_real >
- struct pade_log_denom< dd_real >
- struct pade_log_numer< qd_real >
- struct pade_log_denom< qd_real >

Variables

- template<typename Scalar_T>
  const pade_sqrt_numer< Scalar_T >::array pade_sqrt_numer< Scalar_T >::numer
- template<typename Scalar_T>
  const pade_sqrt_denom< Scalar_T >::array pade_sqrt_denom< Scalar_T >::denom
- const pade_sqrt_numer< float >::array pade_sqrt_numer< float >::numer
- const pade_sqrt_denom< float >::array pade_sqrt_denom< float >::denom
- const pade_sqrt_numer< longdouble >::array pade_sqrt_numer< longdouble >::numer
- const pade_sqrt_denom< longdouble >::array pade_sqrt_denom< longdouble >::denom
- const pade_sqrt_numer< dd_real >::array pade_sqrt_numer< dd_real >::numer
- const pade_sqrt_denom< dd_real >::array pade_sqrt_denom< dd_real >::denom
- const pade_sqrt_numer< qd_real >::array pade_sqrt_numer< qd_real >::numer
- const pade_sqrt_denom< qd_real >::array pade_sqrt_denom< qd_real >::denom
- template<typename Scalar_T>
  const pade_log_numer< Scalar_T >::array pade_log_numer< Scalar_T >::numer
- template<typename Scalar_T>
  const pade_log_denom< Scalar_T >::array pade_log_denom< Scalar_T >::denom
- const pade_log_numer< float >::array pade_log_numer< float >::numer
- const pade_log_denom< float >::array pade_log_denom< float >::denom
- const pade_log_numer< longdouble >::array pade_log_numer< longdouble >::numer
- const pade_log_denom< longdouble >::array pade_log_denom< longdouble >::denom
- const pade_log_numer< dd_real >::array pade_log_numer< dd_real >::numer
- const pade_log_denom< dd_real >::array pade_log_denom< dd_real >::denom
- const pade_log_numer< qd_real >::array pade_log_numer< qd_real >::numer
- const pade_log_denom< qd_real >::array pade_log_denom< qd_real >::denom

7.6.1 Variable Documentation

7.6.1.1 pade_log_denom< dd_real >::denom

```
const pade_log_denom<dd_real>::array pade::pade_log_denom< dd_real >::denom
```

Initial value:

```
                                                                =
{
        dd_real("1"),                                              dd_real("21")/dd_real("2"),
     dd_real("2100")/dd_real("41"),                              dd_real("12635")/dd_real("82"),
   dd_real("341145")/dd_real("1066"),                           dd_real("1037799")/dd_real("2132"),
 dd_real("11069856")/dd_real("19721"),                          dd_real("9883800")/dd_real("19721"),
  dd_real("6918660")/dd_real("19721"),                           dd_real("293930")/dd_real("1517"),
  dd_real("1410864")/dd_real("16687"),                            dd_real("88179")/dd_real("3034"),
   dd_real("734825")/dd_real("94054"),                           dd_real("305235")/dd_real("188108"),
   dd_real("348840")/dd_real("1363783"),                          dd_real("40698")/dd_real("1363783"),
     dd_real("6783")/dd_real("2727566"),                           dd_real("9975")/dd_real("70916716"),
      dd_real("266")/dd_real("53187537"),                             dd_real("7")/dd_real("70916716"),
        dd_real("7")/dd_real("8155422340"),                           dd_real("1")/dd_real("538257874440")
}
```

Definition at line 1815 of file matrix_multi_imp.h.

7.6.1.2 pade_log_denom< float >::denom

```
const pade_log_denom<float>::array pade::pade_log_denom< float >::denom
```

Initial value:

```
                                                    =
{
    1.0,            9.0/2.0,       144.0/17.0,   147.0/17.0,
  441.0/85.0,      63.0/34.0,       84.0/221.0,    9.0/221.0,
    9.0/4862.0,     1.0/48620.0
}
```

Definition at line 1753 of file matrix_multi_imp.h.

7.6.1.3 pade_log_denom< longdouble >::denom

```
const pade_log_denom<longdouble>::array pade::pade_log_denom< longdouble >::denom
```

Initial value:

```
                                                                =
{
       1.0L,                      17.0L/2.0L,                   1088.0L/33.0L,              850.0L/11.0L,
   41650.0L/341.0L,           140777.0L/1023.0L,            1126216.0L/9889.0L,         63206.0L/899.0L,
  790075.0L/24273.0L,          60775.0L/5394.0L,              38896.0L/13485.0L,
      21658.0L/40455.0L,
   21658.0L/310155.0L,          4165.0L/682341.0L,             680.0L/2047023.0L,
      34.0L/3411705.0L,
      17.0L/129644790.0L,          1.0L/2333606220
}
```

Definition at line 1780 of file matrix_multi_imp.h.

7.6.1.4 pade_log_denom< qd_real >::denom

```
const pade_log_denom<qd_real>::array pade::pade_log_denom< qd_real >::denom
```

Initial value:

```
                                                                =
{
             qd_real("1"),
      qd_real("33")/qd_real("2"),
           qd_real("8448")/qd_real("65"),
      qd_real("42284")/qd_real("65"),
          qd_real("211420")/qd_real("91"),
      qd_real("573562")/qd_real("91"),
        qd_real("32119472")/qd_real("2379"),
      qd_real("92917044")/qd_real("3965"),
       qd_real("603960786")/qd_real("17995"),
      qd_real("144626625")/qd_real("3599"),
      qd_real("2776831200")/qd_real("68381"),
      qd_real("16692542100")/qd_real("478667"),
     qd_real("12241197540")/qd_real("478667"),
      qd_real("1098569010")/qd_real("68381"),
     qd_real("31387686000")/qd_real("3624193"),
      qd_real("9939433900")/qd_real("2479711"),
     qd_real("67091178825")/qd_real("42155087"),
      qd_real("2683647153")/qd_real("4959422"),
     qd_real("19083713088")/qd_real("121505839"),
      qd_real("4708152900")/qd_real("121505839"),
       qd_real("941630580")/qd_real("116546417"),
      qd_real("88704330")/qd_real("62755763"),
        qd_real("12902448")/qd_real("62755763"),
      qd_real("1542684")/qd_real("62755763"),
         qd_real("6427850")/qd_real("2698497809"),
      qd_real("3471039")/qd_real("18889484663"),
         qd_real("8544096")/qd_real("774468871183"),
      qd_real("39556")/qd_real("79027435835"),
          qd_real("118668")/qd_real("7191496660985"),
      qd_real("10230")/qd_real("27327687311743"),
            qd_real("5456")/qd_real("1011124430534491"),
      qd_real("44")/qd_real("1011124430534491"),
             qd_real("11")/qd_real("70778710137414370"),
      qd_real("1")/qd_real("7219428434016265740")
}
```

Definition at line 1862 of file matrix_multi_imp.h.

7.6.1.5 pade_log_denom< Scalar_T >::denom

```
template<typename Scalar_T>
const pade_log_denom<Scalar_T>::array pade::pade_log_denom< Scalar_T >::denom
```

Initial value:

```
                                                                =
{
       1.0,                    13.0/2.0,              468.0/25.0,         1573.0/50.0,
    1573.0/46.0,            11583.0/460.0,         10296.0/805.0,         2574.0/575.0,
   11583.0/10925.0,           143.0/874.0,           572.0/37145.0,       117.0/148580.0,
      13.0/742900.0,            1.0/10400600.0
}
```

Definition at line 1727 of file matrix_multi_imp.h.

7.6.1.6 pade_log_numer< dd_real >::numer

```
const pade_log_numer<dd_real>::array pade::pade_log_numer< dd_real >::numer
```

Initial value:

```
                                                                =
{
        dd_real("0"),                                  dd_real("1"),
```

```
       dd_real("10"),                                dd_real("22781")/dd_real("492"),
    dd_real("21603")/dd_real("164"),               dd_real("5492649")/dd_real("21320"),
   dd_real("978724")/dd_real("2665"),              dd_real("4191605")/dd_real("10619"),
 dd_real("12874933")/dd_real("39442"),            dd_real("11473457")/dd_real("54612"),
  dd_real("2406734")/dd_real("22755"),           dd_real("166770367")/dd_real("4004880"),
 dd_real("30653165")/dd_real("2402928"),         dd_real("647746389")/dd_real("215195552"),
 dd_real("25346331")/dd_real("47074027"),        dd_real("278270613")/dd_real("3900419380"),
dd_real("105689791")/dd_real("15601677520"),     dd_real("606046475")/dd_real("1379188292768"),
   dd_real("969715")/dd_real("53502994116"),      dd_real("11098301")/dd_real("26204577562592"),
   dd_real("118999")/dd_real("26204577562592"), dd_real("18858053")/dd_real("1392249205900512960")
}
```

Definition at line 1795 of file matrix_multi_imp.h.

7.6.1.7 pade_log_numer< float >::numer

```
const pade_log_numer<float>::array pade::pade_log_numer< float >::numer
```

Initial value:

```
                                                                =
{
    0.0,            1.0,            4.0,       1337.0/204.0,
  385.0/68.0,    1879.0/680.0,    193.0/255.0,  197.0/1820.0,
  419.0/61880.0, 7129.0/61261200.0
}
```

Definition at line 1741 of file matrix_multi_imp.h.

7.6.1.8 pade_log_numer< longdouble >::numer

```
const pade_log_numer<longdouble>::array pade::pade_log_numer< longdouble >::numer
```

Initial value:

```
                                                                =
{
       0.0L,                        1.0L,                             8.0L,                      3835.0L/132.0L,
    8365.0L/132.0L,          11363807.0L/122760.0L,             162981.0L/1705.0L,
      9036157.0L/125860.0L,
18009875.0L/453096.0L,       44211925.0L/2718576.0L,           4149566.0L/849555.0L,
      16973929.0L/16020180.0L,
  172459.0L/1068012.0L,     116317061.0L/7025382936.0L,      19679783.0L/18441630207.0L,
      23763863.0L/614721006900.0L,
   50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
}
```

Definition at line 1766 of file matrix_multi_imp.h.

7.6.1.9 pade_log_numer< qd_real >::numer

```
const pade_log_numer<qd_real>::array pade::pade_log_numer< qd_real >::numer
```

Initial value:

```
                                                                =
{
             qd_real("0"),                                                          qd_real("1"),
             qd_real("16"),
      qd_real("95201")/qd_real("780"),
           qd_real("30721")/qd_real("52"),
      qd_real("7416257")/qd_real("3640"),
         qd_real("1039099")/qd_real("195"),
      qd_real("6097772319")/qd_real("555100"),
      qd_real("1564058073")/qd_real("85400"),
      qd_real("30404640205")/qd_real("1209264"),
       qd_real("725351278")/qd_real("25193"),
      qd_real("4092322670789")/qd_real("147429436"),
```

```
   qd_real("4559713849589")/qd_real("201040140"),
      qd_real("5049361751189")/qd_real("320023080"),
     qd_real("74979677195")/qd_real("8000577"),
      qd_real("16569850691873")/qd_real("3481514244"),
   qd_real("1065906022369")/qd_real("515779888"),
      qd_real("335956770855841")/qd_real("438412904800"),
qd_real("1462444287585964")/qd_real("6041877844275"),
      qd_real("397242326339851")/qd_real("6122436215532"),
  qd_real("64211291334131")/qd_real("4373168725380"),
      qd_real("142322343550859")/qd_real("51080680851480"),
 qd_real("154355972958659")/qd_real("351179680853925"),
      qd_real("167483568676259")/qd_real("2937139148960100"),
   qd_real("4230788929433")/qd_real("704913395750424"),
      qd_real("197968763176019")/qd_real("392923948371995600"),
  qd_real("10537522306718")/qd_real("319250708052246425"),
      qd_real("236648286272519")/qd_real("144249197475035425500"),
 qd_real("260715545088119")/qd_real("4375558990076074573500"),
      qd_real("289596255666839")/qd_real("1928746402825533671998 80"),
   qd_real("8802625510547")/qd_real("3616399505297875634997 75"),
      qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
 qd_real("446033437968239")/qd_real("4645771460485862955743509 56000"),
      qd_real("53676090078349")/qd_real("4738686889695580214858 3797512000")
    }
```

Definition at line 1836 of file matrix_multi_imp.h.

7.6.1.10 pade_log_numer< Scalar_T >::numer

```
template<typename Scalar_T>
const pade_log_numer<Scalar_T>::array pade::pade_log_numer< Scalar_T >::numer
```

Initial value:

```
                                                                =
{
       0.0,                    1.0,                    6.0,            4741.0/300.0,
    1441.0/60.0,           107091.0/4600.0,          8638.0/575.0,    263111.0/40250.0,
  153081.0/80500.0,        395243.0/1101240.0,      28549.0/688275.0, 605453.0/228813200.0,
  785633.0/10296594000.0, 1145993.0/1873980108000.0
}
```

Definition at line 1710 of file matrix_multi_imp.h.

7.6.1.11 pade_sqrt_denom< dd_real >::denom

```
const pade_sqrt_denom<dd_real>::array pade::pade_sqrt_denom< dd_real >::denom
```

Initial value:

```
                                                                =
{
        dd_real("1"),                              dd_real("41")/dd_real("4"),
      dd_real("195")/dd_real("4"),                dd_real("9139")/dd_real("64"),
    dd_real("73815")/dd_real("256"),             dd_real("435897")/dd_real("1024"),
   dd_real("121737")/dd_real("256"),             dd_real("840565")/dd_real("2048"),
  dd_real("4539051")/dd_real("16384"),          dd_real("9641775")/dd_real("65536"),
  dd_real("4032015")/dd_real("65536"),         dd_real("84672315")/dd_real("4194304"),
 dd_real("86493225")/dd_real("16777216"),      dd_real("67863915")/dd_real("67108864"),
  dd_real("5014575")/dd_real("33554432"),       dd_real("4345965")/dd_real("268435456"),
  dd_real("5311735")/dd_real("4294967296"),     dd_real("1081575")/dd_real("17179869184"),
    dd_real("33649")/dd_real("17179869184"),       dd_real("8855")/dd_real("274877906944"),
      dd_real("231")/dd_real("1099511627776"),        dd_real("1")/dd_real("4398046511104")
}
```

Definition at line 1488 of file matrix_multi_imp.h.

7.6.1.12 pade_sqrt_denom< float >::denom

```
const pade_sqrt_denom<float>::array pade::pade_sqrt_denom< float >::denom
```

Initial value:

```
                                                                =
{
     1.0,             17.0/4.0,        15.0/2.0,      455.0/64.0,
  1001.0/256.0,    1287.0/1024.0,    231.0/1024.0,   165.0/8192.0,
    45.0/65536,       1.0/262144.0
}
```

Definition at line 1425 of file matrix_multi_imp.h.

7.6.1.13 pade_sqrt_denom< longdouble >::denom

```
const pade_sqrt_denom<longdouble>::array pade::pade_sqrt_denom< longdouble >::denom
```

Initial value:

```
                                                                =
{
      1.0L,                   33.0L/4.0L,             31.0L,               4495.0L/64.0L,
  27405.0L/256.0L,         118755.0L/1024.0L,       94185.0L/1024.0L,     444015.0L/8192.0L,
1562275.0L/65536.0L,     2042975.0L/262144.0L,    245157.0L/131072.0L,   676039.0L/2097152.0L,
 323323.0L/8388608.0L,    101745.0L/33554432.0L,    4845.0L/33554432.0L,    969.0L/268435456.0L,
    153.0L/4294967296.0L,      1.0L/17179869184.0L
}
```

Definition at line 1452 of file matrix_multi_imp.h.

7.6.1.14 pade_sqrt_denom< qd_real >::denom

```
const pade_sqrt_denom<qd_real>::array pade::pade_sqrt_denom< qd_real >::denom
```

Initial value:

```
  {                                                                 =
           qd_real("1"),                                                qd_real("65")/qd_real("4"),
         qd_real("126"),                                              qd_real("39711")/qd_real("64"),
      qd_real("557845")/qd_real("256"),                             qd_real("5949147")/qd_real("1024"),
    qd_real("12515965")/qd_real("1024"),                           qd_real("170574723")/qd_real("8192"),
   qd_real("1916797311")/qd_real("65536"),                         qd_real("8996462475")/qd_real("262144"),
   qd_real("4450881435")/qd_real("131072"),                       qd_real("59826782925")/qd_real("2097152"),
 qd_real("171503444385")/qd_real("8388608"),                     qd_real("420696483235")/qd_real("33554432"),
 qd_real("221120793075")/qd_real("33554432"),                    qd_real("797168807855")/qd_real("268435456"),
qd_real("4923689695575")/qd_real("4294967296"),                 qd_real("6499270398159")/qd_real("17179869184"),
 qd_real("456864812569")/qd_real("4294967296"),
      qd_real("3486599885395")/qd_real("137438953472"),
qd_real("2804116503573")/qd_real("549755813888"),
      qd_real("1886827875075")/qd_real("2199023255552"),
 qd_real("263012370465")/qd_real("2199023255552"),
      qd_real("2401417295555")/qd_real("17592186044416"),
 qd_real("176848560525")/qd_real("140737488355328"),
      qd_real("51538723353")/qd_real("562949953421312"),
   qd_real("1450433115")/qd_real("281474976710656"),
      qd_real("977699359")/qd_real("4503599627370496"),
    qd_real("118183439")/qd_real("18014398509481984"),
      qd_real("9652005")/qd_real("72057594037927936"),
       qd_real("121737")/qd_real("72057594037927936"),
      qd_real("6545")/qd_real("576460752303423488"),
          qd_real("561")/qd_real("1844674407370955161 6"),
      qd_real("1")/qd_real("73786976294838206464")
  }
```

Definition at line 1535 of file matrix_multi_imp.h.

7.6.1.15 pade_sqrt_denom< Scalar_T >::denom

```
template<typename Scalar_T>
const pade_sqrt_denom<Scalar_T>::array pade::pade_sqrt_denom< Scalar_T >::denom
```

Initial value:

```
                                                                =
{
      1.0,               25.0/4.0,          69.0/4.0,       1771.0/64.0,
   7315.0/256.0,      20349.0/1024.0,    4845.0/512.0,    12597.0/4096.0,
  21879.0/32768.0,    12155.0/131072.0,  1001.0/131072.0,  1365.0/4194304.0,
     91.0/16777216.0,     1.0/67108864.0
}
```

Definition at line 1399 of file matrix_multi_imp.h.

7.6.1.16 pade_sqrt_numer< dd_real >::numer

```
const pade_sqrt_numer<dd_real>::array pade::pade_sqrt_numer< dd_real >::numer
```

Initial value:

```
                                                                =
{
        dd_real("1"),                                  dd_real("43")/dd_real("4"),
      dd_real("215")/dd_real("4"),                   dd_real("10621")/dd_real("64"),
    dd_real("90687")/dd_real("256"),               dd_real("567987")/dd_real("1024"),
   dd_real("168861")/dd_real("256"),              dd_real("1246355")/dd_real("2048"),
  dd_real("7228859")/dd_real("16384"),           dd_real("16583853")/dd_real("65536"),
  dd_real("7538115")/dd_real("65536"),          dd_real("173376645")/dd_real("4194304"),
dd_real("195747825")/dd_real("16777216"),       dd_real("171655785")/dd_real("67108864"),
 dd_real("14375115")/dd_real("33554432"),       dd_real("14375115")/dd_real("268435456"),
 dd_real("20764055")/dd_real("4294967296"),      dd_real("5167525")/dd_real("17179869184"),
   dd_real("206701")/dd_real("17179869184"),       dd_real("76153")/dd_real("274877906944"),
     dd_real("3311")/dd_real("1099511627776") ,      dd_real("43")/dd_real("4398046511104")
}
```

Definition at line 1468 of file matrix_multi_imp.h.

7.6.1.17 pade_sqrt_numer< float >::numer

```
const pade_sqrt_numer<float>::array pade::pade_sqrt_numer< float >::numer
```

Initial value:

```
                                                                =
{
     1.0,            19.0/4.0,         19.0/2.0,       665.0/64.0,
  1729.0/256.0,    2717.0/1024.0,    627.0/1024.0,    627.0/8192.0,
   285.0/65536.0,    19.0/262144.0
}
```

Definition at line 1413 of file matrix_multi_imp.h.

7.6.1.18 pade_sqrt_numer< longdouble >::numer

```
const pade_sqrt_numer<longdouble>::array pade::pade_sqrt_numer< longdouble >::numer
```

Initial value:

```
                                                                =
{
      1.0L,                   35.0L/4.0L,             35.0L,               5425.0L/64.0L,
  35525.0L/256.0L,         166257.0L/1024.0L,      143325.0L/1024.0L,     740025.0L/8192.0L,
2877875.0L/65536.0L,     4206125.0L/262144.0L,    572033.0L/131072.0L,  1820105.0L/2097152.0L,
1028755.0L/8388608.0L,    395675.0L/33554432.0L,   24225.0L/33554432.0L,   6783.0L/268435456.0L,
   1785.0L/4294967296.0L,     35.0L/17179869184.0L
}
```

Definition at line 1438 of file matrix_multi_imp.h.

7.6.1.19 pade_sqrt_numer< qd_real >::numer

```
const pade_sqrt_numer<qd_real>::array pade::pade_sqrt_numer< qd_real >::numer
```

Initial value:

```
                                                                    =
{
           qd_real("1"),                                                qd_real("67")/qd_real("4"),
         qd_real("134"),                                              qd_real("43617")/qd_real("64"),
       qd_real("633485")/qd_real("256"),                            qd_real("6992857")/qd_real("1024"),
     qd_real("15246721")/qd_real("1024"),                         qd_real("215632197")/qd_real("8192"),
   qd_real("2518145487")/qd_real("65536"),                      qd_real("12301285425")/qd_real("262144"),
   qd_real("6344873535")/qd_real("131072"),                     qd_real("89075432355")/qd_real("2097152"),
 qd_real("267226297065")/qd_real("8388608"),                   qd_real("687479618945")/qd_real("33554432"),
 qd_real("379874182975")/qd_real("33554432"),                 qd_real("1443521895305")/qd_real("268435456"),
qd_real("9425348845815")/qd_real("4294967296"),              qd_real("13195488384141")/qd_real("17179869184"),
 qd_real("987417498133")/qd_real("4294967296"),               qd_real("8055248011085")/qd_real("137438953472"),
qd_real("6958363175533")/qd_real("549755813888"),             qd_real("5056698705201")/qd_real("2199023255552"),
 qd_real("766166470485")/qd_real("2199023255552"),
      qd_real("766166470485")/qd_real("17592186044416"),
 qd_real("623623871325")/qd_real("140737488355328"),
      qd_real("203123203803")/qd_real("562949953421312"),
   qd_real("6478601247")/qd_real("281474976710656"),
      qd_real("5038912081")/qd_real("4503599627370496"),
    qd_real("719844583")/qd_real("18014398509481984"),
      qd_real("71853815")/qd_real("72057594037927936"),
      qd_real("1165197")/qd_real("72057594037927936"),
      qd_real("87703")/qd_real("576460752303423488"),
        qd_real("12529")/qd_real("18446744073709551616"),
      qd_real("67")/qd_real("73786976294838206464")
}
```

Definition at line 1509 of file matrix_multi_imp.h.

7.6.1.20 pade_sqrt_numer< Scalar_T >::numer

```
template<typename Scalar_T>
const pade_sqrt_numer<Scalar_T>::array pade::pade_sqrt_numer< Scalar_T >::numer
```

Initial value:

```
                                                    =
{
      1.0,              27.0/4.0,         81.0/4.0,       2277.0/64.0,
  10395.0/256.0,      32319.0/1024.0,    8721.0/512.0,    26163.0/4096.0,
  53703.0/32768.0,    36465.0/131072.0,  3861.0/131072.0,  7371.0/4194304.0,
    819.0/16777216.0,    27.0/67108864.0
}
```

Definition at line 1382 of file matrix_multi_imp.h.

7.7 PyClical Namespace Reference

Classes

- class index_set
- class clifford

Functions

- index_set_hidden_doctests ()
- clifford_hidden_doctests ()
- e (obj)
- istpq (p, q)
- _test ()

Variables

- __version__ = str(glucat_package_version,'utf-8')
- lhs
- rhs
- threshold = error_squared_tol(rhs) if threshold is None else threshold
- None
- tol = error_squared_tol(rhs) if tol is None else tol
- obj
- i
- ixt
- fill
- scalar_epsilon = epsilon
- float pi = atan(clifford(1.0)) * 4.0
- float tau = atan(clifford(1.0)) * 8.0
- cl = clifford
- ist = index_set
- ninf3 = e(4) + e(-1)
- nbar3 = e(4) - e(-1)

7.7.1 Function Documentation

7.7.1.1 _test()

```
_test () [protected]
```

Definition at line 2025 of file PyClical.pyx.

References _test().

Referenced by _test().

7.7.1.2 clifford_hidden_doctests()

```
clifford_hidden_doctests ()
```

```
Tests for functions that Doctest cannot see.

For clifford.__cinit__: Construct an object of type clifford.

>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2,index_set({1,2})))
2{1,2}
```

```
>>> print(clifford([2,3],index_set({1,2})))
2{1}+3{2}
>>> print(clifford([1,2]))
Traceback (most recent call last):
  ...
TypeError: Cannot initialize clifford object from <class 'list'>.
>>> print(clifford(None))
Traceback (most recent call last):
  ...
TypeError: Cannot initialize clifford object from <class 'NoneType'>.
>>> print(clifford(None,[1,2]))
Traceback (most recent call last):
  ...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
>>> print(clifford([1,2],[1,2]))
Traceback (most recent call last):
  ...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
>>> print(clifford(""))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string ''.
>>> print(clifford("{"))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string '{'.
>>> print(clifford("{1"))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string '{1'.
>>> print(clifford("+"))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string '+'.
>>> print(clifford("-"))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string '-'.
>>> print(clifford("{1}+"))
Traceback (most recent call last):
  ...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.

For clifford.__richcmp__: Compare objects of type clifford.

>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 1316 of file PyClical.pyx.

7.7.1.3 e()

```
e (
              obj)
```

```
Abbreviation for clifford(index_set(obj)).

>>> print(e(1))
{1}
>>> print(e(-1))
{-1}
>>> print(e(0))
1
```

Definition at line 1999 of file PyClical.pyx.

7.7.1.4 index_set_hidden_doctests()

```
index_set_hidden_doctests ()


Tests for functions that Doctest cannot see.

For index_set.__cinit__: Construct index_set.

>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set(""))
{}
>>> print(index_set("{"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{'.
>>> print(index_set("{1"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
>>> print(index_set("{1,2,100}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
>>> print(index_set({1,2,100}))
Traceback (most recent call last):
...
IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
>>> print(index_set([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize index_set object from <class 'list'>.

For index_set.__richcmp__: Compare two objects of class index_set.

>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
```

```
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
>>> None == index_set({1,2})
False
>>> None != index_set({1,2})
True
>>> None < index_set({1,2})
False
>>> None <= index_set({1,2})
False
>>> None > index_set({1,2})
False
>>> None >= index_set({1,2})
False
>>> index_set({1,2}) == None
False
>>> index_set({1,2}) != None
True
>>> index_set({1,2}) < None
False
>>> index_set({1,2}) <= None
False
>>> index_set({1,2}) > None
False
>>> index_set({1,2}) >= None
False
```

Definition at line 406 of file PyClical.pyx.

7.7.1.5 istpq()

```
istpq (
              p,
              q)
```

```
Abbreviation for index_set({-q,...p}).

>>> print(istpq(2,3))
{-3,-2,-1,1,2}
```

Definition at line 2012 of file PyClical.pyx.

7.7.2 Variable Documentation

7.7.2.1 __version__

```
PyClical.__version__ = str(glucat_package_version,'utf-8')  [private]
```

Definition at line 35 of file PyClical.pyx.

7.7.2.2 cl

```
PyClical.cl = clifford
```

Definition at line 1973 of file PyClical.pyx.

7.7.2.3 fill

```
PyClical.fill
```

Definition at line 1927 of file PyClical.pyx.

7.7.2.4 i

```
PyClical.i
```

Definition at line 1654 of file PyClical.pyx.

7.7.2.5 ist

```
PyClical.ist = index_set
```

Definition at line 1991 of file PyClical.pyx.

7.7.2.6 ixt

```
PyClical.ixt
```

Definition at line 1927 of file PyClical.pyx.

7.7.2.7 lhs

```
PyClical.lhs
```

Definition at line 1422 of file PyClical.pyx.

7.7.2.8 nbar3

```
PyClical.nbar3 = e(4) - e(-1)
```

Definition at line 2022 of file PyClical.pyx.

7.7.2.9 ninf3

```
PyClical.ninf3 = e(4) + e(-1)
```

Definition at line 2021 of file PyClical.pyx.

7.7.2.10 None

```
PyClical.None
```

Definition at line 1422 of file PyClical.pyx.

7.7.2.11 obj

```
PyClical.obj
```

Definition at line 1654 of file PyClical.pyx.

7.7.2.12 pi

```
float PyClical.pi = atan(clifford(1.0)) * 4.0
```

Definition at line 1970 of file PyClical.pyx.

7.7.2.13 rhs

```
PyClical.rhs
```

Definition at line 1422 of file PyClical.pyx.

7.7.2.14 scalar_epsilon

```
PyClical.scalar_epsilon = epsilon
```

Definition at line 1968 of file PyClical.pyx.

7.7.2.15 tau

```
float PyClical.tau = atan(clifford(1.0)) * 8.0
```

Definition at line 1971 of file PyClical.pyx.

7.7.2.16 threshold

```
PyClical.threshold = error_squared_tol(rhs) if threshold is None else threshold
```

Definition at line 1422 of file PyClical.pyx.

7.7.2.17 tol

```
PyClical.tol = error_squared_tol(rhs) if tol is None else tol
```

Definition at line 1422 of file PyClical.pyx.

7.8 std Namespace Reference

Classes

- struct numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >

  Numeric limits for framed_multi inherit limits for the corresponding scalar type.
- struct numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >

  Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

Chapter 8

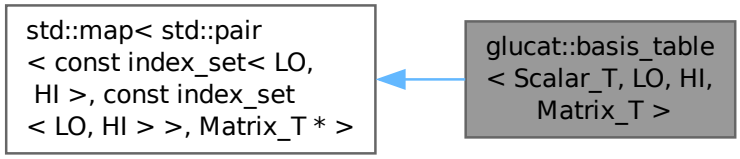
Class Documentation

8.1 glucat::basis_table< Scalar_T, LO, HI, Matrix_T > Class Template Reference

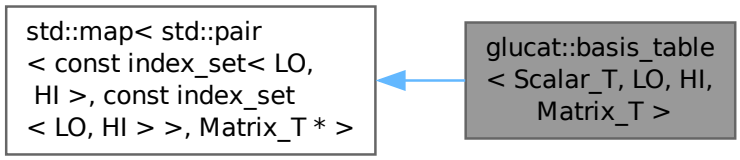
Table of basis elements used as a cache by basis_element().

`#include <matrix_multi_imp.h>`

Inheritance diagram for glucat::basis_table< Scalar_T, LO, HI, Matrix_T >:



Collaboration diagram for glucat::basis_table< Scalar_T, LO, HI, Matrix_T >:



Public Member Functions

- basis_table (const basis_table &)=delete
- auto operator= (const basis_table &) -> basis_table &=delete

Static Public Member Functions

- static auto basis () -> basis_table &

  Single instance of basis table.

Private Member Functions

- basis_table ()=default
- ~basis_table ()=default

Friends

- class friend_for_private_destructor

8.1.1 Detailed Description

template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >

Table of basis elements used as a cache by basis_element().

Definition at line 1159 of file matrix_multi_imp.h.

8.1.2 Constructor & Destructor Documentation

8.1.2.1 basis_table() [1/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis_table ()  [private], [default]
```

Referenced by basis(), basis_table(), and operator=().

8.1.2.2 ~basis_table()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::~basis_table ()  [private], [default]
```

8.1.2.3 basis_table() [2/2]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis_table (
            const basis_table< Scalar_T, LO, HI, Matrix_T > & )  [delete]
```

References basis_table().

8.1.3 Member Function Documentation

8.1.3.1 basis()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis () -> basis_table &  [inline],
[static]
```

Single instance of basis table.

Definition at line 1165 of file matrix_multi_imp.h.

References basis_table().

8.1.3.2 operator=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::operator= (
            const basis_table< Scalar_T, LO, HI, Matrix_T > & ) -> basis_table &=delete
[delete]
```

References basis_table().

8.1.4 Friends And Related Symbol Documentation

8.1.4.1 friend_for_private_destructor

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T>
friend class friend_for_private_destructor  [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE http://doc.ece.uci.edu

Definition at line 1170 of file matrix_multi_imp.h.

References friend_for_private_destructor.

Referenced by friend_for_private_destructor.

The documentation for this class was generated from the following file:

- glucat/matrix_multi_imp.h

8.2 glucat::bool_to_type< truth_value > Class Template Reference

Bool to type.

```
#include <global.h>
```

Private Types

- enum { value = truth_value }

8.2.1 Detailed Description

template<bool truth_value>
class glucat::bool_to_type< truth_value >

Bool to type.

Definition at line 69 of file global.h.

8.2.2 Member Enumeration Documentation

8.2.2.1 anonymous enum

```
template<bool truth_value>
anonymous enum  [private]
```

Enumerator

value	

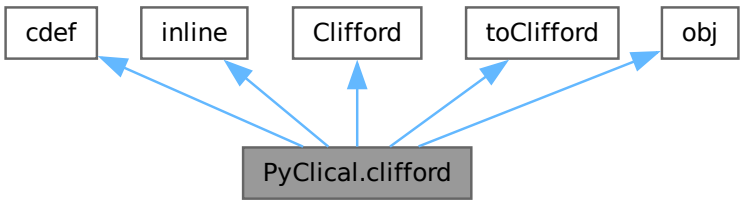
Definition at line 72 of file global.h.

The documentation for this class was generated from the following file:

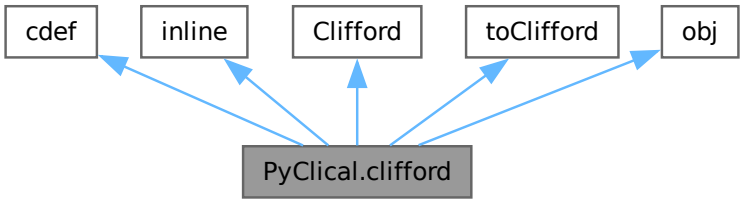
- glucat/global.h

8.3 PyClical.clifford Class Reference

Inheritance diagram for PyClical.clifford:



Collaboration diagram for PyClical.clifford:



Public Member Functions

- __cinit__ (self, other=0, ixt=None)
- __dealloc__ (self)
- __contains__ (self, x)
- __iter__ (self)
- reframe (self, ixt)
- __richcmp__ (lhs, rhs, int, op)
- __getitem__ (self, ixt)
- __neg__ (self)
- __pos__ (self)
- __add__ (lhs, rhs)
- __radd__ (rhs, lhs)
- __iadd__ (self, rhs)
- __sub__ (lhs, rhs)
- __rsub__ (rhs, lhs)
- __isub__ (self, rhs)
- __mul__ (lhs, rhs)
- __rmul__ (rhs, lhs)
- __imul__ (self, rhs)
- __mod__ (lhs, rhs)
- __rmod__ (rhs, lhs)
- __imod__ (self, rhs)
- __and__ (lhs, rhs)
- __rand__ (rhs, lhs)
- __iand__ (self, rhs)
- __xor__ (lhs, rhs)
- __rxor__ (rhs, lhs)
- __ixor__ (self, rhs)
- __truediv__ (lhs, rhs)
- __rtruediv__ (rhs, lhs)
- __idiv__ (self, rhs)
- inv (self)
- __or__ (lhs, rhs)
- __ior__ (self, rhs)
- __pow__ (self, m, dummy)
- pow (self, m)
- outer_pow (self, m)

- __call__ (self, grade)
- scalar (self)
- pure (self)
- even (self)
- odd (self)
- vector_part (self, frm=None)
- involute (self)
- reverse (self)
- conj (self)
- quad (self)
- norm (self)
- abs (self)
- max_abs (self)
- truncated (self, limit)
- isinf (self)
- isnan (self)
- frame (self)
- __repr__ (self)
- __str__ (self)

Public Attributes

- instance = new Clifford((<clifford>other).unwrap())

8.3.1 Detailed Description

```
Python class clifford wraps C++ class Clifford.
```

Definition at line 532 of file PyClical.pyx.

8.3.2 Member Function Documentation

8.3.2.1 __add__()

```
PyClical.clifford.__add__ (
              lhs,
              rhs)
```

```
Geometric sum.

>>> print(clifford(1) + clifford("{2}"))
1+{2}
>>> print(clifford("{1}") + clifford("{2}"))
{1}+{2}
```

Definition at line 740 of file PyClical.pyx.

8.3.2.2 __and__()

```
PyClical.clifford.__and__ (
              lhs,
              rhs)
```

Inner product.

```
>>> print(clifford("{1}") & clifford("{2}"))
0
>>> print(clifford(2) & clifford("{2}"))
0
>>> print(clifford("{1}") & clifford("{1}"))
1
>>> print(clifford("{1}") & clifford("{1,2}"))
{2}
```

Definition at line 872 of file PyClical.pyx.

8.3.2.3 __call__()

```
PyClical.clifford.__call__ (
              self,
              grade)
```

Pure grade-vector part.

```
>>> print(clifford("{1}")(1))
{1}
>>> print(clifford("{1}")(0))
0
>>> print(clifford("1+{1}+{1,2}")(0))
1
>>> print(clifford("1+{1}+{1,2}")(1))
{1}
>>> print(clifford("1+{1}+{1,2}")(2))
{1,2}
>>> print(clifford("1+{1}+{1,2}")(3))
0
```

Definition at line 1083 of file PyClical.pyx.

References instance, and PyClical.index_set.instance.

8.3.2.4 __cinit__()

```
PyClical.clifford.__cinit__ (
              self,
              other = 0,
              ixt = None)
```

```
Construct an object of type clifford.

>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2,index_set({1,2})))
2{1,2}
>>> print(clifford([2,3],index_set({1,2})))
2{1}+3{2}
```

Definition at line 565 of file PyClical.pyx.

8.3.2.5 __contains__()

```
PyClical.clifford.__contains__ (
              self,
              x)


Not applicable.

>>> x=clifford(index_set({-3,4,7})); -3 in x
Traceback (most recent call last):
  ...
TypeError: Not applicable.
```

Definition at line 627 of file PyClical.pyx.

8.3.2.6 __dealloc__()

```
PyClical.clifford.__dealloc__ (
              self)


Clean up by deallocating the instance of C++ class Clifford.
```

Definition at line 621 of file PyClical.pyx.

References instance, and PyClical.index_set.instance.

8.3.2.7 __getitem__()

```
PyClical.clifford.__getitem__ (
              self,
              ixt)
```

```
Subscripting: map from index set to scalar coordinate.
```

```
>>> clifford("{1}")[index_set(1)]
1.0
>>> clifford("{1}")[index_set({1})]
1.0
>>> clifford("{1}")[index_set({1,2})]
0.0
>>> clifford("2{1,2}")[index_set({1,2})]
2.0
```

Definition at line 707 of file PyClical.pyx.

References instance, and PyClical.index_set.instance.

8.3.2.8 __iadd__()

```
PyClical.clifford.__iadd__ (
              self,
              rhs)
```

```
Geometric sum.
```

```
>>> x = clifford(1); x += clifford("{2}"); print(x)
1+{2}
```

Definition at line 760 of file PyClical.pyx.

8.3.2.9 __iand__()

```
PyClical.clifford.__iand__ (
              self,
              rhs)
```

```
Inner product.
```

```
>>> x = clifford("{1}"); x &= clifford("{2}"); print(x)
0
>>> x = clifford(2); x &= clifford("{2}"); print(x)
0
>>> x = clifford("{1}"); x &= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
{2}
```

Definition at line 896 of file PyClical.pyx.

8.3.2.10 __idiv__()

```
PyClical.clifford.__idiv__ (
              self,
              rhs)
```

Geometric quotient.

```
>>> x = clifford("{1}"); x /= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x /= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x /= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
-{2}
```

Definition at line 974 of file PyClical.pyx.

8.3.2.11 __imod__()

```
PyClical.clifford.__imod__ (
              self,
              rhs)
```

Contraction.

```
>>> x = clifford("{1}"); x %= clifford("{2}"); print(x)
0
>>> x = clifford(2); x %= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x %= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
{2}
```

Definition at line 857 of file PyClical.pyx.

8.3.2.12 __imul__()

```
PyClical.clifford.__imul__ (
              self,
              rhs)
```

Geometric product.

```
>>> x = clifford(2); x *= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x *= clifford("{2}"); print(x)
{1,2}
>>> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
{2}
```

Definition at line 820 of file PyClical.pyx.

8.3.2.13 __ior__()

```
PyClical.clifford.__ior__ (
              self,
              rhs)
```

```
Transform left hand side, using right hand side as a transformation.
```

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
-{1}
```

Definition at line 1013 of file PyClical.pyx.

8.3.2.14 __isub__()

```
PyClical.clifford.__isub__ (
              self,
              rhs)
```

```
Geometric difference.
```

```
>>> x = clifford(1); x -= clifford("{2}"); print(x)
1-{2}
```

Definition at line 789 of file PyClical.pyx.

8.3.2.15 __iter__()

```
PyClical.clifford.__iter__ (
              self)
```

```
Not applicable.
```

```
>>> for a in clifford(index_set({-3,4,7})):print(a, end=",")
Traceback (most recent call last):
  ...
TypeError: Not applicable.
```

Definition at line 638 of file PyClical.pyx.

8.3.2.16 __ixor__()

```
PyClical.clifford.__ixor__ (
              self,
              rhs)
```

```
Outer product.
```

```
>>> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x ^= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
0
>>> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
0
```

Definition at line 935 of file PyClical.pyx.

8.3.2.17 __mod__()

```
PyClical.clifford.__mod__ (
              lhs,
              rhs)
```

Contraction.

```
>>> print(clifford("{1}") % clifford("{2}"))
0
>>> print(clifford(2) % clifford("{2}"))
2{2}
>>> print(clifford("{1}") % clifford("{1}"))
1
>>> print(clifford("{1}") % clifford("{1,2}"))
{2}
```

Definition at line 833 of file PyClical.pyx.

8.3.2.18 __mul__()

```
PyClical.clifford.__mul__ (
              lhs,
              rhs)
```

Geometric product.

```
>>> print(clifford("{1}") * clifford("{2}"))
{1,2}
>>> print(clifford(2) * clifford("{2}"))
2{2}
>>> print(clifford("{1}") * clifford("{1,2}"))
{2}
```

Definition at line 798 of file PyClical.pyx.

8.3.2.19 __neg__()

```
PyClical.clifford.__neg__ (
              self)
```

Unary -.

```
>>> print(-clifford("{1}"))
-{1}
```

Definition at line 722 of file PyClical.pyx.

References instance, and PyClical.index_set.instance.

8.3.2.20 __or__()

```
PyClical.clifford.__or__ (
              lhs,
              rhs)
```

```
Transform left hand side, using right hand side as a transformation.
```

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
-{1}
```

Definition at line 1002 of file PyClical.pyx.

8.3.2.21 __pos__()

```
PyClical.clifford.__pos__ (
              self)
```

```
Unary +.
```

```
>>> print(+clifford("{1}"))
{1}
```

Definition at line 731 of file PyClical.pyx.

8.3.2.22 __pow__()

```
PyClical.clifford.__pow__ (
              self,
              m,
              dummy)
```

```
Power: self to the m.
```

```
>>> x=clifford("{1}"); print(x ** 2)
1
>>> x=clifford("2"); print(x ** 2)
4
>>> x=clifford("2+{1}"); print(x ** 0)
1
>>> x=clifford("2+{1}"); print(x ** 1)
2+{1}
>>> x=clifford("2+{1}"); print(x ** 2)
5+4{1}
>>> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
1
```

Definition at line 1024 of file PyClical.pyx.

References pow().

8.3.2.23 __radd__()

```
PyClical.clifford.__radd__ (
              rhs,
              lhs)
```

Geometric sum.

```
>>> print(1 + clifford("{2}"))
1+{2}
```

Definition at line 751 of file PyClical.pyx.

8.3.2.24 __rand__()

```
PyClical.clifford.__rand__ (
              rhs,
              lhs)
```

Inner product.

```
>>> print(2 & clifford("{2}"))
0
```

Definition at line 887 of file PyClical.pyx.

8.3.2.25 __repr__()

```
PyClical.clifford.__repr__ (
              self)
```

The "official" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
```

Definition at line 1298 of file PyClical.pyx.

References clifford_to_repr().

8.3.2.26 __richcmp__()

```
PyClical.clifford.__richcmp__ (
              lhs,
              rhs,
              int,
              op)
```

Compare objects of type clifford.

```
>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 672 of file PyClical.pyx.

8.3.2.27 __rmod__()

```
PyClical.clifford.__rmod__ (
              rhs,
              lhs)
```

Contraction.

```
>>> print(2 % clifford("{2}"))
2{2}
```

Definition at line 848 of file PyClical.pyx.

8.3.2.28 __rmul__()

```
PyClical.clifford.__rmul__ (
              rhs,
              lhs)
```

Geometric product.

```
>>> print(2 * clifford("{2}"))
2{2}
```

Definition at line 811 of file PyClical.pyx.

8.3.2.29 __rsub__()

```
PyClical.clifford.__rsub__ (
              rhs,
              lhs)
```

Geometric difference.

```
>>> print(1 - clifford("{2}"))
1-{2}
```

Definition at line 780 of file PyClical.pyx.

8.3.2.30 __rtruediv__()

```
PyClical.clifford.__rtruediv__ (
              rhs,
              lhs)
```

Geometric quotient.

```
>>> print(2 / clifford("{2}"))
2{2}
```

Definition at line 965 of file PyClical.pyx.

8.3.2.31 __rxor__()

```
PyClical.clifford.__rxor__ (
              rhs,
              lhs)
```

Outer product.

```
>>> print(2 ^ clifford("{2}"))
2{2}
```

Definition at line 926 of file PyClical.pyx.

8.3.2.32 __str__()

```
PyClical.clifford.__str__ (
              self)
```

The "informal" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
'1+3{-1}+2{1,2}+4{-2,7}'
```

Definition at line 1307 of file PyClical.pyx.

References clifford_to_str().

8.3.2.33 __sub__()

```
PyClical.clifford.__sub__ (
              lhs,
              rhs)
```

Geometric difference.

```
>>> print(clifford(1) - clifford("{2}"))
1-{2}
>>> print(clifford("{1}") - clifford("{2}"))
{1}-{2}
```

Definition at line 769 of file PyClical.pyx.

8.3.2.34 __truediv__()

```
PyClical.clifford.__truediv__ (
              lhs,
              rhs)
```

Geometric quotient.

```
>>> print(clifford("{1}") / clifford("{2}"))
{1,2}
>>> print(clifford(2) / clifford("{2}"))
2{2}
>>> print(clifford("{1}") / clifford("{1}"))
1
>>> print(clifford("{1}") / clifford("{1,2}"))
-{2}
```

Definition at line 950 of file PyClical.pyx.

8.3.2.35 __xor__()

```
PyClical.clifford.__xor__ (
              lhs,
              rhs)
```

Outer product.

```
>>> print(clifford("{1}") ^ clifford("{2}"))
{1,2}
>>> print(clifford(2) ^ clifford("{2}"))
2{2}
>>> print(clifford("{1}") ^ clifford("{1}"))
0
>>> print(clifford("{1}") ^ clifford("{1,2}"))
0
```

Definition at line 911 of file PyClical.pyx.

8.3.2.36 abs()

```
PyClical.clifford.abs (
              self)
```

```
Absolute value: square root of norm.
```

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
2.0
```

Definition at line 1238 of file PyClical.pyx.

References glucat.abs().

8.3.2.37 conj()

```
PyClical.clifford.conj (
              self)
```

```
Conjugation, reverse o involute == involute o reverse.
```

```
>>> print((clifford("{1}")).conj())
-{1}
>>> print((clifford("{2}") * clifford("{1}")).conj())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).conj())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").conj())
1-{1}-{1,2}
```

Definition at line 1201 of file PyClical.pyx.

References conj(), instance, and PyClical.index_set.instance.

Referenced by conj().

8.3.2.38 even()

```
PyClical.clifford.even (
              self)
```

```
Even part of multivector, sum of even grade terms.
```

```
>>> print(clifford("1+{1}+{1,2}").even())
1+{1,2}
```

Definition at line 1124 of file PyClical.pyx.

References even(), instance, and PyClical.index_set.instance.

Referenced by even().

8.3.2.39 frame()

```
PyClical.clifford.frame (
              self)
```

```
Subalgebra generated by all generators of terms of given multivector.

>>> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
{-2,-1,1,2,7}
>>> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
<class 'PyClical.index_set'>
```

Definition at line 1287 of file PyClical.pyx.

References frame(), instance, and PyClical.index_set.instance.

Referenced by frame().

8.3.2.40 inv()

```
PyClical.clifford.inv (
              self)
```

```
Geometric multiplicative inverse.

>>> x = clifford("{1}"); print(x.inv())
{1}
>>> x = clifford(2); print(x.inv())
0.5
>>> x = clifford("{1,2}"); print(x.inv())
-{1,2}
```

Definition at line 989 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and inv().

Referenced by inv().

8.3.2.41 involute()

```
PyClical.clifford.involute (
              self)
```

```
Main involution, each {i} is replaced by -{i} in each term,
eg. clifford("{1}") -> -clifford("{1}").

>>> print(clifford("{1}").involute())
-{1}
>>> print((clifford("{2}") * clifford("{1}")).involute())
-{1,2}
>>> print((clifford("{1}") * clifford("{2}")).involute())
{1,2}
>>> print(clifford("1+{1}+{1,2}").involute())
1-{1}+{1,2}
```

Definition at line 1170 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and involute().

Referenced by involute().

8.3.2.42 isinf()

```
PyClical.clifford.isinf (
              self)
```

Check if a multivector contains any infinite values.

```
>>> clifford().isinf()
False
```

Definition at line 1269 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and isnan().

8.3.2.43 isnan()

```
PyClical.clifford.isnan (
              self)
```

Check if a multivector contains any IEEE NaN values.

```
>>> clifford().isnan()
False
```

Definition at line 1278 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and isnan().

Referenced by isinf(), and isnan().

8.3.2.44 max_abs()

```
PyClical.clifford.max_abs (
              self)
```

Maximum of absolute values of components of multivector: multivector infinity norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
1.0
>>> clifford("3+2{1}+{1,2}").max_abs()
3.0
```

Definition at line 1247 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and max_abs().

Referenced by max_abs().

8.3.2.45 norm()

```
PyClical.clifford.norm (
              self)
```

```
Norm == sum of squares of coordinates.

>>> clifford("1+{1}+{1,2}").norm()
3.0
>>> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
4.0
```

Definition at line 1227 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and norm().

Referenced by norm().

8.3.2.46 odd()

```
PyClical.clifford.odd (
              self)
```

```
Odd part of multivector, sum of odd grade terms.

>>> print(clifford("1+{1}+{1,2}").odd())
{1}
```

Definition at line 1133 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and odd().

Referenced by odd().

8.3.2.47 outer_pow()

```
PyClical.clifford.outer_pow (
              self,
              m)
```

```
Outer product power.

>>> x=clifford("2+{1}"); print(x.outer_pow(0))
1
>>> x=clifford("2+{1}"); print(x.outer_pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.outer_pow(2))
4+4{1}
>>> print(clifford("1+{1}+{1,2}").outer_pow(3))
1+3{1}+3{1,2}
```

Definition at line 1067 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and outer_pow().

Referenced by outer_pow().

8.3.2.48 pow()

```
PyClical.clifford.pow (
              self,
              m)

Power: self to the m.

>>> x=clifford("{1}"); print(x.pow(2))
1
>>> x=clifford("2"); print(x.pow(2))
4
>>> x=clifford("2+{1}"); print(x.pow(0))
1
>>> x=clifford("2+{1}"); print(x.pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.pow(2))
5+4{1}
>>> print(clifford("1+{1}+{1,2}").pow(3))
1+3{1}+3{1,2}
>>> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
1
```

Definition at line 1043 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and pow().

Referenced by __pow__(), and pow().

8.3.2.49 pure()

```
PyClical.clifford.pure (
              self)

Pure part.

>>> print(clifford("1+{1}+{1,2}").pure())
{1}+{1,2}
>>> print(clifford("{1,2}").pure())
{1,2}
```

Definition at line 1113 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and pure().

Referenced by pure().

8.3.2.50 quad()

```
PyClical.clifford.quad (
              self)

Quadratic form == (rev(x)*x)(0).

>>> print(clifford("1+{1}+{1,2}").quad())
3.0
>>> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
2.0
```

Definition at line 1216 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and quad().

Referenced by quad().

8.3.2.51 reframe()

```
PyClical.clifford.reframe (
              self,
              ixt)
```

```
Put self into a larger frame, containing the union of self.frame() and index set ixt.
This can be used to make multiplication faster, by multiplying within a common frame.

>>> clifford("2+3{1}").reframe(index_set({1,2,3}))
clifford("2+3{1}")
>>> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);
True
```

Definition at line 649 of file PyClical.pyx.

8.3.2.52 reverse()

```
PyClical.clifford.reverse (
              self)
```

```
Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").

>>> print(clifford("{1}").reverse())
{1}
>>> print((clifford("{2}") * clifford("{1}")).reverse())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).reverse())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").reverse())
1+{1}-{1,2}
```

Definition at line 1186 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and reverse().

Referenced by reverse().

8.3.2.53 scalar()

```
PyClical.clifford.scalar (
              self)
```

```
Scalar part.

>>> clifford("1+{1}+{1,2}").scalar()
1.0
>>> clifford("{1,2}").scalar()
0.0
```

Definition at line 1102 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and scalar().

Referenced by scalar().

8.3.2.54 truncated()

```
PyClical.clifford.truncated (
              self,
              limit)
```

```
Remove all terms of self with relative size smaller than limit.

>>> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
clifford("100000000")
>>> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
clifford("10000+{1}")
```

Definition at line 1258 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and truncated().

Referenced by truncated().

8.3.2.55 vector_part()

```
PyClical.clifford.vector_part (
              self,
              frm = None)
```

```
Vector part of multivector, as a Python list, with respect to frm.

>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
[2.0, 3.0]
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
[0.0, 2.0, 3.0]
```

Definition at line 1142 of file PyClical.pyx.

References instance, PyClical.index_set.instance, and vector_part().

Referenced by vector_part().

8.3.3 Member Data Documentation

8.3.3.1 instance

```
PyClical.clifford.instance = new Clifford((<clifford>other).unwrap())
```

Definition at line 592 of file PyClical.pyx.

Referenced by __call__(), __dealloc__(), __getitem__(), __neg__(), conj(), even(), frame(), inv(), involute(), isinf(), isnan(), max_abs(), norm(), odd(), outer_pow(), pow(), pure(), quad(), reverse(), scalar(), truncated(), and vector_part().

The documentation for this class was generated from the following file:

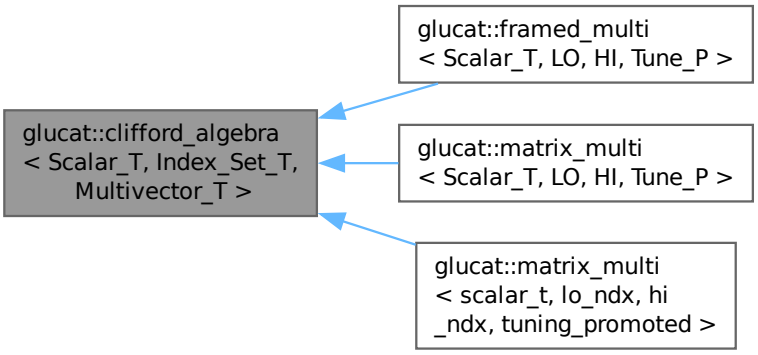
- pyclical/PyClical.pyx

8.4 glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T > Class Template Reference

clifford_algebra<> declares the operations of a Clifford algebra

`#include <clifford_algebra.h>`

Inheritance diagram for glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >:



Public Types

- using scalar_t = Scalar_T
- using index_set_t = Index_Set_T
- using multivector_t = Multivector_T
- using pair_t = std::pair<const index_set_t, Scalar_T>
- using vector_t = std::vector<Scalar_T>

Public Member Functions

- virtual ~clifford_algebra ()=default
- virtual auto operator== (const multivector_t &val) const -> bool=0

  Test for equality of multivectors.
- virtual auto operator== (const Scalar_T &scr) const -> bool=0

  Test for equality of multivector and scalar.
- virtual auto operator+= (const multivector_t &rhs) -> multivector_t &=0

  Geometric sum.
- virtual auto operator+= (const Scalar_T &scr) -> multivector_t &=0

  Geometric sum of multivector and scalar.
- virtual auto operator-= (const multivector_t &rhs) -> multivector_t &=0

  Geometric difference.
- virtual auto operator-= (const Scalar_T &scr) -> multivector_t &=0

  Geometric difference of multivector and scalar.
- virtual auto operator- () const -> const multivector_t=0

  Unary -.

- virtual auto operator*= (const Scalar_T &scr) -> multivector_t &=0

  Product of multivector and scalar.
- virtual auto operator*= (const multivector_t &rhs) -> multivector_t &=0

  Geometric product.
- virtual auto operator%= (const multivector_t &rhs) -> multivector_t &=0

  Contraction.
- virtual auto operator&= (const multivector_t &rhs) -> multivector_t &=0

  Inner product.
- virtual auto operator^= (const multivector_t &rhs) -> multivector_t &=0

  Outer product.
- virtual auto operator/= (const Scalar_T &scr) -> multivector_t &=0

  Quotient of multivector and scalar.
- virtual auto operator/= (const multivector_t &rhs) -> multivector_t &=0

  Geometric quotient.
- virtual auto operator|= (const multivector_t &rhs) -> multivector_t &=0

  Transformation via twisted adjoint action.
- virtual auto inv () const -> const multivector_t=0

  Geometric multiplicative inverse.
- virtual auto pow (int m) const -> const multivector_t=0

  **this to the m*
- virtual auto outer_pow (int m) const -> const multivector_t=0

  Outer product power.
- virtual auto frame () const -> const index_set_t=0

  Subalgebra generated by all generators of terms of given multivector.
- virtual auto grade () const -> index_t=0

  Maximum of the grades of each term.
- virtual auto operator[ ] (const index_set_t ist) const -> Scalar_T=0

  Subscripting: map from index set to scalar coordinate.
- virtual auto operator() (index_t grade) const -> const multivector_t=0

  Pure grade-vector part.
- virtual auto scalar () const -> Scalar_T=0

  Scalar part.
- virtual auto pure () const -> const multivector_t=0

  Pure part.
- virtual auto even () const -> const multivector_t=0

  Even part of multivector, sum of even grade terms.
- virtual auto odd () const -> const multivector_t=0

  Odd part of multivector, sum of odd grade terms.
- virtual auto vector_part () const -> const vector_t=0

  Vector part of multivector, as a vector_t with respect to frame().
- virtual auto vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t=0

  Vector part of multivector, as a vector_t with respect to frm.
- virtual auto involute () const -> const multivector_t=0

  Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.
- virtual auto reverse () const -> const multivector_t=0

  Reversion, eg. {1}{2} -> {2}*{1}.*
- virtual auto conj () const -> const multivector_t=0

  Conjugation, reverse o involute == involute o reverse.
- virtual auto quad () const -> Scalar_T=0

  *Scalar_T quadratic form == (rev(x)*x)(0).*
- virtual auto norm () const -> Scalar_T=0

Scalar_T norm == sum of norm of coordinates.

- virtual auto max_abs () const -> Scalar_T=0

  Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto truncated (const Scalar_T &limit=default_truncation) const -> const multivector_t=0

  Remove all terms with relative size smaller than limit.
- virtual auto isinf () const -> bool=0

  Check if a multivector contains any infinite values.
- virtual auto isnan () const -> bool=0

  Check if a multivector contains any IEEE NaN values.
- virtual void write (const std::string &msg="") const =0

  Write formatted multivector to output.
- virtual void write (std::ofstream &ofile, const std::string &msg="") const =0

  Write formatted multivector to file.

Static Public Member Functions

- static auto classname () -> const std::string

Static Public Attributes

- static const index_t v_lo = index_set_t::v_lo
- static const index_t v_hi = index_set_t::v_hi
- static const Scalar_T default_truncation

  Default for truncation.

8.4.1 Detailed Description

template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
class glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >

clifford_algebra<> declares the operations of a Clifford algebra

Definition at line 45 of file clifford_algebra.h.

8.4.2 Member Typedef Documentation

8.4.2.1 index_set_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::index_set_t = Index_↩
Set_T
```

Definition at line 49 of file clifford_algebra.h.

8.4.2.2 multivector_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::multivector_t = Multivector↩
_T
```

Definition at line 52 of file clifford_algebra.h.

8.4.2.3 pair_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pair_t = std::pair<const
index_set_t, Scalar_T>
```

Definition at line 53 of file clifford_algebra.h.

8.4.2.4 scalar_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar_t = Scalar_T
```

Definition at line 48 of file clifford_algebra.h.

8.4.2.5 vector_t

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_t = std::↩
vector<Scalar_T>
```

Definition at line 54 of file clifford_algebra.h.

8.4.3 Constructor & Destructor Documentation

8.4.3.1 ~clifford_algebra()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::~clifford_algebra
() [virtual], [default]
```

8.4.4 Member Function Documentation

8.4.4.1 classname()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::classname () -> const
std::string [static]
```

Definition at line 66 of file clifford_algebra_imp.h.

8.4.4.2 conj()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::conj () const
-> const multivector_t  [pure virtual]
```

Conjugation, reverse o involute == involute o reverse.

References conj().

Referenced by conj().

8.4.4.3 even()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::even () const
-> const multivector_t  [pure virtual]
```

Even part of multivector, sum of even grade terms.

References even().

Referenced by even().

8.4.4.4 frame()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame () const
-> const index_set_t  [pure virtual]
```

Subalgebra generated by all generators of terms of given multivector.

References frame().

Referenced by frame(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, T and glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi().

8.4.4.5 grade()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::grade () const
-> index_t  [pure virtual]
```

Maximum of the grades of each term.

References grade().

Referenced by grade(), and operator()().

8.4.4.6 inv()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::inv () const
-> const multivector_t  [pure virtual]
```

Geometric multiplicative inverse.

References inv().

Referenced by inv().

8.4.4.7 involute()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::involute ()
const -> const multivector_t  [pure virtual]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

References involute().

Referenced by involute().

8.4.4.8 isinf()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isinf () const
-> bool  [pure virtual]
```

Check if a multivector contains any infinite values.

References isinf().

Referenced by isinf().

8.4.4.9 isnan()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan () const
-> bool  [pure virtual]
```

Check if a multivector contains any IEEE NaN values.

References isnan().

Referenced by glucat::cascade_log(), glucat::exp(), isnan(), glucat::log(), glucat::matrix_log(), glucat::matrix_sqrt(), glucat::pade_log(), and glucat::sqrt().

8.4.4.10 max_abs()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::max_abs ()
const -> Scalar_T  [pure virtual]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

References max_abs().

Referenced by max_abs().

8.4.4.11 norm()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::norm () const
-> Scalar_T  [pure virtual]
```

Scalar_T norm == sum of norm of coordinates.

References norm().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), and norm().

8.4.4.12 odd()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::odd () const
-> const multivector_t  [pure virtual]
```

Odd part of multivector, sum of odd grade terms.

References odd().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast(), and odd().

8.4.4.13 operator%=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator%= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Contraction.

8.4.4.14 operator&=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator&= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Inner product.

8.4.4.15 operator()()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator() (
            index_t grade) const -> const multivector_t  [pure virtual]
```

Pure grade-vector part.

References grade().

8.4.4.16 operator*=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Geometric product.

8.4.4.17 operator*=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
            const Scalar_T & scr) -> multivector_t &  [pure virtual]
```

Product of multivector and scalar.

8.4.4.18 operator+=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Geometric sum.

8.4.4.19 operator+=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
            const Scalar_T & scr) -> multivector_t &  [pure virtual]
```

Geometric sum of multivector and scalar.

8.4.4.20 operator-()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator- ()
const -> const multivector_t  [pure virtual]
```

Unary -.

8.4.4.21 operator-=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Geometric difference.

8.4.4.22 operator-=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
            const Scalar_T & scr) -> multivector_t &  [pure virtual]
```

Geometric difference of multivector and scalar.

8.4.4.23 operator/=() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Geometric quotient.

8.4.4.24 operator/=() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
            const Scalar_T & scr) -> multivector_t &  [pure virtual]
```

Quotient of multivector and scalar.

8.4.4.25 operator==() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
            const multivector_t & val) const -> bool  [pure virtual]
```

Test for equality of multivectors.

8.4.4.26 operator==() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator== (
            const Scalar_T & scr) const -> bool  [pure virtual]
```

Test for equality of multivector and scalar.

8.4.4.27 operator[]()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator[] (
            const index_set_t ist) const -> Scalar_T  [pure virtual]
```

Subscripting: map from index set to scalar coordinate.

8.4.4.28 operator^=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator^= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Outer product.

8.4.4.29 operator"|=()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator|= (
            const multivector_t & rhs) -> multivector_t &  [pure virtual]
```

Transformation via twisted adjoint action.

8.4.4.30 outer_pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::outer_pow (
            int m) const -> const multivector_t  [pure virtual]
```

Outer product power.

References outer_pow().

Referenced by outer_pow().

8.4.4.31 pow()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pow (
            int m) const -> const multivector_t  [pure virtual]
```

$*$this to the m

References pow().

Referenced by pow().

8.4.4.32 pure()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pure () const
-> const multivector_t  [pure virtual]
```

Pure part.

References pure().

Referenced by pure().

8.4.4.33 quad()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::quad () const
-> Scalar_T  [pure virtual]
```

Scalar_T quadratic form == (rev(x)∗x)(0).

References quad().

Referenced by quad().

8.4.4.34 reverse()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::reverse ()
const -> const multivector_t  [pure virtual]
```

Reversion, eg. {1}∗{2} -> {2}∗{1}.

References reverse().

Referenced by reverse().

8.4.4.35 scalar()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar () const
-> Scalar_T  [pure virtual]
```

Scalar part.

References scalar().

Referenced by glucat::exp(), glucat::matrix_log(), glucat::matrix_sqrt(), and scalar().

8.4.4.36 truncated()

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated (
            const Scalar_T & limit = default_truncation) const -> const multivector_t  [pure
virtual]
```

Remove all terms with relative size smaller than limit.

References default_truncation, and truncated().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::operator<<(), and truncated().

8.4.4.37 vector_part() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part ()
const -> const vector_t  [pure virtual]
```

Vector part of multivector, as a vector_t with respect to frame().

References vector_part().

Referenced by vector_part(), and vector_part().

8.4.4.38 vector_part() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
            const index_set_t frm,
            const bool prechecked) const -> const vector_t  [pure virtual]
```

Vector part of multivector, as a vector_t with respect to frm.

References vector_part().

8.4.4.39 write() [1/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
            const std::string & msg = "") const  [pure virtual]
```

Write formatted multivector to output.

References write().

Referenced by write(), and write().

8.4.4.40 write() [2/2]

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
            std::ofstream & ofile,
            const std::string & msg = "") const  [pure virtual]
```

Write formatted multivector to file.

References write().

8.4.5 Member Data Documentation

8.4.5.1 default_truncation

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↩
truncation  [static]
```

Default for truncation.

Definition at line 59 of file clifford_algebra.h.

Referenced by truncated().

8.4.5.2 v_hi

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_hi = index↩
_set_t::v_hi  [static]
```

Definition at line 51 of file clifford_algebra.h.

8.4.5.3 v_lo

```
template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_lo = index↩
_set_t::v_lo  [static]
```

Definition at line 50 of file clifford_algebra.h.

The documentation for this class was generated from the following files:

- glucat/clifford_algebra.h
- glucat/clifford_algebra_imp.h

8.5 glucat::compare_types< LHS_T, RHS_T > Class Template Reference

Type comparison.

```
#include <global.h>
```

Public Types

- enum { are_same = false }

8.5.1 Detailed Description

template<typename LHS_T, typename RHS_T>
class glucat::compare_types< LHS_T, RHS_T >

Type comparison.

Definition at line 54 of file global.h.

8.5.2 Member Enumeration Documentation

8.5.2.1 anonymous enum

```
template<typename LHS_T, typename RHS_T>
anonymous enum
```

Enumerator

are_same	

Definition at line 57 of file global.h.

The documentation for this class was generated from the following file:

- glucat/global.h

8.6 glucat::compare_types< T, T > Class Template Reference

```
#include <global.h>
```

Public Types

- enum { are_same = true }
- enum

8.6.1 Detailed Description

template<typename T>
class glucat::compare_types< T, T >

Definition at line 60 of file global.h.

8.6.2 Member Enumeration Documentation

8.6.2.1 anonymous enum

```
anonymous enum
```

Definition at line 57 of file global.h.

8.6.2.2 anonymous enum

```
template<typename T>
anonymous enum
```

Enumerator

are_same	

Definition at line 63 of file global.h.

The documentation for this class was generated from the following file:

- glucat/global.h

8.7 glucat::control_t Class Reference

Parameters to control tests.

```
#include <control.h>
```

Public Member Functions

- int call (intfn f) const

  Call a function that returns int.
- int call (intintfn f, int arg) const

  Call a function of int that returns int.

Static Public Member Functions

- static const control_t & control (int argc, char **argv)
- static bool verbose ()

  Produce more detailed output from tests.

Private Member Functions

- bool valid () const
- bool catch_exceptions () const
- control_t (int argc, char **argv)

  Constructor from program arguments.
- control_t ()=default
- ~control_t ()=default
- control_t (const control_t &)=delete
- control_t & operator= (const control_t &)=delete

Private Attributes

- bool m_valid

  Test parameters are valid.
- bool m_catch_exceptions

  Catch exceptions.

Static Private Attributes

- static bool m_verbose_output = false

  Produce more detailed output from tests.

Friends

- class friend_for_private_destructor

8.7.1 Detailed Description

Parameters to control tests.

Definition at line 39 of file control.h.

8.7.2 Constructor & Destructor Documentation

8.7.2.1 control_t() [1/3]

```
glucat::control_t::control_t (
            int argc,
            char ** argv)  [private]
```

Constructor from program arguments.

Test control constructor from program arguments.

Definition at line 88 of file control.h.

References GLUCAT_PACKAGE_NAME, GLUCAT_VERSION, m_catch_exceptions, m_valid, m_verbose_output, and valid().

Referenced by control(), control_t(), and operator=().

8.7.2.2 control_t() [2/3]

```
glucat::control_t::control_t ()  [private], [default]
```

8.7.2.3 ~control_t()

```
glucat::control_t::~control_t ()  [private], [default]
```

8.7.2.4 control_t() [3/3]

```
glucat::control_t::control_t (
            const control_t & )  [private], [delete]
```

References control_t().

8.7.3 Member Function Documentation

8.7.3.1 call() [1/2]

```
int glucat::control_t::call (
            intfn f) const  [inline]
```

Call a function that returns int.

Definition at line 136 of file control.h.

References catch_exceptions(), glucat::try_catch(), and valid().

8.7.3.2 call() [2/2]

```
int glucat::control_t::call (
            intintfn f,
            int arg) const  [inline]
```

Call a function of int that returns int.

Definition at line 150 of file control.h.

References catch_exceptions(), glucat::try_catch(), and valid().

8.7.3.3 catch_exceptions()

```
bool glucat::control_t::catch_exceptions () const  [inline], [private]
```

Definition at line 49 of file control.h.

References m_catch_exceptions.

Referenced by call(), and call().

8.7.3.4 control()

```
const control_t & glucat::control_t::control (
            int argc,
            char ** argv)  [inline], [static]
```

Single instance Ref: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.

Definition at line 71 of file control.h.

References control_t().

8.7.3.5 operator=()

```
control_t & glucat::control_t::operator= (
            const control_t & )  [private], [delete]
```

References control_t().

8.7.3.6 valid()

```
bool glucat::control_t::valid () const  [inline], [private]
```

Definition at line 44 of file control.h.

References m_valid.

Referenced by call(), call(), and control_t().

8.7.3.7 verbose()

```
bool glucat::control_t::verbose ()  [inline], [static]
```

Produce more detailed output from tests.

Definition at line 80 of file control.h.

References m_verbose_output.

8.7.4 Friends And Related Symbol Documentation

8.7.4.1 friend_for_private_destructor

```
friend class friend_for_private_destructor  [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE http://doc.ece.uci.edu

Definition at line 67 of file control.h.

References friend_for_private_destructor.

Referenced by friend_for_private_destructor.

8.7.5 Member Data Documentation

8.7.5.1 m_catch_exceptions

```
bool glucat::control_t::m_catch_exceptions  [private]
```

Catch exceptions.

Definition at line 48 of file control.h.

Referenced by catch_exceptions(), and control_t().

8.7.5.2 m_valid

```
bool glucat::control_t::m_valid  [private]
```

Test parameters are valid.

Definition at line 43 of file control.h.

Referenced by control_t(), and valid().

8.7.5.3 m_verbose_output

```
bool glucat::control_t::m_verbose_output = false  [static], [private]
```

Produce more detailed output from tests.

Definition at line 53 of file control.h.

Referenced by control_t(), and verbose().

The documentation for this class was generated from the following file:

- test/control.h

8.8 glucat::CTAssertion< bool > Struct Template Reference

Compile time assertion.

8.8.1 Detailed Description

template<bool>
struct glucat::CTAssertion< bool >

Compile time assertion.

Definition at line 46 of file global.h.

The documentation for this struct was generated from the following file:

- glucat/global.h

8.9 glucat::CTAssertion< true > Struct Reference

`#include <global.h>`

8.9.1 Detailed Description

Definition at line 47 of file global.h.

The documentation for this struct was generated from the following file:

- glucat/global.h

8.10 glucat::numeric_traits< Scalar_T >::demoted Struct Reference

Demoted type for long double.

`#include <promotion.h>`

Public Types

- using type = float
- using type = float

8.10.1 Detailed Description

template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::demoted

Demoted type for long double.

Demoted type.

Definition at line 76 of file promotion.h.

8.10.2 Member Typedef Documentation

8.10.2.1 type [1/2]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 78 of file promotion.h.

8.10.2.2 type [2/2]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 148 of file scalar.h.

The documentation for this struct was generated from the following files:

- glucat/promotion.h
- glucat/scalar.h

8.11 glucat::matrix::eig_genus< Matrix_T > Struct Template Reference

Structure containing classification of eigenvalues.

```
#include <matrix.h>
```

Public Types

- using Scalar_T = typename Matrix_T::value_type

Public Attributes

- bool m_is_singular = false

  Is the matrix singular?
- eig_case_t m_eig_case = safe_eigs

  What kind of eigenvalues does the matrix contain?
- Scalar_T m_safe_arg = Scalar_T(0)

  Argument such that exp(pi-m_safe_arg) lies between arguments of eigenvalues.

8.11.1 Detailed Description

template<typename Matrix_T>
struct glucat::matrix::eig_genus< Matrix_T >

Structure containing classification of eigenvalues.

Definition at line 140 of file matrix.h.

8.11.2 Member Typedef Documentation

8.11.2.1 Scalar_T

```
template<typename Matrix_T>
using glucat::matrix::eig_genus< Matrix_T >::Scalar_T = typename Matrix_T::value_type
```

Definition at line 142 of file matrix.h.

8.11.3 Member Data Documentation

8.11.3.1 m_eig_case

```
template<typename Matrix_T>
eig_case_t glucat::matrix::eig_genus< Matrix_T >::m_eig_case = safe_eigs
```

What kind of eigenvalues does the matrix contain?

Definition at line 146 of file matrix.h.

Referenced by glucat::matrix::classify_eigenvalues().

8.11.3.2 m_is_singular

```
template<typename Matrix_T>
bool glucat::matrix::eig_genus< Matrix_T >::m_is_singular = false
```

Is the matrix singular?

Definition at line 144 of file matrix.h.

Referenced by glucat::matrix::classify_eigenvalues().

8.11.3.3 m_safe_arg

```
template<typename Matrix_T>
Scalar_T glucat::matrix::eig_genus< Matrix_T >::m_safe_arg = Scalar_T(0)
```

Argument such that exp(pi-m_safe_arg) lies between arguments of eigenvalues.

Definition at line 148 of file matrix.h.

Referenced by glucat::matrix::classify_eigenvalues().

The documentation for this struct was generated from the following file:

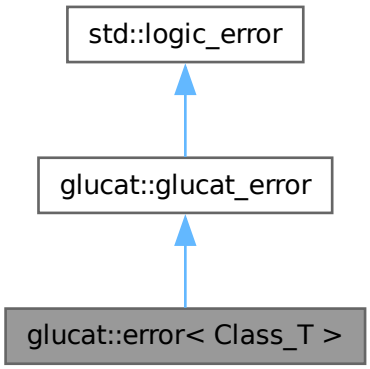
- glucat/matrix.h

8.12 glucat::error< Class_T > Class Template Reference

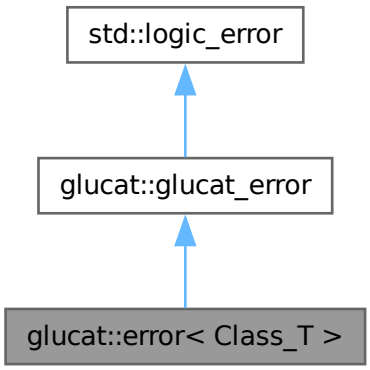
Specific exception class.

`#include <errors.h>`

Inheritance diagram for glucat::error< Class_T >:



Collaboration diagram for glucat::error< Class_T >:



Public Member Functions

- error (const std::string &msg)
  Specific exception class.
- error (const std::string &context, const std::string &msg)
- auto heading () const noexcept -> const std::string override
- auto classname () const noexcept -> const std::string override
- void print_error_msg () const override

Public Member Functions inherited from glucat::glucat_error

- glucat_error (const std::string &context, const std::string &msg)
- ~glucat_error () noexcept override=default

Additional Inherited Members

Public Attributes inherited from glucat::glucat_error

- std::string name

8.12.1 Detailed Description

template<class Class_T>
class glucat::error< Class_T >

Specific exception class.

Definition at line 56 of file errors.h.

8.12.2 Constructor & Destructor Documentation

8.12.2.1 error() [1/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
            const std::string & msg)
```

Specific exception class.

Definition at line 44 of file errors_imp.h.

References classname(), and glucat::glucat_error::glucat_error().

8.12.2.2 error() [2/2]

```
template<class Class_T>
glucat::error< Class_T >::error (
            const std::string & context,
            const std::string & msg)
```

Definition at line 50 of file errors_imp.h.

References glucat::glucat_error::glucat_error().

8.12.3 Member Function Documentation

8.12.3.1 classname()

```
template<class Class_T>
auto glucat::error< Class_T >::classname () const -> const std::string  [override], [virtual],
[noexcept]
```

Implements glucat::glucat_error.

Definition at line 63 of file errors_imp.h.

References glucat::glucat_error::name.

Referenced by error(), and print_error_msg().

8.12.3.2 heading()

```
template<class Class_T>
auto glucat::error< Class_T >::heading () const -> const std::string  [override], [virtual],
[noexcept]
```

Implements glucat::glucat_error.

Definition at line 57 of file errors_imp.h.

Referenced by print_error_msg().

8.12.3.3 print_error_msg()

```
template<class Class_T>
void glucat::error< Class_T >::print_error_msg () const  [override], [virtual]
```

Implements glucat::glucat_error.

Definition at line 69 of file errors_imp.h.

References classname(), and heading().

The documentation for this class was generated from the following files:

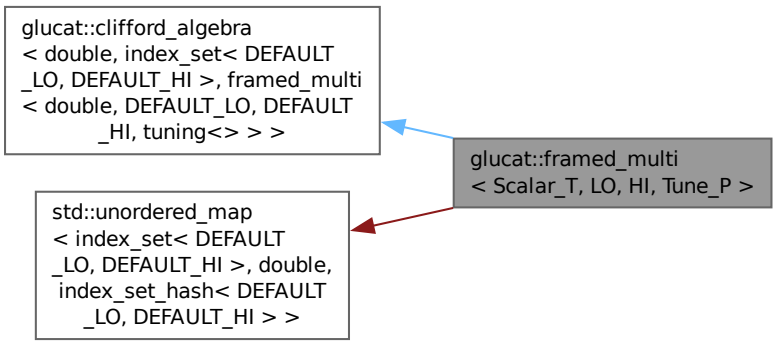
- glucat/errors.h
- glucat/errors_imp.h

8.13 glucat::framed_multi< Scalar_T, LO, HI, Tune_P > Class Template Reference

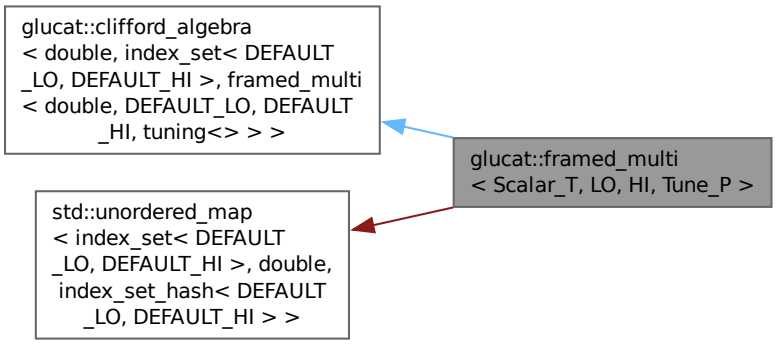
A framed_multi<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector.

`#include <framed_multi.h>`

Inheritance diagram for glucat::framed_multi< Scalar_T, LO, HI, Tune_P >:



Collaboration diagram for glucat::framed_multi< Scalar_T, LO, HI, Tune_P >:



Classes

- class hash_size_t
- class var_term

  Variable term.

Public Types

- using multivector_t = framed_multi
- using framed_multi_t = multivector_t
- using scalar_t = Scalar_T
- using tune_p = Tune_P
- using index_set_t = index_set<LO, HI>
- using term_t = std::pair<const index_set_t, Scalar_T>
- using vector_t = std::vector<Scalar_T>
- using error_t = error<multivector_t>
- using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >

Public Types inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- using scalar_t
- using index_set_t
- using multivector_t
- using pair_t
- using vector_t

Public Member Functions

- ~framed_multi () override=default

  Destructor.
- framed_multi ()

  Default constructor.
- template<typename Other_Scalar_T>
  framed_multi (const framed_multi< Other_Scalar_T, LO, HI, Tune_P > &val)

  Construct a multivector from a multivector with a different scalar type.
- template<typename Other_Scalar_T>
  framed_multi (const framed_multi< Other_Scalar_T, LO, HI, Tune_P > &val, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given multivector.
- framed_multi (const framed_multi_t &val, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given multivector.
- framed_multi (const index_set_t ist, const Scalar_T &crd=Scalar_T(1))

  Construct a multivector from an index set and a scalar coordinate.
- framed_multi (const index_set_t ist, const Scalar_T &crd, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from an index set and a scalar coordinate.
- framed_multi (const Scalar_T &scr, const index_set_t frm=index_set_t())

  Construct a multivector from a scalar (within a frame, if given).
- framed_multi (const int scr, const index_set_t frm=index_set_t())

  Construct a multivector from an int (within a frame, if given).
- framed_multi (const vector_t &vec, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given vector.
- framed_multi (const std::string &str)

  Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- framed_multi (const std::string &str, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- framed_multi (const char *str)

Construct a multivector from a char∗: eg: "3+2{1,2}-6.1e-2{2,3}".

- framed_multi (const char ∗str, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a char∗: eg: "3+2{1,2}-6.1e-2{2,3}".

- template<typename Other_Scalar_T>
  framed_multi (const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > &val)

  Construct a multivector from a matrix_multi_t.

- template<typename Other_Scalar_T>
  auto fast_matrix_multi (const index_set_t frm) const -> const matrix_multi< Other_Scalar_T, LO, HI, Tune_P >

  Use generalized FFT to construct a matrix_multi_t.

- auto fast_framed_multi () const -> const framed_multi_t

  Use inverse generalized FFT to construct a framed_multi_t.

- _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS auto nbr_terms () const -> unsigned long

  Number of terms.

- auto operator+= (const term_t &term) -> multivector_t &

  Add a term, if non-zero.

Public Member Functions inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- virtual ~clifford_algebra ()=default
- virtual auto operator== (const multivector_t &val) const -> bool=0

  Test for equality of multivectors.

- virtual auto operator+= (const multivector_t &rhs) -> multivector_t &=0

  Geometric sum.

- virtual auto operator-= (const multivector_t &rhs) -> multivector_t &=0

  Geometric difference.

- virtual auto operator- () const -> const multivector_t=0

  Unary -.

- virtual auto operator∗= (const double &scr) -> multivector_t &=0

  Product of multivector and scalar.

- virtual auto operator%= (const multivector_t &rhs) -> multivector_t &=0

  Contraction.

- virtual auto operator&= (const multivector_t &rhs) -> multivector_t &=0

  Inner product.

- virtual auto operator^= (const multivector_t &rhs) -> multivector_t &=0

  Outer product.

- virtual auto operator/= (const double &scr) -> multivector_t &=0

  Quotient of multivector and scalar.

- virtual auto operator|= (const multivector_t &rhs) -> multivector_t &=0

  Transformation via twisted adjoint action.

- virtual auto inv () const -> const multivector_t=0

  Geometric multiplicative inverse.

- virtual auto pow (int m) const -> const multivector_t=0

  ∗*this to the m*

- virtual auto outer_pow (int m) const -> const multivector_t=0

  Outer product power.

- virtual auto frame () const -> const index_set_t=0

  Subalgebra generated by all generators of terms of given multivector.

- virtual auto grade () const -> index_t=0

  Maximum of the grades of each term.
- virtual auto operator[ ] (const index_set_t ist) const -> double=0

  Subscripting: map from index set to scalar coordinate.
- virtual auto operator() (index_t grade) const -> const multivector_t=0

  Pure grade-vector part.
- virtual auto scalar () const -> double=0

  Scalar part.
- virtual auto pure () const -> const multivector_t=0

  Pure part.
- virtual auto even () const -> const multivector_t=0

  Even part of multivector, sum of even grade terms.
- virtual auto odd () const -> const multivector_t=0

  Odd part of multivector, sum of odd grade terms.
- virtual auto vector_part () const -> const vector_t=0

  Vector part of multivector, as a vector_t with respect to frame().
- virtual auto involute () const -> const multivector_t=0

  Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.
- virtual auto reverse () const -> const multivector_t=0

  Reversion, eg. {1}∗{2} -> {2}∗{1}.
- virtual auto conj () const -> const multivector_t=0

  Conjugation, reverse o involute == involute o reverse.
- virtual auto quad () const -> double=0

  Scalar_T quadratic form == (rev(x)∗x)(0).
- virtual auto norm () const -> double=0

  Scalar_T norm == sum of norm of coordinates.
- virtual auto max_abs () const -> double=0

  Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto truncated (const double &limit=default_truncation) const -> const multivector_t=0

  Remove all terms with relative size smaller than limit.
- virtual auto isinf () const -> bool=0

  Check if a multivector contains any infinite values.
- virtual auto isnan () const -> bool=0

  Check if a multivector contains any IEEE NaN values.
- virtual void write (const std::string &msg="") const=0

  Write formatted multivector to output.

Static Public Member Functions

- static auto classname () -> const std::string

  Class name used in messages.
- static auto random (const index_set_t frm, Scalar_T fill=Scalar_T(1)) -> const multivector_t

  Random multivector within a frame.

Static Public Member Functions inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- static auto classname () -> const std::string

Private Types

- using var_term_t = class var_term
- using matrix_t = typename matrix_multi_t::matrix_t
- using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >
- using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>>
- using framed_pair_t = std::pair<const multivector_t, const multivector_t>
- using size_type = typename map_t::size_type
- using iterator = typename map_t::iterator
- using const_iterator = typename map_t::const_iterator

Private Member Functions

- framed_multi (const hash_size_t &hash_size)

  Private constructor using hash_size.
- auto fold (const index_set_t frm) const -> multivector_t

  Subalgebra isomorphism: fold each term within the given frame.
- auto unfold (const index_set_t frm) const -> multivector_t

  Subalgebra isomorphism: unfold each term within the given frame.
- auto centre_pm4_qp4 (index_t &p, index_t &q) -> multivector_t &

  Subalgebra isomorphism: R_{p,q} to R_{p-4,q+4}.
- auto centre_pp4_qm4 (index_t &p, index_t &q) -> multivector_t &

  Subalgebra isomorphism: R_{p,q} to R_{p+4,q-4}.
- auto centre_qp1_pm1 (index_t &p, index_t &q) -> multivector_t &

  Subalgebra isomorphism: R_{p,q} to R_{q+1,p-1}.
- auto divide (const index_set_t ist) const -> const framed_pair_t

  Divide multivector into part divisible by index_set and remainder.
- auto fast (const index_t level, const bool odd) const -> const matrix_t

  Generalized FFT from multivector_t to matrix_t.

Friends

- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P> class matrix_multi
- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P> class framed_multi
- auto operator∗ (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto operator^ (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto operator& (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto operator% (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto star (const multivector_t &lhs, const multivector_t &rhs) -> Scalar_T
- auto operator/ (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto operator| (const multivector_t &lhs, const multivector_t &rhs) -> const multivector_t
- auto operator>> (std::istream &s, multivector_t &val) -> std::istream &
- auto operator<< (std::ostream &os, const multivector_t &val) -> std::ostream &
- auto operator<< (std::ostream &os, const term_t &term) -> std::ostream &
- auto exp (const multivector_t &val) -> const multivector_t

Additional Inherited Members

Static Public Attributes inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- static const index_t v_lo
- static const index_t v_hi
- static const double default_truncation

  Default for truncation.

8.13.1 Detailed Description

template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >

A framed_multi<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector.

Definition at line 126 of file framed_multi.h.

8.13.2 Member Typedef Documentation

8.13.2.1 const_iterator

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::const_iterator = typename map_t::↩
const_iterator  [private]
```

Definition at line 167 of file framed_multi.h.

8.13.2.2 error_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 138 of file framed_multi.h.

8.13.2.3 framed_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = multivector_t
```

Definition at line 132 of file framed_multi.h.

8.13.2.4 framed_pair_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_pair_t = std::pair<const multivector_t,
const multivector_t>  [private]
```

Definition at line 164 of file framed_multi.h.

8.13.2.5 index_set_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 135 of file framed_multi.h.

8.13.2.6 iterator

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::iterator = typename map_t::iterator
[private]
```

Definition at line 166 of file framed_multi.h.

8.13.2.7 map_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::map_t = std::unordered_map<index_set_t,
Scalar_T, index_set_hash<LO, HI>>  [private]
```

Definition at line 150 of file framed_multi.h.

8.13.2.8 matrix_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = matrix_multi<Scalar↩
_T,LO,HI,Tune_P >
```

Definition at line 139 of file framed_multi.h.

8.13.2.9 matrix_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = typename matrix_multi_t::matrix_t
[private]
```

Definition at line 148 of file framed_multi.h.

8.13.2.10 multivector_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = framed_multi
```

Definition at line 131 of file framed_multi.h.

8.13.2.11 scalar_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 133 of file framed_multi.h.

8.13.2.12 size_type

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::size_type = typename map_t::size_type
[private]
```

Definition at line 165 of file framed_multi.h.

8.13.2.13 sorted_map_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::sorted_map_t = std::map< index_set_t,
Scalar_T, std::less<const index_set_t> > [private]
```

Definition at line 149 of file framed_multi.h.

8.13.2.14 term_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 136 of file framed_multi.h.

8.13.2.15 tune_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 134 of file framed_multi.h.

8.13.2.16 var_term_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term_t = class var_term  [private]
```

Definition at line 147 of file framed_multi.h.

8.13.2.17 vector_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 137 of file framed_multi.h.

8.13.3 Constructor & Destructor Documentation

8.13.3.1 ~framed_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::~framed_multi ()  [override], [default]
```

Destructor.

8.13.3.2 framed_multi() [1/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi ()
```

Default constructor.

Definition at line 59 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.3 framed_multi() [2/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const hash_size_t & hash_size)  [private]
```

Private constructor using hash_size.

Definition at line 66 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.4 framed_multi() [3/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 74 of file framed_multi_imp.h.

References _GLUCAT_HASH_N, framed_multi, and glucat::numeric_traits< Scalar_T >::to_scalar_t().

8.13.3.5 framed_multi() [4/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 85 of file framed_multi_imp.h.

References _GLUCAT_HASH_N, glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame(), framed_multi, and glucat::numeric_traits< Scalar_T >::to_scalar_t().

8.13.3.6 framed_multi() [5/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const framed_multi_t & val,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 98 of file framed_multi_imp.h.

References _GLUCAT_HASH_N, and glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame().

8.13.3.7 framed_multi() [6/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const index_set_t ist,
            const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 111 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.8 framed_multi() [7/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const index_set_t ist,
            const Scalar_T & crd,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 121 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.9 framed_multi() [8/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const Scalar_T & scr,
            const index_set_t frm = index_set_t())
```

Construct a multivector from a scalar (within a frame, if given).

Definition at line 134 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.10 framed_multi() [9/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const int scr,
            const index_set_t frm = index_set_t())
```

Construct a multivector from an int (within a frame, if given).

Definition at line 144 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.11 framed_multi() [10/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const vector_t & vec,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 154 of file framed_multi_imp.h.

References _GLUCAT_HASH_N, glucat::index_set< LO, HI >::count(), glucat::index_set< LO, HI >::max(), and glucat::index_set< LO, HI >::min().

8.13.3.12 framed_multi() [11/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 176 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.13 framed_multi() [12/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const std::string & str,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 192 of file framed_multi_imp.h.

References _GLUCAT_HASH_N.

8.13.3.14 framed_multi() [13/15]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const char * str)  [inline]
```

Construct a multivector from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 209 of file framed_multi.h.

8.13.3.15 framed_multi() [14/15]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const char * str,
            const index_set_t frm,
            const bool prechecked = false)  [inline]
```

Construct a multivector, within a given frame, from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 212 of file framed_multi.h.

8.13.3.16 framed_multi() [15/15]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
            const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a matrix_multi_t.

Definition at line 205 of file framed_multi_imp.h.

References _GLUCAT_HASH_N, _GLUCAT_HASH_SIZE_T, glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element(), fast_framed_multi(), glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame(), glucat::matrix::inner(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix, matrix_multi, glucat::matrix::nnz(), glucat::clifford_algebra< Scalar_T, I and glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::truncated().

8.13.4 Member Function Documentation

8.13.4.1 centre_pm4_qp4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4 (
            index_t & p,
            index_t & q) -> multivector_t &  [private]
```

Subalgebra isomorphism: R_{p,q} to R_{p-4,q+4}.

Definition at line 1464 of file framed_multi_imp.h.

References glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::frame().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi().

8.13.4.2 centre_pp4_qm4()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pp4_qm4 (
            index_t & p,
            index_t & q) -> multivector_t &  [private]
```

Subalgebra isomorphism: R_{p,q} to R_{p+4,q-4}.

Definition at line 1506 of file framed_multi_imp.h.

References glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::frame().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi().

8.13.4.3 centre_qp1_pm1()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1 (
             index_t & p,
             index_t & q) -> multivector_t &  [private]
```

Subalgebra isomorphism: R_{p,q} to R_{q+1,p-1}.

Definition at line 1548 of file framed_multi_imp.h.

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi().

8.13.4.4 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 50 of file framed_multi_imp.h.

8.13.4.5 divide()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::divide (
             const index_set_t ist) const -> const framed_pair_t  [private]
```

Divide multivector into part divisible by index_set and remainder.

Divide multivector into quotient with terms divisible by index set, and remainder.

Definition at line 1581 of file framed_multi_imp.h.

Referenced by fast().

8.13.4.6 fast()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast (
             const index_t level,
             const bool odd) const -> const matrix_t  [private]
```

Generalized FFT from multivector_t to matrix_t.

Definition at line 1597 of file framed_multi_imp.h.

References divide(), glucat::matrix::kron(), glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::odd(), glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, framed_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::scalar(), and glucat::matrix::unit().

8.13.4.7 fast_framed_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi_t  [inline]
```

Use inverse generalized FFT to construct a framed_multi_t.

Definition at line 1695 of file framed_multi_imp.h.

Referenced by framed_multi().

8.13.4.8 fast_matrix_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
             const index_set_t frm) const -> const matrix_multi< Other_Scalar_T, LO, HI, Tune↩
_P >
```

Use generalized FFT to construct a matrix_multi_t.

Definition at line 1663 of file framed_multi_imp.h.

References fold(), matrix_multi, glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi, and glucat::pos_mod().

8.13.4.9 fold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fold (
             const index_set_t frm) const -> multivector_t  [private]
```

Subalgebra isomorphism: fold each term within the given frame.

Definition at line 1429 of file framed_multi_imp.h.

Referenced by fast_matrix_multi().

8.13.4.10 nbr_terms()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::nbr_terms () const -> unsigned long
```

Number of terms.

Definition at line 1351 of file framed_multi_imp.h.

8.13.4.11 operator+=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
            const term_t & term) -> multivector_t &  [inline]
```

Add a term, if non-zero.

Insert a term into a multivector, add terms with same index set.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 295 of file framed_multi_imp.h.

8.13.4.12 random()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random (
            const index_set_t frm,
            Scalar_T fill = Scalar_T(1)) -> const multivector_t  [static]
```

Random multivector within a frame.

Definition at line 1054 of file framed_multi_imp.h.

References framed_multi.

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::random().

8.13.4.13 unfold()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold (
            const index_set_t frm) const -> multivector_t  [private]
```

Subalgebra isomorphism: unfold each term within the given frame.

Definition at line 1446 of file framed_multi_imp.h.

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi().

8.13.5 Friends And Related Symbol Documentation

8.13.5.1 exp

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto exp (
            const multivector_t & val) -> const multivector_t  [friend]
```

8.13.5.2 framed_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class framed_multi  [friend]
```

Definition at line 143 of file framed_multi.h.

Referenced by framed_multi(), framed_multi(), framed_multi(), framed_multi(), and random().

8.13.5.3 matrix_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class matrix_multi  [friend]
```

Definition at line 141 of file framed_multi.h.

Referenced by fast_matrix_multi(), and framed_multi().

8.13.5.4 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator% (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.5 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator& (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.6 operator∗

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator* (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.7 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator/ (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.8 operator<< [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator<< (
            std::ostream & os,
            const multivector_t & val) -> std::ostream &  [friend]
```

8.13.5.9 operator<< [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator<< (
            std::ostream & os,
            const term_t & term) -> std::ostream &  [friend]
```

8.13.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator>> (
            std::istream & s,
            multivector_t & val) -> std::istream &  [friend]
```

8.13.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator^ (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.12 operator"|

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator| (
            const multivector_t & lhs,
            const multivector_t & rhs) -> const multivector_t  [friend]
```

8.13.5.13 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto star (
            const multivector_t & lhs,
            const multivector_t & rhs) -> Scalar_T  [friend]
```

The documentation for this class was generated from the following files:

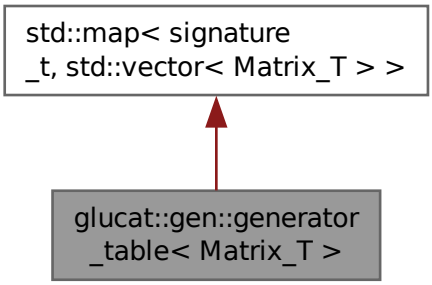
- glucat/framed_multi.h
- glucat/framed_multi_imp.h

8.14 glucat::gen::generator_table< Matrix_T > Class Template Reference

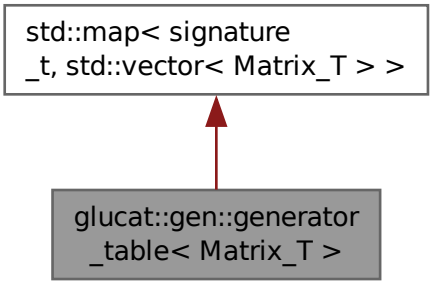
Table of generators for specific signatures.

`#include <generation.h>`

Inheritance diagram for glucat::gen::generator_table< Matrix_T >:



Collaboration diagram for glucat::gen::generator_table< Matrix_T >:



Public Member Functions

- auto operator() (const index_t p, const index_t q) -> const Matrix_T *
  Pointer to generators for a specific signature.
- generator_table (const generator_table &)=delete
- auto operator= (const generator_table &) -> generator_table &=delete

Static Public Member Functions

- static auto generator () -> generator_table< Matrix_T > &
  Single instance of generator table.

Private Member Functions

- auto gen_vector (const index_t p, const index_t q) -> const std::vector< Matrix_T > &
  Construct a vector of generators for a specific signature.
- void gen_from_pm1_qm1 (const std::vector< Matrix_T > &old, const signature_t sig)
  Construct generators for p,q given generators for p-1,q-1.
- void gen_from_pm4_qp4 (const std::vector< Matrix_T > &old, const signature_t sig)
  Construct generators for p,q given generators for p-4,q+4.
- void gen_from_pp4_qm4 (const std::vector< Matrix_T > &old, const signature_t sig)
  Construct generators for p,q given generators for p+4,q-4.
- void gen_from_qp1_pm1 (const std::vector< Matrix_T > &old, const signature_t sig)
  Construct generators for p,q given generators for q+1,p-1.
- generator_table ()=default
- ~generator_table ()=default

Friends

- class friend_for_private_destructor

8.14.1 Detailed Description

template<class Matrix_T>
class glucat::gen::generator_table< Matrix_T >

Table of generators for specific signatures.

Definition at line 52 of file generation.h.

8.14.2 Constructor & Destructor Documentation

8.14.2.1 generator_table() [1/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table ()  [private], [default]
```

Referenced by generator(), generator_table(), and operator=().

8.14.2.2 ~generator_table()

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::~generator_table ()  [private], [default]
```

8.14.2.3 generator_table() [2/2]

```
template<class Matrix_T>
glucat::gen::generator_table< Matrix_T >::generator_table (
            const generator_table< Matrix_T > & )  [delete]
```

References generator_table().

8.14.3 Member Function Documentation

8.14.3.1 gen_from_pm1_qm1()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1 (
            const std::vector< Matrix_T > & old,
            const signature_t sig)  [private]
```

Construct generators for p,q given generators for p-1,q-1.

Definition at line 127 of file generation_imp.h.

References glucat::matrix::mono_kron(), and glucat::matrix::unit().

Referenced by gen_vector().

8.14.3.2 gen_from_pm4_qp4()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pm4_qp4 (
            const std::vector< Matrix_T > & old,
            const signature_t sig)  [private]
```

Construct generators for p,q given generators for p-4,q+4.

Definition at line 165 of file generation_imp.h.

References glucat::matrix::mono_prod().

Referenced by gen_vector().

8.14.3.3 gen_from_pp4_qm4()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4 (
            const std::vector< Matrix_T > & old,
            const signature_t sig)  [private]
```

Construct generators for p,q given generators for p+4,q-4.

Definition at line 198 of file generation_imp.h.

References glucat::matrix::mono_prod().

Referenced by gen_vector().

8.14.3.4 gen_from_qp1_pm1()

```
template<class Matrix_T>
void glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1 (
            const std::vector< Matrix_T > & old,
            const signature_t sig)  [private]
```

Construct generators for p,q given generators for q+1,p-1.

Definition at line 231 of file generation_imp.h.

References glucat::matrix::mono_prod().

Referenced by gen_vector().

8.14.3.5 gen_vector()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::gen_vector (
            const index_t p,
            const index_t q) -> const std::vector< Matrix_T > &  [private]
```

Construct a vector of generators for a specific signature.

Definition at line 79 of file generation_imp.h.

References gen_from_pm1_qm1(), gen_from_pm4_qp4(), gen_from_pp4_qm4(), gen_from_qp1_pm1(), gen_vector(), glucat::pos_mod(), and glucat::matrix::unit().

Referenced by gen_vector(), and operator()().

8.14.3.6 generator()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::generator () -> generator_table< Matrix_T > &
[static]
```

Single instance of generator table.

Definition at line 49 of file generation_imp.h.

References generator_table().

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element().

8.14.3.7 operator()()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator() (
             const index_t p,
             const index_t q) -> const Matrix_T *  [inline]
```

Pointer to generators for a specific signature.

Definition at line 58 of file generation_imp.h.

References gen_vector(), and glucat::pos_mod().

8.14.3.8 operator=()

```
template<class Matrix_T>
auto glucat::gen::generator_table< Matrix_T >::operator= (
             const generator_table< Matrix_T > & ) -> generator_table &=delete  [delete]
```

References generator_table().

8.14.4 Friends And Related Symbol Documentation

8.14.4.1 friend_for_private_destructor

```
template<class Matrix_T>
friend class friend_for_private_destructor  [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE http://doc.ece.uci.edu

Definition at line 75 of file generation.h.

References friend_for_private_destructor.

Referenced by friend_for_private_destructor.

The documentation for this class was generated from the following files:

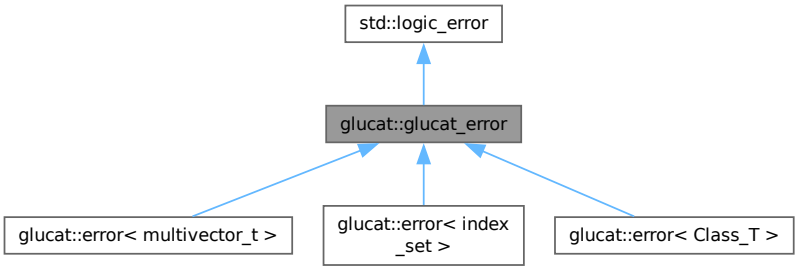
- glucat/generation.h
- glucat/generation_imp.h

8.15 glucat::glucat_error Class Reference

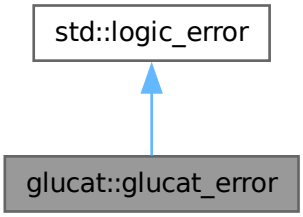
Abstract exception class.

`#include <errors.h>`

Inheritance diagram for glucat::glucat_error:



Collaboration diagram for glucat::glucat_error:



Public Member Functions

- glucat_error (const std::string &context, const std::string &msg)
- ~glucat_error () noexcept override=default
- virtual auto heading () const noexcept -> const std::string=0
- virtual auto classname () const noexcept -> const std::string=0
- virtual void print_error_msg () const =0

Public Attributes

- std::string name

8.15.1 Detailed Description

Abstract exception class.

Definition at line 41 of file errors.h.

8.15.2 Constructor & Destructor Documentation

8.15.2.1 glucat_error()

```
glucat::glucat_error::glucat_error (
            const std::string & context,
            const std::string & msg)  [inline]
```

Definition at line 44 of file errors.h.

References name.

Referenced by glucat::error< Class_T >::error(), and glucat::error< Class_T >::error().

8.15.2.2 ~glucat_error()

```
glucat::glucat_error::~glucat_error ()  [override], [default], [noexcept]
```

8.15.3 Member Function Documentation

8.15.3.1 classname()

```
virtual auto glucat::glucat_error::classname () const -> const std::string  [pure virtual],
[noexcept]
```

Implemented in glucat::error< Class_T >, glucat::error< index_set >, and glucat::error< multivector_t >.

References classname().

Referenced by classname().

8.15.3.2 heading()

```
virtual auto glucat::glucat_error::heading () const -> const std::string  [pure virtual],
[noexcept]
```

Implemented in glucat::error< Class_T >, glucat::error< index_set >, and glucat::error< multivector_t >.

References heading().

Referenced by heading().

8.15.3.3 print_error_msg()

```
virtual void glucat::glucat_error::print_error_msg () const  [pure virtual]
```

Implemented in glucat::error< Class_T >, glucat::error< index_set >, and glucat::error< multivector_t >.

References print_error_msg().

Referenced by print_error_msg().

8.15.4 Member Data Documentation

8.15.4.1 name

```
std::string glucat::glucat_error::name
```

Definition at line 51 of file errors.h.

Referenced by glucat::error< Class_T >::classname(), and glucat_error().

The documentation for this class was generated from the following file:

- glucat/errors.h

8.16 glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t Class Reference

Public Member Functions

- hash_size_t (size_t hash_size)
- auto operator() () const -> size_t

Private Attributes

- size_t n

8.16.1 Detailed Description

template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t

Definition at line 152 of file framed_multi.h.

8.16.2 Constructor & Destructor Documentation

8.16.2.1 hash_size_t()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::hash_size_t (
            size_t hash_size)  [inline]
```

Definition at line 155 of file framed_multi.h.

References n.

8.16.3 Member Function Documentation

8.16.3.1 operator()()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::operator() () const ->
size_t  [inline]
```

Definition at line 158 of file framed_multi.h.

References n.

8.16.4 Member Data Documentation

8.16.4.1 n

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
size_t glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::n  [private]
```

Definition at line 161 of file framed_multi.h.

Referenced by hash_size_t(), and operator()().

The documentation for this class was generated from the following file:

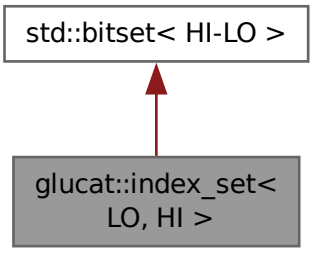
- glucat/framed_multi.h

8.17 glucat::index_set< LO, HI > Class Template Reference

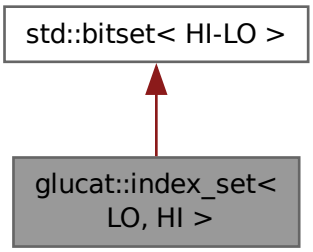
Index set class based on std::bitset<> in Gnu standard C++ library.

`#include <index_set.h>`

Inheritance diagram for glucat::index_set< LO, HI >:



Collaboration diagram for glucat::index_set< LO, HI >:



Classes

- class reference

  Index set member reference.

Public Types

- using index_set_t = index_set
- using index_pair_t = std::pair<index_t, index_t>

Public Member Functions

- index_set ()=default

  Default constructor creates an empty set.
- index_set (const bitset_t bst)

  Constructor from bitset_t.
- index_set (const index_t idx)

  Constructor from index.
- index_set (const set_value_t folded_val, const index_set_t frm, const bool prechecked=false)

  Constructor from set value of an index set folded within the given frame.
- index_set (const index_pair_t &range, const bool prechecked=false)

  Constructor from range of indices from range.first to range.second.
- index_set (const std::string &str)

  Constructor from string.
- auto operator== (const index_set_t rhs) const -> bool

  Equality.
- auto operator!= (const index_set_t rhs) const -> bool

  Inequality.
- auto operator~ () const -> index_set_t

  Set complement: not.
- auto operator^= (const index_set_t rhs) -> index_set_t &

  Symmetric set difference: exclusive or.
- auto operator&= (const index_set_t rhs) -> index_set_t &

  Set intersection: and.
- auto operator|= (const index_set_t rhs) -> index_set_t &

  Set union: or.
- auto operator[ ] (const index_t idx) const -> bool

  Subscripting: Test idx for membership: test value of bit idx.
- auto test (const index_t idx) const -> bool

  Test idx for membership: test value of bit idx.
- auto set () -> index_set_t &

  Include all indices except 0: set all bits except 0.
- auto set (const index_t idx) -> index_set_t &

  Include idx: Set bit at idx if idx != 0.
- auto set (const index_t idx, const int val) -> index_set_t &

  Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.
- auto reset () -> index_set_t &

  Make set empty: Set all bits to 0.
- auto reset (const index_t idx) -> index_set_t &

  Exclude idx: Set bit at idx to 0.
- auto flip () -> index_set_t &

  Set complement, except 0: flip all bits, except 0.
- auto flip (const index_t idx) -> index_set_t &

  Complement membership of idx if idx != 0: flip bit at idx if idx != 0.
- auto count () const -> index_t

  Cardinality: Number of indices included in set.
- auto count_neg () const -> index_t

  Number of negative indices included in set.
- auto count_pos () const -> index_t

  Number of positive indices included in set.
- auto min () const -> index_t

Minimum member.

- auto max () const -> index_t

  Maximum member.

- auto operator< (const index_set_t rhs) const -> bool

  Less than operator used for comparisons, map, etc.

- auto is_contiguous () const -> bool

  Determine if the index set is contiguous, ie. has no gaps.

- auto fold () const -> const index_set_t

  Fold this index set within itself as a frame.

- auto fold (const index_set_t frm, const bool prechecked=false) const -> const index_set_t

  Fold this index set within the given frame.

- auto unfold (const index_set_t frm, const bool prechecked=false) const -> const index_set_t

  Unfold this index set within the given frame.

- auto value_of_fold (const index_set_t frm) const -> set_value_t

  The set value of the fold of this index set within the given frame.

- auto sign_of_mult (const index_set_t ist) const -> int

  Sign of geometric product of two Clifford basis elements.

- auto sign_of_square () const -> int

  Sign of geometric square of a Clifford basis element.

- auto hash_fn () const -> size_t

  Hash function.

- auto operator[ ] (index_t idx) -> reference

  Subscripting: Element access.

Static Public Member Functions

- static auto classname () -> const std::string

Static Public Attributes

- static const index_t v_lo = LO
- static const index_t v_hi = HI

Private Types

- using bitset_t = std::bitset<HI - LO>
- using error_t = error<index_set>

Private Member Functions

- BOOST_STATIC_ASSERT ((LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< _GLUCAT_BITS_PER_ULONG) &&(HI< _GLUCAT_BITS_PER_ULONG) &&(HI-LO<=_GLUCAT_BITS_PER_ULONG))
- auto lex_less_than (const index_set_t rhs) const -> bool

  Lexicographic ordering of two sets: $*this < rhs$.

Friends

- class reference
- auto operator^ (const index_set_t &lhs, const index_set_t &rhs) -> const index_set_t
- auto operator& (const index_set_t &lhs, const index_set_t &rhs) -> const index_set_t
- auto operator| (const index_set_t &lhs, const index_set_t &rhs) -> const index_set_t
- auto compare (const index_set_t &lhs, const index_set_t &rhs) -> int

8.17.1 Detailed Description

template<const index_t LO, const index_t HI>
class glucat::index_set< LO, HI >

Index set class based on std::bitset<> in Gnu standard C++ library.

Definition at line 73 of file index_set.h.

8.17.2 Member Typedef Documentation

8.17.2.1 bitset_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::bitset_t = std::bitset<HI - LO>  [private]
```

Definition at line 81 of file index_set.h.

8.17.2.2 error_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::error_t = error<index_set>  [private]
```

Definition at line 82 of file index_set.h.

8.17.2.3 index_pair_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_pair_t = std::pair<index_t, index_t>
```

Definition at line 85 of file index_set.h.

8.17.2.4 index_set_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_set_t = index_set
```

Definition at line 84 of file index_set.h.

8.17.3 Constructor & Destructor Documentation

8.17.3.1 index_set() [1/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set ()  [default]
```

Default constructor creates an empty set.

Referenced by glucat::operator>>().

8.17.3.2 index_set() [2/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
            const bitset_t bst)
```

Constructor from bitset_t.

Definition at line 61 of file index_set_imp.h.

8.17.3.3 index_set() [3/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
            const index_t idx)
```

Constructor from index.

Constructor from index value.

Definition at line 55 of file index_set_imp.h.

References set().

8.17.3.4 index_set() [4/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
            const set_value_t folded_val,
            const index_set_t frm,
            const bool prechecked = false)
```

Constructor from set value of an index set folded within the given frame.

Definition at line 68 of file index_set_imp.h.

References count(), fold(), min(), and unfold().

8.17.3.5 index_set() [5/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
            const index_pair_t & range,
            const bool prechecked = false)
```

Constructor from range of indices from range.first to range.second.

Definition at line 82 of file index_set_imp.h.

8.17.3.6 index_set() [6/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
            const std::string & str)
```

Constructor from string.

Definition at line 102 of file index_set_imp.h.

8.17.4 Member Function Documentation

8.17.4.1 BOOST_STATIC_ASSERT()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::BOOST_STATIC_ASSERT (
            (LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< _GLUCAT_BITS_PER_ULONG) &&(HI< _GLUCAT_↩
BITS_PER_ULONG) &&(HI-LO<=_GLUCAT_BITS_PER_ULONG) )  [private]
```

8.17.4.2 classname()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::classname () -> const std::string  [inline], [static]
```

Definition at line 49 of file index_set_imp.h.

8.17.4.3 count()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count () const -> index_t  [inline]
```

Cardinality: Number of indices included in set.

Definition at line 344 of file index_set_imp.h.

Referenced by count_neg(), count_pos(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), index_set(), is_contiguous(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), operator<(), and sign_of_square().

8.17.4.4 count_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_neg () const -> index_t  [inline]
```

Number of negative indices included in set.

Definition at line 364 of file index_set_imp.h.

References count().

Referenced by sign_of_square().

8.17.4.5 count_pos()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_pos () const -> index_t  [inline]
```

Number of positive indices included in set.

Definition at line 376 of file index_set_imp.h.

References count().

8.17.4.6 flip() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip () -> index_set_t &
```

Set complement, except 0: flip all bits, except 0.

8.17.4.7 flip() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip (
            const index_t idx) -> index_set_t &  [inline]
```

Complement membership of idx if idx != 0: flip bit at idx if idx != 0.

Definition at line 330 of file index_set_imp.h.

8.17.4.8 fold() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold () const -> const index_set_t
```

Fold this index set within itself as a frame.

Referenced by index_set(), and value_of_fold().

8.17.4.9 fold() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold (
            const index_set_t frm,
            const bool prechecked = false) const -> const index_set_t
```

Fold this index set within the given frame.

8.17.4.10 hash_fn()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::hash_fn () const -> size_t  [inline]
```

Hash function.

Definition at line 950 of file index_set_imp.h.

8.17.4.11 is_contiguous()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::is_contiguous () const -> bool  [inline]
```

Determine if the index set is contiguous, ie. has no gaps.

Determine if the index set is contiguous, ie. has no gaps when 0 is included.

Definition at line 732 of file index_set_imp.h.

References count(), max(), and min().

8.17.4.12 lex_less_than()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::lex_less_than (
            const index_set_t rhs) const -> bool  [inline], [private]
```

Lexicographic ordering of two sets: $*$this $<$ rhs.

Definition at line 588 of file index_set_imp.h.

Referenced by operator<().

8.17.4.13 max()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::max () const -> index_t
```

Maximum member.

Maximum member, or 0 if none.

Definition at line 550 of file index_set_imp.h.

References test().

Referenced by PyClical.index_set::__iter__(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), is_contiguous(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), and unfold().

8.17.4.14 min()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::min () const -> index_t
```

Minimum member.

Minimum member, or 0 if none.

Definition at line 461 of file index_set_imp.h.

References test().

Referenced by PyClical.index_set::__iter__(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), index_set(), is_contiguous(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), and unfold().

8.17.4.15 operator"!=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator!= (
            const index_set_t rhs) const -> bool  [inline]
```

Inequality.

Definition at line 130 of file index_set_imp.h.

8.17.4.16 operator&=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator&= (
            const index_set_t rhs) -> index_set_t &  [inline]
```

Set intersection: and.

Definition at line 174 of file index_set_imp.h.

8.17.4.17 operator<()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator< (
            const index_set_t rhs) const -> bool  [inline]
```

Less than operator used for comparisons, map, etc.

Definition at line 596 of file index_set_imp.h.

References count(), and lex_less_than().

8.17.4.18 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator== (
            const index_set_t rhs) const -> bool  [inline]
```

Equality.

Definition at line 119 of file index_set_imp.h.

8.17.4.19 operator[]() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
            const index_t idx) const -> bool  [inline]
```

Subscripting: Test idx for membership: test value of bit idx.

Definition at line 232 of file index_set_imp.h.

References test().

8.17.4.20 operator[]() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
            index_t idx) -> reference  [inline]
```

Subscripting: Element access.

Definition at line 224 of file index_set_imp.h.

8.17.4.21 operator^=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator^= (
            const index_set_t rhs) -> index_set_t &  [inline]
```

Symmetric set difference: exclusive or.

Definition at line 149 of file index_set_imp.h.

8.17.4.22 operator"|=()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator|= (
            const index_set_t rhs) -> index_set_t &  [inline]
```

Set union: or.

Definition at line 199 of file index_set_imp.h.

8.17.4.23 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator~ () const -> index_set_t  [inline]
```

Set complement: not.

Definition at line 141 of file index_set_imp.h.

8.17.4.24 reset() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset () -> index_set_t &  [inline]
```

Make set empty: Set all bits to 0.

Definition at line 294 of file index_set_imp.h.

8.17.4.25 reset() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset (
            const index_t idx) -> index_set_t &  [inline]
```

Exclude idx: Set bit at idx to 0.

Definition at line 305 of file index_set_imp.h.

8.17.4.26 set() [1/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set () -> index_set_t &  [inline]
```

Include all indices except 0: set all bits except 0.

Definition at line 255 of file index_set_imp.h.

Referenced by index_set().

8.17.4.27 set() [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
            const index_t idx) -> index_set_t &  [inline]
```

Include idx: Set bit at idx if idx != 0.

Definition at line 266 of file index_set_imp.h.

8.17.4.28 set() [3/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
             const index_t idx,
             const int val) -> index_set_t &  [inline]
```

Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.

Definition at line 280 of file index_set_imp.h.

8.17.4.29 sign_of_mult()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_mult (
             const index_set_t ist) const -> int
```

Sign of geometric product of two Clifford basis elements.

Definition at line 880 of file index_set_imp.h.

References glucat::inverse_gray(), and glucat::inverse_reversed_gray().

8.17.4.30 sign_of_square()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_square () const -> int  [inline]
```

Sign of geometric square of a Clifford basis element.

Definition at line 930 of file index_set_imp.h.

References count(), and count_neg().

8.17.4.31 test()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::test (
             const index_t idx) const -> bool  [inline]
```

Test idx for membership: test value of bit idx.

Definition at line 240 of file index_set_imp.h.

Referenced by max(), min(), operator[]() , and unfold().

8.17.4.32 unfold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::unfold (
             const index_set_t frm,
             const bool prechecked = false) const -> const index_set_t
```

Unfold this index set within the given frame.

Definition at line 794 of file index_set_imp.h.

References max(), min(), and test().

Referenced by index_set().

8.17.4.33 value_of_fold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::value_of_fold (
             const index_set_t frm) const -> set_value_t  [inline]
```

The set value of the fold of this index set within the given frame.

Definition at line 829 of file index_set_imp.h.

References fold().

8.17.5 Friends And Related Symbol Documentation

8.17.5.1 compare

```
template<const index_t LO, const index_t HI>
auto compare (
             const index_set_t & lhs,
             const index_set_t & rhs) -> int  [friend]
```

8.17.5.2 operator&

```
template<const index_t LO, const index_t HI>
auto operator& (
             const index_set_t & lhs,
             const index_set_t & rhs) -> const index_set_t  [friend]
```

8.17.5.3 operator^

```
template<const index_t LO, const index_t HI>
auto operator^ (
             const index_set_t & lhs,
             const index_set_t & rhs) -> const index_set_t  [friend]
```

8.17.5.4 operator" |

```
template<const index_t LO, const index_t HI>
auto operator| (
             const index_set_t & lhs,
             const index_set_t & rhs) -> const index_set_t  [friend]
```

8.17.5.5 reference

```
template<const index_t LO, const index_t HI>
friend class reference  [friend]
```

Definition at line 174 of file index_set.h.

8.17.6 Member Data Documentation

8.17.6.1 v_hi

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_hi = HI  [static]
```

Definition at line 88 of file index_set.h.

8.17.6.2 v_lo

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_lo = LO  [static]
```

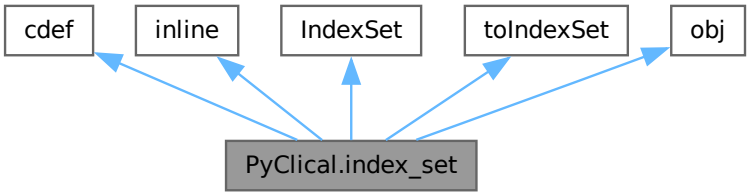
Definition at line 87 of file index_set.h.

The documentation for this class was generated from the following files:

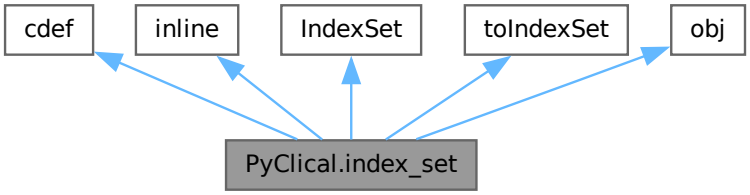
- glucat/index_set.h
- glucat/index_set_imp.h

8.18 PyClical.index_set Class Reference

Inheritance diagram for PyClical.index_set:



Collaboration diagram for PyClical.index_set:



Public Member Functions

- __cinit__ (self, other=0)
- __dealloc__ (self)
- __richcmp__ (lhs, rhs, int, op)
- __setitem__ (self, idx, val)
- __getitem__ (self, idx)
- __contains__ (self, idx)
- __iter__ (self)
- __invert__ (self)
- __xor__ (lhs, rhs)
- __ixor__ (self, rhs)
- __and__ (lhs, rhs)
- __iand__ (self, rhs)
- __or__ (lhs, rhs)
- __ior__ (self, rhs)
- count (self)
- count_neg (self)
- count_pos (self)
- min (self)
- max (self)

- hash_fn (self)
- sign_of_mult (self, rhs)
- sign_of_square (self)
- __repr__ (self)
- __str__ (self)

Public Attributes

- instance = new IndexSet((<index_set>other).unwrap())

8.18.1 Detailed Description

```
Return the C++ IndexSet instance wrapped by index_set(obj).

Python class index_set wraps C++ class IndexSet.
```

Definition at line 38 of file PyClical.pyx.

8.18.2 Member Function Documentation

8.18.2.1 __and__()

```
PyClical.index_set.__and__ (
              lhs,
              rhs)

Set intersection: and.

>>> print(index_set({1}) & index_set({2}))
{}
>>> print(index_set({1,2}) & index_set({2}))
{2}
```

Definition at line 271 of file PyClical.pyx.

8.18.2.2 __cinit__()

```
PyClical.index_set.__cinit__ (
              self,
              other = 0)

Construct an object of type index_set.

>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set("{1,2,1}"))
{1,2}
>>> print(index_set(""))
{}
```

Definition at line 74 of file PyClical.pyx.

8.18.2.3 __contains__()

```
PyClical.index_set.__contains__ (
              self,
              idx)
```

Check that an index_set object contains the index idx: idx in self.

```
>>> 1 in index_set({1})
True
>>> 2 in index_set({1})
False
>>> -1 in index_set({2})
False
>>> 1 in index_set({2})
False
>>> 2 in index_set({2})
True
>>> 33 in index_set({2})
False
```

Definition at line 210 of file PyClical.pyx.

References instance.

8.18.2.4 __dealloc__()

```
PyClical.index_set.__dealloc__ (
              self)
```

Clean up by deallocating the instance of C++ class IndexSet.

Definition at line 116 of file PyClical.pyx.

References instance.

8.18.2.5 __getitem__()

```
PyClical.index_set.__getitem__ (
              self,
              idx)
```

Get the value of an index_set object at an index.

```
>>> index_set({1})[1]
True
>>> index_set({1})[2]
False
>>> index_set({2})[-1]
False
>>> index_set({2})[1]
False
>>> index_set({2})[2]
True
>>> index_set({2})[33]
False
```

Definition at line 191 of file PyClical.pyx.

References instance.

8.18.2.6 __iand__()

```
PyClical.index_set.__iand__ (
              self,
              rhs)
```

```
Set intersection: and.

>>> x = index_set({1}); x &= index_set({2}); print(x)
{}
>>> x = index_set({1,2}); x &= index_set({2}); print(x)
{2}
```

Definition at line 282 of file PyClical.pyx.

8.18.2.7 __invert__()

```
PyClical.index_set.__invert__ (
              self)
```

```
Set complement: not.

>>> print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,
```

Definition at line 240 of file PyClical.pyx.

References instance.

8.18.2.8 __ior__()

```
PyClical.index_set.__ior__ (
              self,
              rhs)
```

```
Set union: or.

>>> x = index_set({1}); x |= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x |= index_set({2}); print(x)
{1,2}
```

Definition at line 304 of file PyClical.pyx.

8.18.2.9 __iter__()

```
PyClical.index_set.__iter__ (
              self)
```

```
Iterate over the indices of an index_set.

>>> for i in index_set({-3,4,7}):print(i, end=",")
-3,4,7,
```

Definition at line 229 of file PyClical.pyx.

References glucat::index_set< LO, HI >.max(), glucat::index_set< lo_ndx, hi_ndx >.max(), max(), glucat::index_set< LO, HI >.min(
glucat::index_set< lo_ndx, hi_ndx >.min(), and min().

8.18.2.10 __ixor__()

```
PyClical.index_set.__ixor__ (
              self,
              rhs)
```

```
Symmetric set difference: exclusive or.

>>> x = index_set({1}); x ^= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x ^= index_set({2}); print(x)
{1}
```

Definition at line 260 of file PyClical.pyx.

8.18.2.11 __or__()

```
PyClical.index_set.__or__ (
              lhs,
              rhs)
```

```
Set union: or.

>>> print(index_set({1}) | index_set({2}))
{1,2}
>>> print(index_set({1,2}) | index_set({2}))
{1,2}
```

Definition at line 293 of file PyClical.pyx.

8.18.2.12 __repr__()

```
PyClical.index_set.__repr__ (
              self)
```

```
The "official" string representation of self.

>>> index_set({1,2}).__repr__()
'index_set({1,2})'
>>> repr(index_set({1,2}))
'index_set({1,2})'
```

Definition at line 384 of file PyClical.pyx.

References index_set_to_repr().

8.18.2.13 __richcmp__()

```
PyClical.index_set.__richcmp__ (
              lhs,
              rhs,
              int,
              op)
```

```
Compare two objects of class index_set.

>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
```

Definition at line 122 of file PyClical.pyx.

8.18.2.14 __setitem__()

```
PyClical.index_set.__setitem__ (
              self,
              idx,
              val)
```

```
Set the value of an index_set object at index idx to value val.

>>> s=index_set({1}); s[2] = True; print(s)
{1,2}
>>> s=index_set({1,2}); s[1] = False; print(s)
{2}
```

Definition at line 179 of file PyClical.pyx.

References instance.

8.18.2.15 __str__()

```
PyClical.index_set.__str__ (
              self)
```

```
The "informal" string representation of self.

>>> index_set({1,2}).__str__()
'{1,2}'
>>> str(index_set({1,2}))
'{1,2}'
```

Definition at line 395 of file PyClical.pyx.

References index_set_to_str().

8.18.2.16 __xor__()

```
PyClical.index_set.__xor__ (
              lhs,
              rhs)
```

```
Symmetric set difference: exclusive or.
```

```
>>> print(index_set({1}) ^ index_set({2}))
{1,2}
>>> print(index_set({1,2}) ^ index_set({2}))
{1}
```

Definition at line 249 of file PyClical.pyx.

8.18.2.17 count()

```
PyClical.index_set.count (
              self)
```

```
Cardinality: Number of indices included in set.
```

```
>>> index_set({-1,1,2}).count()
3
```

Definition at line 315 of file PyClical.pyx.

References count(), and instance.

Referenced by count().

8.18.2.18 count_neg()

```
PyClical.index_set.count_neg (
              self)
```

```
Number of negative indices included in set.
```

```
>>> index_set({-1,1,2}).count_neg()
1
```

Definition at line 324 of file PyClical.pyx.

References count_neg(), and instance.

Referenced by count_neg().

8.18.2.19 count_pos()

```
PyClical.index_set.count_pos (
              self)
```

```
Number of positive indices included in set.

>>> index_set({-1,1,2}).count_pos()
2
```

Definition at line 333 of file PyClical.pyx.

References count_pos(), and instance.

Referenced by count_pos().

8.18.2.20 hash_fn()

```
PyClical.index_set.hash_fn (
              self)
```

```
Hash function.
```

Definition at line 360 of file PyClical.pyx.

References hash_fn(), and instance.

Referenced by hash_fn().

8.18.2.21 max()

```
PyClical.index_set.max (
              self)
```

```
Maximum member.

>>> index_set({-1,1,2}).max()
2
```

Definition at line 351 of file PyClical.pyx.

References instance, and max().

Referenced by __iter__(), and max().

8.18.2.22 min()

```
PyClical.index_set.min (
              self)
```

```
Minimum member.

>>> index_set({-1,1,2}).min()
-1
```

Definition at line 342 of file PyClical.pyx.

References instance, and min().

Referenced by __iter__(), and min().

8.18.2.23 sign_of_mult()

```
PyClical.index_set.sign_of_mult (
              self,
              rhs)
```

```
Sign of geometric product of two Clifford basis elements.

>>> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
1
```

Definition at line 366 of file PyClical.pyx.

References instance, and sign_of_mult().

Referenced by sign_of_mult().

8.18.2.24 sign_of_square()

```
PyClical.index_set.sign_of_square (
              self)
```

```
Sign of geometric square of a Clifford basis element.

>>> s = index_set({1,2}); s.sign_of_square()
-1
```

Definition at line 375 of file PyClical.pyx.

References instance, and sign_of_square().

Referenced by sign_of_square().

8.18.3 Member Data Documentation

8.18.3.1 instance

```
PyClical.index_set.instance = new IndexSet((<index_set>other).unwrap())
```

Definition at line 95 of file PyClical.pyx.

Referenced by PyClical.clifford.__call__(), __contains__(), PyClical.clifford.__dealloc__(), __dealloc__(), PyClical.clifford.__getitem__(), __getitem__(), __invert__(), PyClical.clifford.__neg__(), __setitem__(), PyClical.clifford.conj(), count(), count_neg(), count_pos(), PyClical.clifford.even(), PyClical.clifford.frame(), hash_fn(), PyClical.clifford.inv(), PyClical.clifford.involute(), PyClical.clifford.isinf(), PyClical.clifford.isnan(), max(), PyClical.clifford.max_abs(), min(), PyClical.clifford.norm(), PyClical.clifford.odd(), PyClical.clifford.outer_pow(), PyClical.clifford.pow(), PyClical.clifford.pure(), PyClical.clifford.quad(), PyClical.clifford.reverse(), PyClical.clifford.scalar(), sign_of_mult(), sign_of_square(), PyClical.clifford.truncated(), and PyClical.clifford.vector_part().

The documentation for this class was generated from the following file:

- pyclical/PyClical.pyx

8.19 glucat::index_set_hash< LO, HI > Class Template Reference

```
#include <framed_multi.h>
```

Public Types

- using index_set_t = index_set<LO, HI>

Public Member Functions

- auto operator() (index_set_t val) const -> size_t

8.19.1 Detailed Description

template<const index_t LO, const index_t HI>
class glucat::index_set_hash< LO, HI >

Definition at line 117 of file framed_multi.h.

8.19.2 Member Typedef Documentation

8.19.2.1 index_set_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set_hash< LO, HI >::index_set_t = index_set<LO, HI>
```

Definition at line 120 of file framed_multi.h.

8.19.3 Member Function Documentation

8.19.3.1 operator()()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set_hash< LO, HI >::operator() (
             index_set_t val) const -> size_t  [inline]
```

Definition at line 121 of file framed_multi.h.

The documentation for this class was generated from the following file:

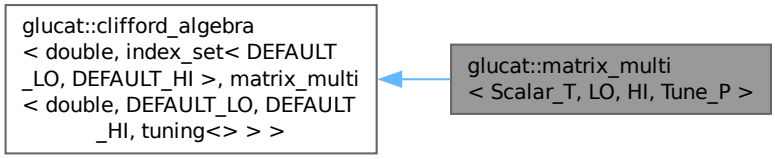
- glucat/framed_multi.h

8.20 glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > Class Template Reference

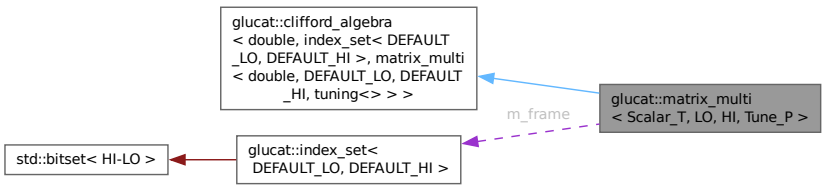
A matrix_multi<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector.

`#include <matrix_multi.h>`

Inheritance diagram for glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >:



Collaboration diagram for glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >:



Public Types

- using multivector_t = matrix_multi
- using matrix_multi_t = multivector_t
- using scalar_t = Scalar_T
- using tune_p = Tune_P
- using index_set_t = index_set<LO, HI>
- using term_t = std::pair<const index_set_t, Scalar_T>
- using vector_t = std::vector<Scalar_T>
- using error_t = error<multivector_t>
- using framed_multi_t = framed_multi<Scalar_T,LO,HI,Tune_P>

Public Types inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- using scalar_t
- using index_set_t
- using multivector_t
- using pair_t
- using vector_t

Public Member Functions

- ~matrix_multi () override=default

  Destructor.
- matrix_multi ()

  Default constructor.
- template<typename Other_Scalar_T>
  matrix_multi (const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > &val)

  Construct a multivector from a multivector with a different scalar type.
- template<typename Other_Scalar_T>
  matrix_multi (const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > &val, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given multivector.
- matrix_multi (const multivector_t &val, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given multivector.
- matrix_multi (const index_set_t ist, const Scalar_T &crd=Scalar_T(1))

  Construct a multivector from an index set and a scalar coordinate.
- matrix_multi (const index_set_t ist, const Scalar_T &crd, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from an index set and a scalar coordinate.
- matrix_multi (const Scalar_T &scr, const index_set_t frm=index_set_t())

  Construct a multivector from a scalar (within a frame, if given).
- matrix_multi (const int scr, const index_set_t frm=index_set_t())

  Construct a multivector from an int (within a frame, if given).
- matrix_multi (const vector_t &vec, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a given vector.
- matrix_multi (const std::string &str)

  Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- matrix_multi (const std::string &str, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".
- matrix_multi (const char *str)

Construct a multivector from a char: eg: "3+2{1,2}-6.1e-2{2,3}".*

- matrix_multi (const char *str, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a char: eg: "3+2{1,2}-6.1e-2{2,3}".*

- template<typename Other_Scalar_T>
  matrix_multi (const framed_multi< Other_Scalar_T, LO, HI, Tune_P > &val)

  Construct a multivector from a framed_multi_t.

- template<typename Other_Scalar_T>
  matrix_multi (const framed_multi< Other_Scalar_T, LO, HI, Tune_P > &val, const index_set_t frm, const bool prechecked=false)

  Construct a multivector, within a given frame, from a framed_multi_t.

- auto fast_matrix_multi (const index_set_t frm) const -> const matrix_multi_t

  Use generalized FFT to construct a matrix_multi_t.

- template<typename Other_Scalar_T>
  auto fast_framed_multi () const -> const framed_multi< Other_Scalar_T, LO, HI, Tune_P >

  Use inverse generalized FFT to construct a framed_multi_t.

- _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS auto operator= (const multivector_t &rhs) -> multivector_t &

  Assignment operator.

- auto operator+= (const term_t &rhs) -> multivector_t &

  Add a term, if non-zero.

Public Member Functions inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- virtual ~clifford_algebra ()=default
- virtual auto operator== (const multivector_t &val) const -> bool=0

  Test for equality of multivectors.

- virtual auto operator+= (const multivector_t &rhs) -> multivector_t &=0

  Geometric sum.

- virtual auto operator-= (const multivector_t &rhs) -> multivector_t &=0

  Geometric difference.

- virtual auto operator- () const -> const multivector_t=0

  Unary -.

- virtual auto operator*= (const double &scr) -> multivector_t &=0

  Product of multivector and scalar.

- virtual auto operator%= (const multivector_t &rhs) -> multivector_t &=0

  Contraction.

- virtual auto operator&= (const multivector_t &rhs) -> multivector_t &=0

  Inner product.

- virtual auto operator^= (const multivector_t &rhs) -> multivector_t &=0

  Outer product.

- virtual auto operator/= (const double &scr) -> multivector_t &=0

  Quotient of multivector and scalar.

- virtual auto operator|= (const multivector_t &rhs) -> multivector_t &=0

  Transformation via twisted adjoint action.

- virtual auto inv () const -> const multivector_t=0

  Geometric multiplicative inverse.

- virtual auto pow (int m) const -> const multivector_t=0

  **this to the m*

- virtual auto outer_pow (int m) const -> const multivector_t=0

Outer product power.

- virtual auto frame () const -> const index_set_t=0

  Subalgebra generated by all generators of terms of given multivector.
- virtual auto grade () const -> index_t=0

  Maximum of the grades of each term.
- virtual auto operator[ ] (const index_set_t ist) const -> double=0

  Subscripting: map from index set to scalar coordinate.
- virtual auto operator() (index_t grade) const -> const multivector_t=0

  Pure grade-vector part.
- virtual auto scalar () const -> double=0

  Scalar part.
- virtual auto pure () const -> const multivector_t=0

  Pure part.
- virtual auto even () const -> const multivector_t=0

  Even part of multivector, sum of even grade terms.
- virtual auto odd () const -> const multivector_t=0

  Odd part of multivector, sum of odd grade terms.
- virtual auto vector_part () const -> const vector_t=0

  Vector part of multivector, as a vector_t with respect to frame().
- virtual auto involute () const -> const multivector_t=0

  Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.
- virtual auto reverse () const -> const multivector_t=0

  Reversion, eg. {1}∗{2} -> {2}∗{1}.
- virtual auto conj () const -> const multivector_t=0

  Conjugation, reverse o involute == involute o reverse.
- virtual auto quad () const -> double=0

  Scalar_T quadratic form == (rev(x)∗x)(0).
- virtual auto norm () const -> double=0

  Scalar_T norm == sum of norm of coordinates.
- virtual auto max_abs () const -> double=0

  Maximum of absolute values of components of multivector: multivector infinity norm.
- virtual auto truncated (const double &limit=default_truncation) const -> const multivector_t=0

  Remove all terms with relative size smaller than limit.
- virtual auto isinf () const -> bool=0

  Check if a multivector contains any infinite values.
- virtual auto isnan () const -> bool=0

  Check if a multivector contains any IEEE NaN values.
- virtual void write (const std::string &msg="") const=0

  Write formatted multivector to output.

Static Public Member Functions

- static auto classname () -> const std::string

  Class name used in messages.
- static auto random (const index_set_t frm, Scalar_T fill=Scalar_T(1)) -> const matrix_multi_t

  Random multivector within a frame.

Static Public Member Functions inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- static auto classname () -> const std::string

Private Types

- using orientation_t = ublas::row_major
- using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>
- using matrix_t = ublas::matrix<Scalar_T, orientation_t>
- using matrix_index_t = typename matrix_t::size_type

Private Member Functions

- template<typename Matrix_T>
  matrix_multi (const Matrix_T &mtx, const index_set_t frm)

  Construct a multivector within a given frame from a given matrix.
- matrix_multi (const matrix_t &mtx, const index_set_t frm)

  Construct a multivector within a given frame from a given matrix.
- auto basis_element (const index_set< LO, HI > &ist) const -> const basis_matrix_t

  Create a basis element matrix within the current frame.

Private Attributes

- index_set_t m_frame

  Index set representing the frame for the subalgebra which contains the multivector.
- matrix_t m_matrix

  Matrix value representing the multivector within the folded frame.

Friends

- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
  class framed_multi
- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
  class matrix_multi
- auto operator* (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto operator^ (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto operator& (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto operator% (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto star (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> Scalar_T
- auto operator/ (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto operator| (const matrix_multi_t &lhs, const matrix_multi_t &rhs) -> const matrix_multi_t
- auto operator>> (std::istream &s, multivector_t &val) -> std::istream &
- auto operator<< (std::ostream &os, const multivector_t &val) -> std::ostream &
- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
  auto reframe (const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs, const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs, matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &lhs_reframed, matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &rhs_reframed) -> const index_set< Other_LO, Other_HI >
- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
  auto matrix_sqrt (const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >
- template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P>
  auto matrix_log (const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &val, const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > &i, const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P >

Additional Inherited Members

Static Public Attributes inherited from glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >

- static const index_t v_lo
- static const index_t v_hi
- static const double default_truncation

  Default for truncation.

8.20.1 Detailed Description

template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
class glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >

A matrix_multi<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector.

Definition at line 137 of file matrix_multi.h.

8.20.2 Member Typedef Documentation

8.20.2.1 basis_matrix_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_matrix_t = ublas::compressed_↩
matrix<int, orientation_t>  [private]
```

Definition at line 157 of file matrix_multi.h.

8.20.2.2 error_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 148 of file matrix_multi.h.

8.20.2.3 framed_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = framed_multi<Scalar↩
_T,LO,HI,Tune_P>
```

Definition at line 149 of file matrix_multi.h.

8.20.2.4 index_set_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 145 of file matrix_multi.h.

8.20.2.5 matrix_index_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_index_t = typename matrix_t::↩
size_type  [private]
```

Definition at line 159 of file matrix_multi.h.

8.20.2.6 matrix_multi_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = multivector_t
```

Definition at line 142 of file matrix_multi.h.

8.20.2.7 matrix_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = ublas::matrix<Scalar_T,
orientation_t>  [private]
```

Definition at line 158 of file matrix_multi.h.

8.20.2.8 multivector_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = matrix_multi
```

Definition at line 141 of file matrix_multi.h.

8.20.2.9 orientation_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::orientation_t = ublas::row_major
[private]
```

Definition at line 156 of file matrix_multi.h.

8.20.2.10 scalar_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 143 of file matrix_multi.h.

8.20.2.11 term_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 146 of file matrix_multi.h.

8.20.2.12 tune_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 144 of file matrix_multi.h.

8.20.2.13 vector_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 147 of file matrix_multi.h.

8.20.3 Constructor & Destructor Documentation

8.20.3.1 ~matrix_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::~matrix_multi () [override], [default]
```

Destructor.

8.20.3.2 matrix_multi() [1/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi ()
```

Default constructor.

Definition at line 106 of file matrix_multi_imp.h.

References m_frame, and m_matrix.

8.20.3.3 matrix_multi() [2/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 115 of file matrix_multi_imp.h.

References m_frame, m_matrix, matrix_multi, and glucat::numeric_traits< Scalar_T >::to_scalar_t().

8.20.3.4 matrix_multi() [3/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const matrix_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 134 of file matrix_multi_imp.h.

References glucat::folded_dim(), m_frame, m_matrix, matrix_multi, and glucat::numeric_traits< Scalar_T >::to_scalar_t().

8.20.3.5 matrix_multi() [4/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const multivector_t & val,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 159 of file matrix_multi_imp.h.

References m_frame, and m_matrix.

8.20.3.6 matrix_multi() [5/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const index_set_t ist,
            const Scalar_T & crd = Scalar_T(1))
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 171 of file matrix_multi_imp.h.

References glucat::folded_dim(), m_frame, and m_matrix.

8.20.3.7 matrix_multi() [6/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const index_set_t ist,
            const Scalar_T & crd,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 183 of file matrix_multi_imp.h.

References glucat::folded_dim(), m_frame, and m_matrix.

8.20.3.8 matrix_multi() [7/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const Scalar_T & scr,
            const index_set_t frm = index_set_t())
```

Construct a multivector from a scalar (within a frame, if given).

Definition at line 197 of file matrix_multi_imp.h.

References glucat::folded_dim(), m_frame, and m_matrix.

8.20.3.9 matrix_multi() [8/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const int scr,
            const index_set_t frm = index_set_t())
```

Construct a multivector from an int (within a frame, if given).

Definition at line 209 of file matrix_multi_imp.h.

8.20.3.10 matrix_multi() [9/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const vector_t & vec,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 215 of file matrix_multi_imp.h.

References glucat::index_set< LO, HI >::count(), glucat::folded_dim(), m_frame, m_matrix, glucat::index_set< LO, HI >::max(), and glucat::index_set< LO, HI >::min().

8.20.3.11 matrix_multi() [10/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const std::string & str)
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 240 of file matrix_multi_imp.h.

8.20.3.12 matrix_multi() [11/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const std::string & str,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 246 of file matrix_multi_imp.h.

8.20.3.13 matrix_multi() [12/17]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const char * str)  [inline]
```

Construct a multivector from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 196 of file matrix_multi.h.

8.20.3.14 matrix_multi() [13/17]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const char * str,
            const index_set_t frm,
            const bool prechecked = false)  [inline]
```

Construct a multivector, within a given frame, from a char*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 199 of file matrix_multi.h.

8.20.3.15 matrix_multi() [14/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val)
```

Construct a multivector from a framed_multi_t.

Definition at line 253 of file matrix_multi_imp.h.

References glucat::folded_dim(), glucat::clifford_algebra< double, index_set< DEFAULT_LO, DEFAULT_HI >, matrix_multi< double, DEFAULT_LO, DEFAULT_HI, tuning<> > >::frame(), framed_multi, m_frame, and m_matrix.

8.20.3.16 matrix_multi() [15/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
            const index_set_t frm,
            const bool prechecked = false)
```

Construct a multivector, within a given frame, from a framed_multi_t.

Definition at line 276 of file matrix_multi_imp.h.

References fast_matrix_multi(), glucat::folded_dim(), framed_multi, m_frame, m_matrix, and glucat::clifford_algebra< Scalar_T, Index

8.20.3.17 matrix_multi() [16/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Matrix_T>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const Matrix_T & mtx,
            const index_set_t frm)  [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 301 of file matrix_multi_imp.h.

References m_frame, m_matrix, and glucat::numeric_traits< Scalar_T >::to_scalar_t().

8.20.3.18 matrix_multi() [17/17]

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
            const matrix_t & mtx,
            const index_set_t frm)  [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 320 of file matrix_multi_imp.h.

References m_frame, and m_matrix.

8.20.4 Member Function Documentation

8.20.4.1 basis_element()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element (
            const index_set< LO, HI > & ist) const -> const basis_matrix_t  [private]
```

Create a basis element matrix within the current frame.

Definition at line 1183 of file matrix_multi_imp.h.

References glucat::gen::generator_table< Matrix_T >::generator(), m_frame, glucat::matrix::mono_prod(), glucat::offset_level(), and glucat::matrix::unit().

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi().

8.20.4.2 classname()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::classname () -> const std::string
[static]
```

Class name used in messages.

Definition at line 78 of file matrix_multi_imp.h.

8.20.4.3 fast_framed_multi()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
template<typename Other_Scalar_T>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi () const -> const
framed_multi< Other_Scalar_T, LO, HI, Tune_P >
```

Use inverse generalized FFT to construct a framed_multi_t.

Definition at line 1106 of file matrix_multi_imp.h.

References glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1(), glucat::fast(), framed_multi, m_frame, m_matrix, glucat::pos_mod(), and glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold().

Referenced by fast_matrix_multi().

8.20.4.4 fast_matrix_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
            const index_set_t frm) const -> const matrix_multi_t  [inline]
```

Use generalized FFT to construct a matrix_multi_t.

Definition at line 1093 of file matrix_multi_imp.h.

References fast_framed_multi(), fast_matrix_multi(), and m_frame.

Referenced by fast_matrix_multi(), and matrix_multi().

8.20.4.5 operator+=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
            const term_t & rhs) -> multivector_t &  [inline]
```

Add a term, if non-zero.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 414 of file matrix_multi_imp.h.

8.20.4.6 operator=()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator= (
            const multivector_t & rhs) -> multivector_t &
```

Assignment operator.

Definition at line 328 of file matrix_multi_imp.h.

References m_frame, and m_matrix.

8.20.4.7 random()

```
template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::random (
            const index_set_t frm,
            Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t  [static]
```

Random multivector within a frame.

Definition at line 923 of file matrix_multi_imp.h.

References glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random().

8.20.5 Friends And Related Symbol Documentation

8.20.5.1 framed_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class framed_multi  [friend]
```

Definition at line 151 of file matrix_multi.h.

Referenced by fast_framed_multi(), matrix_multi(), and matrix_multi().

8.20.5.2 matrix_log

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
auto matrix_log (
            const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
            const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
            const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI,
Other_Tune_P >  [friend]
```

8.20.5.3 matrix_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
friend class matrix_multi  [friend]
```

Definition at line 153 of file matrix_multi.h.

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), and matrix_multi().

8.20.5.4 matrix_sqrt

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
auto matrix_sqrt (
            const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
            const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
            const index_t level) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI,
Other_Tune_P >  [friend]
```

8.20.5.5 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator% (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.6 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator& (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.7 operator*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator* (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.8 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator/ (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.9 operator<<

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator<< (
            std::ostream & os,
            const multivector_t & val) -> std::ostream &  [friend]
```

8.20.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator>> (
            std::istream & s,
            multivector_t & val) -> std::istream &  [friend]
```

8.20.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator^ (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.12 operator"|

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto operator| (
            const matrix_multi_t & lhs,
            const matrix_multi_t & rhs) -> const matrix_multi_t  [friend]
```

8.20.5.13 reframe

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P>
auto reframe (
             const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs,
             const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs,
             matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs_reframed,
             matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs_reframed)
-> const index_set< Other_LO, Other_HI >  [friend]
```

8.20.5.14 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto star (
             const matrix_multi_t & lhs,
             const matrix_multi_t & rhs) -> Scalar_T  [friend]
```

8.20.6 Member Data Documentation

8.20.6.1 m_frame

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
index_set_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_frame  [private]
```

Index set representing the frame for the subalgebra which contains the multivector.

Definition at line 278 of file matrix_multi.h.

Referenced by basis_element(), fast_framed_multi(), fast_matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), and operator=().

8.20.6.2 m_matrix

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
matrix_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix  [private]
```

Matrix value representing the multivector within the folded frame.

Definition at line 280 of file matrix_multi.h.

Referenced by fast_framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), matrix_multi(), and operator=().

The documentation for this class was generated from the following files:

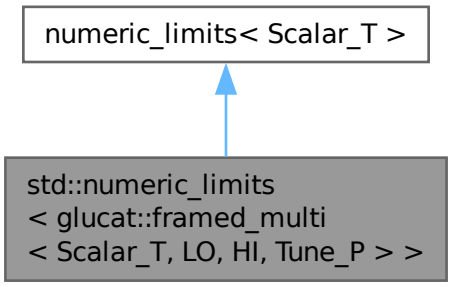
- glucat/framed_multi.h
- glucat/matrix_multi.h
- glucat/matrix_multi_imp.h

8.21 std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > > Struct Template Reference

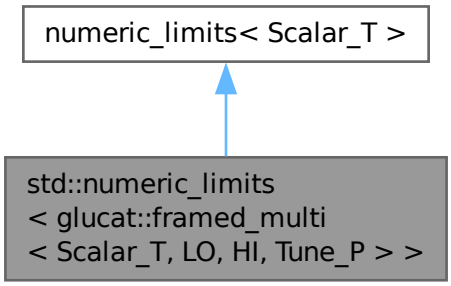
Numeric limits for framed_multi inherit limits for the corresponding scalar type.

`#include <framed_multi.h>`

Inheritance diagram for std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >:



Collaboration diagram for std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >:



8.21.1 Detailed Description

template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >

Numeric limits for framed_multi inherit limits for the corresponding scalar type.

Definition at line 345 of file framed_multi.h.

The documentation for this struct was generated from the following file:

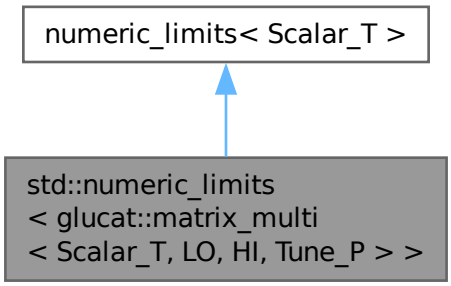
- glucat/framed_multi.h

8.22 std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > > Struct Template Reference

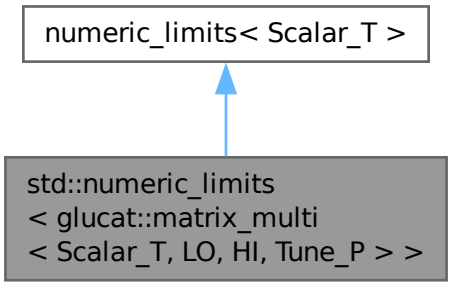
Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

`#include <matrix_multi.h>`

Inheritance diagram for std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >:



Collaboration diagram for std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >:



8.22.1 Detailed Description

template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >

Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

Definition at line 296 of file matrix_multi.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi.h

8.23 glucat::numeric_traits< Scalar_T > Class Template Reference

Extra traits which extend numeric limits.

`#include <scalar.h>`

Classes

- struct promoted

  Extra traits which extend numeric limits.
- struct demoted

  Demoted type for long double.

Public Member Functions

- auto pi () -> long double

  Pi for long double.
- auto ln_2 () -> long double

  log(2) for long double
- auto to_scalar_t (const Other_Scalar_T &val) -> float

  Extra traits which extend numeric limits.
- auto to_scalar_t (const Other_Scalar_T &val) -> double

  Cast to double.
- auto to_scalar_t (const dd_real &val) -> long double

  Cast to long double.
- auto to_scalar_t (const qd_real &val) -> long double

  Cast to long double.
- auto to_scalar_t (const long double &val) -> dd_real

  Cast to dd_real.
- auto to_scalar_t (const qd_real &val) -> dd_real

  Cast to dd_real.
- auto to_scalar_t (const long double &val) -> qd_real

  Cast to qd_real.
- auto to_scalar_t (const dd_real &val) -> qd_real

  Cast to qd_real.

Static Public Member Functions

- static auto isInf (const Scalar_T &val) -> bool

  Smart isinf.
- static auto isNaN (const Scalar_T &val) -> bool

  Smart isnan.
- static auto isNaN_or_isInf (const Scalar_T &val) -> bool

  Smart isnan or isinf.
- static auto NaN () -> Scalar_T

  Smart NaN.
- static auto to_int (const Scalar_T &val) -> int

  Cast to int.
- static auto to_double (const Scalar_T &val) -> double

Cast to double.

- template<typename Other_Scalar_T>
  static auto to_scalar_t (const Other_Scalar_T &val) -> Scalar_T

  Cast to Scalar_T.
- static auto fmod (const Scalar_T &lhs, const Scalar_T &rhs) -> Scalar_T

  Modulo function for scalar.
- static auto conj (const Scalar_T &val) -> Scalar_T

  Complex conjugate of scalar.
- static auto real (const Scalar_T &val) -> Scalar_T

  Real part of scalar.
- static auto imag (const Scalar_T &val) -> Scalar_T

  Imaginary part of scalar.
- static auto abs (const Scalar_T &val) -> Scalar_T

  Absolute value of scalar.
- static auto pi () -> Scalar_T

  Pi.
- static auto ln_2 () -> Scalar_T

  log(2)
- static auto pow (const Scalar_T &val, int n) -> Scalar_T

  Integer power.
- static auto sqrt (const Scalar_T &val) -> Scalar_T

  Square root of scalar.
- static auto exp (const Scalar_T &val) -> Scalar_T

  Exponential.
- static auto log (const Scalar_T &val) -> Scalar_T

  Logarithm of scalar.
- static auto log2 (const Scalar_T &val) -> Scalar_T

  Log base 2.
- static auto cos (const Scalar_T &val) -> Scalar_T

  Cosine of scalar.
- static auto acos (const Scalar_T &val) -> Scalar_T

  Inverse cosine of scalar.
- static auto cosh (const Scalar_T &val) -> Scalar_T

  Hyperbolic cosine of scalar.
- static auto sin (const Scalar_T &val) -> Scalar_T

  Sine of scalar.
- static auto asin (const Scalar_T &val) -> Scalar_T

  Inverse sine of scalar.
- static auto sinh (const Scalar_T &val) -> Scalar_T

  Hyperbolic sine of scalar.
- static auto tan (const Scalar_T &val) -> Scalar_T

  Tangent of scalar.
- static auto atan (const Scalar_T &val) -> Scalar_T

  Inverse tangent of scalar.
- static auto tanh (const Scalar_T &val) -> Scalar_T

  Hyperbolic tangent of scalar.

Static Private Member Functions

- static auto isInf (const Scalar_T &val, bool_to_type< false >) -> bool

  Smart isinf specialised for Scalar_T without infinity.
- static auto isInf (const Scalar_T &val, bool_to_type< true >) -> bool

  Smart isinf specialised for Scalar_T with infinity.
- static auto isNaN (const Scalar_T &val, bool_to_type< false >) -> bool

  Smart isnan specialised for Scalar_T without quiet NaN.
- static auto isNaN (const Scalar_T &val, bool_to_type< true >) -> bool

  Smart isnan specialised for Scalar_T with quiet NaN.

8.23.1 Detailed Description

template<typename Scalar_T>
class glucat::numeric_traits< Scalar_T >

Extra traits which extend numeric limits.

Definition at line 47 of file scalar.h.

8.23.2 Member Function Documentation

8.23.2.1 abs()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::abs (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Absolute value of scalar.

Definition at line 182 of file scalar.h.

8.23.2.2 acos()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::acos (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Inverse cosine of scalar.

Definition at line 245 of file scalar.h.

8.23.2.3 asin()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::asin (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Inverse sine of scalar.

Definition at line 266 of file scalar.h.

8.23.2.4 atan()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::atan (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Inverse tangent of scalar.

Definition at line 287 of file scalar.h.

8.23.2.5 conj()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::conj (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Complex conjugate of scalar.

Definition at line 161 of file scalar.h.

8.23.2.6 cos()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::cos (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Cosine of scalar.

Definition at line 238 of file scalar.h.

8.23.2.7 cosh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::cosh (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Hyperbolic cosine of scalar.

Definition at line 252 of file scalar.h.

8.23.2.8 exp()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::exp (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Exponential.

Definition at line 217 of file scalar.h.

8.23.2.9 fmod()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::fmod (
             const Scalar_T & lhs,
             const Scalar_T & rhs) -> Scalar_T  [inline], [static]
```

Modulo function for scalar.

Definition at line 154 of file scalar.h.

8.23.2.10 imag()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::imag (
             const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Imaginary part of scalar.

Definition at line 175 of file scalar.h.

8.23.2.11 isInf() [1/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
             const Scalar_T & val) -> bool  [inline], [static]
```

Smart isinf.

Definition at line 83 of file scalar.h.

References isInf().

8.23.2.12 isInf() [2/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
             const Scalar_T & val,
             bool_to_type< false > ) -> bool  [inline], [static], [private]
```

Smart isinf specialised for Scalar_T without infinity.

Definition at line 54 of file scalar.h.

Referenced by isInf(), glucat::matrix::isinf(), and isNaN_or_isInf().

8.23.2.13 isInf() [3/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isInf (
            const Scalar_T & val,
            bool_to_type< true > ) -> bool  [inline], [static], [private]
```

Smart isinf specialised for Scalar_T with infinity.

Definition at line 61 of file scalar.h.

References _GLUCAT_ISINF.

8.23.2.14 isNaN() [1/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
            const Scalar_T & val) -> bool  [inline], [static]
```

Smart isnan.

Definition at line 93 of file scalar.h.

References isNaN().

8.23.2.15 isNaN() [2/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
            const Scalar_T & val,
            bool_to_type< false > ) -> bool  [inline], [static], [private]
```

Smart isnan specialised for Scalar_T without quiet NaN.

Definition at line 68 of file scalar.h.

Referenced by isNaN(), glucat::matrix::isnan(), isNaN_or_isInf(), glucat::matrix::norm_frob2(), and glucat::matrix::trace().

8.23.2.16 isNaN() [3/3]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN (
            const Scalar_T & val,
            bool_to_type< true > ) -> bool  [inline], [static], [private]
```

Smart isnan specialised for Scalar_T with quiet NaN.

Definition at line 75 of file scalar.h.

References _GLUCAT_ISNAN.

8.23.2.17 isNaN_or_isInf()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::isNaN_or_isInf (
            const Scalar_T & val) -> bool  [inline], [static]
```

Smart isnan or isinf.

Definition at line 103 of file scalar.h.

References isInf(), and isNaN().

8.23.2.18 ln_2() [1/2]

```
auto glucat::numeric_traits< longdouble >::ln_2 () -> long double  [inline]
```

log(2) for long double

Definition at line 59 of file long_double.h.

References glucat::l_ln2.

8.23.2.19 ln_2() [2/2]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::ln_2 () -> Scalar_T  [inline], [static]
```

log(2)

Definition at line 196 of file scalar.h.

Referenced by log2().

8.23.2.20 log()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::log (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Logarithm of scalar.

Definition at line 224 of file scalar.h.

Referenced by log2().

8.23.2.21 log2()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::log2 (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Log base 2.

Definition at line 231 of file scalar.h.

References ln_2(), and log().

Referenced by glucat::log2().

8.23.2.22 NaN()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::NaN () -> Scalar_T  [inline], [static]
```

Smart NaN.

Definition at line 115 of file scalar.h.

Referenced by glucat::cr_sqrt(), glucat::db_sqrt(), glucat::matrix::norm_frob2(), glucat::operator*(), and glucat::matrix::trace().

8.23.2.23 pi() [1/2]

```
auto glucat::numeric_traits< longdouble >::pi () -> long double  [inline]
```

Pi for long double.

Definition at line 51 of file long_double.h.

References glucat::l_pi.

8.23.2.24 pi() [2/2]

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::pi () -> Scalar_T  [inline], [static]
```

Pi.

Definition at line 189 of file scalar.h.

Referenced by glucat::matrix::classify_eigenvalues().

8.23.2.25 pow()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::pow (
            const Scalar_T & val,
            int n) -> Scalar_T  [inline], [static]
```

Integer power.

Definition at line 203 of file scalar.h.

Referenced by glucat::error_squared_tol().

8.23.2.26 real()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::real (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Real part of scalar.

Definition at line 168 of file scalar.h.

8.23.2.27 sin()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sin (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Sine of scalar.

Definition at line 259 of file scalar.h.

8.23.2.28 sinh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sinh (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Hyperbolic sine of scalar.

Definition at line 273 of file scalar.h.

8.23.2.29 sqrt()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::sqrt (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Square root of scalar.

Definition at line 210 of file scalar.h.

Referenced by glucat::abs().

8.23.2.30 tan()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::tan (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Tangent of scalar.

Definition at line 280 of file scalar.h.

8.23.2.31 tanh()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::tanh (
            const Scalar_T & val) -> Scalar_T  [inline], [static]
```

Hyperbolic tangent of scalar.

Definition at line 294 of file scalar.h.

8.23.2.32 to_double()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_double (
            const Scalar_T & val) -> double  [inline], [static]
```

Cast to double.

Definition at line 133 of file scalar.h.

Referenced by glucat::matrix::classify_eigenvalues(), glucat::operator<<(), PyFloat_FromDouble(), to_scalar_t(), and to_scalar_t().

8.23.2.33 to_int()

```
template<typename Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_int (
            const Scalar_T & val) -> int  [inline], [static]
```

Cast to int.

Definition at line 126 of file scalar.h.

8.23.2.34 to_scalar_t() [1/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
            const dd_real & val) -> long double  [inline]
```

Cast to long double.

Definition at line 71 of file scalar_imp.h.

8.23.2.35 to_scalar_t() [2/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
            const dd_real & val) -> qd_real  [inline]
```

Cast to qd_real.

Definition at line 116 of file scalar_imp.h.

8.23.2.36 to_scalar_t() [3/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
            const long double & val) -> dd_real  [inline]
```

Cast to dd_real.

Definition at line 89 of file scalar_imp.h.

8.23.2.37 to_scalar_t() [4/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
            const long double & val) -> qd_real  [inline]
```

Cast to qd_real.

Definition at line 107 of file scalar_imp.h.

8.23.2.38 to_scalar_t() [5/9]

```
auto glucat::numeric_traits< double >::to_scalar_t (
            const Other_Scalar_T & val) -> double  [inline]
```

Cast to double.

Definition at line 61 of file scalar_imp.h.

References to_double().

8.23.2.39 to_scalar_t() [6/9]

```
auto glucat::numeric_traits< float >::to_scalar_t (
            const Other_Scalar_T & val) -> float  [inline]
```

Extra traits which extend numeric limits.

Cast to float

Definition at line 52 of file scalar_imp.h.

References to_double().

8.23.2.40 to_scalar_t() [7/9]

```
template<typename Scalar_T>
template<typename Other_Scalar_T>
auto glucat::numeric_traits< Scalar_T >::to_scalar_t (
            const Other_Scalar_T & val) -> Scalar_T  [inline], [static]
```

Cast to Scalar_T.

Definition at line 141 of file scalar.h.

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(),
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi(), glucat::matrix::nork_range(), glucat::to_demote(),
and glucat::to_promote().

8.23.2.41 to_scalar_t() [8/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
            const qd_real & val) -> dd_real  [inline]
```

Cast to dd_real.

Definition at line 98 of file scalar_imp.h.

8.23.2.42 to_scalar_t() [9/9]

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
            const qd_real & val) -> long double  [inline]
```

Cast to long double.

Definition at line 80 of file scalar_imp.h.

The documentation for this class was generated from the following file:

- glucat/scalar.h

8.24 pade::pade_log_denom< Scalar_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<Scalar_T, 14>

Static Public Attributes

- static const array denom

8.24.1 Detailed Description

template<typename Scalar_T>
struct pade::pade_log_denom< Scalar_T >

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

Definition at line 1721 of file matrix_multi_imp.h.

8.24.2 Member Typedef Documentation

8.24.2.1 array

```
template<typename Scalar_T>
using pade::pade_log_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1723 of file matrix_multi_imp.h.

8.24.3 Member Data Documentation

8.24.3.1 denom

```
template<typename Scalar_T>
const array pade::pade_log_denom< Scalar_T >::denom  [static]
```

Definition at line 1724 of file matrix_multi_imp.h.

Referenced by glucat::pade_log().

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.25 pade::pade_log_denom< dd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<dd_real, 22>

Static Public Attributes

- static const array denom

8.25.1 Detailed Description

Definition at line 1810 of file matrix_multi_imp.h.

8.25.2 Member Typedef Documentation

8.25.2.1 array

```
using pade::pade_log_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1812 of file matrix_multi_imp.h.

8.25.3 Member Data Documentation

8.25.3.1 denom

```
const array pade::pade_log_denom< dd_real >::denom  [static]
```

Definition at line 1813 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.26 pade::pade_log_denom< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<float, 10>

Static Public Attributes

- static const array denom

8.26.1 Detailed Description

Definition at line 1748 of file matrix_multi_imp.h.

8.26.2 Member Typedef Documentation

8.26.2.1 array

```
using pade::pade_log_denom< float >::array = std::array<float, 10>
```

Definition at line 1750 of file matrix_multi_imp.h.

8.26.3 Member Data Documentation

8.26.3.1 denom

```
const array pade::pade_log_denom< float >::denom  [static]
```

Definition at line 1751 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.27 pade::pade_log_denom< long double > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<long double, 18>

Static Public Attributes

- static const array denom

8.27.1 Detailed Description

Definition at line 1775 of file matrix_multi_imp.h.

8.27.2 Member Typedef Documentation

8.27.2.1 array

```
using pade::pade_log_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1777 of file matrix_multi_imp.h.

8.27.3 Member Data Documentation

8.27.3.1 denom

```
const array pade::pade_log_denom< long double >::denom  [static]
```

Definition at line 1778 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.28 pade::pade_log_denom< qd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<qd_real, 34>

Static Public Attributes

- static const array denom

8.28.1 Detailed Description

Definition at line 1857 of file matrix_multi_imp.h.

8.28.2 Member Typedef Documentation

8.28.2.1 array

```
using pade::pade_log_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1859 of file matrix_multi_imp.h.

8.28.3 Member Data Documentation

8.28.3.1 denom

```
const array pade::pade_log_denom< qd_real >::denom  [static]
```

Definition at line 1860 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.29 pade::pade_log_numer< Scalar_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<Scalar_T, 14>

Static Public Attributes

- static const array numer

8.29.1 Detailed Description

template<typename Scalar_T>
struct pade::pade_log_numer< Scalar_T >

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).

Definition at line 1704 of file matrix_multi_imp.h.

8.29.2 Member Typedef Documentation

8.29.2.1 array

```
template<typename Scalar_T>
using pade::pade_log_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1706 of file matrix_multi_imp.h.

8.29.3 Member Data Documentation

8.29.3.1 numer

```
template<typename Scalar_T>
const array pade::pade_log_numer< Scalar_T >::numer  [static]
```

Definition at line 1707 of file matrix_multi_imp.h.

Referenced by glucat::pade_log().

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.30 pade::pade_log_numer< dd_real > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<dd_real, 22>

Static Public Attributes

- static const array numer

8.30.1 Detailed Description

Definition at line 1790 of file matrix_multi_imp.h.

8.30.2 Member Typedef Documentation

8.30.2.1 array

```
using pade::pade_log_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1792 of file matrix_multi_imp.h.

8.30.3 Member Data Documentation

8.30.3.1 numer

```
const array pade::pade_log_numer< dd_real >::numer  [static]
```

Definition at line 1793 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.31 pade::pade_log_numer< float > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<float, 10>

Static Public Attributes

- static const array numer

8.31.1 Detailed Description

Definition at line 1736 of file matrix_multi_imp.h.

8.31.2 Member Typedef Documentation

8.31.2.1 array

```
using pade::pade_log_numer< float >::array = std::array<float, 10>
```

Definition at line 1738 of file matrix_multi_imp.h.

8.31.3 Member Data Documentation

8.31.3.1 numer

```
const array pade::pade_log_numer< float >::numer  [static]
```

Definition at line 1739 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.32 pade::pade_log_numer< long double > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<long double, 18>

Static Public Attributes

- static const array numer

8.32.1 Detailed Description

Definition at line 1761 of file matrix_multi_imp.h.

8.32.2 Member Typedef Documentation

8.32.2.1 array

```
using pade::pade_log_numer< long double >::array = std::array<long double, 18>
```

Definition at line 1763 of file matrix_multi_imp.h.

8.32.3 Member Data Documentation

8.32.3.1 numer

```
const array pade::pade_log_numer< long double >::numer  [static]
```

Definition at line 1764 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.33 pade::pade_log_numer< qd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<qd_real, 34>

Static Public Attributes

- static const array numer

8.33.1 Detailed Description

Definition at line 1831 of file matrix_multi_imp.h.

8.33.2 Member Typedef Documentation

8.33.2.1 array

```
using pade::pade_log_numer< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1833 of file matrix_multi_imp.h.

8.33.3 Member Data Documentation

8.33.3.1 numer

```
const array pade::pade_log_numer< qd_real >::numer  [static]
```

Definition at line 1834 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.34 pade::pade_sqrt_denom< Scalar_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<Scalar_T, 14>

Static Public Attributes

- static const array denom

8.34.1 Detailed Description

template<typename Scalar_T>
struct pade::pade_sqrt_denom< Scalar_T >

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

Definition at line 1393 of file matrix_multi_imp.h.

8.34.2 Member Typedef Documentation

8.34.2.1 array

```
template<typename Scalar_T>
using pade::pade_sqrt_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1395 of file matrix_multi_imp.h.

8.34.3 Member Data Documentation

8.34.3.1 denom

```
template<typename Scalar_T>
const array pade::pade_sqrt_denom< Scalar_T >::denom  [static]
```

Definition at line 1396 of file matrix_multi_imp.h.

Referenced by glucat::matrix_sqrt().

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.35 pade::pade_sqrt_denom< dd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<dd_real, 22>

Static Public Attributes

- static const array denom

8.35.1 Detailed Description

Definition at line 1483 of file matrix_multi_imp.h.

8.35.2 Member Typedef Documentation

8.35.2.1 array

```
using pade::pade_sqrt_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1485 of file matrix_multi_imp.h.

8.35.3 Member Data Documentation

8.35.3.1 denom

```
const array pade::pade_sqrt_denom< dd_real >::denom  [static]
```

Definition at line 1486 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.36 pade::pade_sqrt_denom< float > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<float, 10>

Static Public Attributes

- static const array denom

8.36.1 Detailed Description

Definition at line 1420 of file matrix_multi_imp.h.

8.36.2 Member Typedef Documentation

8.36.2.1 array

`using pade::pade_sqrt_denom< float >::array = std::array<float, 10>`

Definition at line 1422 of file matrix_multi_imp.h.

8.36.3 Member Data Documentation

8.36.3.1 denom

`const array pade::pade_sqrt_denom< float >::denom [static]`

Definition at line 1423 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.37 pade::pade_sqrt_denom< long double > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<long double, 18>

Static Public Attributes

- static const array denom

8.37.1 Detailed Description

Definition at line 1447 of file matrix_multi_imp.h.

8.37.2 Member Typedef Documentation

8.37.2.1 array

```
using pade::pade_sqrt_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1449 of file matrix_multi_imp.h.

8.37.3 Member Data Documentation

8.37.3.1 denom

```
const array pade::pade_sqrt_denom< long double >::denom  [static]
```

Definition at line 1450 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.38 pade::pade_sqrt_denom< qd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<qd_real, 34>

Static Public Attributes

- static const array denom

8.38.1 Detailed Description

Definition at line 1530 of file matrix_multi_imp.h.

8.38.2 Member Typedef Documentation

8.38.2.1 array

```
using pade::pade_sqrt_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1532 of file matrix_multi_imp.h.

8.38.3 Member Data Documentation

8.38.3.1 denom

```
const array pade::pade_sqrt_denom< qd_real >::denom  [static]
```

Definition at line 1533 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.39 pade::pade_sqrt_numer< Scalar_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<Scalar_T, 14>

Static Public Attributes

- static const array numer

8.39.1 Detailed Description

template<typename Scalar_T>
struct pade::pade_sqrt_numer< Scalar_T >

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).

Definition at line 1376 of file matrix_multi_imp.h.

8.39.2 Member Typedef Documentation

8.39.2.1 array

```
template<typename Scalar_T>
using pade::pade_sqrt_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1378 of file matrix_multi_imp.h.

8.39.3 Member Data Documentation

8.39.3.1 numer

```
template<typename Scalar_T>
const array pade::pade_sqrt_numer< Scalar_T >::numer  [static]
```

Definition at line 1379 of file matrix_multi_imp.h.

Referenced by glucat::matrix_sqrt().

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.40 pade::pade_sqrt_numer< dd_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<dd_real, 22>

Static Public Attributes

- static const array numer

8.40.1 Detailed Description

Definition at line 1463 of file matrix_multi_imp.h.

8.40.2 Member Typedef Documentation

8.40.2.1 array

```
using pade::pade_sqrt_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1465 of file matrix_multi_imp.h.

8.40.3 Member Data Documentation

8.40.3.1 numer

```
const array pade::pade_sqrt_numer< dd_real >::numer  [static]
```

Definition at line 1466 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.41 pade::pade_sqrt_numer< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

Public Types

- using array = std::array<float, 10>

Static Public Attributes

- static const array numer

8.41.1 Detailed Description

Definition at line 1408 of file matrix_multi_imp.h.

8.41.2 Member Typedef Documentation

8.41.2.1 array

```
using pade::pade_sqrt_numer< float >::array = std::array<float, 10>
```

Definition at line 1410 of file matrix_multi_imp.h.

8.41.3 Member Data Documentation

8.41.3.1 numer

```
const array pade::pade_sqrt_numer< float >::numer  [static]
```

Definition at line 1411 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.42 pade::pade_sqrt_numer< long double > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<long double, 18>

Static Public Attributes

- static const array numer

8.42.1 Detailed Description

Definition at line 1433 of file matrix_multi_imp.h.

8.42.2 Member Typedef Documentation

8.42.2.1 array

`using pade::pade_sqrt_numer< long double >::array = std::array<long double, 18>`

Definition at line 1435 of file matrix_multi_imp.h.

8.42.3 Member Data Documentation

8.42.3.1 numer

`const array pade::pade_sqrt_numer< long double >::numer [static]`

Definition at line 1436 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.43 pade::pade_sqrt_numer< qd_real > Struct Reference

`#include <matrix_multi_imp.h>`

Public Types

- using array = std::array<qd_real, 34>

Static Public Attributes

- static const array numer

8.43.1 Detailed Description

Definition at line 1504 of file matrix_multi_imp.h.

8.43.2 Member Typedef Documentation

8.43.2.1 array

```
using pade::pade_sqrt_numer< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1506 of file matrix_multi_imp.h.

8.43.3 Member Data Documentation

8.43.3.1 numer

```
const array pade::pade_sqrt_numer< qd_real >::numer  [static]
```

Definition at line 1507 of file matrix_multi_imp.h.

The documentation for this struct was generated from the following file:

- glucat/matrix_multi_imp.h

8.44 glucat::numeric_traits< Scalar_T >::promoted Struct Reference

Extra traits which extend numeric limits.

```
#include <promotion.h>
```

Public Types

- using type = double
- using type = long double
- using type = double

8.44.1 Detailed Description

template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::promoted

Extra traits which extend numeric limits.

Promoted type.

Promoted type for long double.

Promoted type for double

Definition at line 70 of file promotion.h.

8.44.2 Member Typedef Documentation

8.44.2.1 type [1/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = long double
```

Definition at line 86 of file promotion.h.

8.44.2.2 type [2/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 72 of file promotion.h.

8.44.2.3 type [3/3]

```
template<typename Scalar_T>
using glucat::numeric_traits< Scalar_T >::promoted::type = double
```

Definition at line 145 of file scalar.h.

The documentation for this struct was generated from the following files:

- glucat/promotion.h
- glucat/scalar.h

8.45 glucat::random_generator< Scalar_T > Class Template Reference

Random number generator with single instance per Scalar_T.

```
#include <random.h>
```

Public Member Functions

- random_generator (const random_generator &)=delete
- auto operator= (const random_generator &) -> random_generator &=delete
- auto uniform () -> Scalar_T
- auto normal () -> Scalar_T

Static Public Member Functions

- static auto generator () -> random_generator &

  Single instance of Random number generator.

Private Member Functions

- random_generator ()
- ~random_generator ()=default

Private Attributes

- std::mt19937 uint_gen
- std::uniform_real_distribution< double > uniform_dist
- std::normal_distribution< double > normal_dist

Static Private Attributes

- static const unsigned long seed = 19590921UL

Friends

- class friend_for_private_destructor

8.45.1 Detailed Description

template<typename Scalar_T>
class glucat::random_generator< Scalar_T >

Random number generator with single instance per Scalar_T.

Definition at line 42 of file random.h.

8.45.2 Constructor & Destructor Documentation

8.45.2.1 random_generator() [1/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator (
            const random_generator< Scalar_T > & )  [delete]
```

References random_generator().

Referenced by generator(), operator=(), and random_generator().

8.45.2.2 random_generator() [2/2]

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::random_generator ()  [inline], [private]
```

Definition at line 61 of file random.h.

References normal_dist, seed, uint_gen, and uniform_dist.

8.45.2.3 ~random_generator()

```
template<typename Scalar_T>
glucat::random_generator< Scalar_T >::~random_generator ()  [private], [default]
```

8.45.3 Member Function Documentation

8.45.3.1 generator()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::generator () -> random_generator &  [inline],
[static]
```

Single instance of Random number generator.

Definition at line 51 of file random.h.

References random_generator().

8.45.3.2 normal()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::normal () -> Scalar_T  [inline]
```

Definition at line 70 of file random.h.

8.45.3.3 operator=()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::operator= (
            const random_generator< Scalar_T > & ) -> random_generator &=delete  [delete]
```

References random_generator().

8.45.3.4 uniform()

```
template<typename Scalar_T>
auto glucat::random_generator< Scalar_T >::uniform () -> Scalar_T  [inline]
```

Definition at line 68 of file random.h.

8.45.4 Friends And Related Symbol Documentation

8.45.4.1 friend_for_private_destructor

```
template<typename Scalar_T>
friend class friend_for_private_destructor  [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE http://doc.ece.uci.edu

Definition at line 48 of file random.h.

References friend_for_private_destructor.

Referenced by friend_for_private_destructor.

8.45.5 Member Data Documentation

8.45.5.1 normal_dist

```
template<typename Scalar_T>
std::normal_distribution<double> glucat::random_generator< Scalar_T >::normal_dist  [private]
```

Definition at line 59 of file random.h.

Referenced by random_generator().

8.45.5.2 seed

```
template<typename Scalar_T>
const unsigned long glucat::random_generator< Scalar_T >::seed = 19590921UL  [static], [private]
```

Definition at line 55 of file random.h.

Referenced by random_generator().

8.45.5.3 uint_gen

```
template<typename Scalar_T>
std::mt19937 glucat::random_generator< Scalar_T >::uint_gen  [private]
```

Definition at line 57 of file random.h.

Referenced by random_generator().

8.45.5.4 uniform_dist

```
template<typename Scalar_T>
std::uniform_real_distribution<double> glucat::random_generator< Scalar_T >::uniform_dist
[private]
```

Definition at line 58 of file random.h.

Referenced by random_generator().

The documentation for this class was generated from the following file:

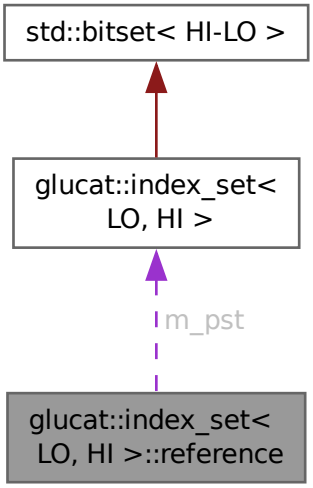
- glucat/random.h

8.46 glucat::index_set< LO, HI >::reference Class Reference

Index set member reference.

```
#include <index_set.h>
```

Collaboration diagram for glucat::index_set< LO, HI >::reference:



Public Member Functions

- reference ()=delete

  Default constructor is deleted.
- reference (index_set_t &ist, index_t idx)

  index_set reference
- ~reference ()=default
- auto operator== (const reference &c_j) const -> bool

  for b[i] == c[j];
- auto operator= (const bool x) -> reference &

  for b[i] = x;
- auto operator= (const reference &c_j) -> reference &

  for b[i] = c[j];
- auto operator~ () const -> bool

  Flips a bit.
- operator bool () const

  for x = b[i];
- auto flip () -> reference &

  for b[i].flip();

Private Attributes

- index_set_t * m_pst
- index_t m_idx

Friends

- class index_set

8.46.1 Detailed Description

template<const index_t LO, const index_t HI>
class glucat::index_set< LO, HI >::reference

Index set member reference.

Definition at line 177 of file index_set.h.

8.46.2 Constructor & Destructor Documentation

8.46.2.1 reference() [1/2]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::reference ()  [delete]
```

Default constructor is deleted.

Referenced by flip(), operator=(), operator=(), operator==(), and ~reference().

8.46.2.2 reference() [2/2]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::reference (
            index_set_t & ist,
            index_t idx)  [inline]
```

index_set reference

Definition at line 985 of file index_set_imp.h.

References m_idx, and m_pst.

8.46.2.3 ~reference()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::~reference ()  [default]
```

References flip(), and reference().

8.46.3 Member Function Documentation

8.46.3.1 flip()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::flip () -> reference &  [inline]
```

for b[i].flip();

Definition at line 1049 of file index_set_imp.h.

References m_idx, m_pst, and reference().

Referenced by ~reference().

8.46.3.2 operator bool()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::operator bool () const  [inline]
```

for x = b[i];

Definition at line 1041 of file index_set_imp.h.

References m_idx, and m_pst.

8.46.3.3 operator=() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
             const bool x) -> reference &  [inline]
```

for b[i] = x;

Definition at line 1003 of file index_set_imp.h.

References m_idx, m_pst, and reference().

8.46.3.4 operator=() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
             const reference & c_j) -> reference &  [inline]
```

for b[i] = c[j];

Definition at line 1017 of file index_set_imp.h.

References m_idx, m_pst, and reference().

8.46.3.5 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator== (
             const reference & c_j) const -> bool  [inline]
```

for b[i] == c[j];

Definition at line 995 of file index_set_imp.h.

References m_idx, m_pst, and reference().

8.46.3.6 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator~ () const -> bool  [inline]
```

Flips a bit.

flips the bit

Definition at line 1034 of file index_set_imp.h.

References m_idx, and m_pst.

8.46.4 Friends And Related Symbol Documentation

8.46.4.1 index_set

```
template<const index_t LO, const index_t HI>
friend class index_set  [friend]
```

Definition at line 178 of file index_set.h.

References index_set.

Referenced by index_set.

8.46.5 Member Data Documentation

8.46.5.1 m_idx

```
template<const index_t LO, const index_t HI>
index_t glucat::index_set< LO, HI >::reference::m_idx  [private]
```

Definition at line 200 of file index_set.h.

Referenced by flip(), operator bool(), operator=(), operator=(), operator==(), operator~(), and reference().

8.46.5.2 m_pst

```
template<const index_t LO, const index_t HI>
index_set_t* glucat::index_set< LO, HI >::reference::m_pst  [private]
```

Definition at line 199 of file index_set.h.

Referenced by flip(), operator bool(), operator=(), operator=(), operator==(), operator~(), and reference().

The documentation for this class was generated from the following files:

- glucat/index_set.h
- glucat/index_set_imp.h

8.47 glucat::sorted_range< Map_T, Sorted_Map_T > Class Template Reference

Sorted range for use with output.

```
#include <framed_multi_imp.h>
```

Public Types

- using map_t = Map_T
- using sorted_map_t = Sorted_Map_T
- using sorted_iterator = typename Sorted_Map_T::const_iterator

Public Member Functions

- sorted_range (Sorted_Map_T &sorted_val, const Map_T &val)

Public Attributes

- sorted_iterator sorted_begin
- sorted_iterator sorted_end

8.47.1 Detailed Description

template<typename Map_T, typename Sorted_Map_T>
class glucat::sorted_range< Map_T, Sorted_Map_T >

Sorted range for use with output.

Definition at line 1108 of file framed_multi_imp.h.

8.47.2 Member Typedef Documentation

8.47.2.1 map_t

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::map_t = Map_T
```

Definition at line 1111 of file framed_multi_imp.h.

8.47.2.2 sorted_iterator

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T::↩
const_iterator
```

Definition at line 1113 of file framed_multi_imp.h.

8.47.2.3 sorted_map_t

```
template<typename Map_T, typename Sorted_Map_T>
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1112 of file framed_multi_imp.h.

8.47.3 Constructor & Destructor Documentation

8.47.3.1 sorted_range()

```
template<typename Map_T, typename Sorted_Map_T>
glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range (
            Sorted_Map_T & sorted_val,
            const Map_T & val)  [inline]
```

Definition at line 1115 of file framed_multi_imp.h.

References sorted_begin, and sorted_end.

Referenced by glucat::operator<<().

8.47.4 Member Data Documentation

8.47.4.1 sorted_begin

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1122 of file framed_multi_imp.h.

Referenced by sorted_range().

8.47.4.2 sorted_end

```
template<typename Map_T, typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1123 of file framed_multi_imp.h.

Referenced by sorted_range().

The documentation for this class was generated from the following file:

- glucat/framed_multi_imp.h

8.48 glucat::sorted_range< Sorted_Map_T, Sorted_Map_T > Class Template Reference

```
#include <framed_multi_imp.h>
```

Public Types

- using map_t = Sorted_Map_T
- using sorted_map_t = Sorted_Map_T
- using sorted_iterator = typename Sorted_Map_T::const_iterator

Public Member Functions

- sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T &val)

Public Attributes

- sorted_iterator sorted_begin
- sorted_iterator sorted_end

8.48.1 Detailed Description

template<typename Sorted_Map_T>
class glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >

Definition at line 1127 of file framed_multi_imp.h.

8.48.2 Member Typedef Documentation

8.48.2.1 map_t

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::map_t = Sorted_Map_T
```

Definition at line 1130 of file framed_multi_imp.h.

8.48.2.2 sorted_iterator

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_↩
Map_T::const_iterator
```

Definition at line 1132 of file framed_multi_imp.h.

8.48.2.3 sorted_map_t

```
template<typename Sorted_Map_T>
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1131 of file framed_multi_imp.h.

8.48.3 Constructor & Destructor Documentation

8.48.3.1 sorted_range()

```
template<typename Sorted_Map_T>
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_range (
            Sorted_Map_T & sorted_val,
            const Sorted_Map_T & val)  [inline]
```

Definition at line 1134 of file framed_multi_imp.h.

References sorted_begin, and sorted_end.

8.48.4 Member Data Documentation

8.48.4.1 sorted_begin

```
template<typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1138 of file framed_multi_imp.h.

Referenced by sorted_range().

8.48.4.2 sorted_end

```
template<typename Sorted_Map_T>
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1139 of file framed_multi_imp.h.

Referenced by sorted_range().

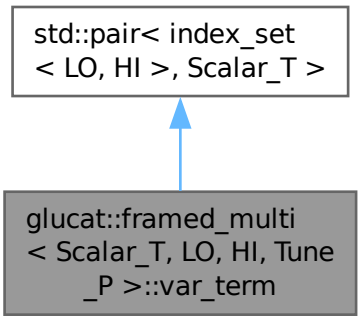
The documentation for this class was generated from the following file:

- glucat/framed_multi_imp.h

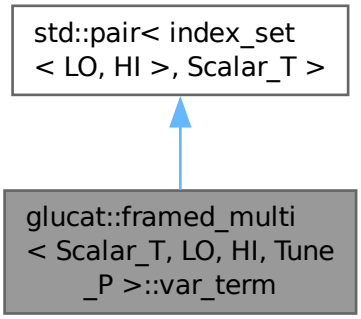
8.49 glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term Class Reference

Variable term.

Inheritance diagram for glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term:



Collaboration diagram for glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term:



Public Types

- using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>

Public Member Functions

- ~var_term ()=default

  Destructor.
- var_term ()

  Default constructor.
- var_term (const index_set_t ist, const Scalar_T &crd=Scalar_T(1))

  Construct a variable term from an index set and a scalar coordinate.
- auto operator*= (const term_t &rhs) -> var_term_t &

  Product of variable term and term.

Static Public Member Functions

- static auto classname () -> const std::string

  Class name used in messages.

8.49.1 Detailed Description

template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term

Variable term.

Definition at line 279 of file framed_multi.h.

8.49.2 Member Typedef Documentation

8.49.2.1 var_pair_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_pair_t = std::pair<index_set<LO,
HI>, Scalar_T>
```

Definition at line 283 of file framed_multi.h.

8.49.3 Constructor & Destructor Documentation

8.49.3.1 ~var_term()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::~var_term ()  [default]
```

Destructor.

8.49.3.2 var_term() [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term ()  [inline]
```

Default constructor.

Definition at line 291 of file framed_multi.h.

8.49.3.3 var_term() [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term (
            const index_set_t ist,
            const Scalar_T & crd = Scalar_T(1))  [inline]
```

Construct a variable term from an index set and a scalar coordinate.

Definition at line 295 of file framed_multi.h.

8.49.4 Member Function Documentation

8.49.4.1 classname()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::classname () -> const std::↩
string  [inline], [static]
```

Class name used in messages.

Definition at line 286 of file framed_multi.h.

8.49.4.2 operator*=()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT↩
_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::operator*= (
            const term_t & rhs) -> var_term_t &  [inline]
```

Product of variable term and term.

Definition at line 299 of file framed_multi.h.

The documentation for this class was generated from the following file:

- glucat/framed_multi.h

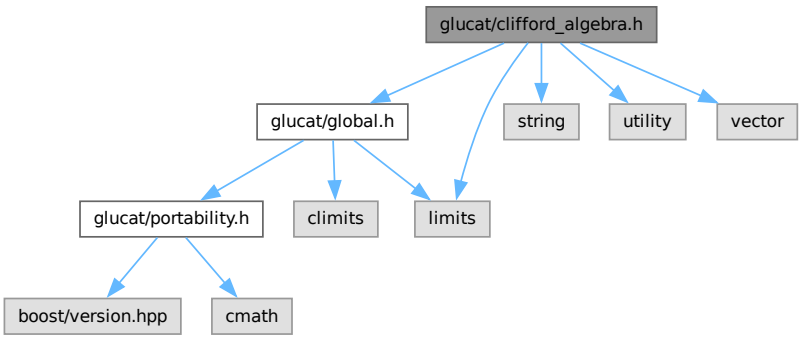
Chapter 9

File Documentation

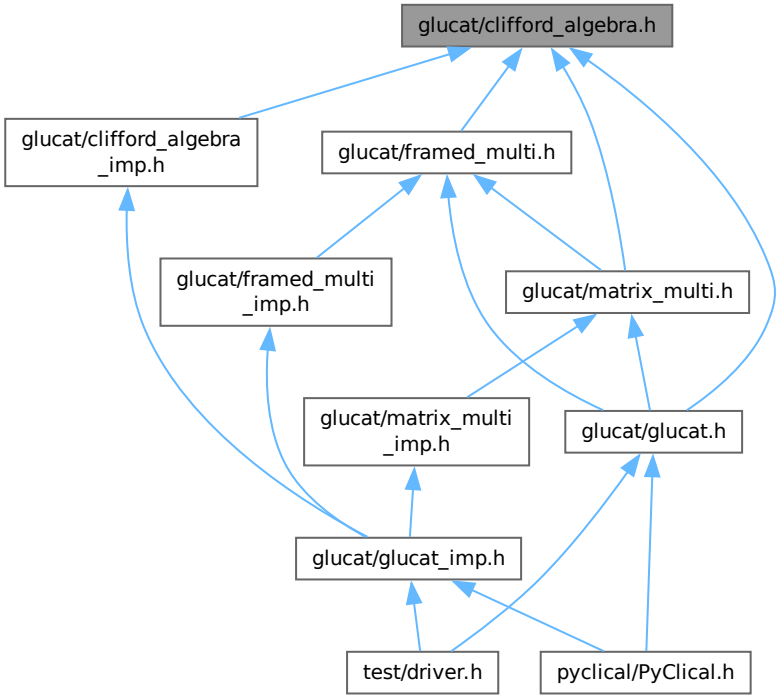
9.1 glucat/clifford_algebra.h File Reference

```
#include "glucat/global.h"
#include <limits>
#include <string>
#include <utility>
#include <vector>
```

Include dependency graph for clifford_algebra.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >
  clifford_algebra<> declares the operations of a Clifford algebra

Namespaces

- namespace glucat

Macros

- #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS

Functions

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool
  Test for inequality of multivectors.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool

Test for inequality of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for inequality of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::error_squared_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Quadratic norm error tolerance relative to a specific multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::error_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T

  Relative or absolute error using the quadratic norm.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool

  Test for approximate equality of multivectors.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for approximate equality of multivectors.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inner product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune↩
  _P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric multiplicative inverse.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Integer power of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune↩_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Multivector power of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::outer_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Outer product power of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Real part: synonym for scalar part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Imaginary part: deprecated (always 0).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Pure part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Even part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Odd part.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::vector_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar↩_T >

  Vector part of multivector, as a vector_t with respect to frame().
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Main involution, each {i} is replaced by -{i} in each term, eg. {1}∗{2} -> (-{2})∗(-{1}).

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Reversion, eg. {1}∗{2} -> {2}∗{1}.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Conjugation, rev o invo == invo o rev.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar_T quadratic form == (rev(x)∗x)(0).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar_T norm == sum of norm of coordinates.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Absolute value == sqrt(norm).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::max_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Maximum of absolute values of components of multivector: multivector infinity norm.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of -1 which commutes with all members of the frame of the given multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::clifford_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Natural logarithm of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Inverse sine of multivector with specified complexifier.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector.

9.1.1 Macro Definition Documentation

9.1.1.1 _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS

```
#define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
```

Definition at line 145 of file clifford_algebra.h.

Referenced by glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::basis_element(), and glucat::framed_multi< Scalar_T, LO, HI, Tun

9.2 clifford_algebra.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     clifford_algebra.h : Declare the operations of a Clifford algebra
00006                          -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035
00036 #include <limits>
00037 #include <string>
00038 #include <utility>
00039 #include <vector>
00040
00041 namespace glucat
00042 {
00044   template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00045   class clifford_algebra
00046   {
00047   public:
00048     using scalar_t = Scalar_T;
00049     using index_set_t = Index_Set_T;
00050     static const index_t v_lo = index_set_t::v_lo;
00051     static const index_t v_hi = index_set_t::v_hi;
00052     using multivector_t = Multivector_T;
00053     using pair_t = std::pair<const index_set_t, Scalar_T>;
00054     using vector_t = std::vector<Scalar_T>;
00055
00056     static  auto   classname() -> const std::string;
00057
00059     static const Scalar_T default_truncation;
00060
00061     virtual ~clifford_algebra() = default;
00062
00063   // clifford_algebra operations
00065     virtual auto operator==  (const multivector_t& val) const -> bool = 0;
```

```
00067     virtual auto operator==  (const Scalar_T& scr) const -> bool = 0;
00069     virtual auto operator+=  (const multivector_t& rhs) -> multivector_t& = 0;
00071     virtual auto operator+=  (const Scalar_T& scr) -> multivector_t& = 0;
00073     virtual auto operator-=  (const multivector_t& rhs) -> multivector_t& = 0;
00075     virtual auto operator-=  (const Scalar_T& scr) -> multivector_t& = 0;
00077     virtual auto operator-   () const -> const multivector_t = 0;
00079     virtual auto operator*=  (const Scalar_T& scr) -> multivector_t& = 0;
00081     virtual auto operator*=  (const multivector_t& rhs) -> multivector_t& = 0;
00083     virtual auto operator%=  (const multivector_t& rhs) -> multivector_t& = 0;
00085     virtual auto operator&=  (const multivector_t& rhs) -> multivector_t& = 0;
00087     virtual auto operator^=  (const multivector_t& rhs) -> multivector_t& = 0;
00089     virtual auto operator/=  (const Scalar_T& scr) -> multivector_t& = 0;
00091     virtual auto operator/=  (const multivector_t& rhs) -> multivector_t& = 0;
00093     virtual auto operator|=  (const multivector_t& rhs) -> multivector_t& = 0;
00095     virtual auto inv         () const -> const multivector_t = 0;
00097     virtual auto pow         (int m) const -> const multivector_t = 0;
00099     virtual auto outer_pow   (int m) const -> const multivector_t = 0;
00101     virtual auto frame       () const -> const index_set_t = 0;
00103     virtual auto grade       () const -> index_t = 0;
00105     virtual auto operator[]  (const index_set_t ist) const -> Scalar_T = 0;
00107     virtual auto operator()  (index_t grade) const -> const multivector_t = 0;
00109     virtual auto scalar      () const -> Scalar_T = 0;
00111     virtual auto pure        () const -> const multivector_t = 0;
00113     virtual auto even        () const -> const multivector_t = 0;
00115     virtual auto odd         () const -> const multivector_t = 0;
00117     virtual auto vector_part () const -> const vector_t = 0;
00119     virtual auto vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t =
    0;
00121     virtual auto involute    () const -> const multivector_t = 0;
00123     virtual auto reverse     () const -> const multivector_t = 0;
00125     virtual auto conj        () const -> const multivector_t = 0;
00127     virtual auto quad        () const -> Scalar_T = 0;
00129     virtual auto norm        () const -> Scalar_T = 0;
00131     virtual auto max_abs     () const -> Scalar_T = 0;
00133     virtual auto truncated   (const Scalar_T& limit = default_truncation) const -> const multivector_t
    = 0;
00135     virtual auto isinf       () const -> bool = 0;
00137     virtual auto isnan       () const -> bool = 0;
00139     virtual void write       (const std::string& msg="") const = 0;
00141     virtual void write       (std::ofstream& ofile, const std::string& msg="") const = 0;
00142   };
00143
00144 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00145 #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS \
00146     auto operator==  (const multivector_t& val) const  -> bool               override;\
00147     auto operator==  (const Scalar_T& scr) const       -> bool               override;\
00148     auto operator+=  (const multivector_t& rhs)        -> multivector_t&     override;\
00149     auto operator+=  (const Scalar_T& scr)             -> multivector_t&     override;\
00150     auto operator-=  (const multivector_t& rhs)        -> multivector_t&     override;\
00151     auto operator-=  (const Scalar_T& scr)             -> multivector_t&     override;\
00152     auto operator-   () const                          -> const multivector_t override;\
00153     auto operator*=  (const Scalar_T& scr)             -> multivector_t&     override;\
00154     auto operator*=  (const multivector_t& rhs)        -> multivector_t&     override;\
00155     auto operator%=  (const multivector_t& rhs)        -> multivector_t&     override;\
00156     auto operator&=  (const multivector_t& rhs)        -> multivector_t&     override;\
00157     auto operator^=  (const multivector_t& rhs)        -> multivector_t&     override;\
00158     auto operator/=  (const Scalar_T& scr)             -> multivector_t&     override;\
00159     auto operator/=  (const multivector_t& rhs)        -> multivector_t&     override;\
00160     auto operator|=  (const multivector_t& rhs)        -> multivector_t&     override;\
00161     auto inv         () const                          -> const multivector_t override;\
00162     auto pow         (int m) const                     -> const multivector_t override;\
00163     auto outer_pow   (int m) const                     -> const multivector_t override;\
00164     auto frame       () const                          -> const index_set_t  override;\
00165     auto grade       () const                          -> index_t            override;\
00166     auto operator[]  (const index_set_t ist) const     -> Scalar_T           override;\
00167     auto operator()  (index_t grade) const             -> const multivector_t override;\
00168     auto scalar      () const                          -> Scalar_T           override;\
00169     auto pure        () const                          -> const multivector_t override;\
00170     auto even        () const                          -> const multivector_t override;\
00171     auto odd         () const                          -> const multivector_t override;\
00172     auto vector_part () const                          -> const vector_t     override;\
00173     auto vector_part (const index_set_t frm, const bool prechecked = false) const      \
00174                                                        -> const vector_t     override;\
00175     auto involute    () const                          -> const multivector_t override;\
00176     auto reverse     () const                          -> const multivector_t override;\
00177     auto conj        () const                          -> const multivector_t override;\
00178     auto quad        () const                          -> Scalar_T           override;\
00179     auto norm        () const                          -> Scalar_T           override;\
00180     auto max_abs     () const                          -> Scalar_T           override;\
00181     auto truncated   (const Scalar_T& limit = multivector_t::default_truncation) const \
00182                                                        -> const multivector_t override;\
00183     auto isinf       () const                          -> bool               override;\
00184     auto isnan       () const                          -> bool               override;\
00185     void write       (const std::string& msg="") const                       override;\
00186     void write       (std::ofstream& ofile, const std::string& msg="") const  override;
00187 #endif // _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00188
```

```
00190   template
00191   <
00192     template<typename, const index_t, const index_t, typename> class Multivector,
00193     template<typename, const index_t, const index_t, typename> class RHS,
00194     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00195   >
00196   auto
00197   operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      bool;
00198
00200   template
00201   <
00202     template<typename, const index_t, const index_t, typename> class Multivector,
00203     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00204   >
00205   auto
00206   operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool;
00207
00209   template
00210   <
00211     template<typename, const index_t, const index_t, typename> class Multivector,
00212     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00213   >
00214   auto
00215   operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00216
00218   template
00219   <
00220     template<typename, const index_t, const index_t, typename> class Multivector,
00221     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00222   >
00223   auto
00224   error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00225
00227   template
00228   <
00229     template<typename, const index_t, const index_t, typename> class Multivector,
00230     template<typename, const index_t, const index_t, typename> class RHS,
00231     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232   >
00233   auto
00234   error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00235                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00236                 const Scalar_T threshold) -> Scalar_T;
00237
00239   template
00240   <
00241     template<typename, const index_t, const index_t, typename> class Multivector,
00242     template<typename, const index_t, const index_t, typename> class RHS,
00243     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00244   >
00245   auto
00246   approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00247                const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00248                const Scalar_T threshold,
00249                const Scalar_T tolerance) -> bool;
00250
00252   template
00253   <
00254     template<typename, const index_t, const index_t, typename> class Multivector,
00255     template<typename, const index_t, const index_t, typename> class RHS,
00256     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00257   >
00258   auto
00259   approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00260                const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00261
00263   template
00264   <
00265     template<typename, const index_t, const index_t, typename> class Multivector,
00266     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00267   >
00268   auto
00269   operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00270
00272   template
00273   <
00274     template<typename, const index_t, const index_t, typename> class Multivector,
00275     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00276   >
00277   auto
00278   operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00279
00281   template
00282   <
```

```
00283     template<typename, const index_t, const index_t, typename> class Multivector,
00284     template<typename, const index_t, const index_t, typename> class RHS,
00285     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00286   >
00287   auto
00288   operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00289
00291   template
00292   <
00293     template<typename, const index_t, const index_t, typename> class Multivector,
00294     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00295   >
00296   auto
00297   operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00298
00300   template
00301   <
00302     template<typename, const index_t, const index_t, typename> class Multivector,
00303     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304   >
00305   auto
00306   operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00307
00309   template
00310   <
00311     template<typename, const index_t, const index_t, typename> class Multivector,
00312     template<typename, const index_t, const index_t, typename> class RHS,
00313     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00314   >
00315   auto
00316   operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00317
00319   template
00320   <
00321     template<typename, const index_t, const index_t, typename> class Multivector,
00322     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00323   >
00324   auto
00325   operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00326
00328   template
00329   <
00330     template<typename, const index_t, const index_t, typename> class Multivector,
00331     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00332   >
00333   auto
00334   operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00335
00337   template
00338   <
00339     template<typename, const index_t, const index_t, typename> class Multivector,
00340     template<typename, const index_t, const index_t, typename> class RHS,
00341     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00342   >
00343   auto
00344   operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00345
00347   template
00348   <
00349     template<typename, const index_t, const index_t, typename> class Multivector,
00350     template<typename, const index_t, const index_t, typename> class RHS,
00351     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00352   >
00353   auto
00354   operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00355
00357   template
00358   <
00359     template<typename, const index_t, const index_t, typename> class Multivector,
00360     template<typename, const index_t, const index_t, typename> class RHS,
00361     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00362   >
00363   auto
00364   operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00365
00367   template
00368   <
00369     template<typename, const index_t, const index_t, typename> class Multivector,
```

```
00370     template<typename, const index_t, const index_t, typename> class RHS,
00371     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00372   >
00373   auto
00374   operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00375
00377   template
00378   <
00379     template<typename, const index_t, const index_t, typename> class Multivector,
00380     template<typename, const index_t, const index_t, typename> class RHS,
00381     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00382   >
00383   auto
00384   star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      Scalar_T;
00385
00387   template
00388   <
00389     template<typename, const index_t, const index_t, typename> class Multivector,
00390     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00391   >
00392   auto
00393   operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00394
00396   template
00397   <
00398     template<typename, const index_t, const index_t, typename> class Multivector,
00399     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00400   >
00401   auto
00402   operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00403
00405   template
00406   <
00407     template<typename, const index_t, const index_t, typename> class Multivector,
00408     template<typename, const index_t, const index_t, typename> class RHS,
00409     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00410   >
00411   auto
00412   operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00413
00415   template
00416   <
00417     template<typename, const index_t, const index_t, typename> class Multivector,
00418     template<typename, const index_t, const index_t, typename> class RHS,
00419     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00420   >
00421   auto
00422   operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>;
00423
00425   template
00426   <
00427     template<typename, const index_t, const index_t, typename> class Multivector,
00428     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00429   >
00430   auto
00431   inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00432
00434   template
00435   <
00436     template<typename, const index_t, const index_t, typename> class Multivector,
00437     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00438   >
00439   auto
00440   pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00441
00443   template
00444   <
00445     template<typename, const index_t, const index_t, typename> class Multivector,
00446     template<typename, const index_t, const index_t, typename> class RHS,
00447     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00448   >
00449   auto
00450   pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00451
00453   template< template<typename, const index_t, const index_t, typename> class Multivector,
00454             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00455   auto
00456   outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
```

```
00457
00459   template
00460   <
00461     template<typename, const index_t, const index_t, typename> class Multivector,
00462     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00463   >
00464   auto
00465   scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00466
00468   template
00469   <
00470     template<typename, const index_t, const index_t, typename> class Multivector,
00471     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00472   >
00473   auto
00474   real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00475
00477   template
00478   <
00479     template<typename, const index_t, const index_t, typename> class Multivector,
00480     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00481   >
00482   auto
00483   imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00484
00486   template
00487   <
00488     template<typename, const index_t, const index_t, typename> class Multivector,
00489     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00490   >
00491   auto
00492   pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00493
00495   template
00496   <
00497     template<typename, const index_t, const index_t, typename> class Multivector,
00498     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00499   >
00500   auto
00501   even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00502
00504   template
00505   <
00506     template<typename, const index_t, const index_t, typename> class Multivector,
00507     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00508   >
00509   auto
00510   odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00511
00513   template
00514   <
00515     template<typename, const index_t, const index_t, typename> class Multivector,
00516     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00517   >
00518   auto
00519   vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>;
00520
00522   template
00523   <
00524     template<typename, const index_t, const index_t, typename> class Multivector,
00525     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00526   >
00527   auto
00528   involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00529
00531   template
00532   <
00533     template<typename, const index_t, const index_t, typename> class Multivector,
00534     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00535   >
00536   auto
00537   reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00538
00540   template
00541   <
00542     template<typename, const index_t, const index_t, typename> class Multivector,
00543     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00544   >
00545   auto
00546   conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00547
00549   template
00550   <
00551     template<typename, const index_t, const index_t, typename> class Multivector,
00552     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00553   >
00554   auto
```

```
00555   quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00556
00558   template
00559   <
00560     template<typename, const index_t, const index_t, typename> class Multivector,
00561     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00562   >
00563   auto
00564   norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00565
00567   template
00568   <
00569     template<typename, const index_t, const index_t, typename> class Multivector,
00570     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00571   >
00572   auto
00573   abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00574
00576   template
00577   <
00578     template<typename, const index_t, const index_t, typename> class Multivector,
00579     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00580   >
00581   auto
00582   max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00583
00585   template
00586   <
00587     template<typename, const index_t, const index_t, typename> class Multivector,
00588     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00589   >
00590   auto
00591   complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00592
00595   template
00596   <
00597     template<typename, const index_t, const index_t, typename> class Multivector,
00598     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00599   >
00600   auto
00601   elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00602
00604   template
00605   <
00606     template<typename, const index_t, const index_t, typename> class Multivector,
00607     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00608   >
00609   auto
00610   sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00611        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00612        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00613
00615   template
00616   <
00617     template<typename, const index_t, const index_t, typename> class Multivector,
00618     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00619   >
00620   auto
00621   sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00622
00623   // Transcendental functions
00624
00626   template
00627   < template<typename, const index_t, const index_t, typename> class Multivector,
00628     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00629   >
00630   auto
00631   clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>;
00632
00634   template
00635   <
00636     template<typename, const index_t, const index_t, typename> class Multivector,
00637     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00638   >
00639   auto
00640   log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00641       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00642       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00643
00645   template
00646   <
00647     template<typename, const index_t, const index_t, typename> class Multivector,
00648     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00649   >
00650   auto
```

```
00651   log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00652
00654   template
00655   <
00656     template<typename, const index_t, const index_t, typename> class Multivector,
00657     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00658   >
00659   auto
00660   cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00661       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00662       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00663
00665   template
00666   <
00667     template<typename, const index_t, const index_t, typename> class Multivector,
00668     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00669   >
00670   auto
00671   cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00672
00674   template
00675   <
00676     template<typename, const index_t, const index_t, typename> class Multivector,
00677     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00678   >
00679   auto
00680   acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00681        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00682        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00683
00685   template
00686   <
00687     template<typename, const index_t, const index_t, typename> class Multivector,
00688     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00689   >
00690   auto
00691   acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00692
00694   template
00695   <
00696     template<typename, const index_t, const index_t, typename> class Multivector,
00697     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00698   >
00699   auto
00700   cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00701
00703   template
00704   <
00705     template<typename, const index_t, const index_t, typename> class Multivector,
00706     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00707   >
00708   auto
00709   acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00710         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00711         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00712
00714   template
00715   <
00716     template<typename, const index_t, const index_t, typename> class Multivector,
00717     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00718   >
00719   auto
00720   acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00721
00723   template
00724   <
00725     template<typename, const index_t, const index_t, typename> class Multivector,
00726     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00727   >
00728   auto
00729   sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00730       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00731       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00732
00734   template
00735   <
00736     template<typename, const index_t, const index_t, typename> class Multivector,
00737     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00738   >
00739   auto
00740   sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00741
00743   template
00744   <
00745     template<typename, const index_t, const index_t, typename> class Multivector,
00746     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00747   >
```

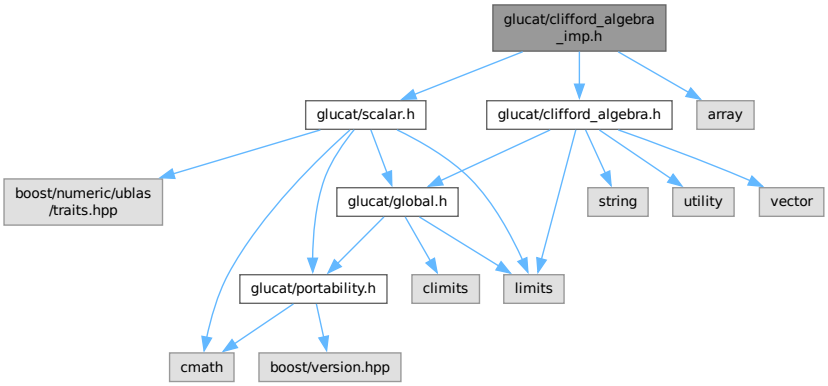
```
00748   auto
00749   asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00750        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00751        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00752
00754   template
00755   <
00756     template<typename, const index_t, const index_t, typename> class Multivector,
00757     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00758   >
00759   auto
00760   asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00761
00763   template
00764   <
00765     template<typename, const index_t, const index_t, typename> class Multivector,
00766     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00767   >
00768   auto
00769   sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00770
00772   template
00773   <
00774     template<typename, const index_t, const index_t, typename> class Multivector,
00775     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00776   >
00777   auto
00778   asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00779         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00780         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00781
00783   template
00784   <
00785     template<typename, const index_t, const index_t, typename> class Multivector,
00786     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00787   >
00788   auto
00789   asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00790
00792   template
00793   <
00794     template<typename, const index_t, const index_t, typename> class Multivector,
00795     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00796   >
00797   auto
00798   tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00799       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00800       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00801
00803   template
00804   <
00805     template<typename, const index_t, const index_t, typename> class Multivector,
00806     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00807   >
00808   auto
00809   tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00810
00812   template
00813   <
00814     template<typename, const index_t, const index_t, typename> class Multivector,
00815     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00816   >
00817   auto
00818   atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00819        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00820        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00821
00823   template
00824   <
00825     template<typename, const index_t, const index_t, typename> class Multivector,
00826     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00827   >
00828   auto
00829   atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00830
00832   template
00833   <
00834     template<typename, const index_t, const index_t, typename> class Multivector,
00835     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00836   >
00837   auto
00838   tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00839
00841   template
00842   <
00843     template<typename, const index_t, const index_t, typename> class Multivector,
00844     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
```

```
00845   >
00846   auto
00847   atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00848         const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00849         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00850
00852   template
00853   <
00854     template<typename, const index_t, const index_t, typename> class Multivector,
00855     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00856   >
00857   auto
00858   atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00859 }
00860 #endif  // _GLUCAT_CLIFFORD_ALGEBRA_H
```

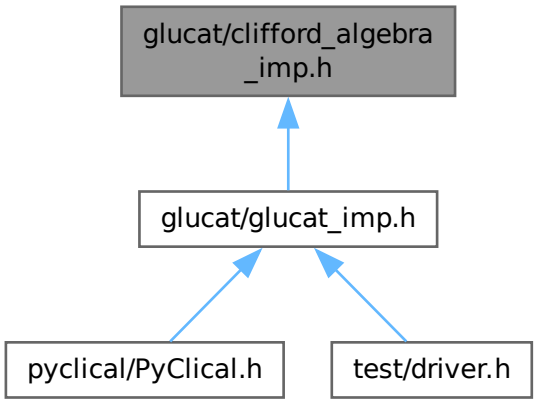
9.3 glucat/clifford_algebra_imp.h File Reference

```
#include "glucat/clifford_algebra.h"
#include "glucat/scalar.h"
#include <array>
```

Include dependency graph for clifford_algebra_imp.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Functions

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for inequality of multivectors.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool

  Test for inequality of multivector and scalar.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for inequality of scalar and multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::error_squared_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Quadratic norm error tolerance relative to a specific multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::error_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T

  Relative or absolute error using the quadratic norm.
- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool

Test for approximate equality of multivectors.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::approx_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool

  Test for approximate equality of multivectors.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric sum.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric difference.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Product of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric product.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Outer product.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inner product.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Left contraction.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune↩_P > &rhs) -> Scalar_T

  Hestenes scalar product.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of multivector and scalar.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Quotient of scalar and multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Geometric multiplicative inverse.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Integer power of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune↩_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Multivector power of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::outer_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >

Outer product power of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar part.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Real part: synonym for scalar part.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Imaginary part: deprecated (always 0).

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Pure part.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Even part.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Odd part.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::vector_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >

  Vector part of multivector, as a vector_t with respect to frame().

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Main involution, each {i} is replaced by -{i} in each term, eg. {1}∗{2} -> (-{2})∗(-{1}).

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Reversion, eg. {1}∗{2} -> {2}∗{1}.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Conjugation, rev o invo == invo o rev.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Scalar_T quadratic form == (rev(x)∗x)(0).

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

Scalar_T norm == sum of norm of coordinates.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Absolute value == sqrt(norm).
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::max_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T

  Maximum of absolute values of components of multivector: multivector infinity norm.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of -1 which commutes with all members of the frame of the given multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static void glucat::check_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)

  Check that i is a valid complexifier for val.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Square root of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::clifford_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic cosine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic cosine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Cosine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Cosine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse cosine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Sine of multivector.

- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse sine of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::asin (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse sine of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Hyperbolic tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse hyperbolic tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Tangent of multivector.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector with specified complexifier.
- template<template< typename, const index_t, const index_t, typename > class Multivector, typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >

  Inverse tangent of multivector.

Variables

- template<typename Scalar_T, typename Index_Set_T, typename Multivector_T>
  const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_truncation = std::numeric_limits<Scalar_T>::epsilon()

  Default for truncation.

9.4 clifford_algebra_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     clifford_algebra_imp.h : Implement common Clifford algebra functions
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 // References for algorithms:
00035 // [AS]:
00036 // Milton Abramowicz and Irene A. Stegun, "Handbook of mathematical functions",
00037 // Dover 1972, first published 1965.
00038 // [CHKL]:
00039 // Sheung Hun Cheng, Nicholas J. Higham, Charles S. Kenney and Alan J. Laub,
00040 // "Approximating the Logarithm of a Matrix to Specified Accuracy", 1999.
00041 // ftp://ftp.ma.man.ac.uk/pub/narep/narep353.ps.gz
00042 // [GL]:
00043 // Gene H. Golub and Charles F. van Loan,
00044 // "Matrix Computations", 3rd ed., Johns Hopkins UP, 1996.
00045 // [GW]:
00046 // C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis",
00047 // 6th Edition, Addison-Wesley, 1999.
00048 // [H]:
00049 // Nicholas J. Higham
00050 // "The Scaling and Squaring Method for the Matrix Exponential Revisited",
00051 // SIAM Journal on Matrix Analysis and Applications,
00052 // Vol. 26,  Issue 4 (2005), pp. 1179-1193.
00053 // [Z]:
00054 // Doron Zeilberger, "PADE" (Maple code), 2002.
00055 // http://www.math.rutgers.edu/~zeilberg/tokhniot/PADE
00056
00057 #include "glucat/clifford_algebra.h"
00058 #include "glucat/scalar.h"
00059
00060 #include <array>
00061
00062 namespace glucat
00063 {
00064   template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00065   auto
00066   clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00067   classname() -> const std::string
00068   { return "clifford_algebra"; }
00069
00071   template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00072   const
00073   Scalar_T
00074   clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00075   default_truncation = std::numeric_limits<Scalar_T>::epsilon();
00076
00078   template
00079   <
00080     template<typename, const index_t, const index_t, typename> class Multivector,
00081     template<typename, const index_t, const index_t, typename> class RHS,
00082     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00083   >
00084   inline
```

```
00085   auto
00086   operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      bool
00087   { return !(lhs == rhs); }
00088
00090   template< template<typename, const index_t, const index_t, typename> class Multivector,
00091             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092   inline
00093   auto
00094   operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool
00095   { return !(lhs == scr); }
00096
00098   template< template<typename, const index_t, const index_t, typename> class Multivector,
00099             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00100   inline
00101   auto
00102   operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00103   { return !(rhs == scr); }
00104
00106   template
00107   <
00108     template<typename, const index_t, const index_t, typename> class Multivector,
00109     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00110   >
00111   auto
00112   error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00113   {
00114     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00115     static const auto scalar_eps  = std::numeric_limits<Scalar_T>::epsilon();
00116     static const auto nbr_different_bits =
00117       std::numeric_limits<Scalar_T>::digits / Tune_P::denom_different_bits +
      Tune_P::extra_different_bits;
00118     static const auto abs_tol = scalar_eps *
00119       numeric_traits<Scalar_T>::pow(Scalar_T(2), nbr_different_bits);
00120     using framed_multi_t = typename multivector_t::framed_multi_t;
00121     const auto nbr_terms = double(framed_multi_t(val).truncated(scalar_eps).nbr_terms());
00122     return abs_tol * abs_tol * std::max(Scalar_T(nbr_terms), Scalar_T(1));
00123   }
00124
00126   template
00127   <
00128     template<typename, const index_t, const index_t, typename> class Multivector,
00129     template<typename, const index_t, const index_t, typename> class RHS,
00130     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00131   >
00132   inline
00133   auto
00134   error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00135                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00136                 const Scalar_T threshold) -> Scalar_T
00137   {
00138     const auto relative = norm(rhs) > threshold;
00139     const auto abs_norm_diff = norm(rhs-lhs);
00140     return (relative)
00141       ? abs_norm_diff/norm(rhs)
00142       : abs_norm_diff;
00143   }
00144
00146   template
00147   <
00148     template<typename, const index_t, const index_t, typename> class Multivector,
00149     template<typename, const index_t, const index_t, typename> class RHS,
00150     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00151   >
00152   inline
00153   auto
00154   approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00155                const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00156                const Scalar_T threshold,
00157                const Scalar_T tolerance) -> bool
00158   { return error_squared(lhs, rhs, threshold) < tolerance; }
00159
00161   template
00162   <
00163     template<typename, const index_t, const index_t, typename> class Multivector,
00164     template<typename, const index_t, const index_t, typename> class RHS,
00165     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00166   >
00167   inline
00168   auto
00169   approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00170                const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00171   {
00172     const Scalar_T rhs_tol = error_squared_tol(rhs);
00173     return approx_equal(lhs, rhs, rhs_tol, rhs_tol);
00174   }
00175
```

```
00177   template< template<typename, const index_t, const index_t, typename> class Multivector,
00178             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00179   inline
00180   auto
00181   operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00182   {
00183     auto result = lhs;
00184     return result += scr;
00185   }
00186
00188   template< template<typename, const index_t, const index_t, typename> class Multivector,
00189             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00190   inline
00191   auto
00192   operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00193   {
00194     return rhs + scr;
00195   }
00196
00198   template
00199   <
00200     template<typename, const index_t, const index_t, typename> class Multivector,
00201     template<typename, const index_t, const index_t, typename> class RHS,
00202     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00203   >
00204   inline
00205   auto
00206   operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00207   {
00208     auto result = lhs;
00209     return result += rhs;
00210   }
00211
00213   template< template<typename, const index_t, const index_t, typename> class Multivector,
00214             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215   inline
00216   auto
00217   operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00218   {
00219     auto result = lhs;
00220     return result -= scr;
00221   }
00222
00224   template< template<typename, const index_t, const index_t, typename> class Multivector,
00225             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00226   inline
00227   auto
00228   operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00229   { return -rhs + scr; }
00230
00232   template
00233   <
00234     template<typename, const index_t, const index_t, typename> class Multivector,
00235     template<typename, const index_t, const index_t, typename> class RHS,
00236     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00237   >
00238   inline
00239   auto
00240   operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00241   {
00242     auto result = lhs;
00243     return result -= rhs;
00244   }
00245
00247   template< template<typename, const index_t, const index_t, typename> class Multivector,
00248             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00249   inline
00250   auto
00251   operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00252   {
00253     auto result = lhs;
00254     return result *= scr;
00255   }
00256
00258   template< template<typename, const index_t, const index_t, typename> class Multivector,
00259             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00260   inline
00261   auto
00262   operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
```

```
00263   { // Note: this assumes that scalar commutes with multivector.
00264     // This excludes Clifford algebras over non-commuting rings.
00265     return rhs * scr;
00266   }
00267
00269   template
00270   <
00271     template<typename, const index_t, const index_t, typename> class Multivector,
00272     template<typename, const index_t, const index_t, typename> class RHS,
00273     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00274   >
00275   inline
00276   auto
00277   operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00278   {
00279     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00280     return lhs * multivector_t(rhs);
00281   }
00282
00284   template
00285   <
00286     template<typename, const index_t, const index_t, typename> class Multivector,
00287     template<typename, const index_t, const index_t, typename> class RHS,
00288     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00289   >
00290   inline
00291   auto
00292   operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00293   {
00294     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00295     return lhs ^ multivector_t(rhs);
00296   }
00297
00299   template
00300   <
00301     template<typename, const index_t, const index_t, typename> class Multivector,
00302     template<typename, const index_t, const index_t, typename> class RHS,
00303     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304   >
00305   inline
00306   auto
00307   operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00308   {
00309     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00310     return lhs & multivector_t(rhs);
00311   }
00312
00314   template
00315   <
00316     template<typename, const index_t, const index_t, typename> class Multivector,
00317     template<typename, const index_t, const index_t, typename> class RHS,
00318     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00319   >
00320   inline
00321   auto
00322   operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00323   {
00324     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00325     return lhs % multivector_t(rhs);
00326   }
00327
00329   template
00330   <
00331     template<typename, const index_t, const index_t, typename> class Multivector,
00332     template<typename, const index_t, const index_t, typename> class RHS,
00333     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00334   >
00335   inline
00336   auto
00337   star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      Scalar_T
00338   {
00339     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00340     return star(lhs, multivector_t(rhs));
00341   }
00342
00344   template< template<typename, const index_t, const index_t, typename> class Multivector,
00345             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00346   inline
00347   auto
00348   operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00349   {
```

```
00350     auto result = lhs;
00351     return result /= scr;
00352   }
00353
00355   template< template<typename, const index_t, const index_t, typename> class Multivector,
00356             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00357   inline
00358   auto
00359   operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00360   {
00361     Multivector<Scalar_T,LO,HI,Tune_P> result = scr;
00362     return result /= rhs;
00363   }
00364
00366   template
00367   <
00368     template<typename, const index_t, const index_t, typename> class Multivector,
00369     template<typename, const index_t, const index_t, typename> class RHS,
00370     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00371   >
00372   inline
00373   auto
00374   operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00375   {
00376     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00377     return lhs / multivector_t(rhs);
00378   }
00379
00381   template
00382   <
00383     template<typename, const index_t, const index_t, typename> class Multivector,
00384     template<typename, const index_t, const index_t, typename> class RHS,
00385     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00386   >
00387   inline
00388   auto
00389   operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
      const Multivector<Scalar_T,LO,HI,Tune_P>
00390   {
00391     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00392     return lhs | multivector_t(rhs);
00393   }
00394
00396   template< template<typename, const index_t, const index_t, typename> class Multivector,
00397             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00398   inline
00399   auto
00400   inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00401   { return val.inv(); }
00402
00404   template< template<typename, const index_t, const index_t, typename> class Multivector,
00405             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00406   auto
00407   pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00408   {
00409     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00410     if (lhs == Scalar_T(0))
00411     {
00412       using traits_t = numeric_traits<Scalar_T>;
00413       return
00414         (rhs < 0)
00415         ? traits_t::NaN()
00416         : (rhs == 0)
00417           ? Scalar_T(1)
00418           : Scalar_T(0);
00419     }
00420     auto result = multivector_t(Scalar_T(1));
00421     auto power =
00422       (rhs < 0)
00423       ? lhs.inv()
00424       : lhs;
00425     for (auto
00426         k = std::abs(rhs);
00427         k != 0;
00428         k /= 2)
00429     {
00430       if (k % 2)
00431         result *= power;
00432       power *= power;
00433     }
00434     return result;
00435   }
00436
00438   template
```

```
00439   <
00440     template<typename, const index_t, const index_t, typename> class Multivector,
00441     template<typename, const index_t, const index_t, typename> class RHS,
00442     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00443   >
00444   inline
00445   auto
00446   pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00447   {
00448     using traits_t = numeric_traits<Scalar_T>;
00449
00450     if (lhs == Scalar_T(0))
00451     {
00452       const Scalar_T m = rhs.scalar();
00453       if (rhs == m)
00454         return
00455           (m < 0)
00456             ? traits_t::NaN()
00457             : (m == 0)
00458               ? Scalar_T(1)
00459               : Scalar_T(0);
00460       else
00461         return Scalar_T(0);
00462     }
00463     return exp(log(lhs) * rhs);
00464   }
00465
00467   template< template<typename, const index_t, const index_t, typename> class Multivector,
00468             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00469   auto
00470   outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00471   { return lhs.outer_pow(rhs); }
00472
00474   template< template<typename, const index_t, const index_t, typename> class Multivector,
00475             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00476   inline
00477   auto
00478   scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00479   { return val.scalar(); }
00480
00482   template< template<typename, const index_t, const index_t, typename> class Multivector,
00483             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00484   inline
00485   auto
00486   real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00487   { return val.scalar(); }
00488
00490   template
00491   <
00492     template<typename, const index_t, const index_t, typename> class Multivector,
00493     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00494   >
00495   inline
00496   auto
00497   imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00498   { return Scalar_T(0); }
00499
00501   template< template<typename, const index_t, const index_t, typename> class Multivector,
00502             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00503   inline
00504   auto
00505   pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00506   { return val - val.scalar(); }
00507
00509   template< template<typename, const index_t, const index_t, typename> class Multivector,
00510             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00511   inline
00512   auto
00513   even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00514   { return val.even(); }
00515
00517   template< template<typename, const index_t, const index_t, typename> class Multivector,
00518             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00519   inline
00520   auto
00521   odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00522   { return val.odd(); }
00523
00525   template< template<typename, const index_t, const index_t, typename> class Multivector,
00526             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00527   inline
00528   auto
00529   vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00530   { return val.vector_part(); }
00531
```

```
00533   template< template<typename, const index_t, const index_t, typename> class Multivector,
00534             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00535   inline
00536   auto
00537   involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00538   { return val.involute(); }
00539
00541   template< template<typename, const index_t, const index_t, typename> class Multivector,
00542             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00543   inline
00544   auto
00545   reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00546   { return val.reverse(); }
00547
00549   template< template<typename, const index_t, const index_t, typename> class Multivector,
00550             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00551   inline
00552   auto
00553   conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00554   { return val.conj(); }
00555
00557   template< template<typename, const index_t, const index_t, typename> class Multivector,
00558             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00559   inline
00560   auto
00561   quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00562   { return val.quad(); }
00563
00565   template< template<typename, const index_t, const index_t, typename> class Multivector,
00566             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00567   inline
00568   auto
00569   norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00570   { return val.norm(); }
00571
00573   template< template<typename, const index_t, const index_t, typename> class Multivector,
00574             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00575   inline
00576   auto
00577   abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00578   { return numeric_traits<Scalar_T>::sqrt(val.norm()); }
00579
00581   template< template<typename, const index_t, const index_t, typename> class Multivector,
00582             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00583   inline
00584   auto
00585   max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00586   { return val.max_abs(); }
00587
00589   template< template<typename, const index_t, const index_t, typename> class Multivector,
00590             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00591   auto
00592   complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00593   {
00594     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00595     using traits_t = numeric_traits<Scalar_T>;
00596
00597     auto frm = val.frame();
00598     using array_t = std::array<index_t, 4>;
00599     auto incp = array_t{0, 2, 1, 0};
00600     auto incq = array_t{1, 0, 0, 0};
00601     auto bott = pos_mod((frm.count_pos() - frm.count_neg()), 4);
00602     for (auto
00603         k = index_t(0);
00604         k != incp[bott];
00605         k++)
00606       for (auto
00607           idx = index_t(1);
00608           idx != HI+1;
00609           ++idx)
00610         if (!frm[idx])
00611         {
00612           frm.set(idx);
00613           break;
00614         }
00615     for (auto
00616         k = index_t(0);
00617         k != incq[bott];
00618         k++)
00619       for (auto
00620           idx = index_t(-1);
00621           idx != LO-1;
00622           --idx)
00623         if (!frm[idx])
00624         {
00625           frm.set(idx);
```

```
00626           break;
00627         }
00628     auto new_bott = pos_mod(frm.count_pos() - frm.count_neg(), 4);
00629
00630     if ((incp[new_bott] == 0) && (incq[new_bott] == 0))
00631       return multivector_t(frm, Scalar_T(1));
00632     else
00633       // Return IEEE NaN or -Inf
00634       return traits_t::NaN();
00635   }
00636
00639   template< template<typename, const index_t, const index_t, typename> class Multivector,
00640             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00641   inline
00642   auto
00643   elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00644   { return complexifier(val); }
00645
00647   template< template<typename, const index_t, const index_t, typename> class Multivector,
00648             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00649   inline
00650   static
00651   void
00652   check_complex(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00653                 const Multivector<Scalar_T,LO,HI,Tune_P>& i, const bool prechecked = false)
00654   {
00655     if (!prechecked)
00656     {
00657       using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00658       using error_t = typename multivector_t::error_t;
00659
00660       const auto i_frame = i.frame();
00661       // We need i to be a complexifier whose frame is large enough to represent val
00662       if (complexifier(i) != i ||
00663          (val.frame() | i_frame) != i_frame ||
00664           complexifier(val).frame().count() > i_frame.count())
00665         throw error_t("check_complex(val, i): i is not a valid complexifier for val");
00666     }
00667   }
00668
00670   template< template<typename, const index_t, const index_t, typename> class Multivector,
00671             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00672   inline
00673   auto
00674   sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00675   { return sqrt(val, i, prechecked); }
00676
00678   template< template<typename, const index_t, const index_t, typename> class Multivector,
00679             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00680   inline
00681   auto
00682   sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00683   { return sqrt(val, complexifier(val), true); }
00684
00686   template< template<typename, const index_t, const index_t, typename> class Multivector,
00687             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00688   auto
00689   clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
      Multivector<Scalar_T,LO,HI,Tune_P>
00690   {
00691     // Scaling and squaring Pade' approximation of matrix exponential
00692     // Reference: [GL], Section 11.3, p572-576
00693     // Reference: [H]
00694
00695     using traits_t = numeric_traits<Scalar_T>;
00696
00697     const auto scalar_val = val.scalar();
00698     const auto scalar_exp = traits_t::exp(scalar_val);
00699     if (traits_t::isNaN_or_isInf(scalar_exp))
00700       return traits_t::NaN();
00701     if (val == scalar_val)
00702       return scalar_exp;
00703
00704     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00705     auto A = val - scalar_val;
00706     const auto pure_scale2 = A.norm();
00707
00708     if (traits_t::isNaN_or_isInf(pure_scale2))
00709       return traits_t::NaN();
00710     if (pure_scale2 == Scalar_T(0))
00711       return scalar_exp;
00712
00713     const auto ilog2_scale =
00714       std::max(0, traits_t::to_int(ceil((log2(pure_scale2) +
      Scalar_T(A.frame().count()))/Scalar_T(2))) - 3);
00715     const auto i_scale = traits_t::pow(Scalar_T(2), ilog2_scale);
```

```
00716     if (traits_t::isNaN_or_isInf(i_scale))
00717       return traits_t::NaN();
00718
00719     A /= i_scale;
00720     multivector_t pure_exp;
00721     {
00722       using limits_t = std::numeric_limits<Scalar_T>;
00723       const auto nbr_even_powers = 2*(limits_t::digits / 32) + 4;
00724       using nbr_t = decltype(nbr_even_powers);
00725
00726       // Create an array of coefficients
00727       const auto max_power = 2*nbr_even_powers + 1;
00728       static std::array<Scalar_T, max_power+1> c;
00729       if (c[0] != Scalar_T(1))
00730       {
00731         c[0] = Scalar_T(1);
00732         for (auto
00733             k = decltype(max_power)(0);
00734             k != max_power;
00735             ++k)
00736           c[k+1] = c[k]*(max_power-k) / ((2*max_power-k)*(k+1));
00737       }
00738
00739       // Create an array of even powers
00740       std::array<multivector_t, nbr_even_powers> AA;
00741       AA[0] = A * A;
00742       AA[1] = AA[0] * AA[0];
00743       for (auto
00744         k = nbr_t(2);
00745         k != nbr_even_powers;
00746         ++k)
00747         AA[k] = AA[k-2] * AA[1];
00748
00749       // Use compensated summation to calculate U and AV
00750       auto residual = multivector_t();
00751       auto U = multivector_t(c[0]);
00752       for (auto
00753           k = nbr_t(0);
00754           k != nbr_even_powers;
00755           ++k)
00756       {
00757         const auto& term = AA[k]*c[2*k + 2] - residual;
00758         const auto& sum = U + term;
00759         residual = (sum - U) - term;
00760         U = sum;
00761       }
00762       residual = multivector_t();
00763       auto AV = multivector_t(c[1]);
00764       for (auto
00765           k = nbr_t(0);
00766           k != nbr_even_powers;
00767           ++k)
00768       {
00769         const auto& term = AA[k]*c[2*k + 3] - residual;
00770         const auto& sum = AV + term;
00771         residual = (sum - AV) - term;
00772         AV = sum;
00773       }
00774       AV *= A;
00775       pure_exp = (U+AV) / (U-AV);
00776     }
00777     for (auto
00778         k = decltype(ilog2_scale)(0);
00779         k != ilog2_scale;
00780         ++k)
00781       pure_exp *= pure_exp;
00782     return pure_exp * scalar_exp;
00783   }
00784
00786   template< template<typename, const index_t, const index_t, typename> class Multivector,
00787             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788   inline
00789   auto
00790   log(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
      prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00791   { return log(val, i, prechecked); }
00792
00794   template< template<typename, const index_t, const index_t, typename> class Multivector,
00795             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796   inline
00797   auto
00798   log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00799   { return log(val, complexifier(val), true); }
00800
00802   template< template<typename, const index_t, const index_t, typename> class Multivector,
00803             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00804   inline
```

```
00805   auto
00806   cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00807   {
00808     using traits_t = numeric_traits<Scalar_T>;
00809     if (val.isnan())
00810       return traits_t::NaN();
00811
00812     const auto& s = val.scalar();
00813     if (val == s)
00814       return traits_t::cosh(s);
00815     return (exp(val)+exp(-val)) / Scalar_T(2);
00816   }
00817
00819   // Reference: [AS], Section 4.6, p86-89
00820   template< template<typename, const index_t, const index_t, typename> class Multivector,
00821             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00822   inline
00823   auto
00824   acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00825   {
00826     using traits_t = numeric_traits<Scalar_T>;
00827     check_complex(val, i, prechecked);
00828     if (val.isnan())
00829       return traits_t::NaN();
00830
00831     const auto radical = sqrt(val*val - Scalar_T(1), i, true);
00832     return (norm(val + radical) >= norm(val))
00833           ?  log(val + radical, i, true)
00834           : -log(val - radical, i, true);
00835   }
00836
00838   // Reference: [AS], Section 4.6, p86-89
00839   template< template<typename, const index_t, const index_t, typename> class Multivector,
00840             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00841   inline
00842   auto
00843   acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00844   { return acosh(val, complexifier(val), true); }
00845
00847   template< template<typename, const index_t, const index_t, typename> class Multivector,
00848             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00849   auto
00850   cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
      prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00851   {
00852     using traits_t = numeric_traits<Scalar_T>;
00853     if (val.isnan())
00854       return traits_t::NaN();
00855
00856     const auto& s = val.scalar();
00857     if (val == s)
00858       return traits_t::cos(s);
00859
00860     check_complex(val, i, prechecked);
00861
00862     static const auto& twopi = Scalar_T(2) * traits_t::pi();
00863     const auto& z = i *
00864       (val - s + traits_t::fmod(s, twopi));
00865     return (exp(z)+exp(-z)) / Scalar_T(2);
00866   }
00867
00869   template< template<typename, const index_t, const index_t, typename> class Multivector,
00870             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00871   inline
00872   auto
00873   cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00874   { return cos(val, complexifier(val), true); }
00875
00877   // Reference: [AS], Section 4.4, p79-83
00878   template< template<typename, const index_t, const index_t, typename> class Multivector,
00879             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00880   inline
00881   auto
00882   acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00883   {
00884     using traits_t = numeric_traits<Scalar_T>;
00885     if (val.isnan())
00886       return traits_t::NaN();
00887
00888     const auto& s = val.scalar();
00889     if (val == s && traits_t::abs(s) <= Scalar_T(1))
00890       return traits_t::acos(s);
00891
00892     check_complex(val, i, prechecked);
00893     return i * acosh(val, i, true);
```

```
00894   }
00895
00897   // Reference: [AS], Section 4.4, p79-83
00898   template< template<typename, const index_t, const index_t, typename> class Multivector,
00899             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00900   inline
00901   auto
00902   acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00903   { return acos(val, complexifier(val), true); }
00904
00906   template< template<typename, const index_t, const index_t, typename> class Multivector,
00907             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00908   inline
00909   auto
00910   sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00911   {
00912     using traits_t = numeric_traits<Scalar_T>;
00913     if (val.isnan())
00914       return traits_t::NaN();
00915
00916     const auto& s = val.scalar();
00917     if (val == s)
00918       return traits_t::sinh(s);
00919
00920     return (exp(val)-exp(-val)) / Scalar_T(2);
00921   }
00922
00924   // Reference: [AS], Section 4.6, p86-89
00925   template< template<typename, const index_t, const index_t, typename> class Multivector,
00926             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00927   inline
00928   auto
00929   asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00930   {
00931     using traits_t = numeric_traits<Scalar_T>;
00932     check_complex(val, i, prechecked);
00933     if (val.isnan())
00934       return traits_t::NaN();
00935
00936     const auto radical = sqrt(val*val + Scalar_T(1), i, true);
00937     return (norm(val + radical) >= norm(val))
00938           ?  log( val + radical, i, true)
00939           : -log(-val + radical, i, true);
00940   }
00941
00943   // Reference: [AS], Section 4.6, p86-89
00944   template< template<typename, const index_t, const index_t, typename> class Multivector,
00945             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00946   inline
00947   auto
00948   asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00949   { return asinh(val, complexifier(val), true); }
00950
00952   template< template<typename, const index_t, const index_t, typename> class Multivector,
00953             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954   auto
00955   sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
      prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00956   {
00957     using traits_t = numeric_traits<Scalar_T>;
00958     if (val.isnan())
00959       return traits_t::NaN();
00960
00961     const auto& s = val.scalar();
00962     if (val == s)
00963       return traits_t::sin(s);
00964
00965     check_complex(val, i, prechecked);
00966
00967     static const auto& twopi = Scalar_T(2) * traits_t::pi();
00968     const auto& z = i *
00969       (val - s + traits_t::fmod(s, twopi));
00970     return i * (exp(-z)-exp(z)) / Scalar_T(2);
00971   }
00972
00974   template< template<typename, const index_t, const index_t, typename> class Multivector,
00975             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976   inline
00977   auto
00978   sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00979   { return sin(val, complexifier(val), true); }
00980
00982   // Reference: [AS], Section 4.4, p79-83
00983   template< template<typename, const index_t, const index_t, typename> class Multivector,
00984             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00985   inline
```

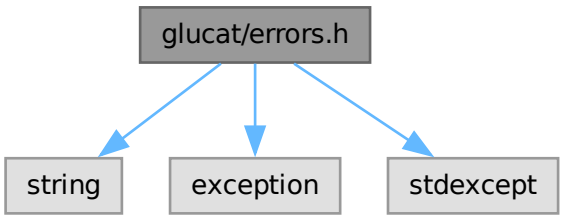
```
00986   auto
00987   asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00988   {
00989     using traits_t = numeric_traits<Scalar_T>;
00990     if (val.isnan())
00991       return traits_t::NaN();
00992
00993     const auto& s = val.scalar();
00994     if (val == s && traits_t::abs(s) <= Scalar_T(1))
00995       return traits_t::asin(s);
00996
00997     check_complex(val, i, prechecked);
00998     return -i * asinh(i * val, i, true);
00999   }
01000
01002   // Reference: [AS], Section 4.4, p79-83
01003   template< template<typename, const index_t, const index_t, typename> class Multivector,
01004             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01005   inline
01006   auto
01007   asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01008   { return asin(val, complexifier(val), true); }
01009
01011   template< template<typename, const index_t, const index_t, typename> class Multivector,
01012             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01013   inline
01014   auto
01015   tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01016   {
01017     using traits_t = numeric_traits<Scalar_T>;
01018     if (val.isnan())
01019       return traits_t::NaN();
01020
01021     const auto& s = val.scalar();
01022     if (val == s)
01023       return traits_t::tanh(s);
01024
01025     return sinh(val) / cosh(val);
01026   }
01027
01029   // Reference: [AS], Section 4.6, p86-89
01030   template< template<typename, const index_t, const index_t, typename> class Multivector,
01031             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01032   inline
01033   auto
01034   atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01035   {
01036     using traits_t = numeric_traits<Scalar_T>;
01037     check_complex(val, i, prechecked);
01038     return val.isnan()
01039         ? traits_t::NaN()
01040         : (norm(val + Scalar_T(1)) > norm(val - Scalar_T(1)))
01041           ? (log(val + Scalar_T(1), i, true) - log(-val + Scalar_T(1), i, true)) / Scalar_T(2)
01042           : log((val + Scalar_T(1)) / (-val + Scalar_T(1)), i, true) / Scalar_T(2);
01043   }
01044
01046   // Reference: [AS], Section 4.6, p86-89
01047   template< template<typename, const index_t, const index_t, typename> class Multivector,
01048             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01049   inline
01050   auto
01051   atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01052   { return atanh(val, complexifier(val), true); }
01053
01055   template< template<typename, const index_t, const index_t, typename> class Multivector,
01056             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01057   inline
01058   auto
01059   tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
      prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01060   {
01061     using traits_t = numeric_traits<Scalar_T>;
01062     if (val.isnan())
01063       return traits_t::NaN();
01064
01065     const auto& s = val.scalar();
01066     if (val == s)
01067       return traits_t::tan(s);
01068
01069     check_complex(val, i, prechecked);
01070     return sin(val, i, true) / cos(val, i, true);
01071   }
01072
01074   template< template<typename, const index_t, const index_t, typename> class Multivector,
01075             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
```

```
01076   inline
01077   auto
01078   tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01079   { return tan(val, complexifier(val), true); }
01080
01082   // Reference: [AS], Section 4.4, p79-83
01083   template< template<typename, const index_t, const index_t, typename> class Multivector,
01084             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01085   inline
01086   auto
01087   atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01088   {
01089     using traits_t = numeric_traits<Scalar_T>;
01090     if (val.isnan())
01091       return traits_t::NaN();
01092
01093     const auto& s = val.scalar();
01094     if (val == s)
01095       return traits_t::atan(s);
01096
01097     check_complex(val, i, prechecked);
01098     return -i * atanh(i * val, i, true);
01099   }
01100
01102   // Reference: [AS], Section 4.4, p79-83
01103   template< template<typename, const index_t, const index_t, typename> class Multivector,
01104             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01105   inline
01106   auto
01107   atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01108   { return atan(val, complexifier(val), true); }
01109
01110 }
01111 #endif  // _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
```

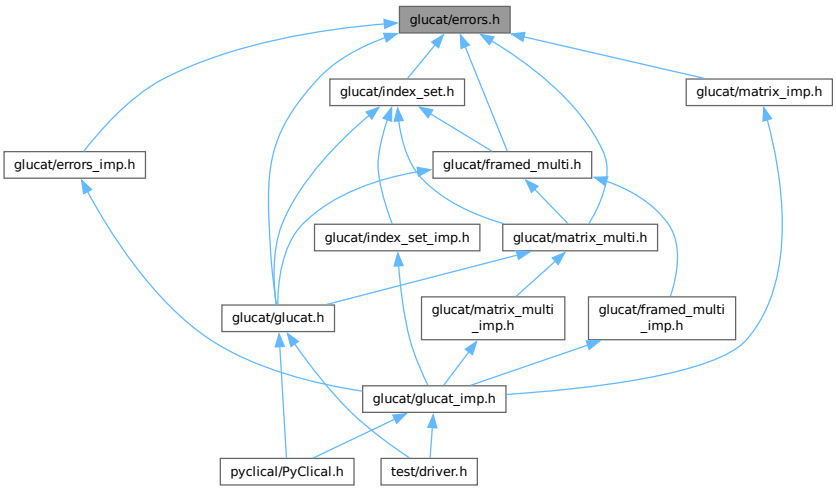
9.5 glucat/errors.h File Reference

```
#include <string>
#include <exception>
#include <stdexcept>
```

Include dependency graph for errors.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::glucat_error

  Abstract exception class.
- class glucat::error< Class_T >

  Specific exception class.

Namespaces

- namespace glucat

9.6 errors.h

Go to the documentation of this file.

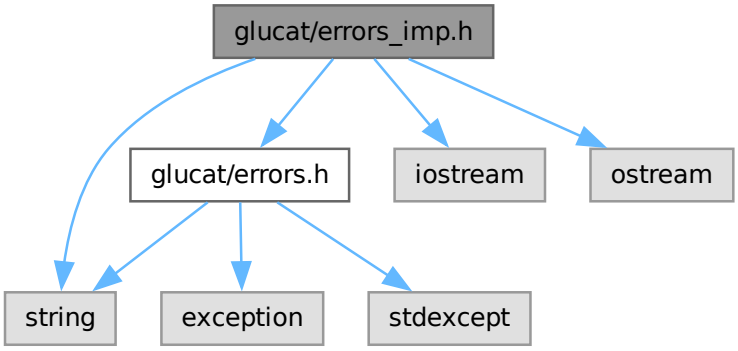
```
00001 #ifndef _GLUCAT_ERRORS_H
00002 #define _GLUCAT_ERRORS_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors.h : Declare error classes and functions
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
```

```
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ********************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ********************************************************************************/
00033
00034 #include <string>
00035 #include <exception>
00036 #include <stdexcept>
00037
00038 namespace glucat
00039 {
00041   class glucat_error : public std::logic_error
00042   {
00043   public:
00044     glucat_error(const std::string& context, const std::string& msg)
00045     : logic_error(msg), name(context)
00046     { }
00047     ~glucat_error() noexcept override = default;
00048     virtual auto heading() const noexcept -> const std::string =0;
00049     virtual auto classname() const noexcept -> const std::string =0;
00050     virtual void print_error_msg() const =0;
00051     std::string name;
00052   };
00053
00055   template< class Class_T >
00056   class error : public glucat_error
00057   {
00058   public:
00059     error(const std::string& msg);
00060     error(const std::string& context, const std::string& msg);
00061     auto heading() const noexcept -> const std::string override;
00062     auto classname() const noexcept -> const std::string override;
00063     void print_error_msg() const override;
00064   };
00065 }
00066 #endif // _GLUCAT_ERRORS_H
```

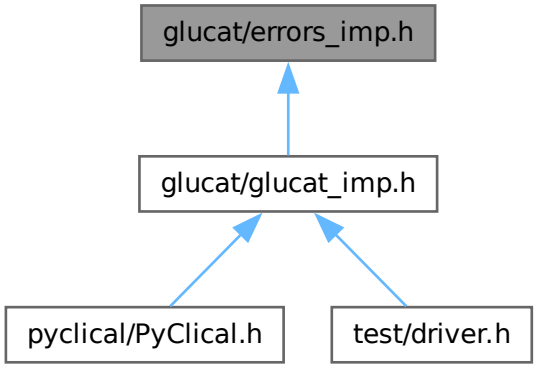
9.7 glucat/errors_imp.h File Reference

```
#include "glucat/errors.h"
#include <string>
#include <iostream>
#include <ostream>
```

Include dependency graph for errors_imp.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

9.8 errors_imp.h

Go to the documentation of this file.

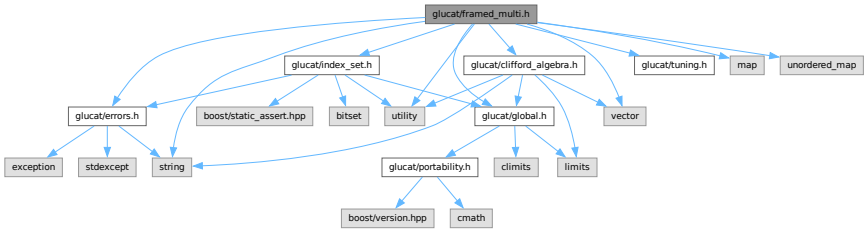
```
00001 #ifndef _GLUCAT_ERRORS_IMP_H
00002 #define _GLUCAT_ERRORS_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors_imp.h : Define error functions
00006                            -------------------
00007     begin                : Sun 2001-12-20
00008     copyright            : (C) 2001-2007 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/errors.h"
00035
00036 #include <string>
00037 #include <iostream>
00038 #include <ostream>
```

```
00039
00040 namespace glucat
00041 {
00043   template< class Class_T >
00044   error<Class_T>::
00045   error(const std::string& msg)
00046   : glucat_error(Class_T::classname(), msg)
00047   { }
00048
00049   template< class Class_T >
00050   error<Class_T>::
00051   error(const std::string& context, const std::string& msg)
00052   : glucat_error(context, msg)
00053   { }
00054
00055   template< class Class_T >
00056   auto
00057   error<Class_T>::
00058   heading() const noexcept -> const std::string
00059   { return "Error in glucat::"; }
00060
00061   template< class Class_T >
00062   auto
00063   error<Class_T>::
00064   classname() const noexcept -> const std::string
00065   { return name; }
00066
00067   template< class Class_T >
00068   void
00069   error<Class_T>::
00070   print_error_msg() const
00071   { std::cerr « heading() « classname() « std::endl « what() « std::endl; }
00072 }
00073 #endif // _GLUCAT_ERRORS_IMP_H
```

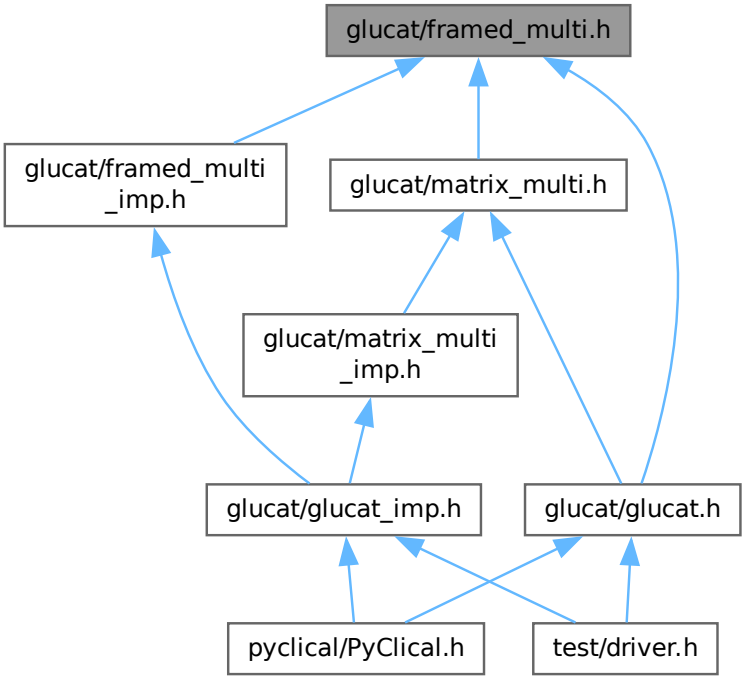
9.9 glucat/framed_multi.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include <string>
#include <utility>
#include <map>
#include <unordered_map>
#include <vector>
```

Include dependency graph for framed_multi.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::index_set_hash< LO, HI >
- class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >

  A framed_multi<Scalar_T,LO,HI,Tune_P> is a framed approximation to a multivector.
- class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t
- class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term

  Variable term.
- struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >

  Numeric limits for framed_multi inherit limits for the corresponding scalar type.

Namespaces

- namespace glucat
- namespace std

Functions

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term) -> std::ostream &

  Write term to output.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  static auto glucat::crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T

  Coordinate of product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >

  Product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.

9.10 framed_multi.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_FRAMED_MULTI_H
00002 #define _GLUCAT_FRAMED_MULTI_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi.h : Declare a class for the framed representation of a multivector
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039
00040 #if defined(_GLUCAT_USE_BOOST_POOL_ALLOC)
00041 // Use the Boost pool allocator
00042 #include <boost/pool/poolfwd.hpp>
00043 #endif
00044
00045 #include <string>
00046 #include <utility>
00047 #include <map>
00048 #include <unordered_map>
00049 #include <vector>
00050
00051 namespace glucat
00052 {
00053   // Forward declarations for friends
00054
00055   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00056   class framed_multi; // forward
00057
00058   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059   class matrix_multi; // forward
00060
00062   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063   auto
00064   operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00065
00067   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00068   auto
00069   operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00070
00072   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00073   auto
00074   operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00075
00077   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00078   auto
00079   operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00080
00082   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00083   auto
```

```
00084   star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
      -> Scalar_T;
00085
00087   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00088   auto
00089   operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00090
00092   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00093   auto
00094   operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00095
00097   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098   auto
00099   operator» (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00100
00102   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00103   auto
00104   operator« (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00105
00107   template< typename Scalar_T, const index_t LO, const index_t HI >
00108   auto
00109   operator« (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
      std::ostream&;
00110
00112   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113   auto
00114   exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00115
00116   template< const index_t LO, const index_t HI>
00117   class index_set_hash
00118   {
00119   public:
00120     using index_set_t = index_set<LO, HI>;
00121     inline auto operator()(index_set_t val) const -> size_t { return val.hash_fn(); }
00122   };
00123
00125   template< typename Scalar_T = double,  const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
      typename Tune_P = tuning<> >
00126   class framed_multi :
00127   public clifford_algebra< Scalar_T, index_set<LO,HI>, framed_multi<Scalar_T,LO,HI,Tune_P> >,
00128   private std::unordered_map< index_set<LO,HI>, Scalar_T, index_set_hash<LO,HI> >
00129   {
00130   public:
00131     using multivector_t = framed_multi;
00132     using framed_multi_t = multivector_t;
00133     using scalar_t = Scalar_T;
00134     using tune_p = Tune_P;
00135     using index_set_t = index_set<LO, HI>;
00136     using term_t = std::pair<const index_set_t, Scalar_T>;
00137     using vector_t = std::vector<Scalar_T>;
00138     using error_t = error<multivector_t>;
00139     using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >;
00140     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
      Other_Tune_P >
00141     friend class matrix_multi;
00142     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
      Other_Tune_P >
00143     friend class framed_multi;
00144
00145   private:
00146     class var_term; // forward
00147     using var_term_t = class var_term;
00148     using matrix_t = typename matrix_multi_t::matrix_t;
00149     using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >;
00150     using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI»;
00151
00152     class hash_size_t
00153     {
00154     public:
00155       hash_size_t(size_t hash_size)
00156       : n(hash_size)
00157       { };
00158       auto operator()() const -> size_t
00159       { return n; }
00160     private:
00161       size_t n;
00162     };
00163
00164     using framed_pair_t = std::pair<const multivector_t, const multivector_t>;
00165     using size_type = typename map_t::size_type;
00166     using iterator = typename map_t::iterator;
00167     using const_iterator = typename map_t::const_iterator;
00168
00169   public:
00171     static auto classname() -> const std::string;
```

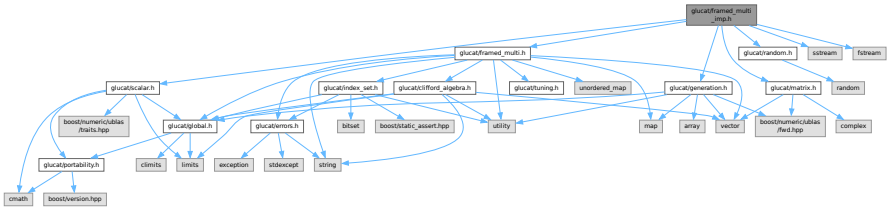
```
00173     ~framed_multi() override = default;
00175     framed_multi();
00176
00177   private:
00179     framed_multi(const hash_size_t& hash_size);
00180   public:
00182     template< typename Other_Scalar_T >
00183     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00185     template< typename Other_Scalar_T >
00186     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00187                  const index_set_t frm, const bool prechecked = false);
00189     framed_multi(const framed_multi_t& val,
00190                  const index_set_t frm, const bool prechecked = false);
00192     framed_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00194     framed_multi(const index_set_t ist, const Scalar_T& crd,
00195                  const index_set_t frm, const bool prechecked = false);
00197     framed_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00199     framed_multi(const int scr, const index_set_t frm = index_set_t());
00201     framed_multi(const vector_t& vec,
00202                  const index_set_t frm, const bool prechecked = false);
00204     framed_multi(const std::string& str);
00206     framed_multi(const std::string& str,
00207                  const index_set_t frm, const bool prechecked = false);
00209     framed_multi(const char* str)
00210     { *this = framed_multi(std::string(str)); };
00212     framed_multi(const char* str,
00213                  const index_set_t frm, const bool prechecked = false)
00214     { *this = framed_multi(std::string(str), frm, prechecked); };
00216     template< typename Other_Scalar_T >
00217     framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P >& val);
00219     template< typename Other_Scalar_T >
00220     auto fast_matrix_multi(const index_set_t frm) const -> const
      matrix_multi<Other_Scalar_T,LO,HI,Tune_P >;
00222     auto fast_framed_multi() const -> const framed_multi_t;
00223
00224     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00225
00227     auto nbr_terms() const -> unsigned long;
00228
00230     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const multivector_t;
00231
00232     // Friend declarations
00233
00234     friend auto
00235       operator* <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00236     friend auto
00237       operator^ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00238     friend auto
00239       operator& <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00240     friend auto
00241       operator% <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00242     friend auto
00243       star      <>(const multivector_t& lhs, const multivector_t& rhs) -> Scalar_T;
00244     friend auto
00245       operator/ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00246     friend auto
00247       operator| <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00248
00249     friend auto
00250       operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00251     friend auto
00252       operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00253     friend auto
00254       operator« <>(std::ostream& os, const term_t& term) -> std::ostream&;
00255
00256     friend auto
00257       exp <>(const multivector_t& val) -> const multivector_t;
00258
00260     auto      operator+= (const term_t& term) -> multivector_t&;
00261
00262   private:
00264     auto       fold(const index_set_t frm) const -> multivector_t;
00266     auto       unfold(const index_set_t frm) const -> multivector_t;
00268     auto      centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&;
00270     auto      centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&;
00272     auto      centre_qp1_pm1(index_t& p, index_t& q) -> multivector_t&;
00274     auto      divide(const index_set_t ist) const -> const framed_pair_t;
00276     auto      fast(const index_t level, const bool odd) const -> const matrix_t;
00277
00279     class var_term :
00280     public std::pair<index_set<LO,HI>, Scalar_T>
00281     {
00282     public:
00283       using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>;
00284
00286       static auto classname() -> const std::string
00287       { return "var_term"; };
```

```
00289       ~var_term() = default;
00291       var_term()
00292       : var_pair_t(index_set_t(), Scalar_T(1))
00293       { };
00295       var_term(const index_set_t ist, const Scalar_T& crd = Scalar_T(1))
00296       : var_pair_t(ist, crd)
00297       { };
00299       auto operator*= (const term_t& rhs) -> var_term_t&
00300       {
00301         this->second *= rhs.second * this->first.sign_of_mult(rhs.first);
00302         this->first  ^= rhs.first;
00303         return *this;
00304       }
00305     };
00306   };
00307
00308   // Non-members
00309
00311   template< typename Scalar_T, const index_t LO, const index_t HI >
00312   inline
00313   static
00314   auto
00315   crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00316               const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T;
00317
00319   template< typename Scalar_T, const index_t LO, const index_t HI >
00320   auto
00321   operator*
00322     (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00323      const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const index_set<LO,HI>,
      Scalar_T>;
00324
00326   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327   auto
00328   sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00329
00331   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00332   auto
00333   exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00334
00336   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00337   auto
00338   log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00339 }
00340
00341 namespace std
00342 {
00344   template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00345   struct numeric_limits< glucat::framed_multi<Scalar_T,LO,HI,Tune_P> > :
00346   public numeric_limits<Scalar_T>
00347   { };
00348 }
00349 #endif  // _GLUCAT_FRAMED_MULTI_H
```

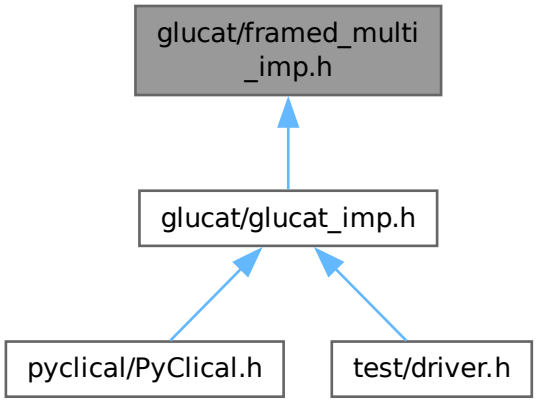
9.11 glucat/framed_multi_imp.h File Reference

```
#include "glucat/framed_multi.h"
#include "glucat/scalar.h"
#include "glucat/random.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <sstream>
#include <fstream>
```

Include dependency graph for framed_multi_imp.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::sorted_range< Map_T, Sorted_Map_T >

  Sorted range for use with output.
- class glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >

Namespaces

- namespace glucat

Macros

- #define _GLUCAT_HASH_N(x)
- #define _GLUCAT_HASH_SIZE_T(x)

Functions

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term) -> std::ostream &

  Write term to output.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  static auto glucat::crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T

  Coordinate of product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI>
  auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >

  Product of terms.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::exp (const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.

9.11.1 Macro Definition Documentation

9.11.1.1 _GLUCAT_HASH_N

```
#define _GLUCAT_HASH_N(
              x)
```

Value:

```
(x)
```

Definition at line 54 of file framed_multi_imp.h.

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_mult
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_mult
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_mult
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_mult
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_mult
and glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi().

9.11.1.2 _GLUCAT_HASH_SIZE_T

```
#define _GLUCAT_HASH_SIZE_T(
              x)
```

Value:

```
(typename multivector_t::hash_size_t)(x)
```

Definition at line 55 of file framed_multi_imp.h.

Referenced by glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi(), glucat::operator%(), glucat::operator&(), glucat::operator*(), and glucat::operator^().

9.12 framed_multi_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_FRAMED_MULTI_IMP_H
00002 #define _GLUCAT_FRAMED_MULTI_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi_imp.h : Implement the coordinate map representation of a
00006     Clifford algebra element
00007                              -------------------
00008     begin                : Sun 2001-12-09
00009     copyright            : (C) 2001-2021 by Paul C. Leopardi
00010  ***************************************************************************
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025  ***************************************************************************
00026  This library is based on a prototype written by Arvind Raja and was
00027  licensed under the LGPL with permission of the author. See Arvind Raja,
00028  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029  in Ablamowicz, Lounesto and Parra (eds.)
00030  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031  ***************************************************************************
00032      See also Arvind Raja's original header comments in glucat.h
00033  ***************************************************************************/
00034
00035 #include "glucat/framed_multi.h"
00036
00037 #include "glucat/scalar.h"
00038 #include "glucat/random.h"
00039 #include "glucat/generation.h"
00040 #include "glucat/matrix.h"
00041
00042 #include <sstream>
00043 #include <fstream>
00044
00045 namespace glucat
00046 {
00048   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00049   auto
00050   framed_multi<Scalar_T,LO,HI,Tune_P>::
00051   classname() -> const std::string
00052   { return "framed_multi"; }
00053
00054 #define _GLUCAT_HASH_N(x) (x)
00055 #define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)
00056
00058   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059   framed_multi<Scalar_T,LO,HI,Tune_P>::
00060   framed_multi()
00061   : map_t(_GLUCAT_HASH_N(0))
00062   { }
00063
00065   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00066   framed_multi<Scalar_T,LO,HI,Tune_P>::
00067   framed_multi(const hash_size_t& hash_size)
00068   : map_t(_GLUCAT_HASH_N(hash_size()))
00069   { }
00070
00072   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00073   template< typename Other_Scalar_T >
00074   framed_multi<Scalar_T,LO,HI,Tune_P>::
00075   framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00076   : map_t(_GLUCAT_HASH_N(val.size()))
00077   {
00078     for (auto& val_term : val)
00079       this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00080   }
00081
00083   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00084   template< typename Other_Scalar_T >
00085   framed_multi<Scalar_T,LO,HI,Tune_P>::
00086   framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00087                const index_set_t frm, const bool prechecked)
```

```
00088   : map_t(_GLUCAT_HASH_N(val.size()))
00089   {
00090     if (!prechecked && (val.frame() | frm) != frm)
00091       throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00092     for (auto& val_term : val)
00093       this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00094   }
00095
00097   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098   framed_multi<Scalar_T,LO,HI,Tune_P>::
00099   framed_multi(const multivector_t& val,
00100                const index_set_t frm, const bool prechecked)
00101   : map_t(_GLUCAT_HASH_N(val.size()))
00102   {
00103     if (!prechecked && (val.frame() | frm) != frm)
00104       throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00105     for (auto& val_term : val)
00106       this->insert(val_term);
00107   }
00108
00110   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111   framed_multi<Scalar_T,LO,HI,Tune_P>::
00112   framed_multi(const index_set_t ist, const Scalar_T& crd)
00113   : map_t(_GLUCAT_HASH_N(1))
00114   {
00115     if (crd != Scalar_T(0))
00116       this->insert(term_t(ist, crd));
00117   }
00118
00120   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00121   framed_multi<Scalar_T,LO,HI,Tune_P>::
00122   framed_multi(const index_set_t ist, const Scalar_T& crd,
00123                const index_set_t frm, const bool prechecked)
00124   : map_t(_GLUCAT_HASH_N(1))
00125   {
00126     if (!prechecked && (ist | frm) != frm)
00127       throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00128     if (crd != Scalar_T(0))
00129       this->insert(term_t(ist, crd));
00130   }
00131
00133   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00134   framed_multi<Scalar_T,LO,HI,Tune_P>::
00135   framed_multi(const Scalar_T& scr, const index_set_t frm)
00136   : map_t(_GLUCAT_HASH_N(1))
00137   {
00138     if (scr != Scalar_T(0))
00139       this->insert(term_t(index_set_t(), scr));
00140   }
00141
00143   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00144   framed_multi<Scalar_T,LO,HI,Tune_P>::
00145   framed_multi(const int scr, const index_set_t frm)
00146   : map_t(_GLUCAT_HASH_N(1))
00147   {
00148     if (scr != Scalar_T(0))
00149       this->insert(term_t(index_set_t(), Scalar_T(scr)));
00150   }
00151
00153   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00154   framed_multi<Scalar_T,LO,HI,Tune_P>::
00155   framed_multi(const vector_t& vec,
00156                const index_set_t frm, const bool prechecked)
00157   : map_t(_GLUCAT_HASH_N(vec.size()))
00158   {
00159     if (!prechecked && index_t(vec.size()) != frm.count())
00160       throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00161     auto idx = frm.min();
00162     const auto frm_end = frm.max()+1;
00163     for (auto& crd : vec)
00164     {
00165       *this += term_t(index_set_t(idx), crd);
00166       for (
00167         ++idx;
00168         idx != frm_end && !frm[idx];
00169         ++idx)
00170         ;
00171     }
00172   }
00173
00175   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00176   framed_multi<Scalar_T,LO,HI,Tune_P>::
00177   framed_multi(const std::string& str)
00178   : map_t(_GLUCAT_HASH_N(0))
00179   {
00180     std::istringstream ss(str);
00181     ss » *this;
```

```
00182     if (!ss)
00183       throw error_t("multivector_t(str): could not parse string");
00184     // Peek to see if the end of the string has been reached.
00185     ss.peek();
00186     if (!ss.eof())
00187       throw error_t("multivector_t(str): could not parse entire string");
00188   }
00189
00191   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00192   framed_multi<Scalar_T,LO,HI,Tune_P>::
00193   framed_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00194   : map_t(_GLUCAT_HASH_N(0))
00195   {
00196     if (prechecked)
00197       *this = multivector_t(str);
00198     else
00199       *this = multivector_t(multivector_t(str), frm, false);
00200   }
00201
00203   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00204   template< typename Other_Scalar_T >
00205   framed_multi<Scalar_T,LO,HI,Tune_P>::
00206   framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00207   : map_t(_GLUCAT_HASH_N(1))
00208   {
00209     if (val == Other_Scalar_T(0))
00210       return;
00211
00212     const auto dim = val.m_matrix.size1();
00213     using traits_t = numeric_traits<Scalar_T>;
00214     if (dim == 1)
00215     {
00216       this->insert(term_t(index_set_t(), traits_t::to_scalar_t(val.m_matrix(0, 0))));
00217       return;
00218     }
00219     if (dim >= Tune_P::inv_fast_dim_threshold)
00220       try
00221       {
00222         *this = (val.template fast_framed_multi<Scalar_T>()).truncated();
00223         return;
00224       }
00225       catch (const glucat_error& e)
00226       { }
00227
00228     const auto val_norm = traits_t::to_scalar_t(val.norm());
00229     if (traits_t::isNaN_or_isInf(val_norm))
00230     {
00231       *this = traits_t::NaN();
00232       return;
00233     }
00234     const auto frm = val.frame();
00235     const auto algebra_dim = set_value_t(1) << frm.count();
00236     auto result = multivector_t(
00237       _GLUCAT_HASH_SIZE_T(std::min<size_t>(algebra_dim, matrix::nnz(val.m_matrix))));
00238     for (auto
00239         stv = set_value_t(0);
00240         stv != algebra_dim;
00241         stv++)
00242     {
00243       const auto ist = index_set_t(stv, frm, true);
00244       const auto crd =
00245         traits_t::to_scalar_t(matrix::inner<Other_Scalar_T>(val.basis_element(ist), val.m_matrix));
00246       if (crd != Scalar_T(0))
00247         result.insert(term_t(ist, crd));
00248     }
00249     *this = result.truncated();
00250   }
00251
00253   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00254   auto
00255   framed_multi<Scalar_T,LO,HI,Tune_P>::
00256   operator==  (const multivector_t& rhs) const -> bool
00257   {
00258     if (this->size() != rhs.size())
00259       return false;
00260     const auto rhs_end = rhs.end();
00261     for (auto& this_term : *this)
00262     {
00263       const const_iterator& rhs_it = rhs.find(this_term.first);
00264       if (rhs_it == rhs_end || rhs_it->second != this_term.second)
00265         return false;
00266     }
00267     return true;
00268   }
00269
00271   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00272   inline
```

```
00273   auto
00274   framed_multi<Scalar_T,LO,HI,Tune_P>::
00275   operator==  (const Scalar_T& scr) const -> bool
00276   {
00277     switch (this->size())
00278     {
00279     case 0:
00280       return scr == Scalar_T(0);
00281     case 1:
00282       {
00283         const auto& this_it = this->begin();
00284         return this_it->first == index_set_t() && this_it->second == scr;
00285       }
00286     default:
00287       return false;
00288     }
00289   }
00290
00292   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00293   inline
00294   auto
00295   framed_multi<Scalar_T,LO,HI,Tune_P>::
00296   operator+= (const Scalar_T& scr) -> multivector_t&
00297   {
00298     *this += term_t(index_set_t(), scr);
00299     return *this;
00300   }
00301
00303   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00304   inline
00305   auto
00306   framed_multi<Scalar_T,LO,HI,Tune_P>::
00307   operator+= (const multivector_t& rhs) -> multivector_t&
00308   { // simply add terms
00309     for (auto& rhs_term : rhs)
00310       *this += rhs_term;
00311     return *this;
00312   }
00313
00315   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00316   inline
00317   auto
00318   framed_multi<Scalar_T,LO,HI,Tune_P>::
00319   operator-= (const Scalar_T& scr) -> multivector_t&
00320   {
00321     *this += term_t(index_set_t(), -scr);
00322     return *this;
00323   }
00324
00326   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327   inline
00328   auto
00329   framed_multi<Scalar_T,LO,HI,Tune_P>::
00330   operator-= (const multivector_t& rhs) -> multivector_t&
00331   {
00332     for (auto& rhs_term : rhs)
00333       *this += term_t(rhs_term.first, -(rhs_term.second));
00334     return *this;
00335   }
00336
00338   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00339   inline
00340   auto
00341   framed_multi<Scalar_T,LO,HI,Tune_P>::
00342   operator- () const -> const multivector_t
00343   { // multiply coordinates of all terms by -1
00344     auto result = *this;
00345     for (auto& result_term : result)
00346       result_term.second *= Scalar_T(-1);
00347     return result;
00348   }
00349
00351   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00352   auto
00353   framed_multi<Scalar_T,LO,HI,Tune_P>::
00354   operator*= (const Scalar_T& scr) -> multivector_t&
00355   { // multiply coordinates of all terms by scalar
00356     using traits_t = numeric_traits<Scalar_T>;
00357
00358     if (traits_t::isNaN_or_isInf(scr))
00359       return *this = traits_t::NaN();
00360     if (scr == Scalar_T(0))
00361       if (this->isnan())
00362         *this = traits_t::NaN();
00363       else
00364         this->clear();
00365     else
```

```
00366       for (auto& this_term : *this)
00367         this_term.second *= scr;
00368     return *this;
00369   }
00370
00372   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00373   auto
00374   operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00375   {
00376     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00377     using traits_t = numeric_traits<Scalar_T>;
00378
00379     if (lhs.isnan() || rhs.isnan())
00380       return traits_t::NaN();
00381
00382     const double lhs_size = lhs.size();
00383     const double rhs_size = rhs.size();
00384     const auto our_frame = lhs.frame() | rhs.frame();
00385     const auto frm_count = our_frame.count();
00386     const auto algebra_dim = set_value_t(1) « frm_count;
00387     const auto direct_mult = lhs_size * rhs_size <= double(algebra_dim);
00388     if (direct_mult)
00389     { // If we have a sparse multiply, store the result directly
00390       auto result = multivector_t(
00391         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00392       for (auto& lhs_term : lhs)
00393         for (auto& rhs_term : rhs)
00394           result += lhs_term * rhs_term;
00395       return result;
00396     }
00397     else
00398     { // Past a certain threshold, the matrix algorithm is fastest
00399       using matrix_multi_t = typename multivector_t::matrix_multi_t;
00400       return matrix_multi_t(lhs, our_frame, true) *
00401              matrix_multi_t(rhs, our_frame, true);
00402     }
00403   }
00404
00406   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00407   inline
00408   auto
00409   framed_multi<Scalar_T,LO,HI,Tune_P>::
00410   operator*= (const multivector_t& rhs) -> multivector_t&
00411   { return *this = *this * rhs; }
00412
00414   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00415   auto
00416   operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00417   { // Arvind Raja's original reference:
00418     // "old clical, outerproduct(p,q:pterm):pterm in file compmod.pas"
00419
00420     if (lhs.empty() || rhs.empty())
00421       return Scalar_T(0);
00422
00423     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00424     using index_set_t = typename multivector_t::index_set_t;
00425     using term_t = typename multivector_t::term_t;
00426
00427     const auto empty_set = index_set_t();
00428
00429     const double lhs_size = lhs.size();
00430     const double rhs_size = rhs.size();
00431     const auto lhs_frame = lhs.frame();
00432     const auto rhs_frame = rhs.frame();
00433     const auto our_frame = lhs_frame | rhs_frame;
00434     const auto algebra_dim = set_value_t(1) « our_frame.count();
00435     auto result = multivector_t(
00436       _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00437     const auto lhs_end = lhs.end();
00438     const auto rhs_end = rhs.end();
00439
00440     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00441     {
00442       for (auto
00443           result_stv = set_value_t(0);
00444           result_stv != algebra_dim;
00445           ++result_stv)
00446       {
00447         const auto result_ist = index_set_t(result_stv, our_frame, true);
00448         const auto lhs_result_frame = lhs_frame & result_ist;
00449         const auto lhs_result_dim = set_value_t(1) « lhs_result_frame.count();
00450         auto result_crd = Scalar_T(0);
00451         for (auto
00452             lhs_stv = set_value_t(0);
00453             lhs_stv != lhs_result_dim;
```

```
00454             ++lhs_stv)
00455         {
00456           const auto lhs_ist = index_set_t(lhs_stv, lhs_result_frame, true);
00457           const auto rhs_ist = result_ist ^ lhs_ist;
00458           if ((rhs_ist | rhs_frame) == rhs_frame)
00459           {
00460             const auto lhs_it = lhs.find(lhs_ist);
00461             if (lhs_it != lhs_end)
00462             {
00463               const auto rhs_it = rhs.find(rhs_ist);
00464               if (rhs_it != rhs_end)
00465                 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00466             }
00467           }
00468         }
00469         if (result_crd != Scalar_T(0))
00470           result.insert(term_t(result_ist, result_crd));
00471       }
00472       return result;
00473     }
00474     else
00475     {
00476       for (auto& lhs_term : lhs)
00477         for (auto& rhs_term : rhs)
00478           if ((lhs_term.first & rhs_term.first) == empty_set)
00479             result += lhs_term * rhs_term;
00480       return result;
00481     }
00482   }
00483
00485   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00486   inline
00487   auto
00488   framed_multi<Scalar_T,LO,HI,Tune_P>::
00489   operator^= (const multivector_t& rhs) -> multivector_t&
00490   { return *this = *this ^ rhs; }
00491
00493   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00494   auto
00495   operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00496   { // Arvind Raja's original reference:
00497     // "old clical, innerproduct(p,q:pterm):pterm in file compmod.pas"
00498
00499     if (lhs.empty() || rhs.empty())
00500       return Scalar_T(0);
00501
00502     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00503     using index_set_t = typename multivector_t::index_set_t;
00504     using term_t = typename multivector_t::term_t;
00505
00506     const auto lhs_end = lhs.end();
00507     const auto rhs_end = rhs.end();
00508     const double lhs_size = lhs.size();
00509     const double rhs_size = rhs.size();
00510
00511     const auto lhs_frame = lhs.frame();
00512     const auto rhs_frame = rhs.frame();
00513
00514     const auto our_frame = lhs_frame | rhs_frame;
00515     const auto algebra_dim = set_value_t(1) « our_frame.count();
00516     auto result = multivector_t(
00517       _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00518     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00519     {
00520       for (auto
00521           result_stv = set_value_t(0);
00522           result_stv != algebra_dim;
00523           ++result_stv)
00524       {
00525         const auto result_ist = index_set_t(result_stv, our_frame, true);
00526         const auto comp_frame = our_frame & ~result_ist;
00527         const auto comp_dim = set_value_t(1) « comp_frame.count();
00528         auto result_crd = Scalar_T(0);
00529         for (auto
00530             comp_stv = set_value_t(1);
00531             comp_stv != comp_dim;
00532             ++comp_stv)
00533         {
00534           const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00535           const auto our_ist = result_ist ^ comp_ist;
00536           if ((our_ist | lhs_frame) == lhs_frame)
00537           {
00538             const auto lhs_it = lhs.find(our_ist);
00539             if (lhs_it != lhs_end)
00540             {
00541               const auto rhs_it = rhs.find(comp_ist);
```

```
00542               if (rhs_it != rhs_end)
00543                 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00544             }
00545           }
00546           if (result_stv != 0)
00547           {
00548             if ((our_ist | rhs_frame) == rhs_frame)
00549             {
00550               const auto rhs_it = rhs.find(our_ist);
00551               if (rhs_it != rhs_end)
00552               {
00553                 const auto lhs_it = lhs.find(comp_ist);
00554                 if (lhs_it != lhs_end)
00555                   result_crd += crd_of_mult(*lhs_it, *rhs_it);
00556               }
00557             }
00558           }
00559         }
00560         if (result_crd != Scalar_T(0))
00561           result.insert(term_t(result_ist, result_crd));
00562       }
00563     }
00564     else
00565     {
00566       const auto empty_set = index_set_t();
00567       for (auto& lhs_term : lhs)
00568       {
00569         const auto lhs_ist = lhs_term.first;
00570         if (lhs_ist != empty_set)
00571           for (auto& rhs_term : rhs)
00572           {
00573             const auto rhs_ist = rhs_term.first;
00574             if (rhs_ist != empty_set)
00575             {
00576               const auto our_ist = lhs_ist | rhs_ist;
00577               if ((lhs_ist == our_ist) || (rhs_ist == our_ist))
00578                 result += lhs_term * rhs_term;
00579             }
00580           }
00581       }
00582     }
00583     return result;
00584   }
00585
00587   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00588   inline
00589   auto
00590   framed_multi<Scalar_T,LO,HI,Tune_P>::
00591   operator&= (const multivector_t& rhs) -> multivector_t&
00592   { return *this = *this & rhs; }
00593
00595   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00596   auto
00597   operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00598   {
00599     // Reference: Leo Dorst, "Honing geometric algebra for its use in the computer sciences",
00600     // in Geometric Computing with Clifford Algebras, ed. G. Sommer,
00601     // Springer 2001, Chapter 6, pp. 127-152.
00602     // http://staff.science.uva.nl/~leo/clifford/index.html
00603
00604     if (lhs.empty() || rhs.empty())
00605       return Scalar_T(0);
00606
00607     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00608     using index_set_t = typename multivector_t::index_set_t;
00609     using term_t = typename multivector_t::term_t;
00610
00611     const auto lhs_end   = lhs.end();
00612     const auto rhs_end   = rhs.end();
00613     const double lhs_size = lhs.size();
00614     const double rhs_size = rhs.size();
00615     const auto lhs_frame = lhs.frame();
00616     const auto rhs_frame = rhs.frame();
00617
00618     const auto our_frame = lhs_frame | rhs_frame;
00619     const auto algebra_dim = set_value_t(1) << our_frame.count();
00620     auto result = multivector_t(
00621       _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00622
00623     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00624     {
00625       for (auto
00626           result_stv = set_value_t(0);
00627           result_stv != algebra_dim;
00628           ++result_stv)
00629       {
```

```
00630         const auto result_ist = index_set_t(result_stv, our_frame, true);
00631         const auto comp_frame = lhs_frame & ~result_ist;
00632         const auto comp_dim = set_value_t(1) << comp_frame.count();
00633         auto result_crd = Scalar_T(0);
00634         for (auto
00635             comp_stv = set_value_t(0);
00636             comp_stv != comp_dim;
00637             ++comp_stv)
00638         {
00639           const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00640           const auto rhs_ist = result_ist ^ comp_ist;
00641           if ((rhs_ist | rhs_frame) == rhs_frame)
00642           {
00643             const auto rhs_it = rhs.find(rhs_ist);
00644             if (rhs_it != rhs_end)
00645             {
00646               const auto lhs_it = lhs.find(comp_ist);
00647               if (lhs_it != lhs_end)
00648                 result_crd += crd_of_mult(*lhs_it, *rhs_it);
00649             }
00650           }
00651         }
00652         if (result_crd != Scalar_T(0))
00653           result.insert(term_t(result_ist, result_crd));
00654       }
00655     }
00656     else
00657     {
00658       for (auto& rhs_term : rhs)
00659       {
00660         const auto rhs_ist = rhs_term.first;
00661         for (auto& lhs_term : lhs)
00662         {
00663           const index_set_t lhs_ist = lhs_term.first;
00664           if ((lhs_ist | rhs_ist) == rhs_ist)
00665             result += lhs_term * rhs_term;
00666         }
00667       }
00668     }
00669     return result;
00670   }
00671
00673   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00674   inline
00675   auto
00676   framed_multi<Scalar_T,LO,HI,Tune_P>::
00677   operator%= (const multivector_t& rhs) -> multivector_t&
00678   { return *this = *this % rhs; }
00679
00681   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00682   auto
00683   star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
      -> Scalar_T
00684   {
00685     auto result = Scalar_T(0);
00686     const auto small_star_large = lhs.size() < rhs.size();
00687     const auto* smallp =
00688       small_star_large
00689       ? &lhs
00690       : &rhs;
00691     const auto* largep =
00692       small_star_large
00693       ? &rhs
00694       : &lhs;
00695
00696     for (auto& small_term : *smallp)
00697     {
00698       const auto small_ist = small_term.first;
00699       const auto large_crd = (*largep)[small_ist];
00700       if (large_crd != Scalar_T(0))
00701         result += small_ist.sign_of_square() * small_term.second * large_crd;
00702     }
00703     return result;
00704   }
00705
00707   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00708   auto
00709   framed_multi<Scalar_T,LO,HI,Tune_P>::
00710   operator/= (const Scalar_T& scr) -> multivector_t&
00711   { // Divide coordinates of all terms by scr
00712     using traits_t = numeric_traits<Scalar_T>;
00713
00714     if (traits_t::isNaN(scr))
00715       return *this = traits_t::NaN();
00716     if (traits_t::isInf(scr))
00717       if (this->isnan())
00718         *this = traits_t::NaN();
```

```
00719       else
00720         this->clear();
00721     else
00722       for (auto& this_term : *this)
00723         this_term.second /= scr;
00724     return *this;
00725   }
00726
00728   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00729   inline
00730   auto
00731   operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00732   {
00733     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00734     using traits_t = numeric_traits<Scalar_T>;
00735     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00736
00737     if (rhs == Scalar_T(0))
00738       return traits_t::NaN();
00739
00740     const auto our_frame = lhs.frame() | rhs.frame();
00741     return matrix_multi_t(lhs, our_frame, true) / matrix_multi_t(rhs, our_frame, true);
00742   }
00743
00745   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00746   inline
00747   auto
00748   framed_multi<Scalar_T,LO,HI,Tune_P>::
00749   operator/= (const multivector_t& rhs) -> multivector_t&
00750   { return *this = *this / rhs; }
00751
00753   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00754   inline
00755   auto
00756   operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00757   {
00758     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00759     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00760
00761     return matrix_multi_t(rhs) * matrix_multi_t(lhs) / matrix_multi_t(rhs.involute());
00762   }
00763
00765   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00766   inline
00767   auto
00768   framed_multi<Scalar_T,LO,HI,Tune_P>::
00769   operator|= (const multivector_t& rhs) -> multivector_t&
00770   { return *this = *this | rhs; }
00771
00773   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00774   inline
00775   auto
00776   framed_multi<Scalar_T,LO,HI,Tune_P>::
00777   inv() const -> const multivector_t
00778   {
00779     auto result = matrix_multi_t(Scalar_T(1), this->frame());
00780     return result /= matrix_multi_t(*this);
00781   }
00782
00784   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00785   auto
00786   framed_multi<Scalar_T,LO,HI,Tune_P>::
00787   pow(int m) const -> const multivector_t
00788   { return glucat::pow(*this, m); }
00789
00791   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00792   auto
00793   framed_multi<Scalar_T,LO,HI,Tune_P>::
00794   outer_pow(int m) const -> const multivector_t
00795   {
00796     if (m < 0)
00797       throw error_t("outer_pow(int): negative exponent");
00798     auto result = multivector_t(Scalar_T(1));
00799     auto a = *this;
00800     for (;
00801         m != 0;
00802         m >>= 1, a = a ^ a)
00803       if (m & 1)
00804         result ^= a;
00805     return result;
00806   }
00807
00809   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00810   inline
00811   auto
```

```
00812   framed_multi<Scalar_T,LO,HI,Tune_P>::
00813   frame() const -> const index_set_t
00814   {
00815     auto result = index_set_t();
00816     for (auto& this_term : *this)
00817       result |= this_term.first;
00818     return result;
00819   }
00820
00822   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00823   inline
00824   auto
00825   framed_multi<Scalar_T,LO,HI,Tune_P>::
00826   grade() const -> index_t
00827   {
00828     auto result = index_t(0);
00829     for (auto& this_term : *this)
00830       result = std::max(result, this_term.first.count());
00831     return result;
00832   }
00833
00835   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00836   inline
00837   auto
00838   framed_multi<Scalar_T,LO,HI,Tune_P>::
00839   operator[] (const index_set_t ist) const -> Scalar_T
00840   {
00841     const auto& this_it = this->find(ist);
00842     if (this_it == this->end())
00843       return Scalar_T(0);
00844     else
00845       return this_it->second;
00846   }
00847
00849   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00850   auto
00851   framed_multi<Scalar_T,LO,HI,Tune_P>::
00852   operator() (index_t grade) const -> const multivector_t
00853   {
00854     if ((grade < 0) || (grade > HI-LO))
00855       return Scalar_T(0);
00856     else
00857     {
00858       auto result = multivector_t();
00859       for (auto& this_term : *this)
00860         if (this_term.first.count() == grade)
00861           result += this_term;
00862       return result;
00863     }
00864   }
00865
00867   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00868   inline
00869   auto
00870   framed_multi<Scalar_T,LO,HI,Tune_P>::
00871   scalar() const -> Scalar_T
00872   { return (*this)[index_set_t()]; }
00873
00875   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00876   inline
00877   auto
00878   framed_multi<Scalar_T,LO,HI,Tune_P>::
00879   pure() const -> const multivector_t
00880   { return *this - this->scalar(); }
00881
00883   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00884   auto
00885   framed_multi<Scalar_T,LO,HI,Tune_P>::
00886   even() const -> const multivector_t
00887   { // even part of x, sum of the pure(count) with even count
00888     auto result = multivector_t();
00889     for (auto& this_term : *this)
00890       if ((this_term.first.count() % 2) == 0)
00891         result.insert(this_term);
00892     return result;
00893   }
00894
00896   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00897   auto
00898   framed_multi<Scalar_T,LO,HI,Tune_P>::
00899   odd() const -> const multivector_t
00900   { // even part of x, sum of the pure(count) with even count
00901     auto result = multivector_t();
00902     for (auto& this_term : *this)
00903       if ((this_term.first.count() % 2) == 1)
00904         result.insert(this_term);
00905     return result;
```

```
00906   }
00907
00909   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00910   auto
00911   framed_multi<Scalar_T,LO,HI,Tune_P>::
00912   vector_part() const -> const vector_t
00913   { return this->vector_part(this->frame(), true); }
00914
00916   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00917   auto
00918   framed_multi<Scalar_T,LO,HI,Tune_P>::
00919   vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00920   {
00921     if (!prechecked && (this->frame() | frm) != frm)
00922       throw error_t("vector_part(frm): value is outside of requested frame");
00923     auto result = vector_t();
00924     result.reserve(frm.count());
00925     const auto frm_end = frm.max()+1;
00926     for (auto
00927         idx  = frm.min();
00928         idx != frm_end;
00929         ++idx)
00930       // Frame may contain indices which do not correspond to a grade 1 term but
00931       // frame cannot omit any index corresponding to a grade 1 term
00932       if (frm[idx])
00933         result.push_back((*this)[index_set_t(idx)]);
00934     return result;
00935   }
00936
00938   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00939   auto
00940   framed_multi<Scalar_T,LO,HI,Tune_P>::
00941   involute() const -> const multivector_t
00942   {
00943     auto result = *this;
00944     for (auto& result_term : result)
00945     { // for a k-vector u, involute(u) == (-1)^k * u
00946       if ((result_term.first.count() % 2) == 1)
00947         result_term.second *= Scalar_T(-1);
00948     }
00949     return result;
00950   }
00951
00953   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954   auto
00955   framed_multi<Scalar_T,LO,HI,Tune_P>::
00956   reverse() const -> const multivector_t
00957   {
00958     auto result = *this;
00959     for (auto& result_term : result)
00960       // For a k-vector u, reverse(u) = { -u, k == 2,3 (mod 4)
00961       //                                {  u, k == 0,1 (mod 4)
00962       switch (result_term.first.count() % 4)
00963       {
00964       case 2:
00965       case 3:
00966         result_term.second *= Scalar_T(-1);
00967         break;
00968       default:
00969         break;
00970       }
00971     return result;
00972   }
00973
00975   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976   auto
00977   framed_multi<Scalar_T,LO,HI,Tune_P>::
00978   conj() const -> const multivector_t
00979   {
00980     auto result = *this;
00981     for (auto& result_term : result)
00982       // For a k-vector u, conj(u) = { -u, k == 1,2 (mod 4)
00983       //                             {  u, k == 0,3 (mod 4)
00984       switch (result_term.first.count() % 4)
00985       {
00986       case 1:
00987       case 2:
00988         result_term.second *= Scalar_T(-1);
00989         break;
00990       default:
00991         break;
00992       }
00993     return result;
00994   }
00995
00997   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00998   auto
```

```
00999   framed_multi<Scalar_T,LO,HI,Tune_P>::
01000   quad() const -> Scalar_T
01001   {
01002     // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
01003     // ref: old clical: quadfunction(p:pter):pterm in file compmod.pas
01004     auto result = Scalar_T(0);
01005     for (auto& this_term : *this)
01006     {
01007       const auto sign =
01008         (this_term.first.count_neg() % 2)
01009         ? -Scalar_T(1)
01010         :  Scalar_T(1);
01011       result += sign * (this_term.second) * (this_term.second);
01012     }
01013     return result;
01014   }
01015
01017   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01018   auto
01019   framed_multi<Scalar_T,LO,HI,Tune_P>::
01020   norm() const -> Scalar_T
01021   {
01022     using traits_t = numeric_traits<Scalar_T>;
01023
01024     auto result = Scalar_T(0);
01025     for (auto& this_term : *this)
01026     {
01027       const auto abs_crd = traits_t::abs(this_term.second);
01028       result +=  abs_crd * abs_crd;
01029     }
01030     return result;
01031   }
01032
01034   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01035   auto
01036   framed_multi<Scalar_T,LO,HI,Tune_P>::
01037   max_abs() const -> Scalar_T
01038   {
01039     using traits_t = numeric_traits<Scalar_T>;
01040
01041     auto result = Scalar_T(0);
01042     for (auto& this_term : *this)
01043     {
01044       const auto abs_crd = traits_t::abs(this_term.second);
01045       if (abs_crd > result)
01046         result = abs_crd;
01047     }
01048     return result;
01049   }
01050
01052   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01053   auto
01054   framed_multi<Scalar_T,LO,HI,Tune_P>::
01055   random(const index_set_t frm, Scalar_T fill) -> const multivector_t
01056   {
01057     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01058     using index_set_t = typename multivector_t::index_set_t;
01059     using term_t = typename multivector_t::term_t;
01060
01061     using random_generator_t = random_generator<Scalar_T>;
01062     auto& generator = random_generator_t::generator();
01063
01064     fill =
01065       (fill < Scalar_T(0))
01066       ? Scalar_T(0)
01067       : (fill > Scalar_T(1))
01068         ? Scalar_T(1)
01069         : fill;
01070     const auto algebra_dim = set_value_t(1) « frm.count();
01071     using traits_t = numeric_traits<Scalar_T>;
01072     const auto mean_abs = traits_t::sqrt(Scalar_T(double(algebra_dim)));
01073     auto result = multivector_t();
01074     for (auto
01075         stv = set_value_t(0);
01076         stv != algebra_dim;
01077         ++stv)
01078       if (generator.uniform() < fill)
01079       {
01080         const auto& result_crd = generator.normal() / mean_abs;
01081         result.insert(term_t(index_set_t(stv, frm, true), result_crd));
01082       }
01083     return result;
01084   }
01085
01087   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01088   inline
01089   void
```

```
01090   framed_multi<Scalar_T,LO,HI,Tune_P>::
01091   write(const std::string& msg) const
01092   { std::cout « msg « std::endl « "  " « (*this) « std::endl; }
01093
01095   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01096   inline
01097   void
01098   framed_multi<Scalar_T,LO,HI,Tune_P>::
01099   write(std::ofstream& ofile, const std::string& msg) const
01100   {
01101     if (!ofile)
01102       throw error_t("write(ofile,msg): cannot write to output file");
01103     ofile « msg « std::endl « "  " « (*this) « std::endl;
01104   }
01105
01107   template< typename Map_T,typename Sorted_Map_T >
01108   class sorted_range
01109   {
01110   public:
01111     using map_t = Map_T;
01112     using sorted_map_t = Sorted_Map_T;
01113     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01114
01115     sorted_range (Sorted_Map_T &sorted_val, const Map_T& val)
01116     {
01117       for (auto& val_term : val)
01118         sorted_val.insert(val_term);
01119       sorted_begin = sorted_val.begin();
01120       sorted_end   = sorted_val.end();
01121     }
01122     sorted_iterator sorted_begin;
01123     sorted_iterator sorted_end;
01124   };
01125
01126   template< typename Sorted_Map_T >
01127   class sorted_range< Sorted_Map_T, Sorted_Map_T >
01128   {
01129   public:
01130     using map_t = Sorted_Map_T;
01131     using sorted_map_t = Sorted_Map_T;
01132     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01133
01134     sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T& val)
01135     : sorted_begin( val.begin() ),
01136       sorted_end( val.end() )
01137     { }
01138     sorted_iterator sorted_begin;
01139     sorted_iterator sorted_end;
01140   };
01141
01143   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01144   auto
01145   operator« (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
01146   {
01147     using limits_t = std::numeric_limits<Scalar_T>;
01148     if (val.empty())
01149       os « 0;
01150     else if (val.isnan())
01151       os « limits_t::quiet_NaN();
01152     else if (val.isinf())
01153     {
01154       const Scalar_T& inf = limits_t::infinity();
01155       os « (scalar(val) < 0.0 ? -inf : inf);
01156     }
01157     else
01158     {
01159       using traits_t = numeric_traits<Scalar_T>;
01160       using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01161       Scalar_T truncation;
01162       switch (os.flags() & std::ios::floatfield)
01163       {
01164         case std::ios_base::scientific:
01165           truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()) + 1);
01166           break;
01167         case std::ios_base::fixed:
01168           truncation = Scalar_T(1) / (traits_t::pow(Scalar_T(10), int(os.precision())) *
      val.max_abs());
01169           break;
01170         case std::ios_base::fixed | std::ios_base::scientific:
01171           truncation = multivector_t::default_truncation;
01172           break;
01173         default:
01174           truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()));
01175           break;
01176       }
01177       auto truncated_val = val.truncated(truncation);
01178       if (truncated_val.empty())
```

```
01179         os « 0;
01180       else
01181       {
01182         using map_t = typename multivector_t::map_t;
01183         using sorted_map_t = typename multivector_t::sorted_map_t;
01184         auto sorted_val = sorted_map_t();
01185         const auto sorted_val_range = sorted_range< map_t, sorted_map_t >(sorted_val, truncated_val);
01186         auto sorted_it = sorted_val_range.sorted_begin;
01187         os « *sorted_it;
01188         for (++sorted_it;
01189             sorted_it != sorted_val_range.sorted_end;
01190             ++sorted_it)
01191         {
01192           const Scalar_T& scr = sorted_it->second;
01193           if (scr >= 0.0)
01194             os « '+';
01195           os « *sorted_it;
01196         }
01197       }
01198     }
01199     return os;
01200   }
01201
01203   template< typename Scalar_T, const index_t LO, const index_t HI >
01204   auto
01205   operator« (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
      std::ostream&
01206   {
01207     const auto second_as_double = numeric_traits<Scalar_T>::to_double(term.second);
01208     const auto use_double =
01209       (os.precision() <= std::numeric_limits<double>::digits10) ||
01210       (term.second == Scalar_T(second_as_double));
01211     if (term.first.count() == 0)
01212       if (use_double)
01213         os « second_as_double;
01214       else
01215         os « term.second;
01216     else if (term.second == Scalar_T(-1))
01217     {
01218       os « '-';
01219       os « term.first;
01220     }
01221     else if (term.second != Scalar_T(1))
01222     {
01223       if (use_double)
01224       {
01225         auto tol = std::pow(10.0,-os.precision());
01226         if ( std::fabs(second_as_double + 1.0) < tol )
01227           os « '-';
01228         else if ( std::fabs(second_as_double - 1.0) >= tol )
01229           os « second_as_double;
01230       }
01231       else
01232         os « term.second;
01233       os « term.first;
01234     }
01235     else
01236       os « term.first;
01237     return os;
01238   }
01239
01241   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01242   auto
01243   operator» (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P> & val) -> std::istream&
01244   { // Input looks like 1.0-2.0{1,2}+3.2{3,4}.
01245     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01246     // Parsing variables.
01247     auto local_val = multivector_t();
01248     auto c = 0;
01249     // Parsing control variables.
01250     auto negative = false;
01251     auto expect_term = true;
01252     // The multivector may begin with '+' or '-'. Check for this.
01253     c = s.peek();
01254     if (s.good() && (c == int('+') || c == int('-')))
01255     { // A '-' here negates the following term.
01256       negative = (c == int('-'));
01257       // Consume the '+' or '-'.
01258       s.get();
01259     }
01260     while (s.good())
01261     { // Parse a term.
01262       // A term consists of an optional scalar, followed by an optional index set.
01263       // At least one of the two must be present.
01264       // Default coordinate is Scalar_T(1).
01265       auto coordinate = Scalar_T(1);
01266       // Default index set is empty.
```

```
01267       auto ist = index_set<LO,HI>();
01268       // First, check for an opening brace.
01269       c = s.peek();
01270       if (s.good())
01271       { // If the character is not an opening brace,
01272         // a coordinate value is expected here.
01273         if (c != int('{'))
01274         { // Try to read a coordinate value.
01275           double coordinate_as_double;
01276           s » coordinate_as_double;
01277           // Reading the coordinate may have resulted in an end of file condition.
01278           // This is not a failure.
01279           if (s)
01280             coordinate = Scalar_T(coordinate_as_double);
01281         }
01282       }
01283       else
01284       { // End of file here ends parsing while a term may still be expected.
01285         break;
01286       }
01287       // Coordinate is now Scalar_T(1) or a Scalar_T value.
01288       // Parse an optional index set.
01289       if (s.good())
01290       {
01291         c = s.peek();
01292         if (s.good() && c == int('{'))
01293         { // Try to read index set.
01294           s » ist;
01295         }
01296       }
01297       // Reading the term may have resulted in an end of file condition.
01298       // This is not a failure.
01299       if (s)
01300       {
01301         // Immediately after parsing a term, another term is not expected.
01302         expect_term = false;
01303         if (coordinate != Scalar_T(0))
01304         {
01305           // Add the term to the local multivector.
01306           coordinate =
01307             negative
01308             ? -coordinate
01309             :  coordinate;
01310           using term_t = typename multivector_t::term_t;
01311           local_val += term_t(ist, coordinate);
01312         }
01313       }
01314       // Check if anything follows the current term.
01315       if (s.good())
01316       {
01317         c = s.peek();
01318         if (s.good())
01319         { // Only '+' and '-' are valid here.
01320           if (c == int('+') || c == int('-'))
01321           { // A '-' here negates the following term.
01322             negative = (c == int('-'));
01323             // Consume the '+' or '-'.
01324             s.get();
01325             // Immediately after '+' or '-',
01326             // expect another term.
01327             expect_term = true;
01328           }
01329           else
01330           { // Any other character here is a not failure,
01331             // but still ends the parsing of the multivector.
01332             break;
01333           }
01334         }
01335       }
01336     }
01337     // If a term is still expected, this is a failure.
01338     if (expect_term)
01339       s.clear(std::istream::failbit);
01340     // End of file is not a failure.
01341     if (s)
01342     { // The multivector has been successfully parsed.
01343       val = local_val;
01344     }
01345     return s;
01346   }
01347
01349   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01350   auto
01351   framed_multi<Scalar_T,LO,HI,Tune_P>::
01352   nbr_terms () const -> unsigned long
01353   { return this->size(); }
01354
```

```
01356   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01357   inline
01358   auto
01359   framed_multi<Scalar_T,LO,HI,Tune_P>::
01360   operator+= (const term_t& term) -> multivector_t&
01361   { // Do not insert terms with 0 coordinate
01362     if (term.second != Scalar_T(0))
01363     {
01364       const auto& this_it = this->find(term.first);
01365       if (this_it == this->end())
01366         this->insert(term);
01367       else if (this_it->second + term.second == Scalar_T(0))
01368         // Erase term if resulting coordinate is 0
01369         this->erase(this_it);
01370       else
01371         this_it->second += term.second;
01372     }
01373     return *this;
01374   }
01375
01377   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01378   auto
01379   framed_multi<Scalar_T,LO,HI,Tune_P>::
01380   isinf() const -> bool
01381   {
01382     using traits_t = numeric_traits<Scalar_T>;
01383
01384     if (std::numeric_limits<Scalar_T>::has_infinity)
01385       for (auto& this_term : *this)
01386         if (traits_t::isInf(this_term.second))
01387           return true;
01388     return false;
01389   }
01390
01392   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01393   auto
01394   framed_multi<Scalar_T,LO,HI,Tune_P>::
01395   isnan() const -> bool
01396   {
01397     using traits_t = numeric_traits<Scalar_T>;
01398
01399     if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
01400       for (auto& this_term : *this)
01401         if (traits_t::isNaN(this_term.second))
01402           return true;
01403     return false;
01404   }
01405
01407   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01408   auto
01409   framed_multi<Scalar_T,LO,HI,Tune_P>::
01410   truncated(const Scalar_T& limit) const -> const multivector_t
01411   {
01412     using traits_t = numeric_traits<Scalar_T>;
01413
01414     if (this->isnan() || this->isinf())
01415       return *this;
01416     const auto truncation = traits_t::abs(limit);
01417     const auto top = max_abs();
01418     auto result = multivector_t();
01419     if (top != Scalar_T(0))
01420       for (auto& this_term : *this)
01421         if (traits_t::abs(this_term.second) > top * truncation)
01422           result.insert(this_term);
01423     return result;
01424   }
01425
01427   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01428   auto
01429   framed_multi<Scalar_T,LO,HI,Tune_P>::
01430   fold(const index_set_t frm) const -> multivector_t
01431   {
01432     if (frm.is_contiguous())
01433       return *this;
01434     else
01435     {
01436       auto result = multivector_t();
01437       for (auto& this_term : *this)
01438         result.insert(term_t(this_term.first.fold(frm), this_term.second));
01439       return result;
01440     }
01441   }
01442
01444   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01445   auto
01446   framed_multi<Scalar_T,LO,HI,Tune_P>::
01447   unfold(const index_set_t frm) const -> multivector_t
```

```
01448   {
01449     if (frm.is_contiguous())
01450       return *this;
01451     else
01452     {
01453       auto result = multivector_t();
01454       for (auto& this_term : *this)
01455         result.insert(term_t(this_term.first.unfold(frm), this_term.second));
01456       return result;
01457     }
01458   }
01459
01461   // Reference: [L] 16.4 Periodicity of 8, p216
01462   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01463   auto
01464   framed_multi<Scalar_T,LO,HI,Tune_P>::
01465   centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&
01466   {
01467     // We add 4 to q by subtracting 4 from p
01468     if (q+4 > -LO)
01469       throw error_t("centre_pm4_qp4(p,q): LO is too high to represent this value");
01470     if (this->frame().max() > p-4)
01471     {
01472       using index_pair_t = typename index_set_t::index_pair_t;
01473       const auto pm3210 = index_set_t(index_pair_t(p-3,p), true);
01474       const auto qm4321 = index_set_t(index_pair_t(-q-4,-q-1), true);
01475       const auto& tqm4321 = term_t(qm4321, Scalar_T(1));
01476       auto result = multivector_t();
01477       for (auto& this_term : *this)
01478       {
01479         const auto ist = this_term.first;
01480         if (ist.max() > p-4)
01481         {
01482           auto var_term = var_term_t();
01483           for (auto
01484               n = index_t(0);
01485               n != index_t(4);
01486               ++n)
01487             if (ist[n+p-3])
01488               var_term *= term_t(index_set_t(n-q-4), Scalar_T(1)) * tqm4321;
01489           // Mask out {p-3}..{p}
01490           result.insert(term_t(ist & ~pm3210, this_term.second) *
01491                         term_t(var_term.first, var_term.second));
01492         }
01493         else
01494           result.insert(this_term);
01495       }
01496       *this = result;
01497     }
01498     p -=4; q += 4;
01499     return *this;
01500   }
01501
01503   // Reference: [L] 16.4 Periodicity of 8, p216
01504   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01505   auto
01506   framed_multi<Scalar_T,LO,HI,Tune_P>::
01507   centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&
01508   {
01509     // We add 4 to p by subtracting 4 from q
01510     if (p+4 > HI)
01511       throw error_t("centre_pp4_qm4(p,q): HI is too low to represent this value");
01512     if (this->frame().min() < -q+4)
01513     {
01514       using index_pair_t = typename index_set_t::index_pair_t;
01515       const auto qp0123 = index_set_t(index_pair_t(-q,-q+3), true);
01516       const auto pp1234 = index_set_t(index_pair_t(p+1,p+4), true);
01517       const auto& tpp1234 = term_t(pp1234, Scalar_T(1));
01518       auto result = multivector_t();
01519       for (auto& this_term : *this)
01520       {
01521         index_set_t ist = this_term.first;
01522         if (ist.min() < -q+4)
01523         {
01524           auto var_term = var_term_t();
01525           for (auto
01526               n = index_t(0);
01527               n != index_t(4);
01528               ++n)
01529             if (ist[n-q])
01530               var_term *= term_t(index_set_t(n+p+1), Scalar_T(1)) * tpp1234;
01531           // Mask out {-q}..{-q+3}
01532           result.insert(term_t(var_term.first, var_term.second) *
01533                         term_t(ist & ~qp0123, this_term.second));
01534         }
01535         else
01536           result.insert(this_term);
```

```
01537       }
01538       *this = result;
01539     }
01540     p +=4; q -= 4;
01541     return *this;
01542   }
01543
01545   // Reference: [P] Proposition 15.20, p 131
01546   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01547   auto
01548   framed_multi<Scalar_T,LO,HI,Tune_P>::
01549   centre_qp1_pm1(index_t& p, index_t& q) -> multivector_t&
01550   {
01551     if (q+1 > HI)
01552       throw error_t("centre_qp1_pm1(p,q): HI is too low to represent this value");
01553     if (p-1 > -LO)
01554       throw error_t("centre_qp1_pm1(p,q): LO is too high to represent this value");
01555     const auto qp1 = index_set_t(q+1);
01556     const auto& tqp1 = term_t(qp1, Scalar_T(1));
01557     auto result = multivector_t();
01558     for (auto& this_term : *this)
01559     {
01560       const auto ist = this_term.first;
01561       auto var_term = var_term_t(index_set_t(), this_term.second);
01562       for (auto
01563           n = -q;
01564           n != p;
01565           ++n)
01566         if (n != 0 && ist[n])
01567           var_term *= term_t(index_set_t(-n) | qp1, Scalar_T(1));
01568       if (p != 0 && ist[p])
01569         var_term *= tqp1;
01570       result.insert(term_t(var_term.first, var_term.second));
01571     }
01572     index_t orig_p = p;
01573     p = q+1;
01574     q = orig_p-1;
01575     return *this = result;
01576   }
01577
01579   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01580   auto
01581   framed_multi<Scalar_T,LO,HI,Tune_P>::
01582   divide(const index_set_t ist) const -> const framed_pair_t
01583   {
01584     auto quo = multivector_t();
01585     auto rem = multivector_t();
01586     for (auto& this_term : *this)
01587       if ((this_term.first | ist) == this_term.first)
01588         quo.insert(term_t(this_term.first ^ ist, this_term.second));
01589       else
01590         rem.insert(this_term);
01591     return framed_pair_t(quo, rem);
01592   }
01593
01595   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01596   auto
01597   framed_multi<Scalar_T,LO,HI,Tune_P>::
01598   fast(const index_t level, const bool odd) const -> const matrix_t
01599   {
01600     // Assume val is already folded and centred
01601     if (this->empty())
01602     {
01603       using matrix_index_t = typename matrix_multi_t::matrix_index_t;
01604       const auto dim = matrix_index_t(1) << level;
01605       auto result =matrix_t(dim, dim);
01606       result.clear();
01607       return result;
01608     }
01609     if (level == 0)
01610       return matrix::unit<matrix_t>(1) * this->scalar();
01611
01612     using basis_matrix_t = typename matrix_multi_t::basis_matrix_t;
01613     using basis_scalar_t = typename basis_matrix_t::value_type;
01614
01615     const auto&  I = matrix::unit<basis_matrix_t>(2);
01616     auto J = basis_matrix_t(2,2,2);
01617     J.clear();
01618     J(0,1)  = basis_scalar_t(-1);
01619     J(1,0)  = basis_scalar_t( 1);
01620     auto K = J;
01621     K(0,1)  = basis_scalar_t( 1);
01622     auto JK = I;
01623     JK(0,0) = basis_scalar_t(-1);
01624
01625     const auto ist_mn = index_set_t(-level);
01626     const auto ist_pn = index_set_t(level);
```

```
01627     if (level == 1)
01628     {
01629       if (odd)
01630         return matrix_t(J) * (*this)[ist_mn] + matrix_t(K)  * (*this)[ist_pn];
01631       else
01632         return matrix_t(I) * this->scalar()  + matrix_t(JK) * (*this)[ist_mn ^ ist_pn];
01633     }
01634     else
01635     {
01636       const auto& pair_mn = this->divide(ist_mn);
01637       const auto& quo_mn = pair_mn.first;
01638       const auto& rem_mn = pair_mn.second;
01639       const auto& pair_quo_mnpn = quo_mn.divide(ist_pn);
01640       const auto& val_mnpn = pair_quo_mnpn.first;
01641       const auto& val_mn   = pair_quo_mnpn.second;
01642       const auto& pair_rem_mnpn = rem_mn.divide(ist_pn);
01643       const auto& val_pn   = pair_rem_mnpn.first;
01644       const auto& val_1    = pair_rem_mnpn.second;
01645       using matrix::kron;
01646       if (odd)
01647         return - kron(JK, val_1.fast   (level-1, 1))
01648                + kron(I,  val_mnpn.fast(level-1, 1))
01649                + kron(J,  val_mn.fast  (level-1, 0))
01650                + kron(K,  val_pn.fast  (level-1, 0));
01651       else
01652         return   kron(I,  val_1.fast   (level-1, 0))
01653                + kron(JK, val_mnpn.fast(level-1, 0))
01654                + kron(K,  val_mn.fast  (level-1, 1))
01655                - kron(J,  val_pn.fast  (level-1, 1));
01656     }
01657   }
01658
01660   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01661   template< typename Other_Scalar_T >
01662   auto
01663   framed_multi<Scalar_T,LO,HI,Tune_P>::
01664   fast_matrix_multi(const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>
01665   {
01666     // Fold val
01667     auto val = this->fold(frm);
01668     auto p = frm.count_pos();
01669     auto q = frm.count_neg();
01670     const auto bott_offset = gen::offset_to_super[pos_mod(p - q, 8)];
01671     p += std::max(bott_offset,index_t(0));
01672     q -= std::min(bott_offset,index_t(0));
01673     if (p > HI)
01674       throw error_t("fast_matrix_multi(frm): HI is too low to represent this value");
01675     if (q > -LO)
01676       throw error_t("fast_matrix_multi(frm): LO is too high to represent this value");
01677     // Centre val
01678     while (p - q > 4)
01679       val.centre_pm4_qp4(p, q);
01680     while (p - q < -3)
01681       val.centre_pp4_qm4(p, q);
01682     if (p - q > 1)
01683       val.centre_qp1_pm1(p, q);
01684     const index_t level = (p + q)/2;
01685
01686     // Do the fast transform
01687     const auto& ev_val = val.even();
01688     const auto& od_val = val.odd();
01689     return matrix_multi<Other_Scalar_T,LO,HI,Tune_P>(ev_val.fast(level, 0) + od_val.fast(level, 1),
      frm);
01690   }
01691
01692   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01693   inline
01694   auto
01695   framed_multi<Scalar_T,LO,HI,Tune_P>::
01696   fast_framed_multi() const -> const multivector_t
01697   { return *this; }
01698
01700   template< typename Scalar_T, const index_t LO, const index_t HI >
01701   inline
01702   static
01703   auto
01704   crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01705               const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T
01706   { return lhs.first.sign_of_mult(rhs.first) * lhs.second * rhs.second; }
01707
01709   template< typename Scalar_T, const index_t LO, const index_t HI >
01710   inline
01711   auto
01712   operator* (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01713              const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const
      index_set<LO,HI>, Scalar_T>
01714   {
```

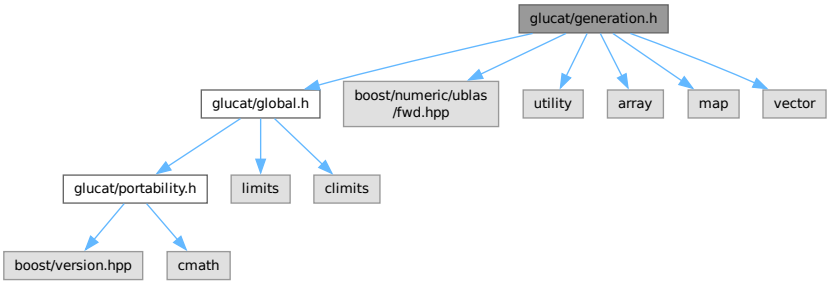
```
01715     using term_t = std::pair<const index_set<LO,HI>, Scalar_T>;
01716     return term_t(lhs.first ^ rhs.first, crd_of_mult(lhs, rhs));
01717   }
01718
01720   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01721   auto
01722   sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01723   {
01724     using traits_t = numeric_traits<Scalar_T>;
01725     if (val.isnan())
01726       return traits_t::NaN();
01727
01728     check_complex(val, i, prechecked);
01729
01730     const auto realval = val.scalar();
01731     if (val == realval)
01732     {
01733       if (realval < Scalar_T(0))
01734         return i * traits_t::sqrt(-realval);
01735       else
01736         return traits_t::sqrt(realval);
01737     }
01738     using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01739     return sqrt(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01740   }
01741
01743   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01744   auto
01745   exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01746   {
01747     using traits_t = numeric_traits<Scalar_T>;
01748     if (val.isnan())
01749       return traits_t::NaN();
01750
01751     const auto s = scalar(val);
01752     if (val == s)
01753       return traits_t::exp(s);
01754
01755     const double size = val.size();
01756     const auto frm_count = val.frame().count();
01757     const auto algebra_dim = set_value_t(1) « frm_count;
01758
01759     if( (size * size <= double(algebra_dim)) || (frm_count < Tune_P::mult_matrix_threshold))
01760     {
01761       switch (Tune_P::function_precision)
01762       {
01763       case precision_demoted:
01764         {
01765           using demoted_scalar_t = typename traits_t::demoted::type;
01766           using demoted_multivector_t = framed_multi<demoted_scalar_t,LO,HI,Tune_P>;
01767
01768           const auto& demoted_val = demoted_multivector_t(val);
01769           return clifford_exp(demoted_val);
01770         }
01771         break;
01772       case precision_promoted:
01773         {
01774           using promoted_scalar_t = typename traits_t::promoted::type;
01775           using promoted_multivector_t = framed_multi<promoted_scalar_t,LO,HI,Tune_P>;
01776
01777           const auto& promoted_val = promoted_multivector_t(val);
01778           return clifford_exp(promoted_val);
01779         }
01780         break;
01781       default:
01782         return clifford_exp(val);
01783       }
01784     }
01785     else
01786     {
01787       using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01788       return exp(matrix_multi_t(val));
01789     }
01790   }
01791
01793   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01794   auto
01795   log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01796   {
01797     using traits_t = numeric_traits<Scalar_T>;
01798     if (val == Scalar_T(0) || val.isnan())
01799       return traits_t::NaN();
01800
01801     check_complex(val, i, prechecked);
01802
```

```
01803     const auto realval = val.scalar();
01804     if (val == realval)
01805     {
01806       if (realval < Scalar_T(0))
01807         return i * traits_t::pi() + traits_t::log(-realval);
01808       else
01809         return traits_t::log(realval);
01810     }
01811     using matrix_multi_t = typename framed_multi<Scalar_T,LO,HI,Tune_P>::matrix_multi_t;
01812     return log(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01813   }
01814 }
01815 #endif  // _GLUCAT_FRAMED_MULTI_IMP_H
```

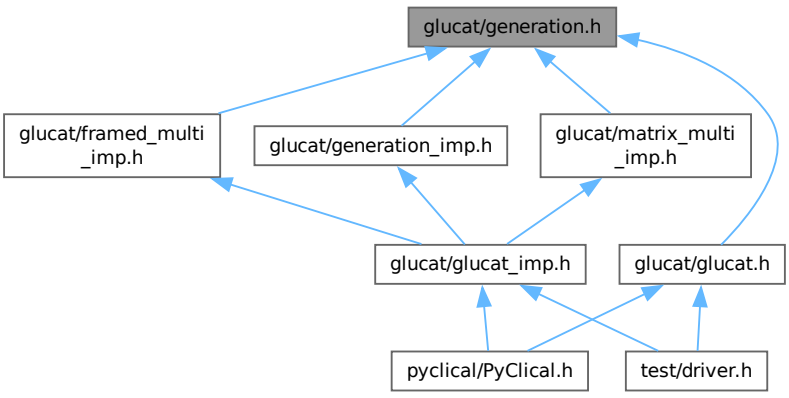
9.13 glucat/generation.h File Reference

```
#include "glucat/global.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <utility>
#include <array>
#include <map>
#include <vector>
```

Include dependency graph for generation.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::gen::generator_table< Matrix_T >

  Table of generators for specific signatures.

Namespaces

- namespace glucat
- namespace glucat::gen

Typedefs

- using glucat::gen::signature_t = std::pair<index_t, index_t>

  A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min().

Variables

- static const std::array< index_t, 8 > glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}

  Offsets between the current signature and that of the real superalgebra.

9.14 generation.h

Go to the documentation of this file.

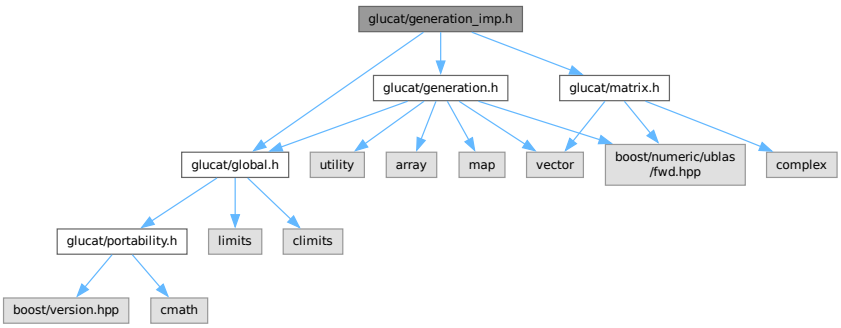
```
00001 #ifndef _GLUCAT_GENERATION_H
00002 #define _GLUCAT_GENERATION_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation.h : Declare functions for generation of the matrix representation
00006                            -------------------
00007     begin                : Wed Jan 23 2002
00008     copyright            : (C) 2002-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035
00036 #include <boost/numeric/ublas/fwd.hpp>
00037
00038 #include <utility>
00039 #include <array>
00040 #include <map>
00041 #include <vector>
00042
```

```
00043 namespace glucat { namespace gen
00044 {
00045   namespace ublas = boost::numeric::ublas;
00046
00048   using signature_t = std::pair<index_t, index_t>;
00049
00051   template< class Matrix_T >
00052   class generator_table :
00053   private std::map< signature_t, std::vector<Matrix_T> >
00054   {
00055   public:
00057     auto operator() (const index_t p, const index_t q) -> const Matrix_T*;
00059     static auto generator() -> generator_table<Matrix_T>&;
00060   private:
00062     auto gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&;
00064     void gen_from_pm1_qm1(const std::vector<Matrix_T>& old, const signature_t sig);
00066     void gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig);
00068     void gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig);
00070     void gen_from_qp1_pm1(const std::vector<Matrix_T>& old, const signature_t sig);
00071
00075     friend class friend_for_private_destructor;
00076     // Enforce singleton
00077     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00078     generator_table() = default;
00079     ~generator_table() = default;
00080   public:
00081     generator_table(const generator_table&) = delete;
00082     auto operator= (const generator_table&) -> generator_table& = delete;
00083   };
00084
00086   static const std::array<index_t, 8> offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1};
00087
00088 } }
00089 #endif  // _GLUCAT_GENERATION_H
```

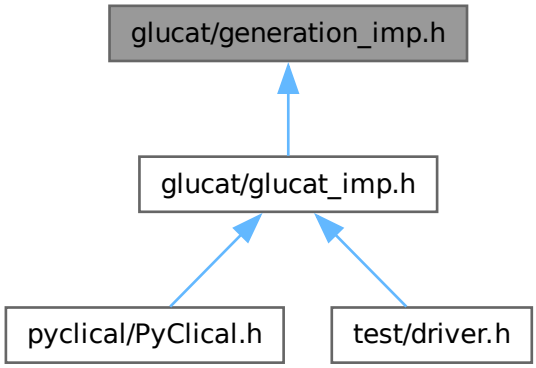
9.15 glucat/generation_imp.h File Reference

```
#include "glucat/global.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
```

Include dependency graph for generation_imp.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat
- namespace glucat::gen

9.16 generation_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_GENERATION_IMP_H
00002 #define _GLUCAT_GENERATION_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation_imp.h : Implement functions for generation of the matrix representation
00006                        -------------------
00007     begin                : Wed Jan 23 2002
00008     copyright            : (C) 2002-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/generation.h"
00036 #include "glucat/matrix.h"
```

```
00037
00038 namespace glucat { namespace gen
00039 {
00040   // References for algorithms:
00041   // [M]: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.
00042   // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00043   // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00044
00046   // Reference: [M] Item 47
00047   template< class Matrix_T >
00048   auto
00049   generator_table<Matrix_T>::
00050   generator() -> generator_table<Matrix_T>&
00051   { static generator_table<Matrix_T> g; return g;}
00052
00054   // Reference: [P] Table 15.27, p 133
00055   template< class Matrix_T >
00056   inline
00057   auto
00058   generator_table<Matrix_T>::
00059   operator() (const index_t p, const index_t q) -> const Matrix_T*
00060   {
00061     const auto bott = pos_mod(p-q, 8);
00062     switch(bott)
00063     {
00064     case 0:
00065     case 2:
00066       // Construct generators
00067       return &(gen_vector(p, q)[q]);
00068     default:
00069       // Select generators from the vector for a larger frame
00070       const auto super_p = p + std::max(offset_to_super[bott],index_t(0));
00071       const auto super_q = q - std::min(offset_to_super[bott],index_t(0));
00072       return &(gen_vector(super_p, super_q)[super_q]);
00073     }
00074   }
00075
00077   template< class Matrix_T >
00078   auto
00079   generator_table<Matrix_T>::
00080   gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&
00081   {
00082     using result_t = std::vector<Matrix_T>;
00083     const auto card = p + q;
00084     const auto bias = p - q;
00085     const auto bott = pos_mod(bias, 8);
00086     const auto sig = signature_t(p, q);
00087     if (this->find(sig) == this->end())
00088       switch(bott)
00089       {
00090       case 0:
00091         if (bias < 0)
00092           // Construct generators for p,q given generators for p+4,q-4
00093           gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00094         else if (bias > 0)
00095           // Construct generators for p,q given generators for p-4,q+4
00096           gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00097         else if (card == 0)
00098         { // Base case. Save a generator vector containing one matrix, size 1.
00099           auto result = result_t(1, matrix::unit<Matrix_T>(1));
00100           this->insert(make_pair(sig, result));
00101         }
00102         else
00103           // Construct generators for p,q given generators for p-1,q-1
00104           gen_from_pm1_qm1(gen_vector(p-1, q-1), sig);
00105         break;
00106       case 2:
00107         if (bias < 2)
00108           // Construct generators for p,q given generators for p+4,q-4
00109           gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00110         else if (bias > 2)
00111           // Construct generators for p,q given generators for p-4,q+4
00112           gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00113         else
00114           // Construct generators for p,q given generators for q+1,p-1
00115           gen_from_qp1_pm1(gen_vector(q+1, p-1), sig);
00116         break;
00117       default:
00118         break;
00119       }
00120     return (*this)[sig];
00121   }
00122
00124   // Reference: [P] Proposition 15.17, p 131
00125   template< class Matrix_T >
00126   void
00127   generator_table<Matrix_T>::
```

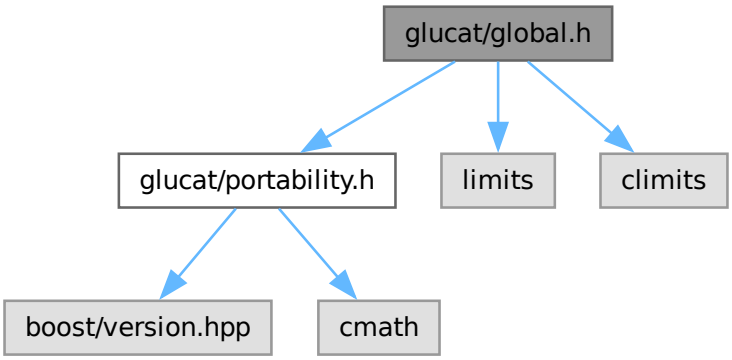
```
00128   gen_from_pm1_qm1(const std::vector<Matrix_T>& old, const signature_t sig)
00129   {
00130     const auto new_size = old.size() + 2;
00131     using size_t = decltype(new_size);
00132     using result_t = std::vector<Matrix_T>;
00133     auto result = result_t(new_size);
00134
00135     const auto old_dim = old[0].size1();
00136     const auto& eye = matrix::unit<Matrix_T>(old_dim);
00137
00138     auto neg = Matrix_T(2,2,2);
00139     neg(0,1) =    -1;
00140     neg(1,0) = 1;
00141
00142     auto pos = neg;
00143     pos(0,1) =     1;
00144
00145     auto dup = Matrix_T(2,2,2);
00146     dup(0,0) = 1;
00147     dup(1,1) =    -1;
00148
00149     result[0] = matrix::mono_kron(neg, eye);
00150     for (auto
00151         k = size_t(1);
00152         k != new_size-1;
00153         ++k)
00154       result[k] = matrix::mono_kron(dup, old[k-1]);
00155     result[new_size-1] = matrix::mono_kron(pos, eye);
00156
00157     // Save the resulting generator array.
00158     this->insert(make_pair(sig, result));
00159   }
00160
00162   // Reference: [L] 16.4 Periodicity of 8, p216
00163   template< class Matrix_T >
00164   void
00165   generator_table<Matrix_T>::
00166   gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig)
00167   {
00168     const auto old_size = old.size();
00169     using size_t = decltype(old_size);
00170     using result_t = std::vector<Matrix_T>;
00171     auto result = result_t(old_size);
00172
00173     auto h = old[0];
00174     for (auto
00175         k = size_t(1);
00176         k != size_t(4);
00177         ++k)
00178       h = matrix::mono_prod(old[k], h);
00179
00180     for (auto
00181         k = size_t(0);
00182         k != old_size-4;
00183         ++k)
00184       result[k] = old[k+4];
00185     for (auto
00186         k = old_size-4;
00187         k != old_size;
00188         ++k)
00189       result[k] = matrix::mono_prod(old[k+4-old_size], h);
00190     // Save the resulting generator array.
00191     this->insert(make_pair(sig, result));
00192   }
00193
00195   // Reference: [L] 16.4 Periodicity of 8, p216
00196   template< class Matrix_T >
00197   void
00198   generator_table<Matrix_T>::
00199   gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig)
00200   {
00201     const auto old_size = old.size();
00202     using size_t = decltype(old_size);
00203     using result_t = std::vector<Matrix_T>;
00204     auto result = result_t(old_size);
00205
00206     auto h = old[old_size-1];
00207     for (auto
00208         k = size_t(1);
00209         k != size_t(4);
00210         ++k)
00211       h = matrix::mono_prod(old[old_size-1-k], h);
00212
00213     for (auto
00214         k = size_t(0);
00215         k != size_t(4);
00216         ++k)
```

```
00217       result[k] = matrix::mono_prod(old[k+old_size-4], h);
00218     for (auto
00219         k = size_t(4);
00220         k != old_size;
00221         ++k)
00222       result[k] = old[k-4];
00223     // Save the resulting generator array.
00224     this->insert(make_pair(sig, result));
00225   }
00226
00228   // Reference: [P] Proposition 15.20, p 131
00229   template< class Matrix_T >
00230   void
00231   generator_table<Matrix_T>::
00232   gen_from_qp1_pm1(const std::vector<Matrix_T>& old, const signature_t sig)
00233   {
00234     const auto old_size = old.size();
00235     using size_t = decltype(old_size);
00236     using result_t = std::vector<Matrix_T>;
00237     auto result = result_t(old_size);
00238
00239     const auto& h = old[old_size-1];
00240     for (auto
00241         k = size_t(0);
00242         k != old_size-1;
00243         ++k)
00244       result[k] = matrix::mono_prod(old[old_size-2-k], h);
00245     result[old_size-1] = h;
00246
00247     // Save the resulting generator array.
00248     this->insert(make_pair(sig, result));
00249   }
00250
00251 } }
00252 #endif  // _GLUCAT_GENERATION_IMP_H
```

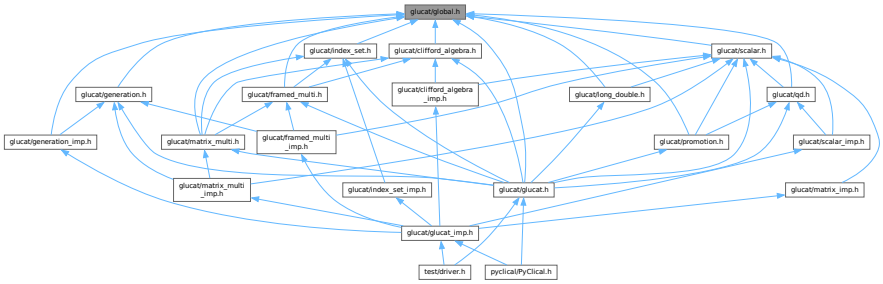
9.17 glucat/global.h File Reference

```
#include "glucat/portability.h"
#include <limits>
#include <climits>
```

Include dependency graph for global.h:



This graph shows which files directly or indirectly include this file:



Classes

- struct glucat::CTAssertion< true >
- class glucat::compare_types< LHS_T, RHS_T >

  Type comparison.
- class glucat::compare_types< T, T >
- class glucat::bool_to_type< truth_value >

  Bool to type.

Namespaces

- namespace glucat

Macros

- #define _GLUCAT_CTAssert(expr, msg)

Typedefs

- using glucat::index_t = int

  Size of index_t should be enough to represent LO, HI.
- using glucat::set_value_t = unsigned long

  Size of set_value_t should be enough to contain index_set<LO,HI>.

Functions

- glucat::_GLUCAT_CTAssert (std::numeric_limits< unsigned char >::radix==2, CannotDetermineBitsPer↩ Char) const index_t BITS_PER_CHAR

  If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.
- glucat::_GLUCAT_CTAssert (_GLUCAT_BITS_PER_ULONG==BITS_PER_SET_VALUE, BitsPerULong↩ DoesNotMatchSetValueT) const index_t DEFAULT_LO

  Default lowest index in an index set.
- template<typename LHS_T, typename RHS_T>
  auto glucat::pos_mod (LHS_T lhs, RHS_T rhs) -> LHS_T

  Modulo function which works reliably for lhs < 0.

Variables

- const double glucat::MS_PER_S = 1000.0

  Timing constant: deprecated here - moved to test/timing.h.
- const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits

  Number of bits in set_value_t.
- const index_t glucat::DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)

  Default highest index in an index set.

9.17.1 Macro Definition Documentation

9.17.1.1 _GLUCAT_CTAssert

```
#define _GLUCAT_CTAssert(
              expr,
              msg)
```

Value:

```
namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
```

Definition at line 48 of file global.h.

Referenced by _GLUCAT_CTAssert().

9.18 global.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_GLOBAL_H
00002 #define _GLUCAT_GLOBAL_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     global.h : Global declarations
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ***************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/portability.h"
00035
00036 #include <limits>
00037 #include <climits>
```

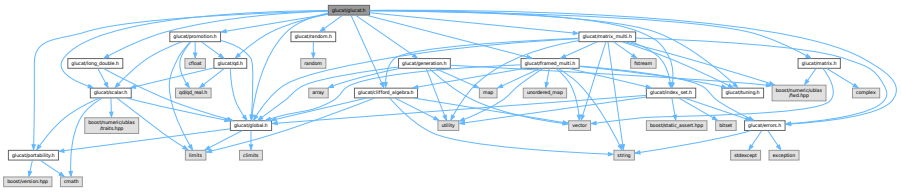
```
00038
00039 namespace glucat
00040 {
00041   // References:
00042   // [AA]: A. Alexandrescu, "Modern C++ Design", Addison-Wesley, 2001.
00043
00045   // Reference: [AA], p. 25
00046   template<bool> struct CTAssertion;
00047   template<> struct CTAssertion<true> { };
00048   #define _GLUCAT_CTAssert(expr, msg) \
00049       namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
00050
00052   // Reference: [AA], pp. 34--37
00053   template < typename LHS_T, typename RHS_T >
00054   class compare_types
00055   {
00056   public:
00057     enum { are_same = false };
00058   };
00059   template < typename T >
00060   class compare_types<T, T>
00061   {
00062   public:
00063     enum { are_same = true };
00064   };
00065
00067   // Reference: [AA], 2.4, p. 29
00068   template< bool truth_value >
00069   class bool_to_type
00070   {
00071   private:
00072     enum { value = truth_value };
00073   };
00074
00075   // Global types which determine sizes
00077   using index_t = int;
00079   using set_value_t = unsigned long;
00080
00081   // Global constants
00083   const double MS_PER_S = 1000.0;
00084
00085   // Constants which determine sizes
00086
00087   // Bits per unsigned long
00088   #if   (ULONG_MAX == (4294967295UL))
00089   #define _GLUCAT_BITS_PER_ULONG 32
00090   #elif (ULONG_MAX == (18446744073709551615UL))
00091   #define _GLUCAT_BITS_PER_ULONG 64
00092   #elif defined(__WORDSIZE)
00093   #define _GLUCAT_BITS_PER_ULONG __WORDSIZE
00094   #endif
00095
00097   _GLUCAT_CTAssert(std::numeric_limits<unsigned char>::radix == 2, CannotDetermineBitsPerChar)
00098
00099
00100   const index_t BITS_PER_CHAR = std::numeric_limits<unsigned char>::digits;
00101
00103   const index_t BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits;
00104
00105   _GLUCAT_CTAssert(_GLUCAT_BITS_PER_ULONG == BITS_PER_SET_VALUE, BitsPerULongDoesNotMatchSetValueT)
00106
00107   // Constants which are determined by size
00109   const index_t DEFAULT_LO = -index_t(BITS_PER_SET_VALUE / 2);
00111   const index_t DEFAULT_HI =  index_t(BITS_PER_SET_VALUE / 2);
00112
00114   template< typename LHS_T, typename RHS_T >
00115   inline
00116   auto
00117   pos_mod(LHS_T lhs, RHS_T rhs) -> LHS_T
00118   { return lhs > 0? lhs % rhs : (-lhs) % rhs == 0 ? 0 : rhs - (-lhs) % rhs; }
00119
00120 }
00121 #endif // _GLUCAT_GLOBAL_H
```

9.19 glucat/glucat.h File Reference

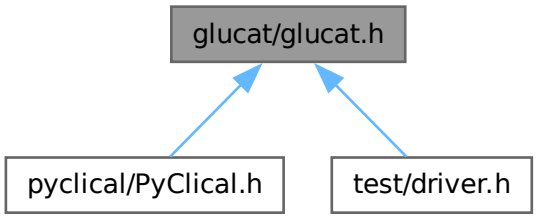
```
#include "glucat/portability.h"
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
```

```
#include "glucat/scalar.h"
#include "glucat/long_double.h"
#include "glucat/qd.h"
#include "glucat/promotion.h"
#include "glucat/random.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include "glucat/matrix_multi.h"
```

Include dependency graph for glucat.h:



This graph shows which files directly or indirectly include this file:



9.20 glucat.h

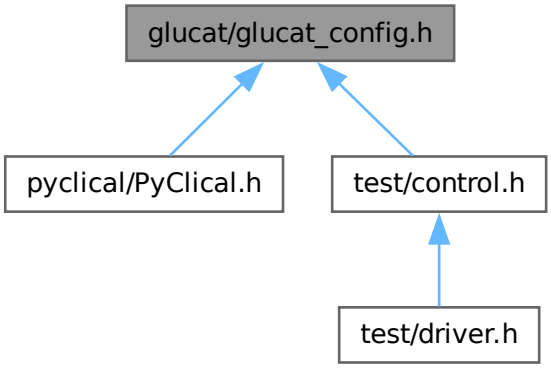
Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_GLUCAT_H
00002 #define _GLUCAT_GLUCAT_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     glucat.h : Organize GluCat header files for applications
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
```

```
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ************************************************************************
00031      Arvind Raja's original header comments and references follow.
00032  ************************************************************************
00033 // clifford algebra package,  Arvind.Raja@hut.fi
00034 // ref: Press et.al. "Numerical Recipes in C", 2nd ed., C.U.P., 1992.
00035 // ref: LEDA, v 3.0, Stefan N\"aher, Max-Planck-Institut f\"ur Informatik
00036 // ref: Stroustrup B., "The C++ Programming Language", 2nd ed.,
00037 //      Addison-Wesley, 1991.
00038 // ref: R. Sedgewick, "Algorithms in C++", Addison-Wesley, 1992.
00039 // ref: S. Meyers, "Effective C++ ", Addison-Wesley, 1992.
00040  ************************************************************************/
00041
00042 #include "glucat/portability.h"
00043
00044 #include "glucat/global.h"
00045
00046 #include "glucat/errors.h"
00047
00048 #include "glucat/index_set.h"
00049
00050 #include "glucat/scalar.h"
00051
00052 #include "glucat/long_double.h"
00053
00054 #include "glucat/qd.h"
00055
00056 #include "glucat/promotion.h"
00057
00058 #include "glucat/random.h"
00059
00060 #include "glucat/clifford_algebra.h"
00061
00062 #include "glucat/tuning.h"
00063
00064 #include "glucat/framed_multi.h"
00065
00066 #include "glucat/generation.h"
00067
00068 #include "glucat/matrix.h"
00069
00070 #include "glucat/matrix_multi.h"
00071
00072 #endif  // _GLUCAT_GLUCAT_H
```

9.21 glucat/glucat_config.h File Reference

This graph shows which files directly or indirectly include this file:



Macros

- #define GLUCAT_HAVE_CXX11 1
- #define GLUCAT_HAVE_INTTYPES_H 1
- #define GLUCAT_HAVE_STDINT_H 1
- #define GLUCAT_HAVE_STDIO_H 1
- #define GLUCAT_HAVE_STDLIB_H 1
- #define GLUCAT_HAVE_STRINGS_H 1
- #define GLUCAT_HAVE_STRING_H 1
- #define GLUCAT_HAVE_SYS_STAT_H 1
- #define GLUCAT_HAVE_SYS_TYPES_H 1
- #define GLUCAT_HAVE_UNISTD_H 1
- #define GLUCAT_PACKAGE "glucat"
- #define GLUCAT_PACKAGE_BUGREPORT ""
- #define GLUCAT_PACKAGE_NAME "glucat"
- #define GLUCAT_PACKAGE_STRING "glucat 0.13.0"
- #define GLUCAT_PACKAGE_TARNAME "glucat"
- #define GLUCAT_PACKAGE_URL ""
- #define GLUCAT_PACKAGE_VERSION "0.13.0"
- #define GLUCAT_STDC_HEADERS 1
- #define GLUCAT_VERSION "0.13.0"

9.21.1 Macro Definition Documentation

9.21.1.1 GLUCAT_HAVE_CXX11

```
#define GLUCAT_HAVE_CXX11 1
```

Definition at line 20 of file glucat_config.h.

9.21.1.2 GLUCAT_HAVE_INTTYPES_H

```
#define GLUCAT_HAVE_INTTYPES_H 1
```

Definition at line 28 of file glucat_config.h.

9.21.1.3 GLUCAT_HAVE_STDINT_H

```
#define GLUCAT_HAVE_STDINT_H 1
```

Definition at line 39 of file glucat_config.h.

9.21.1.4 GLUCAT_HAVE_STDIO_H

```
#define GLUCAT_HAVE_STDIO_H 1
```

Definition at line 44 of file glucat_config.h.

9.21.1.5 GLUCAT_HAVE_STDLIB_H

```
#define GLUCAT_HAVE_STDLIB_H 1
```

Definition at line 49 of file glucat_config.h.

9.21.1.6 GLUCAT_HAVE_STRING_H

```
#define GLUCAT_HAVE_STRING_H 1
```

Definition at line 59 of file glucat_config.h.

9.21.1.7 GLUCAT_HAVE_STRINGS_H

```
#define GLUCAT_HAVE_STRINGS_H 1
```

Definition at line 54 of file glucat_config.h.

9.21.1.8 GLUCAT_HAVE_SYS_STAT_H

```
#define GLUCAT_HAVE_SYS_STAT_H 1
```

Definition at line 64 of file glucat_config.h.

9.21.1.9 GLUCAT_HAVE_SYS_TYPES_H

```
#define GLUCAT_HAVE_SYS_TYPES_H 1
```

Definition at line 69 of file glucat_config.h.

9.21.1.10 GLUCAT_HAVE_UNISTD_H

```
#define GLUCAT_HAVE_UNISTD_H 1
```

Definition at line 74 of file glucat_config.h.

9.21.1.11 GLUCAT_PACKAGE

```
#define GLUCAT_PACKAGE "glucat"
```

Definition at line 79 of file glucat_config.h.

9.21.1.12 GLUCAT_PACKAGE_BUGREPORT

```
#define GLUCAT_PACKAGE_BUGREPORT ""
```

Definition at line 84 of file glucat_config.h.

9.21.1.13 GLUCAT_PACKAGE_NAME

```
#define GLUCAT_PACKAGE_NAME "glucat"
```

Definition at line 89 of file glucat_config.h.

Referenced by glucat::control_t::control_t().

9.21.1.14 GLUCAT_PACKAGE_STRING

```
#define GLUCAT_PACKAGE_STRING "glucat 0.13.0"
```

Definition at line 94 of file glucat_config.h.

9.21.1.15 GLUCAT_PACKAGE_TARNAME

```
#define GLUCAT_PACKAGE_TARNAME "glucat"
```

Definition at line 99 of file glucat_config.h.

9.21.1.16 GLUCAT_PACKAGE_URL

```
#define GLUCAT_PACKAGE_URL ""
```

Definition at line 104 of file glucat_config.h.

9.21.1.17 GLUCAT_PACKAGE_VERSION

```
#define GLUCAT_PACKAGE_VERSION "0.13.0"
```

Definition at line 109 of file glucat_config.h.

9.21.1.18 GLUCAT_STDC_HEADERS

```
#define GLUCAT_STDC_HEADERS 1
```

Definition at line 116 of file glucat_config.h.

9.21.1.19 GLUCAT_VERSION

```
#define GLUCAT_VERSION "0.13.0"
```

Definition at line 121 of file glucat_config.h.

Referenced by glucat::control_t::control_t().

9.22 glucat_config.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_GLUCAT_CONFIG_H
00002 #define _GLUCAT_GLUCAT_CONFIG_H 1
00003
00004 /* glucat/glucat_config.h. Generated automatically at end of configure. */
00005 /* config.h.  Generated from config.h.in by configure.  */
00006 /* config.h.in.  Generated from configure.ac by autoheader.  */
00007
00008 /* Define to dummy 'main' function (if any) required to link to the Fortran
00009    libraries. */
00010 /* #undef F77_DUMMY_MAIN */
00011
00012 /* Define if F77 and FC dummy 'main' functions are identical. */
00013 /* #undef FC_DUMMY_MAIN_EQ_F77 */
00014
00015 /* Define if you have a BLAS library. */
00016 /* #undef HAVE_BLAS */
00017
00018 /* define if the compiler supports basic C++11 syntax */
00019 #ifndef GLUCAT_HAVE_CXX11
00020 #define GLUCAT_HAVE_CXX11 1
00021 #endif
00022
00023 /* define if the compiler supports basic C++14 syntax */
00024 /* #undef HAVE_CXX14 */
00025
00026 /* Define to 1 if you have the <inttypes.h> header file. */
00027 #ifndef GLUCAT_HAVE_INTTYPES_H
00028 #define GLUCAT_HAVE_INTTYPES_H 1
00029 #endif
00030
00031 /* Define if you have LAPACK library. */
00032 /* #undef HAVE_LAPACK */
00033
00034 /* Define to 1 if you have the 'imf' library (-limf). */
00035 /* #undef HAVE_LIBIMF */
00036
00037 /* Define to 1 if you have the <stdint.h> header file. */
00038 #ifndef GLUCAT_HAVE_STDINT_H
00039 #define GLUCAT_HAVE_STDINT_H 1
00040 #endif
00041
00042 /* Define to 1 if you have the <stdio.h> header file. */
00043 #ifndef GLUCAT_HAVE_STDIO_H
```

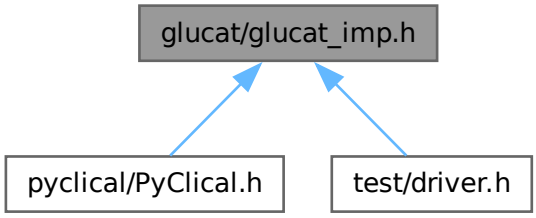
```
00044 #define GLUCAT_HAVE_STDIO_H 1
00045 #endif
00046
00047 /* Define to 1 if you have the <stdlib.h> header file. */
00048 #ifndef GLUCAT_HAVE_STDLIB_H
00049 #define GLUCAT_HAVE_STDLIB_H 1
00050 #endif
00051
00052 /* Define to 1 if you have the <strings.h> header file. */
00053 #ifndef GLUCAT_HAVE_STRINGS_H
00054 #define GLUCAT_HAVE_STRINGS_H 1
00055 #endif
00056
00057 /* Define to 1 if you have the <string.h> header file. */
00058 #ifndef GLUCAT_HAVE_STRING_H
00059 #define GLUCAT_HAVE_STRING_H 1
00060 #endif
00061
00062 /* Define to 1 if you have the <sys/stat.h> header file. */
00063 #ifndef GLUCAT_HAVE_SYS_STAT_H
00064 #define GLUCAT_HAVE_SYS_STAT_H 1
00065 #endif
00066
00067 /* Define to 1 if you have the <sys/types.h> header file. */
00068 #ifndef GLUCAT_HAVE_SYS_TYPES_H
00069 #define GLUCAT_HAVE_SYS_TYPES_H 1
00070 #endif
00071
00072 /* Define to 1 if you have the <unistd.h> header file. */
00073 #ifndef GLUCAT_HAVE_UNISTD_H
00074 #define GLUCAT_HAVE_UNISTD_H 1
00075 #endif
00076
00077 /* Name of package */
00078 #ifndef GLUCAT_PACKAGE
00079 #define GLUCAT_PACKAGE "glucat"
00080 #endif
00081
00082 /* Define to the address where bug reports for this package should be sent. */
00083 #ifndef GLUCAT_PACKAGE_BUGREPORT
00084 #define GLUCAT_PACKAGE_BUGREPORT ""
00085 #endif
00086
00087 /* Define to the full name of this package. */
00088 #ifndef GLUCAT_PACKAGE_NAME
00089 #define GLUCAT_PACKAGE_NAME "glucat"
00090 #endif
00091
00092 /* Define to the full name and version of this package. */
00093 #ifndef GLUCAT_PACKAGE_STRING
00094 #define GLUCAT_PACKAGE_STRING "glucat 0.13.0"
00095 #endif
00096
00097 /* Define to the one symbol short name of this package. */
00098 #ifndef GLUCAT_PACKAGE_TARNAME
00099 #define GLUCAT_PACKAGE_TARNAME "glucat"
00100 #endif
00101
00102 /* Define to the home page for this package. */
00103 #ifndef GLUCAT_PACKAGE_URL
00104 #define GLUCAT_PACKAGE_URL ""
00105 #endif
00106
00107 /* Define to the version of this package. */
00108 #ifndef GLUCAT_PACKAGE_VERSION
00109 #define GLUCAT_PACKAGE_VERSION "0.13.0"
00110 #endif
00111
00112 /* Define to 1 if all of the C89 standard headers exist (not just the ones
00113    required in a freestanding environment). This macro is provided for
00114    backward compatibility; new code need not use it. */
00115 #ifndef GLUCAT_STDC_HEADERS
00116 #define GLUCAT_STDC_HEADERS 1
00117 #endif
00118
00119 /* Version number of package */
00120 #ifndef GLUCAT_VERSION
00121 #define GLUCAT_VERSION "0.13.0"
00122 #endif
00123
00124 /* once: _GLUCAT_GLUCAT_CONFIG_H */
00125 #endif
```

9.23 glucat/glucat_imp.h File Reference

```
#include "glucat/errors_imp.h"
#include "glucat/index_set_imp.h"
#include "glucat/scalar_imp.h"
#include "glucat/clifford_algebra_imp.h"
#include "glucat/random.h"
#include "glucat/framed_multi_imp.h"
#include "glucat/matrix_imp.h"
#include "glucat/generation_imp.h"
#include "glucat/matrix_multi_imp.h"
```

Include dependency graph for glucat_imp.h:

This graph shows which files directly or indirectly include this file:



9.24 glucat_imp.h

Go to the documentation of this file.

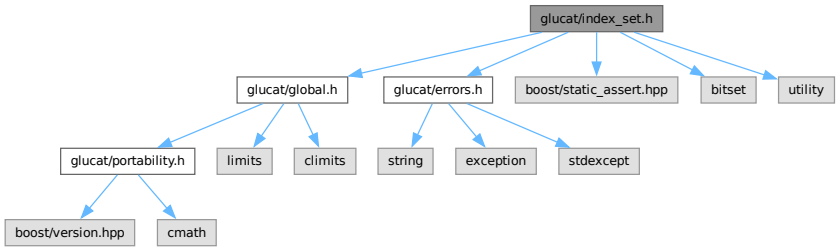
```
00001 #ifndef _GLUCAT_GLUCAT_IMP_H
00002 #define _GLUCAT_GLUCAT_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     glucat_imp.h : Organize GluCat template definitions which cannot be compiled separately
00006                            -------------------
00007     begin                : Sun 2001-12-25
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
```

```
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ****************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ****************************************************************************
00031      For Arvind Raja's original header comments, see glucat.h
00032  ****************************************************************************/
00033
00034 // Template definitions which cannot be compiled separately
00035
00036 #include "glucat/errors_imp.h"
00037
00038 #include "glucat/index_set_imp.h"
00039
00040 #include "glucat/scalar_imp.h"
00041
00042 #include "glucat/clifford_algebra_imp.h"
00043
00044 #include "glucat/random.h"
00045
00046 #include "glucat/framed_multi_imp.h"
00047
00048 #include "glucat/matrix_imp.h"
00049
00050 #include "glucat/generation_imp.h"
00051
00052 #include "glucat/matrix_multi_imp.h"
00053
00054 #endif  // _GLUCAT_GLUCAT_IMP_H
```

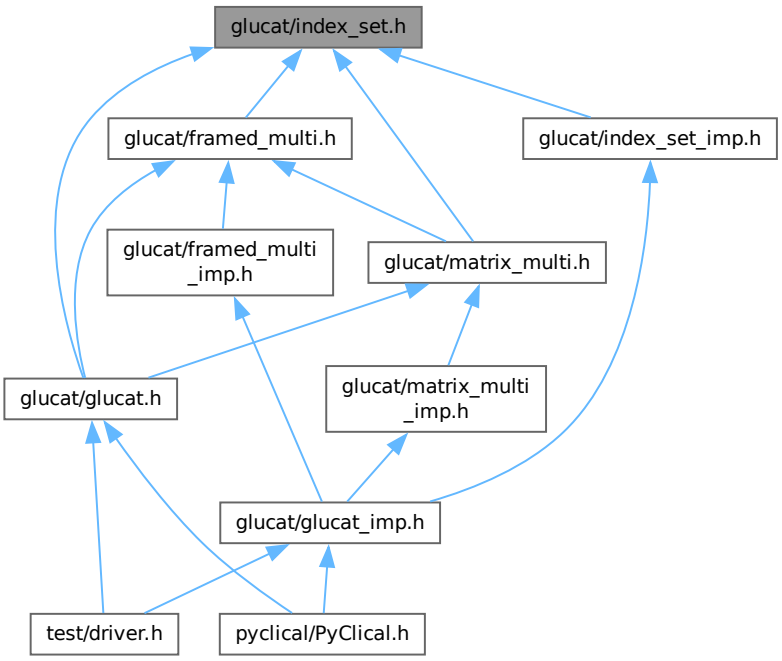
9.25 glucat/index_set.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include <boost/static_assert.hpp>
#include <bitset>
#include <utility>
```

Include dependency graph for index_set.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::index_set< LO, HI >

  Index set class based on std::bitset<> in Gnu standard C++ library.
- class glucat::index_set< LO, HI >::reference

  Index set member reference.

Namespaces

- namespace glucat

Functions

- template<const index_t LO, const index_t HI>
  auto glucat::operator^ (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Symmetric set difference: exclusive or.
- template<const index_t LO, const index_t HI>
  auto glucat::operator& (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Set intersection: and.
- template<const index_t LO, const index_t HI>
  auto glucat::operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

Set union: or.

- template<const index_t LO, const index_t HI>
  auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int

  "lexicographic compare" eg. {3,4,5} is less than {3,7,8}
- glucat::_GLUCAT_CTAssert (sizeof(set_value_t) >=sizeof(std::bitset< DEFAULT_HI-DEFAULT_LO >), Default_index_set_too_big_for_value) template< const index_t LO

  Size of set_value_t should be enough to contain bitset<DEFAULT_HI-DEFAULT_LO>.
- const index_t HI auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &
- template<const index_t LO, const index_t HI>
  auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &

  Read in index set.
- auto glucat::sign_of_square (index_t j) -> int

  Square of generator {j}.
- template<const index_t LO, const index_t HI>
  auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t

  Minimum negative index, or 0 if none.
- template<const index_t LO, const index_t HI>
  auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t

  Maximum positive index, or 0 if none.

9.26 index_set.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_INDEX_SET_H
00002 #define _GLUCAT_INDEX_SET_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set.h : Declare a class for a set of non-zero integer indices
00006                           -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036
00037 #include <boost/static_assert.hpp>
00038
00039 #include <bitset>
00040 #include <utility>
00041
00042 namespace glucat
00043 {
00044   template<const index_t LO, const index_t HI>
```

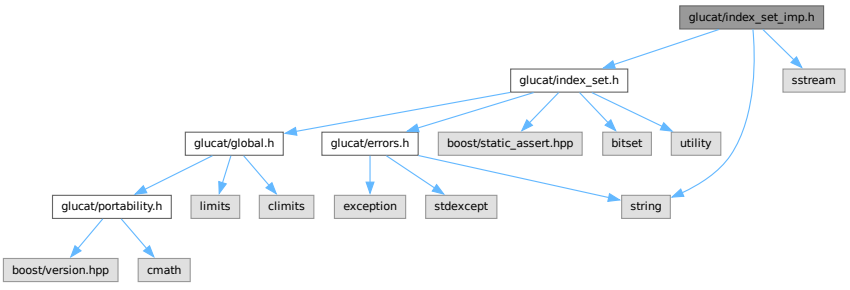
```
00045   class index_set; // forward
00046
00048   template<const index_t LO, const index_t HI>
00049   auto
00050   operator^ (const index_set<LO,HI>& lhs,
00051              const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00052
00054   template<const index_t LO, const index_t HI>
00055   auto
00056   operator& (const index_set<LO,HI>& lhs,
00057              const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00058
00060   template<const index_t LO, const index_t HI>
00061   auto
00062   operator| (const index_set<LO,HI>& lhs,
00063              const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00064
00066   // -1 if a<b, +1 if a>b, 0 if a==b
00067   template<const index_t LO, const index_t HI>
00068   auto
00069   compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int;
00070
00072   template<const index_t LO, const index_t HI>
00073   class index_set :
00074   private std::bitset<HI-LO>
00075   {
00076   private:
00077     BOOST_STATIC_ASSERT((LO <= 0) && (0 <= HI) && (LO < HI) && \
00078                         (-LO    <  _GLUCAT_BITS_PER_ULONG)   && \
00079                         ( HI    <  _GLUCAT_BITS_PER_ULONG)   && \
00080                         ( HI-LO <= _GLUCAT_BITS_PER_ULONG));
00081     using bitset_t = std::bitset<HI - LO>;
00082     using error_t = error<index_set>;
00083   public:
00084     using index_set_t = index_set;
00085     using index_pair_t = std::pair<index_t, index_t>;
00086
00087     static const index_t v_lo = LO;
00088     static const index_t v_hi = HI;
00089
00090     static auto classname() -> const std::string;
00092     index_set    () = default;
00094     index_set    (const bitset_t bst);
00096     index_set    (const index_t idx);
00098     index_set    (const set_value_t folded_val, const index_set_t frm, const bool prechecked = false);
00100     index_set    (const index_pair_t& range, const bool prechecked = false);
00102     index_set    (const std::string& str);
00103
00105     auto operator==  (const index_set_t rhs) const -> bool;
00107     auto operator!=  (const index_set_t rhs) const -> bool;
00109     auto operator~   () const -> index_set_t;
00111     auto operator^=  (const index_set_t rhs) -> index_set_t&;
00113     auto operator&=  (const index_set_t rhs) -> index_set_t&;
00115     auto operator|=  (const index_set_t rhs) -> index_set_t&;
00117     auto operator[]  (const index_t idx) const -> bool;
00119     auto test(const index_t idx) const -> bool;
00121     auto set() -> index_set_t&;
00123     auto set(const index_t idx) -> index_set_t&;
00125     auto set(const index_t idx, const int val) -> index_set_t&;
00127     auto reset() -> index_set_t&;
00129     auto reset(const index_t idx) -> index_set_t&;
00131     auto flip() -> index_set_t&;
00133     auto flip(const index_t idx) -> index_set_t&;
00135     auto count() const -> index_t;
00137     auto count_neg() const -> index_t;
00139     auto count_pos() const -> index_t;
00141     auto min() const -> index_t;
00143     auto max() const -> index_t;
00144
00145   // Functions which support Clifford algebra operations
00147     auto operator<     (const index_set_t rhs) const -> bool;
00149     auto is_contiguous () const -> bool;
00151     auto fold          () const -> const index_set_t;
00153     auto fold          (const index_set_t frm, const bool prechecked = false) const -> const
      index_set_t;
00155     auto unfold        (const index_set_t frm, const bool prechecked = false) const -> const
      index_set_t;
00157     auto value_of_fold (const index_set_t frm) const -> set_value_t;
00159     auto sign_of_mult  (const index_set_t ist) const -> int;
00161     auto sign_of_square()                      const -> int;
00162
00164     auto hash_fn       ()                      const -> size_t;
00165
00166   // Friends
00167     friend auto operator^<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00168     friend auto operator&<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00169     friend auto operator|<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
```

```
00170     friend auto compare<>                 (const index_set_t& lhs, const index_set_t& rhs) -> int;
00171
00172   // Member reference:
00173     class reference;
00174     friend class reference;
00175
00177     class reference {
00178       friend class index_set;
00179
00180     public:
00182       reference() = delete;
00183       reference   (index_set_t& ist, index_t idx);
00184       ~reference  () = default;
00186       auto operator== (const reference& c_j) const -> bool;
00188       auto operator= (const bool x) -> reference&;
00190       auto operator= (const reference& c_j) -> reference&;
00192       auto operator~ () const -> bool;
00194            operator bool () const;
00196       auto flip() -> reference&;
00197
00198     private:
00199       index_set_t* m_pst;
00200       index_t      m_idx;
00201     };
00203     auto operator[](index_t idx) -> reference;
00204   private:
00206     auto lex_less_than (const index_set_t rhs) const -> bool;
00207   };
00208
00210   _GLUCAT_CTAssert(sizeof(set_value_t) >= sizeof(std::bitset<DEFAULT_HI-DEFAULT_LO>),
00211            Default_index_set_too_big_for_value)
00212
00213   // non-members
00214
00216   template<const index_t LO, const index_t HI>
00217   auto
00218   operator« (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&;
00219
00221   template<const index_t LO, const index_t HI>
00222   auto
00223   operator» (std::istream& s, index_set<LO,HI>& ist) -> std::istream&;
00224
00225   // Functions which support Clifford algebra operations
00227   auto sign_of_square(index_t j) -> int;
00228
00230   template<const index_t LO, const index_t HI>
00231   auto
00232   min_neg(const index_set<LO,HI>& ist) -> index_t;
00233
00235   template<const index_t LO, const index_t HI>
00236   auto
00237   max_pos(const index_set<LO,HI>& ist) -> index_t;
00238 }
00239 #endif // _GLUCAT_INDEX_SET_H
```

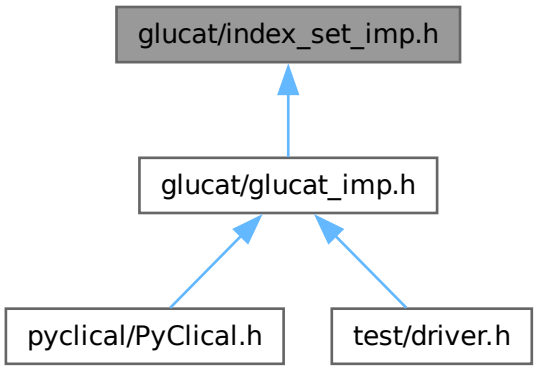
9.27 glucat/index_set_imp.h File Reference

```
#include "glucat/index_set.h"
#include <string>
#include <sstream>
```

Include dependency graph for index_set_imp.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Functions

- template<const index_t LO, const index_t HI>
  auto glucat::operator^ (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Symmetric set difference: exclusive or.
- template<const index_t LO, const index_t HI>
  auto glucat::operator& (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Set intersection: and.
- template<const index_t LO, const index_t HI>
  auto glucat::operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >

  Set union: or.
- template<const index_t LO, const index_t HI>
  auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int

  "lexicographic compare" eg. {3,4,5} is less than {3,7,8}
- template<const index_t LO, const index_t HI>
  auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &

  Write out index set.
- template<const index_t LO, const index_t HI>
  auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &

  Read in index set.
- static auto glucat::inverse_reversed_gray (unsigned long x) -> unsigned long

  Inverse reversed Gray code.
- static auto glucat::inverse_gray (unsigned long x) -> unsigned long

  Inverse Gray code.
- auto glucat::sign_of_square (index_t j) -> int

Square of generator {j}.

- template<const index_t LO, const index_t HI>
  auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t

  Minimum negative index, or 0 if none.

- template<const index_t LO, const index_t HI>
  auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t

  Maximum positive index, or 0 if none.

9.28 index_set_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_INDEX_SET_IMP_H
00002 #define _GLUCAT_INDEX_SET_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set_imp.h : Implement a class for a set of non-zero integer indices
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/index_set.h"
00035
00036 #include <string>
00037 #include <sstream>
00038
00039 namespace glucat
00040 {
00041   // References for algorithms:
00042   // [JA]: Joerg Arndt,  "Algorithms for programmers", http://www.jjj.de/fxt/fxtbook.pdf
00043   //       Chapter 1, Bit wizardry, http://www.jjj.de/bitwizardry/bitwizardrypage.html
00044   // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00045
00046   template<const index_t LO, const index_t HI>
00047   inline
00048   auto
00049   index_set<LO,HI>::
00050   classname() -> const std::string
00051   { return "index_set"; }
00052
00054   template<const index_t LO, const index_t HI>
00055   index_set<LO,HI>::
00056   index_set(const index_t idx)
00057   { this->set(idx); }
00058
00060   template<const index_t LO, const index_t HI>
00061   index_set<LO,HI>::
00062   index_set(const bitset_t bst):
00063   bitset_t(bst)
00064   { }
00065
00067   template<const index_t LO, const index_t HI>
00068   index_set<LO,HI>::
00069   index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked)
```

```
00070   {
00071     if (!prechecked && folded_val >= (set_value_t(1) « frm.count()))
00072         throw error_t("index_set(val,frm): cannot create: value gives an index set outside of frame");
00073     const index_set_t folded_frame = frm.fold();
00074     const index_t min_index = folded_frame.min();
00075     const index_t skip = min_index > 0 ? 1 : 0;
00076     const index_set_t folded_set = index_set_t(bitset_t(folded_val) « (min_index - skip - LO));
00077     *this = folded_set.unfold(frm);
00078   }
00079
00081   template<const index_t LO, const index_t HI>
00082   index_set<LO,HI>::
00083   index_set(const index_pair_t& range, const bool prechecked)
00084   {
00085     if (!prechecked && (range.first < LO || range.second > HI))
00086         throw error_t("index_set(range): cannot create: range is too large");
00087     const index_t begin_bit = (range.first < 0)
00088                               ? range.first-LO
00089                               : range.first-LO-1;
00090     const index_t end_bit = (range.second < 0)
00091                             ? range.second-LO+1
00092                             : range.second-LO;
00093     unsigned long mask = ( (end_bit == _GLUCAT_BITS_PER_ULONG)
00094                           ? -1UL
00095                           : (1UL « end_bit)-1UL)
00096                         & ~((1UL « begin_bit)-1UL);
00097     *this = bitset_t(mask);
00098   }
00099
00101   template<const index_t LO, const index_t HI>
00102   index_set<LO,HI>::
00103   index_set(const std::string& str)
00104   {
00105     std::istringstream ss(str);
00106     ss » *this;
00107     if (!ss)
00108       throw error_t("index_set_t(str): could not parse string");
00109     // Peek to see if the end of the string has been reached.
00110     ss.peek();
00111     if (!ss.eof())
00112       throw error_t("index_set_t(str): could not parse entire string");
00113   }
00114
00116   template<const index_t LO, const index_t HI>
00117   inline
00118   auto
00119   index_set<LO,HI>::
00120   operator== (const index_set_t rhs) const -> bool
00121   {
00122     const auto* pthis = static_cast<const bitset_t*>(this);
00123     return *pthis == static_cast<bitset_t>(rhs);
00124   }
00125
00127   template<const index_t LO, const index_t HI>
00128   inline
00129   auto
00130   index_set<LO,HI>::
00131   operator!= (const index_set_t rhs) const -> bool
00132   {
00133     const auto* pthis = static_cast<const bitset_t*>(this);
00134     return *pthis != static_cast<bitset_t>(rhs);
00135   }
00136
00138   template<const index_t LO, const index_t HI>
00139   inline
00140   auto
00141   index_set<LO,HI>::
00142   operator~ () const -> index_set_t
00143   { return bitset_t::operator~(); }
00144
00146   template<const index_t LO, const index_t HI>
00147   inline
00148   auto
00149   index_set<LO,HI>::
00150   operator^= (const index_set_t rhs) -> index_set_t&
00151   {
00152     bitset_t* pthis = this;
00153     *pthis ^= static_cast<bitset_t>(rhs);
00154     return *this;
00155   }
00156
00158   template<const index_t LO, const index_t HI>
00159   inline
00160   auto
00161   operator^ (const index_set<LO,HI>& lhs,
00162              const index_set<LO,HI>& rhs) -> const
00163   index_set<LO,HI>
```

```
00164   {
00165     using index_set_t = index_set<LO, HI>;
00166     using bitset_t = typename index_set_t::bitset_t;
00167     return static_cast<bitset_t>(lhs) ^ static_cast<bitset_t>(rhs);
00168   }
00169
00171   template<const index_t LO, const index_t HI>
00172   inline
00173   auto
00174   index_set<LO,HI>::
00175   operator&= (const index_set_t rhs) -> index_set_t&
00176   {
00177     bitset_t* pthis = this;
00178     *pthis &= static_cast<bitset_t>(rhs);
00179     return *this;
00180   }
00181
00183   template<const index_t LO, const index_t HI>
00184   inline
00185   auto
00186   operator& (const index_set<LO,HI>& lhs,
00187              const index_set<LO,HI>& rhs) -> const
00188   index_set<LO,HI>
00189   {
00190     using index_set_t = index_set<LO, HI>;
00191     using bitset_t = typename index_set_t::bitset_t;
00192     return static_cast<bitset_t>(lhs) & static_cast<bitset_t>(rhs);
00193   }
00194
00196   template<const index_t LO, const index_t HI>
00197   inline
00198   auto
00199   index_set<LO,HI>::
00200   operator|= (const index_set_t rhs) -> index_set_t&
00201   {
00202     bitset_t* pthis = this;
00203     *pthis |= static_cast<bitset_t>(rhs);
00204     return *this;
00205   }
00206
00208   template<const index_t LO, const index_t HI>
00209   inline
00210   auto
00211   operator| (const index_set<LO,HI>& lhs,
00212              const index_set<LO,HI>& rhs) -> const
00213   index_set<LO,HI>
00214   {
00215     using index_set_t = index_set<LO, HI>;
00216     using bitset_t = typename index_set_t::bitset_t;
00217     return static_cast<bitset_t>(lhs) | static_cast<bitset_t>(rhs);
00218   }
00219
00221   template<const index_t LO, const index_t HI>
00222   inline
00223   auto
00224   index_set<LO,HI>::
00225   operator[] (const index_t idx) -> reference
00226   { return reference(*this, idx); }
00227
00229   template<const index_t LO, const index_t HI>
00230   inline
00231   auto
00232   index_set<LO,HI>::
00233   operator[] (const index_t idx) const -> bool
00234   { return this->test(idx); }
00235
00237   template<const index_t LO, const index_t HI>
00238   inline
00239   auto
00240   index_set<LO,HI>::
00241   test(const index_t idx) const -> bool
00242   {
00243     // Reference: [JA], 1.2.1
00244     return (idx < 0)
00245            ?   bool(bitset_t::to_ulong() & (1UL << (idx - LO)))
00246            : (idx > 0)
00247              ? bool(bitset_t::to_ulong() & (1UL << (idx - LO - 1)))
00248              : false;
00249   }
00250
00252   template<const index_t LO, const index_t HI>
00253   inline
00254   auto
00255   index_set<LO,HI>::
00256   set() -> index_set_t&
00257   {
00258     bitset_t::set();
```

```
00259     return *this;
00260   }
00261
00263   template<const index_t LO, const index_t HI>
00264   inline
00265   auto
00266   index_set<LO,HI>::
00267   set(index_t idx) -> index_set_t&
00268   {
00269     if (idx > 0)
00270       bitset_t::set(idx-LO-1);
00271     else if (idx < 0)
00272       bitset_t::set(idx-LO);
00273     return *this;
00274   }
00275
00277   template<const index_t LO, const index_t HI>
00278   inline
00279   auto
00280   index_set<LO,HI>::
00281   set(const index_t idx, const int val) -> index_set_t&
00282   {
00283     if (idx > 0)
00284       bitset_t::set(idx-LO-1, val);
00285     else if (idx < 0)
00286       bitset_t::set(idx-LO, val);
00287     return *this;
00288   }
00289
00291   template<const index_t LO, const index_t HI>
00292   inline
00293   auto
00294   index_set<LO,HI>::
00295   reset() -> index_set_t&
00296   {
00297     bitset_t::reset();
00298     return *this;
00299   }
00300
00302   template<const index_t LO, const index_t HI>
00303   inline
00304   auto
00305   index_set<LO,HI>::
00306   reset(const index_t idx) -> index_set_t&
00307   {
00308     if (idx > 0)
00309       bitset_t::reset(idx-LO-1);
00310     else if (idx < 0)
00311       bitset_t::reset(idx-LO);
00312     return *this;
00313   }
00314
00316   template<const index_t LO, const index_t HI>
00317   inline
00318   auto
00319   index_set<LO,HI>::
00320   flip() -> index_set<LO,HI>&
00321   {
00322     bitset_t::flip();
00323     return *this;
00324   }
00325
00327   template<const index_t LO, const index_t HI>
00328   inline
00329   auto
00330   index_set<LO,HI>::
00331   flip(const index_t idx) -> index_set_t&
00332   {
00333     if (idx > 0)
00334       bitset_t::flip(idx-LO-1);
00335     else if (idx < 0)
00336       bitset_t::flip(idx-LO);
00337     return *this;
00338   }
00339
00341   template<const index_t LO, const index_t HI>
00342   inline
00343   auto
00344   index_set<LO,HI>::
00345   count() const -> index_t
00346   {
00347     unsigned long val = bitset_t::to_ulong();
00348     // Reference: [JA], 1.3
00349     if (val == 0)
00350       return 0;
00351     else
00352     {
```

```
00353       index_t result = 1;
00354       while (val &= val-1)
00355         ++result;
00356       return result;
00357     }
00358   }
00359
00361   template<const index_t LO, const index_t HI>
00362   inline
00363   auto
00364   index_set<LO,HI>::
00365   count_neg() const -> index_t
00366   {
00367     static const index_set_t lo_mask = bitset_t((1UL << -LO) - 1UL);
00368     const index_set_t neg_part = *this & lo_mask;
00369     return neg_part.count();
00370   }
00371
00373   template<const index_t LO, const index_t HI>
00374   inline
00375   auto
00376   index_set<LO,HI>::
00377   count_pos() const -> index_t
00378   {
00379     const auto* pthis = static_cast<const bitset_t*>(this);
00380     const index_set_t pos_part = *pthis >> -LO;
00381     return pos_part.count();
00382   }
00383
00384 #if (_GLUCAT_BITS_PER_ULONG == 64)
00386   template<const index_t LO, const index_t HI>
00387   inline
00388   auto
00389   index_set<LO,HI>::
00390   min() const -> index_t
00391   {
00392     // Reference: [JA], 1.3
00393     unsigned long val = bitset_t::to_ulong();
00394     if (val == 0)
00395       return 0;
00396     else
00397     {
00398       val -= val & (val-1); // isolate lowest bit
00399
00400       index_t idx = 0;
00401       const index_t nbits = HI - LO;
00402
00403       if (nbits > 8)
00404       {
00405         if (val & 0xffffffff00000000ul)
00406           idx += 32;
00407         if (val & 0xffff0000ffff0000ul)
00408           idx += 16;
00409         if (val & 0xff00ff00ff00ff00ul)
00410           idx +=  8;
00411       }
00412         if (val & 0xf0f0f0f0f0f0f0f0ul)
00413           idx +=  4;
00414         if (val & 0xccccccccccccccccul)
00415           idx +=  2;
00416         if (val & 0xaaaaaaaaaaaaaaaaul)
00417           idx +=  1;
00418
00419       return idx + ((idx < -LO) ? LO : LO+1);
00420     }
00421   }
00422 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00424   template<const index_t LO, const index_t HI>
00425   inline
00426   index_t
00427   index_set<LO,HI>::
00428   min() const
00429   {
00430     // Reference: [JA], 1.3
00431     unsigned long val = bitset_t::to_ulong();
00432     if (val == 0)
00433       return 0;
00434     else
00435     {
00436       val -= val & (val-1); // isolate lowest bit
00437
00438       index_t idx = 0;
00439       const index_t nbits = HI - LO;
00440       if (nbits > 8)
00441       {
00442         if (val & 0xffff0000ul)
00443           idx += 16;
```

```
00444         if (val & 0xff00ff00ul)
00445           idx +=  8;
00446       }
00447         if (val & 0xf0f0f0f0ul)
00448           idx +=  4;
00449         if (val & 0xccccccccul)
00450           idx +=  2;
00451         if (val & 0xaaaaaaaaul)
00452           idx +=  1;
00453
00454       return idx + ((idx < -LO) ? LO : LO+1);
00455     }
00456   }
00457 #else
00459   template<const index_t LO, const index_t HI>
00460   auto
00461   index_set<LO,HI>::
00462   min() const -> index_t
00463   {
00464     for (auto
00465         idx = LO;
00466         idx != 0;
00467         ++idx)
00468       if (this->test(idx))
00469         return idx;
00470     for (auto
00471         idx = index_t(1);
00472         idx <= HI;
00473         ++idx)
00474       if (this->test(idx))
00475         return idx;
00476     return 0;
00477   }
00478 #endif
00479
00480 #if (_GLUCAT_BITS_PER_ULONG == 64)
00482   template<const index_t LO, const index_t HI>
00483   inline
00484   auto
00485   index_set<LO,HI>::
00486   max() const -> index_t
00487   {
00488     // Reference: [JA], 1.6
00489     auto val = bitset_t::to_ulong();
00490     if (val == 0)
00491       return 0;
00492     else
00493     {
00494       auto idx = index_t(0);
00495       const auto nbits = HI - LO;
00496       if (nbits > 8)
00497       {
00498         if (val & 0xffffffff00000000ul)
00499           { val >>= 32; idx += 32; }
00500         if (val & 0x00000000ffff0000ul)
00501           { val >>= 16; idx += 16; }
00502         if (val & 0x000000000000ff00ul)
00503           { val >>=  8; idx +=  8; }
00504       }
00505         if (val & 0x00000000000000f0ul)
00506           { val >>=  4; idx +=  4; }
00507         if (val & 0x000000000000000cul)
00508           { val >>=  2; idx +=  2; }
00509         if (val & 0x0000000000000002ul)
00510           {             idx +=  1; }
00511       return idx + ((idx < -LO) ? LO : LO+1);
00512     }
00513   }
00514 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00516   template<const index_t LO, const index_t HI>
00517   inline
00518   auto
00519   index_set<LO,HI>::
00520   max() const -> index_t
00521   {
00522     // Reference: [JA], 1.6
00523     auto val = bitset_t::to_ulong();
00524     if (val == 0)
00525       return 0;
00526     else
00527     {
00528       auto idx = index_t(0);
00529       const auto nbits = HI - LO;
00530       if (nbits > 8)
00531       {
00532         if (val & 0xffff0000ul)
00533           { val >>= 16; idx += 16; }
```

```
00534         if (val & 0x0000ff00ul)
00535           { val >>=  8; idx +=  8; }
00536       }
00537         if (val & 0x000000f0ul)
00538           { val >>=  4; idx +=  4; }
00539         if (val & 0x00000000cul)
00540           { val >>=  2; idx +=  2; }
00541         if (val & 0x00000002ul)
00542           {             idx +=  1; }
00543       return idx + ((idx < -LO) ? LO : LO+1);
00544     }
00545   }
00546 #else
00548   template<const index_t LO, const index_t HI>
00549   auto
00550   index_set<LO,HI>::
00551   max() const -> index_t
00552   {
00553     for (auto
00554         idx = HI;
00555         idx != 0;
00556         --idx)
00557       if (this->test(idx))
00558         return idx;
00559     for (auto
00560         idx = index_t(-1);
00561         idx >= LO;
00562         --idx)
00563       if (this->test(idx))
00564         return idx;
00565     return 0;
00566   }
00567 #endif
00568
00570   //  eg. {3,4,5} is less than {3,7,8}
00571   template<const index_t LO, const index_t HI>
00572   inline
00573   auto
00574   compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int
00575   {
00576     return (a == b)
00577            ? 0
00578            : a.lex_less_than(b)
00579              ? -1
00580              :  1;
00581   }
00582
00584   //  eg. {3,4,5} is less than {3,7,8}
00585   template<const index_t LO, const index_t HI>
00586   inline
00587   auto
00588   index_set<LO,HI>::
00589   lex_less_than(const index_set_t rhs) const -> bool
00590   { return bitset_t::to_ulong() < rhs.bitset_t::to_ulong(); }
00591
00593   // Order by count, then order lexicographically within the equivalence class of count.
00594   template<const index_t LO, const index_t HI>
00595   inline
00596   auto
00597   index_set<LO,HI>::
00598   operator< (const index_set_t rhs) const -> bool
00599   {
00600     const auto this_grade = this->count();
00601     const auto rhs_grade  = rhs.count();
00602     return (this_grade < rhs_grade)
00603            ? true
00604            : (this_grade > rhs_grade)
00605              ? false
00606              : this->lex_less_than(rhs);
00607   }
00608
00610   template<const index_t LO, const index_t HI>
00611   auto
00612   operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&
00613   {
00614     index_t i;
00615     os << '{';
00616     for (i = LO;
00617         (i <= HI) && !(ist[i]);
00618         ++i)
00619     { }
00620     if (i <= HI)
00621       os << i;
00622     for (++i;
00623         i <= HI;
00624         ++i)
00625       if (ist[i])
```

```
00626         os « ',' « i;
00627     os « '}';
00628     return os;
00629   }
00630
00632   template<const index_t LO, const index_t HI>
00633   auto
00634   operator» (std::istream& s, index_set<LO,HI>& ist) -> std::istream&
00635   {
00636     // Parsing variables.
00637     auto i = index_t(0);
00638     using index_set_t = index_set<LO,HI>;
00639     auto local_ist = index_set_t();
00640     // Parsing control variables.
00641     auto parse_index_list = true;
00642     auto expect_closing_brace = false;
00643     auto expect_index = false;
00644     // Parse an optional opening brace.
00645     auto c = s.peek();
00646     // If there is a failure or end of file, this ends parsing.
00647     if (!s.good())
00648       parse_index_list = false;
00649     else
00650     { // Check for an opening brace.
00651       expect_closing_brace = (c == int('{'));
00652       if (expect_closing_brace)
00653       { // Consume the opening brace.
00654         s.get();
00655         // The next character may be a closing brace,
00656         // indicating the empty index set.
00657         c = s.peek();
00658         if (s.good() && (c == int('}')))
00659         { // A closing brace has been parsed and is no longer expected.
00660           expect_closing_brace = false;
00661           // Consume the closing brace.
00662           s.get();
00663           // This ends parsing.
00664           parse_index_list = false;
00665         }
00666       }
00667     }
00668     if (s.good() && parse_index_list)
00669     { // Parse an optional index list.
00670       // The index list starts with a first index.
00671       for (s » i;
00672           !s.fail();
00673           s » i)
00674       { // An index has been parsed. Check to see if it is in range.
00675         if ((i < LO) || (i > HI))
00676         { // An index out of range is a failure.
00677           s.clear(std::istream::failbit);
00678           break;
00679         }
00680         // Add the index to the index set local_ist.
00681         local_ist.set(i);
00682         // Immediately after parsing an index, an index is no longer expected.
00683         expect_index = false;
00684         // Reading the index may have resulted in an end of file condition.
00685         // If so, this ends the index list.
00686         if (s.eof())
00687           break;
00688         // The index list continues with a comma, and
00689         // may be ended by a closing brace, if it was begun with an opening brace.
00690         // Parse a possible comma or closing brace.
00691         c = s.peek();
00692         if (!s.good())
00693           break;
00694         // First, test for a closing brace, if expected.
00695         if (expect_closing_brace && (c == int('}')))
00696         { // Consume the closing brace.
00697           s.get();
00698           // Immediately after parsing the closing brace, it is no longer expected.
00699           expect_closing_brace = false;
00700           // A closing brace ends the index list.
00701           break;
00702         }
00703         // Now test for a comma.
00704         if (c == int(','))
00705         { // Consume the comma.
00706           s.get();
00707           // A index is expected after the comma.
00708           expect_index = true;
00709         }
00710         else
00711         { // Any other character here is a failure.
00712           s.clear(std::istream::failbit);
00713           break;
```

```
00714         }
00715       }
00716     }
00717     // If an index or a closing brace is still expected, this is a failure.
00718     if (expect_index || expect_closing_brace)
00719       s.clear(std::istream::failbit);
00720     // End of file is not a failure.
00721     if (s)
00722     { // The index set has been successfully parsed.
00723       ist = local_ist;
00724     }
00725     return s;
00726   }
00727
00729   template<const index_t LO, const index_t HI>
00730   inline
00731   auto
00732   index_set<LO,HI>::
00733   is_contiguous () const -> bool
00734   {
00735     const auto min_index = this->min();
00736     const auto max_index = this->max();
00737     return (min_index < 0 && max_index > 0)
00738          ?  max_index - min_index == this->count()
00739          : (min_index == 1 || max_index == -1) &&
00740            (max_index - min_index == this->count() - 1);
00741   }
00742
00744   template<const index_t LO, const index_t HI>
00745   inline
00746   auto
00747   index_set<LO,HI>::
00748   fold() const -> const
00749   index_set<LO,HI>
00750   { return this->fold(*this, true); }
00751
00753   template<const index_t LO, const index_t HI>
00754   auto
00755   index_set<LO,HI>::
00756   fold(const index_set_t frm, const bool prechecked) const -> const
00757   index_set<LO,HI>
00758   {
00759     if (!prechecked && ((*this | frm) != frm))
00760       throw error_t("fold(frm): cannot fold from outside of frame");
00761     const auto frm_min = frm.min();
00762     const auto frm_max = frm.max();
00763     auto result = index_set_t();
00764     auto fold_idx = index_t(-1);
00765     for (auto
00766         unfold_idx = fold_idx;
00767         unfold_idx >= frm_min;
00768         --unfold_idx)
00769       if (frm.test(unfold_idx))
00770         // result.set(fold_idx--, this->test(unfold_idx));
00771       {
00772         if (this->test(unfold_idx))
00773           result.set(fold_idx);
00774         --fold_idx;
00775       }
00776     fold_idx = index_t(1);
00777     for (auto
00778         unfold_idx = fold_idx;
00779         unfold_idx <= frm_max;
00780         ++unfold_idx)
00781       if (frm.test(unfold_idx))
00782         // result.set(fold_idx++, this->test(unfold_idx));
00783       {
00784         if (this->test(unfold_idx))
00785           result.set(fold_idx);
00786         ++fold_idx;
00787       }
00788     return result;
00789   }
00790
00792   template<const index_t LO, const index_t HI>
00793   auto
00794   index_set<LO,HI>::
00795   unfold(const index_set_t frm, const bool prechecked) const -> const index_set_t
00796   {
00797     const char* msg =
00798       "unfold(frm): cannot unfold into a smaller frame";
00799     const auto frm_min = frm.min();
00800     const auto frm_max = frm.max();
00801     auto result = index_set_t();
00802     auto fold_idx = index_t(-1);
00803     for (auto
00804         unfold_idx = fold_idx;
```

```
00805         unfold_idx >= frm_min;
00806         --unfold_idx)
00807       if (frm.test(unfold_idx))
00808         if (this->test(fold_idx--))
00809           result.set(unfold_idx);
00810     if (!prechecked && ((fold_idx+1) > this->min()))
00811       throw error_t(msg);
00812     fold_idx = index_t(1);
00813     for (auto
00814         unfold_idx = fold_idx;
00815         unfold_idx <= frm_max;
00816         ++unfold_idx)
00817       if (frm.test(unfold_idx))
00818         if (this->test(fold_idx++))
00819           result.set(unfold_idx);
00820     if (!prechecked && ((fold_idx-1) < this->max()))
00821       throw error_t(msg);
00822     return result;
00823   }
00824
00826   template<const index_t LO, const index_t HI>
00827   inline
00828   auto
00829   index_set<LO,HI>::
00830   value_of_fold(const index_set_t frm) const -> set_value_t
00831   {
00832     const auto min_index = frm.fold().min();
00833     if (min_index == 0)
00834       return 0;
00835     else
00836     {
00837       const auto folded_set = this->fold(frm);
00838       const auto skip = min_index > 0 ? index_t(1) : index_t(0);
00839       return folded_set.bitset_t::to_ulong() » (min_index-LO-skip);
00840     }
00841   }
00842
00844   inline
00845   static
00846   auto inverse_reversed_gray(unsigned long x) -> unsigned long
00847   {
00848     // Reference: [JA]
00849 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00850     x ^= x « 32; // for 64-bit words
00851 #endif
00852     x ^= x « 16; // reversed_gray ** 16
00853     x ^= x «  8; // reversed_gray **  8
00854     x ^= x «  4; // reversed_gray **  4
00855     x ^= x «  2; // reversed_gray **  2
00856     x ^= x «  1; // reversed_gray **  1
00857     return x;
00858   }
00859
00861   inline
00862   static
00863   auto inverse_gray(unsigned long x) -> unsigned long
00864   {
00865     // Reference: [JA]
00866 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00867     x ^= x » 32; // for 64-bit words
00868 #endif
00869     x ^= x » 16; // gray ** 16
00870     x ^= x »  8; // gray **  8
00871     x ^= x »  4; // gray **  4
00872     x ^= x »  2; // gray **  2
00873     x ^= x »  1; // gray **  1
00874     return x;
00875   }
00876
00878   template<const index_t LO, const index_t HI>
00879   auto
00880   index_set<LO,HI>::
00881   sign_of_mult(const index_set_t rhs) const -> int
00882   {
00883     // Implemented using Walsh functions and Gray codes.
00884     // Reference: [L] Chapter 21, 21.3
00885     // Reference: [JA]
00886     const auto uthis = this->bitset_t::to_ulong();
00887     const auto urhs  =   rhs.bitset_t::to_ulong();
00888     const auto nbits = HI - LO;
00889     auto negative = 0UL;
00890     if (nbits > 8)
00891     {
00892       // Set h to be the inverse reversed Gray code of rhs.
00893       // This sets each bit of h to be the cumulative ^ of
00894       // the same and lower bits of rhs.
00895       const auto h = inverse_reversed_gray(urhs);
```

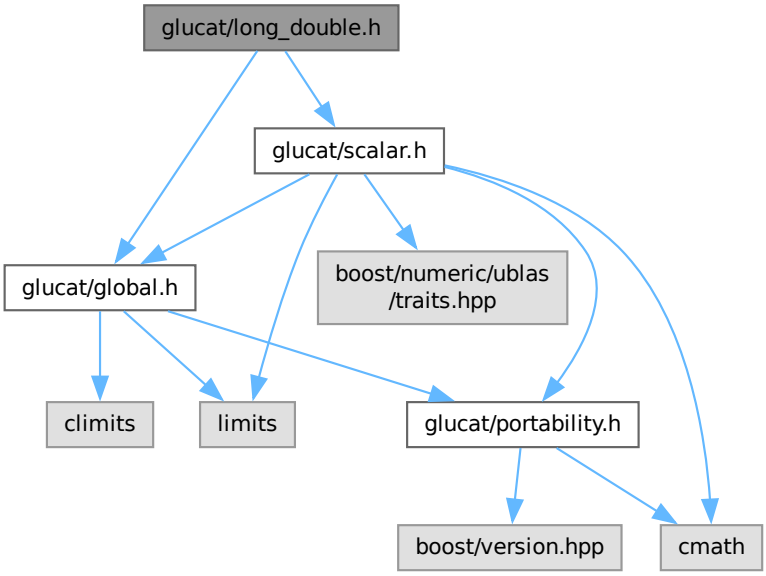
```
00896       // Set k to be the inverse Gray code of *this & h.
00897       // This sets the low bit of k to be parity(*this & h).
00898       const auto k = inverse_gray(uthis & h);
00899       // Set q to be the inverse Gray code of the positive part of *this & rhs.
00900       const auto q = inverse_gray((uthis & urhs) » -LO);
00901       negative = k ^ q;
00902     }
00903     else
00904     {
00905       auto h = 0UL;
00906       for (auto
00907           j = index_t(0);
00908           j < -LO;
00909           ++j)
00910       {
00911         h ^= urhs » j;
00912         negative ^= h & (uthis » j);
00913       }
00914       for (auto
00915           j = index_t(-LO);
00916           j < nbits;
00917           ++j)
00918       {
00919         negative ^= h & (uthis » j);
00920         h ^= urhs » j;
00921       }
00922     }
00923     return 1 - int((negative & 1) « 1);
00924   }
00925
00927   template<const index_t LO, const index_t HI>
00928   inline
00929   auto
00930   index_set<LO,HI>::
00931   sign_of_square() const -> int
00932   {
00933     auto result = 1 - int((this->count_neg() % 2) « 1);
00934     switch (this->count() % 4)
00935     {
00936       case 2:
00937       case 3:
00938         result *= -1;
00939         break;
00940       default:
00941         break;
00942     }
00943     return result;
00944   }
00945
00947   template<const index_t LO, const index_t HI>
00948   inline
00949   auto
00950   index_set<LO,HI>::
00951   hash_fn() const -> size_t
00952   {
00953     static const auto lo_mask = (1UL « -LO) - 1UL;
00954     const auto uthis = bitset_t::to_ulong();
00955     const auto neg_part = uthis & lo_mask;
00956     const auto pos_part = uthis » -LO;
00957     return size_t(neg_part ^ pos_part);
00958   }
00959
00961   inline
00962   auto
00963   sign_of_square(index_t j) -> int
00964   { return (j < 0) ? -1 : 1; }
00965
00967   template<const index_t LO, const index_t HI>
00968   inline
00969   auto
00970   min_neg(const index_set<LO,HI>& ist) -> index_t
00971   { return std::min(ist.min(), 0); }
00972
00974   template<const index_t LO, const index_t HI>
00975   inline
00976   auto
00977   max_pos(const index_set<LO,HI>& ist) -> index_t
00978   { return std::max(ist.max(), 0); }
00979
00980 // index_set reference
00981
00983   template<const index_t LO, const index_t HI>
00984   inline
00985   index_set<LO,HI>::reference::
00986   reference( index_set_t& ist, index_t idx ) :
00987     m_pst(&ist),
00988     m_idx(idx)
```

```
00989   { }
00990
00992   template<const index_t LO, const index_t HI>
00993   inline
00994   auto
00995   index_set<LO,HI>::reference::
00996   operator== (const reference& c_j) const -> bool
00997   { return m_pst == c_j.m_pst && m_idx == c_j.m_idx; }
00998
01000   template<const index_t LO, const index_t HI>
01001   inline
01002   auto
01003   index_set<LO,HI>::reference::
01004   operator= (bool x) -> reference&
01005   {
01006     if ( x )
01007       m_pst->set(m_idx);
01008     else
01009       m_pst->reset(m_idx);
01010     return *this;
01011   }
01012
01014   template<const index_t LO, const index_t HI>
01015   inline
01016   auto
01017   index_set<LO,HI>::reference::
01018   operator= (const reference& c_j) -> reference&
01019   {
01020     if (&c_j != this && c_j != *this)
01021     {
01022       if ( (c_j.m_pst)[c_j.m_idx] )
01023         m_pst->set(m_idx);
01024       else
01025         m_pst->reset(m_idx);
01026     }
01027     return *this;
01028   }
01029
01031   template<const index_t LO, const index_t HI>
01032   inline
01033   auto
01034   index_set<LO,HI>::reference::
01035   operator~ () const -> bool
01036   { return !(m_pst->test(m_idx)); }
01037
01039   template<const index_t LO, const index_t HI>
01040   inline
01041   index_set<LO,HI>::reference::
01042   operator bool () const
01043   { return m_pst->test(m_idx); }
01044
01046   template<const index_t LO, const index_t HI>
01047   inline
01048   auto
01049   index_set<LO,HI>::reference::
01050   flip() -> reference&
01051   {
01052     m_pst->flip(m_idx);
01053     return *this;
01054   }
01055 }
01056 #endif // _GLUCAT_INDEX_SET_IMP_H
```

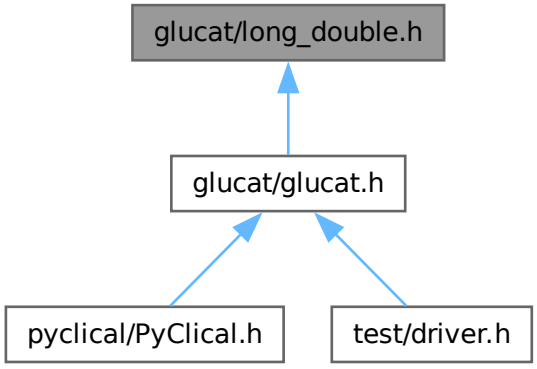
9.29 glucat/long_double.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
```

Include dependency graph for long_double.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Variables

- static const long double glucat::l_pi = 3.14159265358979323846264338327950291
- static const long double glucat::l_ln2 = 0.69314718055994530941723212145817661

9.30 long_double.h

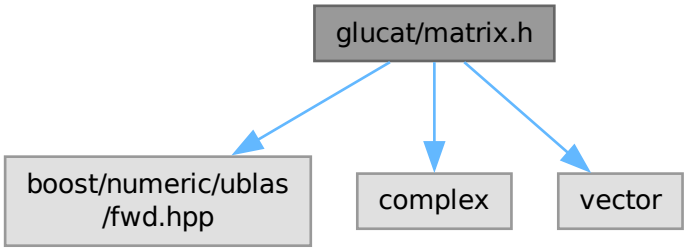
Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_LONG_DOUBLE_H
00002 #define _GLUCAT_LONG_DOUBLE_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     long_double.h : Define std functions for long double
00006                             -------------------
00007     begin                : 2001-12-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ***************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 namespace glucat
00038 {
00039 #if defined(__USE_GNU)
00040   static const long double l_pi   = M_PIl;
00041   static const long double l_ln2 = M_LN2l;
00042 #else
00043   static const long double l_pi = 3.14159265358979323846264338327950 29L;
00044   static const long double l_ln2 = 0.69314718055994530941723212145817 66L;
00045 #endif
00046
00048   template<>
00049   inline
00050   auto
00051   numeric_traits<long double>::
00052   pi() -> long double
00053   { return l_pi; }
00054
00056   template<>
00057   inline
00058   auto
00059   numeric_traits<long double>::
00060   ln_2() -> long double
00061   { return l_ln2; }
00062 }
00063 #endif // _GLUCAT_LONG_DOUBLE_H
```

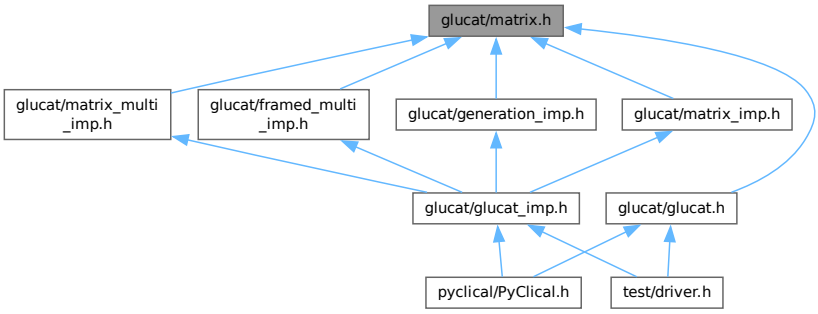
9.31 glucat/matrix.h File Reference

```
#include <boost/numeric/ublas/fwd.hpp>
#include <complex>
#include <vector>
```

Include dependency graph for matrix.h:



This graph shows which files directly or indirectly include this file:



Classes

- struct glucat::matrix::eig_genus< Matrix_T >

  Structure containing classification of eigenvalues.

Namespaces

- namespace glucat
- namespace glucat::matrix

Typedefs

- using glucat::matrix::eig_case_t

  Classification of eigenvalues of a matrix.

Functions

- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Kronecker tensor product of matrices - as per Matlab kron.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Sparse Kronecker tensor product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T

  Left inverse of Kronecker product.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Left inverse of Kronecker product where lhs is a signed permutation matrix.
- template<typename Matrix_T>
  auto glucat::matrix::nnz (const Matrix_T &m) -> typename Matrix_T::size_type

  Number of non-zeros.
- template<typename Matrix_T>
  auto glucat::matrix::isinf (const Matrix_T &m) -> bool

  Infinite.
- template<typename Matrix_T>
  auto glucat::matrix::isnan (const Matrix_T &m) -> bool

  Not a Number.
- template<typename Matrix_T>
  auto glucat::matrix::unit (const typename Matrix_T::size_type n) -> const Matrix_T

  Unit matrix - as per Matlab eye.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::mono_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↩ expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::sparse_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↩ expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of sparse matrices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of matrices.
- template<typename Scalar_T, typename LHS_T, typename RHS_T>
  auto glucat::matrix::inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T

  Inner product: sum(x(i,j)∗y(i,j))/x.nrows().
- template<typename Matrix_T>
  auto glucat::matrix::norm_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type

  Square of Frobenius norm.
- template<typename Matrix_T>
  auto glucat::matrix::trace (const Matrix_T &val) -> typename Matrix_T::value_type

  Matrix trace.
- template<typename Matrix_T>
  auto glucat::matrix::eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >

  Eigenvalues of a matrix.
- template<typename Matrix_T>
  auto glucat::matrix::classify_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >

  Classify the eigenvalues of a matrix.

9.32 matrix.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_MATRIX_H
00002 #define _GLUCAT_MATRIX_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix.h : Declare common matrix functions
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                            : uBLAS interface contributed by Joerg Walter
00010  ***************************************************************************
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025  ***************************************************************************
00026  This library is based on a prototype written by Arvind Raja and was
00027  licensed under the LGPL with permission of the author. See Arvind Raja,
00028  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029  in Ablamowicz, Lounesto and Parra (eds.)
00030  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031  ***************************************************************************
00032      See also Arvind Raja's original header comments in glucat.h
00033  ***************************************************************************/
00034
00035 #include <boost/numeric/ublas/fwd.hpp>
00036
00037 #include <complex>
00038 #include <vector>
00039
00040 namespace glucat
00041 {
00042   namespace ublas = boost::numeric::ublas;
00043
00044   namespace matrix
00045   {
00047     template< typename LHS_T, typename RHS_T >
00048     auto
00049     kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00050     RHS_T;
00051
00053     template< typename LHS_T, typename RHS_T >
00054     auto
00055     mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00056     RHS_T;
00057
00059     template< typename LHS_T, typename RHS_T >
00060     auto
00061     nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono = true) -> const
00062     RHS_T;
00063
00065     template< typename LHS_T, typename RHS_T >
00066     auto
00067     signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00068     RHS_T;
00069
00071     template< typename Matrix_T >
00072     auto
00073     nnz(const Matrix_T& m) -> typename Matrix_T::size_type;
00074
00076     template< typename Matrix_T >
00077     auto
00078     isinf(const Matrix_T& m) -> bool;
00079
00081     template< typename Matrix_T >
00082     auto
00083     isnan(const Matrix_T& m) -> bool;
00084
00086     template< typename Matrix_T >
00087     auto
00088     unit(const typename Matrix_T::size_type n) -> const
00089     Matrix_T;
00090
```

```
00092     template< typename LHS_T, typename RHS_T >
00093     auto
00094     mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00095               const ublas::matrix_expression<RHS_T>& rhs) -> const
00096     typename RHS_T::expression_type;
00097
00099     template< typename LHS_T, typename RHS_T >
00100     auto
00101     sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00102                 const ublas::matrix_expression<RHS_T>& rhs) -> const
00103     typename RHS_T::expression_type;
00104
00106     template< typename LHS_T, typename RHS_T >
00107     auto
00108     prod(const ublas::matrix_expression<LHS_T>& lhs,
00109          const ublas::matrix_expression<RHS_T>& rhs) -> const
00110     typename RHS_T::expression_type;
00111
00113     template< typename Scalar_T, typename LHS_T, typename RHS_T >
00114     auto
00115     inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T;
00116
00118     template< typename Matrix_T >
00119     auto
00120     norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type;
00121
00123     template< typename Matrix_T >
00124     auto
00125     trace(const Matrix_T& val) -> typename Matrix_T::value_type;
00126
00128     template< typename Matrix_T >
00129     auto
00130     eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >;
00131
00133     using eig_case_t = enum {
00134       safe_eigs,
00135       neg_real_eigs,
00136       both_eigs};
00137
00139     template< typename Matrix_T >
00140     struct eig_genus
00141     {
00142       using Scalar_T = typename Matrix_T::value_type;
00144       bool m_is_singular = false;
00146       eig_case_t m_eig_case = safe_eigs;
00148       Scalar_T   m_safe_arg = Scalar_T(0);
00149     };
00150
00152     template< typename Matrix_T >
00153     auto
00154     classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>;
00155   }
00156 }
00157
00158 #endif  // _GLUCAT_MATRIX_H
```

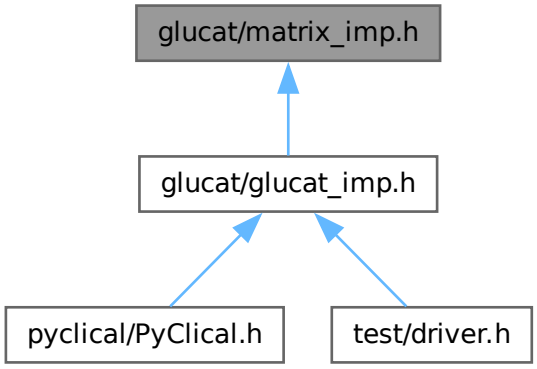
9.33 glucat/matrix_imp.h File Reference

```
#include "glucat/errors.h"
#include "glucat/scalar.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/vector.hpp>
#include <boost/numeric/ublas/vector_proxy.hpp>
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <blaze/Math.h>
#include <blaze/math/DynamicMatrix.h>
#include <blaze/math/DynamicVector.h>
#include <set>
```

```
#include <vector>
```

Include dependency graph for matrix_imp.h:

This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat
- namespace glucat::matrix

Functions

- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Kronecker tensor product of matrices - as per Matlab kron.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

  Sparse Kronecker tensor product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  void glucat::matrix::nork_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)

  Utility routine for nork: calculate result for a range of indices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T

  Left inverse of Kronecker product.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T

Left inverse of Kronecker product where lhs is a signed permutation matrix.

- template<typename Matrix_T>
  auto glucat::matrix::nnz (const Matrix_T &m) -> typename Matrix_T::size_type

  Number of non-zeros.
- template<typename Matrix_T>
  auto glucat::matrix::isinf (const Matrix_T &m) -> bool

  Infinite.
- template<typename Matrix_T>
  auto glucat::matrix::isnan (const Matrix_T &m) -> bool

  Not a Number.
- template<typename Matrix_T>
  auto glucat::matrix::unit (const typename Matrix_T::size_type n) -> const Matrix_T

  Unit matrix - as per Matlab eye.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::mono_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↩ expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of monomial matrices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::sparse_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↩ expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of sparse matrices.
- template<typename LHS_T, typename RHS_T>
  auto glucat::matrix::prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression< RHS_T > &rhs) -> const typename RHS_T::expression_type

  Product of matrices.
- template<typename Scalar_T, typename LHS_T, typename RHS_T>
  auto glucat::matrix::inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T

  Inner product: sum(x(i,j)∗y(i,j))/x.nrows().
- template<typename Matrix_T>
  auto glucat::matrix::norm_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type

  Square of Frobenius norm.
- template<typename Matrix_T>
  auto glucat::matrix::trace (const Matrix_T &val) -> typename Matrix_T::value_type

  Matrix trace.
- template<typename Matrix_T>
  static auto glucat::matrix::to_blaze (const Matrix_T &val) -> blaze::DynamicMatrix< double, blaze::rowMajor >

  Convert matrix to Blaze format.
- template<typename Matrix_T>
  auto glucat::matrix::eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >

  Eigenvalues of a matrix.
- template<typename Matrix_T>
  auto glucat::matrix::classify_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >

  Classify the eigenvalues of a matrix.

9.34 matrix_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_MATRIX_IMP_H
00002 #define _GLUCAT_MATRIX_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_imp.h : Implement common matrix functions
```

```
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                          : uBLAS interface contributed by Joerg Walter
00010  ***************************************************************************
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025  ***************************************************************************
00026  This library is based on a prototype written by Arvind Raja and was
00027  licensed under the LGPL with permission of the author. See Arvind Raja,
00028  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029  in Ablamowicz, Lounesto and Parra (eds.)
00030  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031  ***************************************************************************
00032      See also Arvind Raja's original header comments in glucat.h
00033  ***************************************************************************/
00034
00035 #include "glucat/errors.h"
00036 #include "glucat/scalar.h"
00037 #include "glucat/matrix.h"
00038
00039 # if  defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00040 #  pragma GCC diagnostic push
00041 #  pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00042 # endif
00043 # if  defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00044 #  include <boost/serialization/array_wrapper.hpp>
00045 # endif
00046 #include <boost/numeric/ublas/vector.hpp>
00047 #include <boost/numeric/ublas/vector_proxy.hpp>
00048 #include <boost/numeric/ublas/matrix.hpp>
00049 #include <boost/numeric/ublas/matrix_expression.hpp>
00050 #include <boost/numeric/ublas/matrix_proxy.hpp>
00051 #include <boost/numeric/ublas/matrix_sparse.hpp>
00052 #include <boost/numeric/ublas/operation.hpp>
00053 #include <boost/numeric/ublas/operation_sparse.hpp>
00054
00055 #if defined(_GLUCAT_USE_BLAZE)
00056 #include <blaze/Math.h>
00057 #include <blaze/math/DynamicMatrix.h>
00058 #include <blaze/math/DynamicVector.h>
00059 #endif
00060
00061 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00062 #  pragma GCC diagnostic pop
00063 # endif
00064
00065 #include <set>
00066 #include <vector>
00067
00068 namespace glucat { namespace matrix
00069 {
00070   // References for algorithms:
00071   // [v]: C. F. van Loan and N. Pitsianis, "Approximation with Kronecker products",
00072   // in Linear Algebra for Large Scale and Real-Time Applications, Marc S. Moonen,
00073   // Gene H. Golub, and Bart L. R. Moor (eds.), 1993, pp. 293--314.
00074
00076   template< typename LHS_T, typename RHS_T >
00077   auto
00078   kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00079   RHS_T
00080   {
00081     const auto rhs_s1 = rhs.size1();
00082     const auto rhs_s2 = rhs.size2();
00083     auto result = RHS_T(lhs.size1()*rhs_s1, lhs.size2()*rhs_s2);
00084     result.clear();
00085
00086     for (auto
00087         lhs_it1 = lhs.begin1();
00088         lhs_it1 != lhs.end1();
00089         ++lhs_it1)
00090       for (auto
00091           lhs_it2 = lhs_it1.begin();
00092           lhs_it2 != lhs_it1.end();
00093           ++lhs_it2)
```

```
00094       {
00095         const auto start1 = rhs_s1 * lhs_it2.index1();
00096         const auto start2 = rhs_s2 * lhs_it2.index2();
00097         const auto& lhs_val = *lhs_it2;
00098         for (auto
00099             rhs_it1 = rhs.begin1();
00100             rhs_it1 != rhs.end1();
00101             ++rhs_it1)
00102           for (auto
00103               rhs_it2 = rhs_it1.begin();
00104               rhs_it2 != rhs_it1.end();
00105               ++rhs_it2)
00106             result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00107       }
00108     return result;
00109   }
00110
00112   template< typename LHS_T, typename RHS_T >
00113   auto
00114   mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00115   RHS_T
00116   {
00117     const auto rhs_s1 = rhs.size1();
00118     const auto rhs_s2 = rhs.size2();
00119     const auto dim = lhs.size1()*rhs_s1;
00120     auto result = RHS_T(dim, dim, dim);
00121     result.clear();
00122
00123     for (auto
00124         lhs_it1 = lhs.begin1();
00125         lhs_it1 != lhs.end1();
00126         ++lhs_it1)
00127     {
00128       const auto lhs_it2 = lhs_it1.begin();
00129       const auto start1 = rhs_s1 * lhs_it2.index1();
00130       const auto start2 = rhs_s2 * lhs_it2.index2();
00131       const auto& lhs_val = *lhs_it2;
00132       for (auto
00133           rhs_it1 = rhs.begin1();
00134           rhs_it1 != rhs.end1();
00135           ++rhs_it1)
00136       {
00137         const auto rhs_it2 = rhs_it1.begin();
00138         result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00139       }
00140     }
00141     return result;
00142   }
00143
00145   template< typename LHS_T, typename RHS_T >
00146   void
00147   nork_range(RHS_T& result,
00148              const typename LHS_T::const_iterator2 lhs_it2,
00149              const RHS_T& rhs,
00150              const typename RHS_T::size_type res_s1,
00151              const typename RHS_T::size_type res_s2)
00152   {
00153     // Definition matches [v] Section 4, Theorem 4.1.
00154     const auto start1 = res_s1 * lhs_it2.index1();
00155     const auto start2 = res_s2 * lhs_it2.index2();
00156     using ublas::range;
00157     const auto& range1 = range(start1, start1 + res_s1);
00158     const auto& range2 = range(start2, start2 + res_s2);
00159     using matrix_range_t = ublas::matrix_range<const RHS_T>;
00160     const auto& rhs_range = matrix_range_t(rhs, range1, range2);
00161     using Scalar_T = typename RHS_T::value_type;
00162     const auto lhs_val = numeric_traits<Scalar_T>::to_scalar_t(*lhs_it2);
00163     for (auto
00164         rhs_it1 = rhs_range.begin1();
00165         rhs_it1 != rhs_range.end1();
00166         ++rhs_it1)
00167       for (auto
00168           rhs_it2 = rhs_it1.begin();
00169           rhs_it2 != rhs_it1.end();
00170           ++rhs_it2)
00171           result(rhs_it2.index1(), rhs_it2.index2()) += lhs_val * *rhs_it2;
00172    }
00173
00175   template< typename LHS_T, typename RHS_T >
00176   auto
00177   nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono) -> const
00178   RHS_T
00179   {
00180     // nork(A, kron(A, B)) is close to B
00181     // Definition matches [v] Section 4, Theorem 4.1.
00182     const auto lhs_s1 = lhs.size1();
00183     const auto lhs_s2 = lhs.size2();
```

```
00184     const auto rhs_s1 = rhs.size1();
00185     const auto rhs_s2 = rhs.size2();
00186     const auto res_s1 = rhs_s1 / lhs_s1;
00187     const auto res_s2 = rhs_s2 / lhs_s2;
00188     using Scalar_T = typename RHS_T::value_type;
00189     const auto norm_frob2_lhs = norm_frob2(lhs);
00190     if (!mono)
00191     {
00192       using error_t = error<RHS_T>;
00193       if (rhs_s1 == 0)
00194         throw error_t("matrix", "nork: number of rows must not be 0");
00195       if (rhs_s2 == 0)
00196         throw error_t("matrix", "nork: number of cols must not be 0");
00197       if (res_s1 * lhs_s1 != rhs_s1)
00198         throw error_t("matrix", "nork: incompatible numbers of rows");
00199       if (res_s2 * lhs_s2 != rhs_s2)
00200         throw error_t("matrix", "nork: incompatible numbers of cols");
00201       if (norm_frob2_lhs == Scalar_T(0))
00202         throw error_t("matrix", "nork: LHS must not be 0");
00203     }
00204     auto result = RHS_T(res_s1, res_s2);
00205     result.clear();
00206     for (auto
00207         lhs_it1 = lhs.begin1();
00208         lhs_it1 != lhs.end1();
00209         ++lhs_it1)
00210       for (auto
00211           lhs_it2 = lhs_it1.begin();
00212           lhs_it2 != lhs_it1.end();
00213           ++lhs_it2)
00214         if (*lhs_it2  != Scalar_T(0))
00215           nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00216     result /= norm_frob2_lhs;
00217     return result;
00218   }
00219
00221   template< typename LHS_T, typename RHS_T >
00222   auto
00223   signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00224   RHS_T
00225   {
00226     // signed_perm_nork(A, kron(A, B)) is close to B
00227     // Definition matches [v] Section 4, Theorem 4.1.
00228     const auto lhs_s1 = lhs.size1();
00229     const auto lhs_s2 = lhs.size2();
00230     const auto rhs_s1 = rhs.size1();
00231     const auto rhs_s2 = rhs.size2();
00232     const auto res_s1 = rhs_s1 / lhs_s1;
00233     const auto res_s2 = rhs_s2 / lhs_s2;
00234     using Scalar_T = typename RHS_T::value_type;
00235     const auto norm_frob2_lhs = Scalar_T( double(lhs_s1) );
00236     auto result = RHS_T(res_s1, res_s2);
00237     result.clear();
00238     for (auto
00239         lhs_it1 = lhs.begin1();
00240         lhs_it1 != lhs.end1();
00241         ++lhs_it1)
00242     {
00243       const auto lhs_it2 = lhs_it1.begin();
00244       nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00245     }
00246     result /= norm_frob2_lhs;
00247     return result;
00248   }
00249
00251   template< typename Matrix_T >
00252   auto
00253   nnz(const Matrix_T& m) -> typename Matrix_T::size_type
00254   {
00255     using size_t = typename Matrix_T::size_type;
00256     auto result = size_t(0);
00257     for (auto
00258         it1 = m.begin1();
00259         it1 != m.end1();
00260         ++it1)
00261       for (auto& entry : it1)
00262         if (entry != 0)
00263           ++result;
00264     return result;
00265   }
00266
00268   template< typename Matrix_T >
00269   auto
00270   isinf(const Matrix_T& m) -> bool
00271   {
00272     using Scalar_T = typename Matrix_T::value_type;
00273     for (auto
```

```
00274         it1 = m.begin1();
00275         it1 != m.end1();
00276         ++it1)
00277       for (auto& entry : it1)
00278         if (numeric_traits<Scalar_T>::isInf(entry))
00279           return true;
00280
00281     return false;
00282   }
00283
00285   template< typename Matrix_T >
00286   auto
00287   isnan(const Matrix_T& m) -> bool
00288   {
00289     using Scalar_T = typename Matrix_T::value_type;
00290     for (auto
00291         it1 = m.begin1();
00292         it1 != m.end1();
00293         ++it1)
00294       for (auto& entry : it1)
00295         if (numeric_traits<Scalar_T>::isNaN(entry))
00296           return true;
00297
00298     return false;
00299   }
00300
00302   template< typename Matrix_T >
00303   inline
00304   auto
00305   unit(const typename Matrix_T::size_type dim) -> const
00306   Matrix_T
00307   {
00308     using Scalar_T = typename Matrix_T::value_type;
00309     return ublas::identity_matrix<Scalar_T>(dim);
00310   }
00311
00313   template< typename LHS_T, typename RHS_T >
00314   auto
00315   mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00316             const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00317   {
00318     using rhs_expression_t = const RHS_T;
00319     using matrix_row_t = typename ublas::matrix_row<rhs_expression_t>;
00320
00321     const auto dim = lhs().size1();
00322     // The following assumes that RHS_T is a sparse matrix type.
00323     auto result = RHS_T(dim, dim, dim);
00324     for (auto
00325         lhs_row = lhs().begin1();
00326         lhs_row != lhs().end1();
00327         ++lhs_row)
00328     {
00329       const auto& lhs_it = lhs_row.begin();
00330       if (lhs_it != lhs_row.end())
00331       {
00332         const auto& rhs_row = matrix_row_t(rhs(), lhs_it.index2());
00333         const auto& rhs_it = rhs_row.begin();
00334         if (rhs_it != rhs_row.end())
00335           result(lhs_it.index1(), rhs_it.index()) = (*lhs_it) * (*rhs_it);
00336       }
00337     }
00338     return result;
00339   }
00340
00342   template< typename LHS_T, typename RHS_T >
00343   inline
00344   auto
00345   sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00346               const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00347   {
00348     using expression_t = typename RHS_T::expression_type;
00349     return ublas::sparse_prod<expression_t>(lhs(), rhs());
00350   }
00351
00353   template< typename LHS_T, typename RHS_T >
00354   inline
00355   auto
00356   prod(const ublas::matrix_expression<LHS_T>& lhs,
00357        const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00358   {
00359     const auto dim = lhs().size1();
00360     RHS_T result(dim, dim);
00361     ublas::axpy_prod(lhs, rhs, result, true);
00362     return result;
00363   }
00364
00366   template< typename Scalar_T, typename LHS_T, typename RHS_T >
```

```
00367   auto
00368   inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T
00369   {
00370     auto result = Scalar_T(0);
00371     for (auto
00372         lhs_it1 = lhs.begin1();
00373         lhs_it1 != lhs.end1();
00374         ++lhs_it1)
00375       for (auto
00376           lhs_it2 = lhs_it1.begin();
00377           lhs_it2 != lhs_it1.end();
00378           ++lhs_it2)
00379       {
00380         const auto& rhs_val = rhs(lhs_it2.index1(),lhs_it2.index2());
00381         if (rhs_val != Scalar_T(0))
00382           result += (*lhs_it2) * rhs_val;
00383       }
00384     return result / lhs.size1();
00385   }
00386
00388   template< typename Matrix_T >
00389   auto
00390   norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type
00391   {
00392     using Scalar_T = typename Matrix_T::value_type;
00393
00394     auto result = Scalar_T(0);
00395     for (auto
00396         val_it1 = val.begin1();
00397         val_it1 != val.end1();
00398         ++val_it1)
00399       for (auto& val_entry : val_it1)
00400       {
00401         if (numeric_traits<Scalar_T>::isNaN(val_entry))
00402           return numeric_traits<Scalar_T>::NaN();
00403         result += val_entry * val_entry;
00404       }
00405     return result;
00406   }
00407
00409   template< typename Matrix_T >
00410   auto
00411   trace(const Matrix_T& val) -> typename Matrix_T::value_type
00412   {
00413     using Scalar_T = typename Matrix_T::value_type;
00414
00415     auto result = Scalar_T(0);
00416     auto dim = val.size1();
00417     for (auto
00418         ndx = decltype(dim)(0);
00419         ndx != dim;
00420         ++ndx)
00421     {
00422       const Scalar_T crd = val(ndx, ndx);
00423       if (numeric_traits<Scalar_T>::isNaN(crd))
00424         return numeric_traits<Scalar_T>::NaN();
00425       result += crd;
00426     }
00427     return result;
00428   }
00429
00430 #if defined(_GLUCAT_USE_BLAZE)
00432   template< typename Matrix_T >
00433   static
00434   auto
00435   to_blaze(const Matrix_T& val) -> blaze::DynamicMatrix<double,blaze::rowMajor>
00436   {
00437     const auto s1 = val.size1();
00438     const auto s2 = val.size2();
00439
00440     using blaze_matrix_t = typename blaze::DynamicMatrix<double,blaze::rowMajor>;
00441     auto result = blaze_matrix_t(s1, s2);
00442
00443     using Scalar_T = typename Matrix_T::value_type;
00444     using traits_t = numeric_traits<Scalar_T>;
00445
00446     for (auto
00447         val_it1 = val.begin1();
00448         val_it1 != val.end1();
00449         ++val_it1)
00450       for (auto
00451           val_it2 = val_it1.begin();
00452           val_it2 != val_it1.end();
00453           ++val_it2)
00454         result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00455
00456     return result;
```

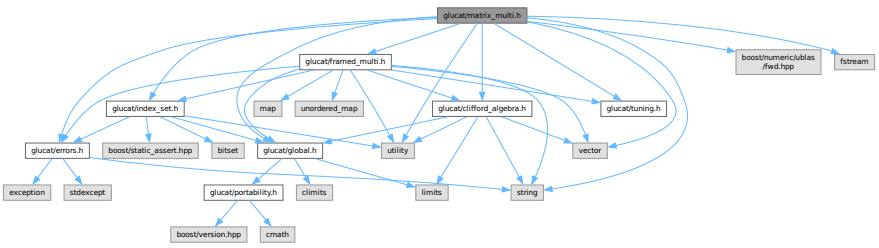
```
00457   }
00458
00459 #endif
00460
00462   template< typename Matrix_T >
00463   auto
00464   eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >
00465   {
00466     using complex_t = std::complex<double>;
00467     using complex_vector_t = typename std::vector<complex_t>;
00468
00469     const auto dim = val.size1();
00470     auto lambda = complex_vector_t(dim);
00471
00472 #if defined(_GLUCAT_USE_BLAZE)
00473     using complex_t = std::complex<double>;
00474     using blaze_complex_vector_t = blaze::DynamicVector<complex_t, blaze::columnVector>;
00475
00476     auto blaze_val = to_blaze(val);
00477     auto blaze_lambda = blaze_complex_vector_t(dim);
00478     blaze::geev(blaze_val, blaze_lambda);
00479
00480     for (auto
00481         k = decltype(dim)(0);
00482         k != dim;
00483         ++k)
00484       lambda[k] = blaze_lambda[k];
00485 #endif
00486     return lambda;
00487   }
00488
00490   template< typename Matrix_T >
00491   auto
00492   classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>
00493   {
00494     using Scalar_T = typename Matrix_T::value_type;
00495     eig_genus<Matrix_T> result;
00496
00497     auto lambda = eigenvalues(val);
00498
00499     std::set<double> arg_set;
00500
00501     const auto dim = lambda.size();
00502     static const auto epsilon =
00503       std::max(std::numeric_limits<double>::epsilon(),
00504                numeric_traits<Scalar_T>::to_double(std::numeric_limits<Scalar_T>::epsilon()));
00505     static const auto zero_eig_tol = 4096.0*epsilon;
00506
00507     bool neg_real_eig_found = false;
00508     bool imag_eig_found = false;
00509     bool zero_eig_found = false;
00510
00511     for (auto
00512         k = decltype(dim)(0);
00513         k != dim;
00514         ++k)
00515     {
00516       const auto lambda_k = lambda[k];
00517       arg_set.insert(std::arg(lambda_k));
00518
00519       const auto real_lambda_k = std::real(lambda_k);
00520       const auto imag_lambda_k = std::imag(lambda_k);
00521       const auto norm_tol = 4096.0*epsilon*std::norm(lambda_k);
00522
00523       if (!neg_real_eig_found &&
00524           real_lambda_k < -epsilon &&
00525          (imag_lambda_k == 0.0 ||
00526           imag_lambda_k * imag_lambda_k < norm_tol))
00527         neg_real_eig_found = true;
00528       if (!imag_eig_found &&
00529           imag_lambda_k > epsilon &&
00530          (real_lambda_k == 0.0 ||
00531           real_lambda_k * real_lambda_k < norm_tol))
00532         imag_eig_found = true;
00533       if (!zero_eig_found &&
00534         std::norm(lambda_k) < zero_eig_tol)
00535         zero_eig_found = true;
00536     }
00537
00538     if (zero_eig_found)
00539         result.m_is_singular = true;
00540
00541     static const auto pi = numeric_traits<double>::pi();
00542     if (neg_real_eig_found)
00543     {
00544       if (imag_eig_found)
00545         result.m_eig_case = both_eigs;
```

```
00546       else
00547       {
00548         result.m_eig_case = neg_real_eigs;
00549         result.m_safe_arg = Scalar_T(-pi / 2.0);
00550       }
00551     }
00552
00553     if (result.m_eig_case == both_eigs)
00554     {
00555       auto arg_it = arg_set.begin();
00556       auto first_arg = *arg_it;
00557       auto best_arg = first_arg;
00558       auto best_diff = 0.0;
00559       auto previous_arg = first_arg;
00560       for (++arg_it;
00561           arg_it != arg_set.end();
00562           ++arg_it)
00563       {
00564         const auto arg_diff = *arg_it - previous_arg;
00565         if (arg_diff > best_diff)
00566         {
00567           best_diff = arg_diff;
00568           best_arg = previous_arg;
00569         }
00570         previous_arg = *arg_it;
00571       }
00572       const auto arg_diff = first_arg + 2.0*pi - previous_arg;
00573       if (arg_diff > best_diff)
00574       {
00575         best_diff = arg_diff;
00576         best_arg = previous_arg;
00577       }
00578       result.m_safe_arg = Scalar_T(pi - (best_arg + best_diff / 2.0));
00579     }
00580     return result;
00581   }
00582 } }
00583
00584 #endif  // _GLUCAT_MATRIX_IMP_H
```

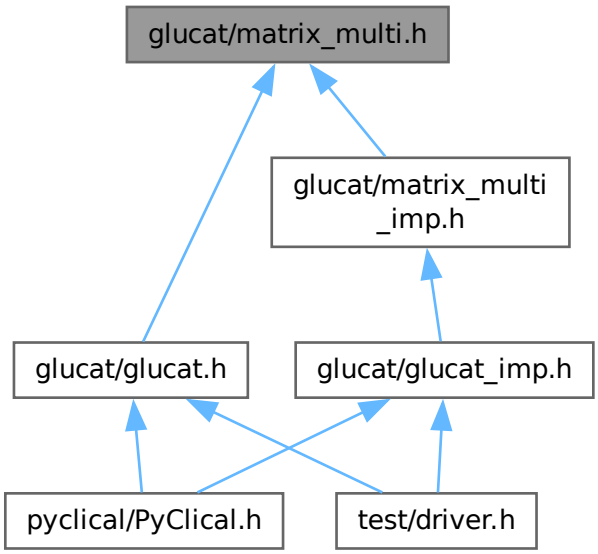
9.35 glucat/matrix_multi.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <fstream>
#include <string>
#include <utility>
#include <vector>
```

Include dependency graph for matrix_multi.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >

  A matrix_multi<Scalar_T,LO,HI,Tune_P> is a matrix approximation to a multivector.
- struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >

  Numeric limits for matrix_multi inherit limits for the corresponding scalar type.

Namespaces

- namespace glucat
- namespace std

Functions

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Geometric product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs_reframed) -> const index_set< LO, HI >

  Find a common frame for operands of a binary operator.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.

9.36 matrix_multi.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_MATRIX_MULTI_H
00002 #define _GLUCAT_MATRIX_MULTI_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi.h : Declare a class for the matrix representation of a multivector
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039 #include "glucat/framed_multi.h"
00040
00041 #include <boost/numeric/ublas/fwd.hpp>
00042
00043 #include <fstream>
00044 #include <string>
00045 #include <utility>
00046 #include <vector>
00047
00048 namespace glucat
00049 {
00050   namespace ublas = boost::numeric::ublas;
00051
00052   // Forward declarations for friends
00053
00054   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00055   class framed_multi;  // forward
00056
00057   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00058   class matrix_multi;  // forward
00059
00061   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00062   auto
00063   operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00064
00066   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00067   auto
00068   operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00069
00071   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00072   auto
00073   operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00074
00076   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00077   auto
00078   operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00079
00081   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00082   auto
00083   star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
      -> Scalar_T;
00084
00086   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00087   auto
00088   operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
```

```
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00089
00091   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092   auto
00093   operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00094
00096   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097   auto
00098   operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00099
00101   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00102   auto
00103   operator« (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00104
00106   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00107   auto
00108   reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs,    const
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00109                  matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>;
00110
00112   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113   auto
00114   sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
    bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00115
00117   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00118   auto
00119   matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
00120               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
00121               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00122
00124   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00125   auto
00126   log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
    bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00127
00129   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00130   auto
00131   matrix_log( const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
00132               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
00133               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00134
00136   template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
    typename Tune_P = tuning<> >
00137   class matrix_multi :
00138   public clifford_algebra< Scalar_T, index_set<LO,HI>, matrix_multi<Scalar_T,LO,HI,Tune_P> >
00139   {
00140   public:
00141     using multivector_t = matrix_multi;
00142     using matrix_multi_t = multivector_t;
00143     using scalar_t = Scalar_T;
00144     using tune_p = Tune_P;
00145     using index_set_t = index_set<LO, HI>;
00146     using term_t = std::pair<const index_set_t, Scalar_T>;
00147     using vector_t = std::vector<Scalar_T>;
00148     using error_t = error<multivector_t>;
00149     using framed_multi_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00150     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
    Other_Tune_P >
00151     friend class framed_multi;
00152     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
    Other_Tune_P >
00153     friend class matrix_multi;
00154
00155   private:
00156     using orientation_t = ublas::row_major;
00157     using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>;
00158     using matrix_t = ublas::matrix<Scalar_T, orientation_t>;
00159     using matrix_index_t = typename matrix_t::size_type;
00160
00161   public:
00163     static auto classname() -> const std::string;
00165     ~matrix_multi() override = default;
00167     matrix_multi();
00169     template< typename Other_Scalar_T >
00170     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00172     template< typename Other_Scalar_T >
00173     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00174                  const index_set_t frm, const bool prechecked = false);
00176     matrix_multi(const multivector_t& val,
00177                  const index_set_t frm, const bool prechecked = false);
00179     matrix_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00181     matrix_multi(const index_set_t ist, const Scalar_T& crd,
00182                  const index_set_t frm, const bool prechecked = false);
00184     matrix_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
```

```
00186     matrix_multi(const int scr, const index_set_t frm = index_set_t());
00188     matrix_multi(const vector_t& vec,
00189                  const index_set_t frm, const bool prechecked = false);
00191     matrix_multi(const std::string& str);
00193     matrix_multi(const std::string& str,
00194                  const index_set_t frm, const bool prechecked = false);
00196     matrix_multi(const char* str)
00197     { *this = matrix_multi(std::string(str)); };
00199     matrix_multi(const char* str,
00200                  const index_set_t frm, const bool prechecked = false)
00201     { *this = matrix_multi(std::string(str), frm, prechecked); };
00203     template< typename Other_Scalar_T >
00204     matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00206     template< typename Other_Scalar_T >
00207     matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00208                  const index_set_t frm, const bool prechecked = false);
00210     auto fast_matrix_multi(const index_set_t frm) const -> const matrix_multi_t;
00212     template< typename Other_Scalar_T >
00213     auto fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00214
00215   private:
00217     template< typename Matrix_T >
00218     matrix_multi(const Matrix_T& mtx, const index_set_t frm);
00220     matrix_multi(const matrix_t& mtx, const index_set_t frm);
00222     auto basis_element(const index_set<LO,HI>& ist) const -> const basis_matrix_t;
00223
00224   public:
00225     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00226
00228     auto     operator= (const multivector_t& rhs) -> multivector_t&;
00229
00231     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t;
00232
00233     // Friend declarations
00234
00235     friend auto
00236       operator* <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00237     friend auto
00238       operator^ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00239     friend auto
00240       operator& <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00241     friend auto
00242       operator% <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00243     friend auto
00244       star      <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> Scalar_T;
00245     friend auto
00246       operator/ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00247     friend auto
00248       operator| <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00249
00250     friend auto
00251       operator» <>(std::istream& s, multivector_t& val) -> std::istream&;
00252     friend auto
00253       operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00254     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
      Other_Tune_P >
00255     friend auto
00256     reframe (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs,    const
      matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs,
00257                  matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs_reframed,
      matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs_reframed) -> const
      index_set<Other_LO,Other_HI>;
00258     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
      Other_Tune_P >
00259     friend auto
00260       matrix_sqrt(const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00261                   const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00262                   const index_t level)
00263                -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00264     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
      Other_Tune_P >
00265     friend auto
00266       matrix_log(const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00267                  const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00268                  const index_t level)
00269               -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00270
00272     auto     operator+= (const term_t& rhs) -> multivector_t&;
00273
00274    private:
00275     // Data members
00276
00278     index_set_t        m_frame;
00280     matrix_t           m_matrix;
00281   };
00282
00283   // Non-members
```

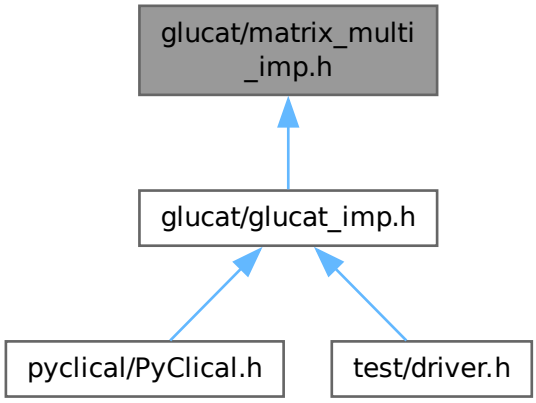
```
00284
00286   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00287   auto
00288   exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00289
00290 }
00291
00292 namespace std
00293 {
00295   template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00296   struct numeric_limits< glucat::matrix_multi<Scalar_T,LO,HI,Tune_P> > :
00297   public numeric_limits<Scalar_T>
00298   { };
00299 }
00300 #endif  // _GLUCAT_MATRIX_MULTI_H
```

9.37 glucat/matrix_multi_imp.h File Reference

```
#include "glucat/matrix_multi.h"
#include "glucat/scalar.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/ublas/triangular.hpp>
#include <boost/numeric/ublas/lu.hpp>
#include <boost/numeric/ublas/io.hpp>
#include <fstream>
#include <iomanip>
#include <array>
#include <iostream>
```

Include dependency graph for matrix_multi_imp.h:

This graph shows which files directly or indirectly include this file:



Classes

- class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >

  Table of basis elements used as a cache by basis_element().
- struct pade::pade_sqrt_numer< Scalar_T >

  Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct pade::pade_sqrt_denom< Scalar_T >

  Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n).
- struct pade::pade_sqrt_numer< float >
- struct pade::pade_sqrt_denom< float >
- struct pade::pade_sqrt_numer< long double >
- struct pade::pade_sqrt_denom< long double >
- struct pade::pade_sqrt_numer< dd_real >
- struct pade::pade_sqrt_denom< dd_real >
- struct pade::pade_sqrt_numer< qd_real >
- struct pade::pade_sqrt_denom< qd_real >
- struct pade::pade_log_numer< Scalar_T >

  Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct pade::pade_log_denom< Scalar_T >

  Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n).
- struct pade::pade_log_numer< float >
- struct pade::pade_log_denom< float >
- struct pade::pade_log_numer< long double >
- struct pade::pade_log_denom< long double >
- struct pade::pade_log_numer< dd_real >
- struct pade::pade_log_denom< dd_real >
- struct pade::pade_log_numer< qd_real >
- struct pade::pade_log_denom< qd_real >

Namespaces

- namespace glucat
- namespace pade

Functions

- auto glucat::offset_level (const index_t p, const index_t q) -> index_t

  Determine the log2 dim corresponding to signature p, q.

- template<typename Matrix_Index_T, const index_t LO, const index_t HI>
  static auto glucat::folded_dim (const index_set< LO, HI > &sub) -> Matrix_Index_T

  Determine the matrix dimension of the fold of a subalegbra.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::reframe (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs, matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs_reframed) -> const index_set< LO, HI >

  Find a common frame for operands of a binary operator.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator* (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Geometric product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator^ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Outer product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator& (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Inner product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator% (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Left contraction.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::star (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T

  Hestenes scalar product.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator/ (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Geometric quotient.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator| (const matrix_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Transformation via twisted adjoint action.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator<< (std::ostream &os, const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &

  Write multivector to output.

- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::operator>> (std::istream &s, matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &

  Read multivector from input.

- template<typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T>
  static auto glucat::fast (const Matrix_T &X, index_t level) -> Multivector_T

  Inverse generalized Fast Fourier Transform.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size>
  static auto glucat::pade_approx (const std::array< Scalar_T, Size > &numer, const std::array< Scalar_T, Size > &denom, const matrix_multi< Scalar_T, LO, HI, Tune_P > &X) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Pade' approximation.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static void glucat::db_step (matrix_multi< Scalar_T, LO, HI, Tune_P > &M, matrix_multi< Scalar_T, LO, HI, Tune_P > &Y)

  Single step of product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto glucat::db_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 4)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Product form of Denman-Beavers square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto glucat::cr_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, Scalar_T norm_Y_tol=std::pow(std::numeric_limits< Scalar_T >::epsilon(), 1)) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Cyclic reduction square root iteration.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Square root of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto glucat::pade_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Pade' approximation of log.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  static auto glucat::cascade_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Incomplete square root cascade and Pade' approximation of log.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Natural logarithm of multivector with specified complexifier.
- template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P>
  auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >

  Exponential of multivector.

Variables

- template<typename Scalar_T>
  const pade_sqrt_numer< Scalar_T >::array pade::pade_sqrt_numer< Scalar_T >::numer

- template<typename Scalar_T>
  const pade_sqrt_denom< Scalar_T >::array pade::pade_sqrt_denom< Scalar_T >::denom
- const pade_sqrt_numer< float >::array pade::pade_sqrt_numer< float >::numer
- const pade_sqrt_denom< float >::array pade::pade_sqrt_denom< float >::denom
- const pade_sqrt_numer< longdouble >::array pade::pade_sqrt_numer< longdouble >::numer
- const pade_sqrt_denom< longdouble >::array pade::pade_sqrt_denom< longdouble >::denom
- const pade_sqrt_numer< dd_real >::array pade::pade_sqrt_numer< dd_real >::numer
- const pade_sqrt_denom< dd_real >::array pade::pade_sqrt_denom< dd_real >::denom
- const pade_sqrt_numer< qd_real >::array pade::pade_sqrt_numer< qd_real >::numer
- const pade_sqrt_denom< qd_real >::array pade::pade_sqrt_denom< qd_real >::denom
- template<typename Scalar_T>
  const pade_log_numer< Scalar_T >::array pade::pade_log_numer< Scalar_T >::numer
- template<typename Scalar_T>
  const pade_log_denom< Scalar_T >::array pade::pade_log_denom< Scalar_T >::denom
- const pade_log_numer< float >::array pade::pade_log_numer< float >::numer
- const pade_log_denom< float >::array pade::pade_log_denom< float >::denom
- const pade_log_numer< longdouble >::array pade::pade_log_numer< longdouble >::numer
- const pade_log_denom< longdouble >::array pade::pade_log_denom< longdouble >::denom
- const pade_log_numer< dd_real >::array pade::pade_log_numer< dd_real >::numer
- const pade_log_denom< dd_real >::array pade::pade_log_denom< dd_real >::denom
- const pade_log_numer< qd_real >::array pade::pade_log_numer< qd_real >::numer
- const pade_log_denom< qd_real >::array pade::pade_log_denom< qd_real >::denom

9.38 matrix_multi_imp.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_MATRIX_MULTI_IMP_H
00002 #define _GLUCAT_MATRIX_MULTI_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi_imp.h : Implement the matrix representation of a multivector
00006                          -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/matrix_multi.h"
00035
00036 #include "glucat/scalar.h"
00037 #include "glucat/generation.h"
00038 #include "glucat/matrix.h"
00039
00040 # if  defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00041 #  pragma GCC diagnostic push
00042 #  pragma GCC diagnostic ignored "-Wunused-local-typedefs"
```

```
00043 # endif
00044 # if  defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00045 #  include <boost/serialization/array_wrapper.hpp>
00046 # endif
00047 #include <boost/numeric/ublas/matrix.hpp>
00048 #include <boost/numeric/ublas/matrix_expression.hpp>
00049 #include <boost/numeric/ublas/matrix_proxy.hpp>
00050 #include <boost/numeric/ublas/matrix_sparse.hpp>
00051 #include <boost/numeric/ublas/operation.hpp>
00052 #include <boost/numeric/ublas/operation_sparse.hpp>
00053 #include <boost/numeric/ublas/triangular.hpp>
00054 #include <boost/numeric/ublas/lu.hpp>
00055 #include <boost/numeric/ublas/io.hpp>
00056 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00057 #  pragma GCC diagnostic pop
00058 # endif
00059
00060 #include <fstream>
00061 #include <iomanip>
00062 #include <array>
00063 #include <iostream>
00064
00065 namespace glucat
00066 {
00067   // References for algorithms:
00068   // [CHKL]:
00069   // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00070   // [MB]: Beatrice Meini, "The Matrix Square Root From a New Functional Perspective:
00071   // Theoretical Results and Computational Issues", SIAM Journal on
00072   // Matrix Analysis and Applications 26(2):362-376, 2004.
00073   // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00074
00076   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00077   auto
00078   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00079   classname() -> const std::string
00080   { return "matrix_multi"; }
00081
00083   // Reference: [P] Table 15.27, p 133
00084   inline
00085   auto
00086   offset_level(const index_t p, const index_t q) -> index_t
00087   {
00088     // Offsets between the log2 of the matrix dimension for the current signature
00089     // and that of the real superalgebra
00090     static const std::array<int, 8> offset_log2_dim = {0, 1, 0, 1, 1, 2, 1, 1};
00091     const index_t bott = pos_mod(p-q, 8);
00092     return (p+q)/2 + offset_log2_dim[bott];
00093   }
00094
00096   // Reference: [P] Table 15.27, p 133
00097   template< typename Matrix_Index_T, const index_t LO, const index_t HI >
00098   inline
00099   static
00100   auto
00101   folded_dim( const index_set<LO,HI>& sub ) -> Matrix_Index_T
00102   { return 1 << offset_level(sub.count_pos(), sub.count_neg()); }
00103
00105   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00106   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00107   matrix_multi()
00108   : m_frame( index_set_t() ),
00109     m_matrix( matrix_t( 1, 1 ) )
00110   { this->m_matrix.clear(); }
00111
00113   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00114   template< typename Other_Scalar_T >
00115   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00116   matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00117   : m_frame( val.m_frame ), m_matrix( val.m_matrix.size1(), val.m_matrix.size2() )
00118   {
00119     this->m_matrix.clear();
00120     for (auto
00121         val_it1 = val.m_matrix.begin1();
00122         val_it1 != val.m_matrix.end1();
00123         ++val_it1)
00124       for (auto
00125           val_it2 = val_it1.begin();
00126           val_it2 != val_it1.end();
00127           ++val_it2)
00128         this->m_matrix(val_it2.index1(), val_it2.index2()) =
      numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00129   }
00130
00132   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00133   template< typename Other_Scalar_T >
00134   matrix_multi<Scalar_T,LO,HI,Tune_P>::
```

```
00135   matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val, const index_set_t frm, const bool
      prechecked)
00136   : m_frame( frm )
00137   {
00138     if (frm != val.m_frame)
00139       *this = multivector_t(framed_multi_t(val), frm);
00140     else
00141     {
00142       const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00143       this->m_matrix.resize(dim, dim, false);
00144       this->m_matrix.clear();
00145       for (auto
00146           val_it1 = val.m_matrix.begin1();
00147           val_it1 != val.m_matrix.end1();
00148           ++val_it1)
00149         for (auto
00150             val_it2 = val_it1.begin();
00151             val_it2 != val_it1.end();
00152             ++val_it2)
00153           this->m_matrix(val_it2.index1(), val_it2.index2()) =
      numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00154     }
00155   }
00156
00158   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00159   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00160   matrix_multi(const multivector_t& val, const index_set_t frm, const bool prechecked)
00161   : m_frame( frm )
00162   {
00163     if (frm != val.m_frame)
00164       *this = multivector_t(framed_multi_t(val), frm);
00165     else
00166       this->m_matrix = val.m_matrix;
00167   }
00168
00170   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00171   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00172   matrix_multi(const index_set_t ist, const Scalar_T& crd)
00173   : m_frame( ist )
00174   {
00175     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00176     this->m_matrix.resize(dim, dim, false);
00177     this->m_matrix.clear();
00178     *this += term_t(ist, crd);
00179   }
00180
00182   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00183   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00184   matrix_multi(const index_set_t ist, const Scalar_T& crd, const index_set_t frm, const bool
      prechecked)
00185   : m_frame( frm )
00186   {
00187     if (!prechecked && (ist | frm) != frm)
00188       throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00189     const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00190     this->m_matrix.resize(dim, dim, false);
00191     this->m_matrix.clear();
00192     *this += term_t(ist, crd);
00193   }
00194
00196   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00197   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00198   matrix_multi(const Scalar_T& scr, const index_set_t frm)
00199   : m_frame( frm )
00200   {
00201     const auto dim = folded_dim<matrix_index_t>(frm);
00202     this->m_matrix.resize(dim, dim, false);
00203     this->m_matrix.clear();
00204     *this += term_t(index_set_t(), scr);
00205   }
00206
00208   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00209   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00210   matrix_multi(const int scr, const index_set_t frm)
00211   { *this = multivector_t(Scalar_T(scr), frm); }
00212
00214   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00216   matrix_multi(const vector_t& vec,
00217                const index_set_t frm, const bool prechecked)
00218   : m_frame( frm )
00219   {
00220     if (!prechecked && index_t(vec.size()) != frm.count())
00221       throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00222     const auto dim = folded_dim<matrix_index_t>(frm);
00223     this->m_matrix.resize(dim, dim, false);
00224     this->m_matrix.clear();
```

```
00225     auto idx = frm.min();
00226     const auto frm_end = frm.max()+1;
00227     for (auto& crd : vec)
00228     {
00229       *this += term_t(index_set_t(idx), crd);
00230       for (
00231         ++idx;
00232         idx != frm_end && !frm[idx];
00233         ++idx)
00234         ;
00235     }
00236   }
00237
00239   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00240   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00241   matrix_multi(const std::string& str)
00242   { *this = framed_multi_t(str); }
00243
00245   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00246   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00247   matrix_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00248   { *this = multivector_t(framed_multi_t(str), frm, prechecked); }
00249
00251   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00252   template< typename Other_Scalar_T >
00253   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00254   matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00255   : m_frame( val.frame() )
00256   {
00257     if (val.size() >= Tune_P::fast_size_threshold)
00258       try
00259       {
00260         *this = val.template fast_matrix_multi<Scalar_T>(this->m_frame);
00261         return;
00262       }
00263       catch (const glucat_error& e)
00264       { }
00265     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00266     this->m_matrix.resize(dim, dim, false);
00267     this->m_matrix.clear();
00268
00269     for (auto& val_term : val)
00270       *this += val_term;
00271   }
00272
00274   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00275   template< typename Other_Scalar_T >
00276   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00277   matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& framed_val, const index_set_t frm,
      const bool prechecked)
00278   {
00279     const auto val = framed_val.truncated();
00280     const auto our_frame = val.frame() | frm;
00281     if (val.size() >= Tune_P::fast_size_threshold)
00282       try
00283       {
00284         *this = val.template fast_matrix_multi<Scalar_T>(our_frame);
00285         return;
00286       }
00287       catch (const glucat_error& e)
00288       { }
00289     this->m_frame = our_frame;
00290     const auto dim = folded_dim<matrix_index_t>(our_frame);
00291     this->m_matrix.resize(dim, dim, false);
00292     this->m_matrix.clear();
00293
00294     for (auto& val_term : val)
00295       *this += val_term;
00296   }
00297
00299   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00300   template< typename Matrix_T >
00301   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00302   matrix_multi(const Matrix_T& mtx, const index_set_t frm)
00303   : m_frame( frm ), m_matrix( mtx.size1(), mtx.size2() )
00304   {
00305     this->m_matrix.clear();
00306
00307     for (auto
00308         mtx_it1 = mtx.begin1();
00309         mtx_it1 != mtx.end1();
00310         ++mtx_it1)
00311       for (auto
00312           mtx_it2 = mtx_it1.begin();
00313           mtx_it2 != mtx_it1.end();
00314           ++mtx_it2)
00315         this->m_matrix(mtx_it2.index1(), mtx_it2.index2()) =
```

```
    numeric_traits<Scalar_T>::to_scalar_t(*mtx_it2);
00316   }
00317
00319   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00320   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00321   matrix_multi(const matrix_t& mtx, const index_set_t frm)
00322   : m_frame( frm ), m_matrix( mtx )
00323   { }
00324
00326   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327   auto
00328   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00329   operator= (const multivector_t& rhs) -> multivector_t&
00330   {
00331     // Check for assignment to self
00332     if (this == &rhs)
00333       return *this;
00334     this->m_frame = rhs.m_frame;
00335     this->m_matrix = rhs.m_matrix;
00336     return *this;
00337   }
00338
00340   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00341   inline
00342   auto
00343   reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs,    const
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00344                  matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
    matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>
00345   {
00346     using index_set_t = index_set<LO, HI>;
00347     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00348     using framed_multi_t = typename multivector_t::framed_multi_t;
00349     // Determine the initial common frame
00350     index_set_t our_frame = lhs.m_frame | rhs.m_frame;
00351     framed_multi_t framed_lhs;
00352     framed_multi_t framed_rhs;
00353     if ((lhs.m_frame != our_frame) || (rhs.m_frame != our_frame))
00354     {
00355       // The common frame may expand as a result of the transform to framed_multi_t
00356       framed_lhs = framed_multi_t(lhs);
00357       framed_rhs = framed_multi_t(rhs);
00358       our_frame |= framed_lhs.frame() | framed_rhs.frame();
00359     }
00360     // Do the reframing only where necessary
00361     if (lhs.m_frame != our_frame)
00362       lhs_reframed = multivector_t(framed_lhs, our_frame, true);
00363     if (rhs.m_frame != our_frame)
00364       rhs_reframed = multivector_t(framed_rhs, our_frame, true);
00365     return our_frame;
00366   }
00367
00369   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00370   auto
00371   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00372   operator== (const multivector_t& rhs) const -> bool
00373   {
00374     // Ensure that there is no aliasing
00375     if (this == &rhs)
00376       return true;
00377
00378     // Operate only within a common frame
00379     multivector_t lhs_reframed;
00380     multivector_t rhs_reframed;
00381     const index_set_t our_frame = reframe(*this, rhs, lhs_reframed, rhs_reframed);
00382     const multivector_t& lhs_ref = (this->m_frame == our_frame)
00383       ? *this
00384       : lhs_reframed;
00385     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00386       ? rhs
00387       : rhs_reframed;
00388
00389     return ublas::norm_inf(lhs_ref.m_matrix - rhs_ref.m_matrix) == 0;
00390   }
00391
00392   // Test for equality of multivector and scalar
00393   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00394   inline
00395   auto
00396   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00397   operator== (const Scalar_T& scr) const -> bool
00398   {
00399     if (scr != Scalar_T(0))
00400       return *this == multivector_t(framed_multi_t(scr), this->m_frame, true);
00401     else if (ublas::norm_inf(this->m_matrix) != 0)
00402       return false;
00403     else
```

```
00404     {
00405       const matrix_index_t dim = this->m_matrix.size1();
00406       return !(dim == 1 && this->isnan());
00407     }
00408   }
00409
00411   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00412   inline
00413   auto
00414   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00415   operator+= (const Scalar_T& scr) -> multivector_t&
00416   { return *this += term_t(index_set_t(), scr); }
00417
00419   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00420   inline
00421   auto
00422   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00423   operator+= (const multivector_t& rhs) -> multivector_t&
00424   {
00425     // Ensure that there is no aliasing
00426     if (this == &rhs)
00427       return *this *= Scalar_T(2);
00428
00429     // Operate only within a common frame
00430     multivector_t rhs_reframed;
00431     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00432     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00433       ? rhs
00434       : rhs_reframed;
00435
00436     noalias(this->m_matrix) += rhs_ref.m_matrix;
00437     return *this;
00438   }
00439
00441   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00442   inline
00443   auto
00444   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00445   operator-= (const Scalar_T& scr) -> multivector_t&
00446   { return *this += term_t(index_set_t(), -scr); }
00447
00449   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00450   inline
00451   auto
00452   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00453   operator-= (const multivector_t& rhs) -> multivector_t&
00454   {
00455     // Ensure that there is no aliasing
00456     if (this == &rhs)
00457       return *this = Scalar_T(0);
00458
00459     // Operate only within a common frame
00460     multivector_t rhs_reframed;
00461     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00462     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00463       ? rhs
00464       : rhs_reframed;
00465
00466     noalias(this->m_matrix) -= rhs_ref.m_matrix;
00467     return *this;
00468   }
00469
00471   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00472   inline
00473   auto
00474   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00475   operator- () const -> const multivector_t
00476   { return multivector_t(-(this->m_matrix), this->m_frame); }
00477
00479   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00480   inline
00481   auto
00482   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00483   operator*= (const Scalar_T& scr) -> multivector_t&
00484   { // multiply coordinates of all terms by scalar
00485
00486     using traits_t = numeric_traits<Scalar_T>;
00487     if (traits_t::isNaN_or_isInf(scr) || this->isnan())
00488       return *this = traits_t::NaN();
00489     if (scr == Scalar_T(0))
00490       *this = Scalar_T(0);
00491     else
00492       this->m_matrix *= scr;
00493     return *this;
00494   }
00495
00497   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
```

```
00498   inline
00499   auto
00500   operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00501   {
00502     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00503     using index_set_t = typename multivector_t::index_set_t;
00504
00505     if (lhs.isnan() || rhs.isnan())
00506       return numeric_traits<Scalar_T>::NaN();
00507
00508     // Operate only within a common frame
00509     multivector_t lhs_reframed;
00510     multivector_t rhs_reframed;
00511     const index_set_t our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00512     const multivector_t& lhs_ref = (lhs.m_frame == our_frame)
00513       ? lhs
00514       : lhs_reframed;
00515     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00516       ? rhs
00517       : rhs_reframed;
00518
00519     using matrix_t = typename multivector_t::matrix_t;
00520     using matrix_index_t = typename matrix_t::size_type;
00521
00522     const matrix_index_t dim = lhs_ref.m_matrix.size1();
00523     multivector_t result = multivector_t(matrix_t(dim, dim), our_frame);
00524     result.m_matrix.clear();
00525     ublas::axpy_prod(lhs_ref.m_matrix, rhs_ref.m_matrix, result.m_matrix, true);
00526     return result;
00527   }
00528
00530   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00531   inline
00532   auto
00533   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00534   operator*= (const multivector_t& rhs) -> multivector_t&
00535   { return *this = *this * rhs; }
00536
00538   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00539   inline
00540   auto
00541   operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00542   {
00543     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00544     using framed_multi_t = typename multivector_t::framed_multi_t;
00545     return framed_multi_t(lhs) ^ framed_multi_t(rhs);
00546   }
00547
00549   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00550   inline
00551   auto
00552   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00553   operator^= (const multivector_t& rhs) -> multivector_t&
00554   { return *this = *this ^ rhs; }
00555
00557   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00558   inline
00559   auto
00560   operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00561   {
00562     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00563     using framed_multi_t = typename multivector_t::framed_multi_t;
00564     return framed_multi_t(lhs) & framed_multi_t(rhs);
00565   }
00566
00568   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00569   inline
00570   auto
00571   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00572   operator&= (const multivector_t& rhs) -> multivector_t&
00573   { return *this = *this & rhs; }
00574
00576   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00577   inline
00578   auto
00579   operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00580   {
00581     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00582     using framed_multi_t = typename multivector_t::framed_multi_t;
00583     return framed_multi_t(lhs) % framed_multi_t(rhs);
00584   }
00585
00587   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
```

```
00588   inline
00589   auto
00590   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00591   operator%= (const multivector_t& rhs) -> multivector_t&
00592   { return *this = *this % rhs; }
00593
00595   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00596   inline
00597   auto
00598   star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
      -> Scalar_T
00599   { return (lhs * rhs).scalar(); }
00600
00602   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00603   inline
00604   auto
00605   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00606   operator/= (const Scalar_T& scr) -> multivector_t&
00607   { return *this *= Scalar_T(1)/scr; }
00608
00610   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00611   auto
00612   operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00613   {
00614     using traits_t = numeric_traits<Scalar_T>;
00615
00616     if (lhs.isnan() || rhs.isnan())
00617       return traits_t::NaN();
00618
00619     if (rhs == Scalar_T(0))
00620       return traits_t::NaN();
00621
00622     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00623
00624     // Operate only within a common frame
00625     multivector_t lhs_reframed;
00626     multivector_t rhs_reframed;
00627     const auto our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00628     const auto& lhs_ref = (lhs.m_frame == our_frame)
00629       ? lhs
00630       : lhs_reframed;
00631     const auto& rhs_ref = (rhs.m_frame == our_frame)
00632       ? rhs
00633       : rhs_reframed;
00634
00635     // Solve result == lhs_ref/rhs_ref <=> result*rhs_ref == lhs_ref
00636     // We now solve X == B/A
00637     // (where X == result, B == lhs_ref.m_matrix and A == rhs_ref.m_matrix)
00638     // X == B/A <=> X*A == B <=> AT*XT == BT
00639     // So, we solve AT*XT == BT
00640
00641     using matrix_t = typename multivector_t::matrix_t;
00642     using matrix_index_t = typename matrix_t::size_type;
00643
00644     const auto& AT = matrix_t(ublas::trans(rhs_ref.m_matrix));
00645     auto LU = AT;
00646
00647     using permutation_t = ublas::permutation_matrix<matrix_index_t>;
00648
00649     auto pvector = permutation_t(AT.size1());
00650     if (! ublas::lu_factorize(LU, pvector))
00651     {
00652       const auto& BT = matrix_t(ublas::trans(lhs_ref.m_matrix));
00653       auto XT = BT;
00654       ublas::lu_substitute(LU, pvector, XT);
00655       if (matrix::isnan(XT))
00656         return traits_t::NaN();
00657
00658       // Iterative refinement.
00659       // Reference: Nicholas J. Higham, "Accuracy and Stability of Numerical Algorithms",
00660       // SIAM, 1996, ISBN 0-89871-355-2, Chapter 11
00661       if (Tune_P::div_max_steps > 0)
00662       {
00663         // matrix_t R = ublas::prod(AT, XT) - BT;
00664         auto R = matrix_t(-BT);
00665         ublas::axpy_prod(AT, XT, R, false);
00666         if (matrix::isnan(R))
00667           return traits_t::NaN();
00668
00669         auto nr = Scalar_T(ublas::norm_inf(R));
00670         if ( nr != Scalar_T(0) && !traits_t::isNaN_or_isInf(nr) )
00671         {
00672           auto XTnew = XT;
00673           auto nrold = nr + Scalar_T(1);
00674           for (auto
00675               step = 0;
```

```
00676               step != Tune_P::div_max_steps &&
00677               nr < nrold &&
00678               nr != Scalar_T(0) &&
00679               nr == nr;
00680               ++step)
00681           {
00682             nrold = nr;
00683             if (step != 0)
00684               XT = XTnew;
00685             auto& D = R;
00686             ublas::lu_substitute(LU, pvector, D);
00687             XTnew -= D;
00688             // noalias(R) = ublas::prod(AT, XTnew) - BT;
00689             R = -BT;
00690             ublas::axpy_prod(AT, XTnew, R, false);
00691             nr = ublas::norm_inf(R);
00692           }
00693         }
00694       }
00695       return multivector_t(ublas::trans(XT), our_frame);
00696     }
00697     else
00698       // AT is singular. Return NaN
00699       return traits_t::NaN();
00700   }
00701
00703   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00704   inline
00705   auto
00706   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00707   operator/= (const multivector_t& rhs) -> multivector_t&
00708   { return *this = *this / rhs; }
00709
00711   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00712   inline
00713   auto
00714   operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
      matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00715   { return rhs * lhs / rhs.involute(); }
00716
00718   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00719   inline
00720   auto
00721   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00722   operator|= (const multivector_t& rhs) -> multivector_t&
00723   { return *this = rhs * *this / rhs.involute(); }
00724
00726   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00727   inline
00728   auto
00729   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00730   inv() const -> const multivector_t
00731   { return multivector_t(Scalar_T(1), this->m_frame) / *this; }
00732
00734   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00735   inline
00736   auto
00737   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00738   pow(int m) const -> const multivector_t
00739   { return glucat::pow(*this, m); }
00740
00742   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00743   auto
00744   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00745   outer_pow(int m) const -> const multivector_t
00746   {
00747     if (m < 0)
00748       throw error_t("outer_pow(m): negative exponent");
00749     framed_multi_t a = *this;
00750     return a.outer_pow(m);
00751   }
00752
00754   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00755   inline
00756   auto
00757   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00758   grade() const -> index_t
00759   { return framed_multi_t(*this).grade(); }
00760
00762   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00763   inline
00764   auto
00765   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00766   frame() const -> const index_set_t
00767   { return this->m_frame; }
00768
00770   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
```

```
00771   inline
00772   auto
00773   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00774   operator[] (const index_set_t ist) const -> Scalar_T
00775   {
00776     // Use matrix inner product only if ist is in frame
00777     if ( (ist | this->m_frame) == this->m_frame)
00778       return matrix::inner<Scalar_T>(this->basis_element(ist), this->m_matrix);
00779     else
00780       return Scalar_T(0);
00781   }
00782
00784   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00785   inline
00786   auto
00787   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00788   operator() (index_t grade) const -> const multivector_t
00789   {
00790     if ((grade < 0) || (grade > HI-LO))
00791       return 0;
00792     else
00793       return (framed_multi_t(*this))(grade);
00794   }
00795
00797   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00798   inline
00799   auto
00800   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00801   scalar() const -> Scalar_T
00802   {
00803     const matrix_index_t dim = this->m_matrix.size1();
00804     return matrix::trace(this->m_matrix) / Scalar_T( double(dim) );
00805   }
00806
00808   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00809   inline
00810   auto
00811   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00812   pure() const -> const multivector_t
00813   { return *this - this->scalar(); }
00814
00816   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00817   inline
00818   auto
00819   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00820   even() const -> const multivector_t
00821   { return framed_multi_t(*this).even(); }
00822
00824   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00825   inline
00826   auto
00827   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00828   odd() const -> const multivector_t
00829   { return framed_multi_t(*this).odd(); }
00830
00832   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00833   auto
00834   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00835   vector_part() const -> const vector_t
00836   { return this->vector_part(this->frame(), true); }
00837
00839   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840   auto
00841   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00842   vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00843   {
00844     if (!prechecked && (this->frame() | frm) != frm)
00845       throw error_t("vector_part(frm): value is outside of requested frame");
00846     vector_t result;
00847     // If we need to enlarge the frame we may as well use a framed_multi_t
00848     if (this->frame() != frm)
00849       return framed_multi_t(*this).vector_part(frm, true);
00850
00851     const auto begin_index = frm.min();
00852     const auto end_index = frm.max()+1;
00853     for (auto
00854         idx = begin_index;
00855         idx != end_index;
00856         ++idx)
00857       if (frm[idx])
00858         // Frame may contain indices which do not correspond to a grade 1 term but
00859         // frame cannot omit any index corresponding to a grade 1 term
00860         result.push_back(
00861           matrix::inner<Scalar_T>(this->basis_element(index_set_t(idx)),
00862           this->m_matrix));
00863     return result;
00864   }
```

```
00865
00867   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00868   inline
00869   auto
00870   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00871   involute() const -> const multivector_t
00872   { return framed_multi_t(*this).involute(); }
00873
00875   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00876   inline
00877   auto
00878   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00879   reverse() const -> const multivector_t
00880   { return framed_multi_t(*this).reverse(); }
00881
00883   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00884   inline
00885   auto
00886   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00887   conj() const -> const multivector_t
00888   { return framed_multi_t(*this).conj(); }
00889
00891   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00892   inline
00893   auto
00894   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00895   quad() const -> Scalar_T
00896   { // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
00897     // Arvind Raja ref: "old clical: quadfunction(p:pter):pterm in file compmod.pas"
00898     return framed_multi_t(*this).quad();
00899   }
00900
00902   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00903   inline
00904   auto
00905   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00906   norm() const -> Scalar_T
00907   {
00908     const matrix_index_t dim = this->m_matrix.size1();
00909     return matrix::norm_frob2(this->m_matrix) / Scalar_T( double(dim) );
00910   }
00911
00913   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00914   inline
00915   auto
00916   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00917   max_abs() const -> Scalar_T
00918   { return framed_multi_t(*this).max_abs(); }
00919
00921   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00922   auto
00923   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00924   random(const index_set<LO,HI> frm, Scalar_T fill) -> const multivector_t
00925   {
00926     return framed_multi<Scalar_T,LO,HI,Tune_P>::random(frm, fill);
00927   }
00928
00930   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00931   inline
00932   void
00933   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00934   write(const std::string& msg) const
00935   { framed_multi_t(*this).write(msg); }
00936
00938   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00939   inline
00940   void
00941   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00942   write(std::ofstream& ofile, const std::string& msg) const
00943   {
00944     if (!ofile)
00945       throw error_t("write(ofile,msg): cannot write to output file");
00946     framed_multi_t(*this).write(ofile, msg);
00947   }
00948
00950   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00951   inline
00952   auto
00953   operator« (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
00954   {
00955     os « typename matrix_multi<Scalar_T,LO,HI,Tune_P>::framed_multi_t(val);
00956     return os;
00957   }
00958
00960   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00961   inline
00962   auto
```

```
00963   operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&
00964   { // Input looks like 1.0-2.0{1,2}+3.2{3,4}
00965     framed_multi<Scalar_T,LO,HI,Tune_P> local;
00966     s » local;
00967     // If s.bad() then we have a corrupt input
00968     // otherwise we are fine and can copy the resulting matrix_multi
00969     if (!s.bad())
00970       val = local;
00971     return s;
00972   }
00973
00975   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976   inline
00977   auto
00978   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00979   isinf() const -> bool
00980   {
00981     if (std::numeric_limits<Scalar_T>::has_infinity)
00982       return matrix::isinf(this->m_matrix);
00983     else
00984       return false;
00985   }
00986
00988   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00989   inline
00990   auto
00991   matrix_multi<Scalar_T,LO,HI,Tune_P>::
00992   isnan() const -> bool
00993   {
00994     if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
00995       return matrix::isnan(this->m_matrix);
00996     else
00997       return false;
00998   }
00999
01001   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01002   inline
01003   auto
01004   matrix_multi<Scalar_T,LO,HI,Tune_P>::
01005   truncated(const Scalar_T& limit) const -> const multivector_t
01006   { return framed_multi_t(*this).truncated(limit); }
01007
01009   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01010   inline
01011   auto
01012   matrix_multi<Scalar_T,LO,HI,Tune_P>::
01013   operator+= (const term_t& term) -> multivector_t&
01014   {
01015     if (term.second != Scalar_T(0))
01016       this->m_matrix.plus_assign(matrix_t(this->basis_element(term.first)) * term.second);
01017     return *this;
01018   }
01019
01021   template< typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T >
01022   static
01023   auto
01024   fast(const Matrix_T& X, index_t level) -> Multivector_T
01025   {
01026     using framed_multi_t = Multivector_T;
01027
01028     using index_set_t = typename framed_multi_t::index_set_t;
01029     using Scalar_T = typename framed_multi_t::scalar_t;
01030     using matrix_t = Matrix_T;
01031     using basis_matrix_t = Basis_Matrix_T;
01032     using basis_scalar_t = typename basis_matrix_t::value_type;
01033     using traits_t = numeric_traits<Scalar_T>;
01034
01035     if (level == 0)
01036       return framed_multi_t(traits_t::to_scalar_t(X(0,0)));
01037
01038     if (ublas::norm_inf(X) == 0)
01039       return Scalar_T(0);
01040
01041     const basis_matrix_t&  I = matrix::unit<basis_matrix_t>(2);
01042     basis_matrix_t J(2,2,2);
01043     J.clear();
01044     J(0,1)  = basis_scalar_t(-1);
01045     J(1,0)  = basis_scalar_t( 1);
01046     basis_matrix_t K = J;
01047     K(0,1)  = basis_scalar_t( 1);
01048     basis_matrix_t JK = I;
01049     JK(0,0) = basis_scalar_t(-1);
01050
01051     using matrix::signed_perm_nork;
01052     const index_set_t ist_mn   = index_set_t(-level);
01053     const index_set_t ist_pn   = index_set_t(level);
01054     const index_set_t ist_mnpn = ist_mn | ist_pn;
```

```
01055     if (level == 1)
01056     {
01057       using term_t = typename framed_multi_t::term_t;
01058       const Scalar_T i_x  = traits_t::to_scalar_t(signed_perm_nork(I, X)(0, 0));
01059       const Scalar_T j_x  = traits_t::to_scalar_t(signed_perm_nork(J, X)(0, 0));
01060       const Scalar_T k_x  = traits_t::to_scalar_t(signed_perm_nork(K, X)(0, 0));
01061       const Scalar_T jk_x = traits_t::to_scalar_t(signed_perm_nork(JK,X)(0, 0));
01062       framed_multi_t
01063             result  = i_x;
01064             result += term_t(ist_mn,   j_x);  // j_x *  mn;
01065             result += term_t(ist_pn,   k_x);  // k_x *  pn;
01066       return result += term_t(ist_mnpn, jk_x); // jk_x * mnpn;
01067     }
01068     else
01069     {
01070       const framed_multi_t& mn   = framed_multi_t(ist_mn);
01071       const framed_multi_t& pn   = framed_multi_t(ist_pn);
01072       const framed_multi_t& mnpn = framed_multi_t(ist_mnpn);
01073       const framed_multi_t& i_x  = fast<framed_multi_t, matrix_t, basis_matrix_t>
01074                                         (signed_perm_nork(I, X), level-1);
01075       const framed_multi_t& j_x  = fast<framed_multi_t, matrix_t, basis_matrix_t>
01076                                         (signed_perm_nork(J, X), level-1);
01077       const framed_multi_t& k_x  = fast<framed_multi_t, matrix_t, basis_matrix_t>
01078                                         (signed_perm_nork(K, X), level-1);
01079       const framed_multi_t& jk_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01080                                         (signed_perm_nork(JK,X), level-1);
01081       framed_multi_t
01082             result  =  i_x.even() - jk_x.odd();
01083             result += (j_x.even() - k_x.odd()) * mn;
01084             result += (k_x.even() - j_x.odd()) * pn;
01085       return result += (jk_x.even() - i_x.odd()) * mnpn;
01086     }
01087   }
01088
01090   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01091   inline
01092   auto
01093   matrix_multi<Scalar_T,LO,HI,Tune_P>::
01094   fast_matrix_multi(const index_set_t frm) const -> const multivector_t
01095   {
01096     if (this->m_frame == frm)
01097       return *this;
01098     else
01099       return (this->template fast_framed_multi<Scalar_T>()).template fast_matrix_multi<Scalar_T>(frm);
01100   }
01101
01103   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01104   template <typename Other_Scalar_T>
01105   auto
01106   matrix_multi<Scalar_T,LO,HI,Tune_P>::
01107   fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>
01108   {
01109     // Determine the amount of off-centering needed
01110     index_t p = this->m_frame.count_pos();
01111     index_t q = this->m_frame.count_neg();
01112
01113     const index_t bott = pos_mod(p-q, 8);
01114     p += std::max(gen::offset_to_super[bott],index_t(0));
01115     q -= std::min(gen::offset_to_super[bott],index_t(0));
01116
01117     const index_t orig_p = p;
01118     const index_t orig_q = q;
01119     while (p-q > 4)
01120       { p -= 4; q += 4; }
01121     while (p-q < -3)
01122       { p += 4; q -= 4; }
01123     if (p-q > 1)
01124     {
01125       index_t old_p = p;
01126       p = q+1;
01127       q = old_p-1;
01128     }
01129     const index_t level = (p+q)/2;
01130
01131     // Do the inverse fast transform
01132     using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
01133     framed_multi_t val = fast<framed_multi_t, matrix_t, basis_matrix_t>(this->m_matrix, level);
01134
01135     // Off-centre val
01136     switch (pos_mod(orig_p-orig_q, 8))
01137     {
01138     case 2:
01139     case 3:
01140     case 4:
01141       val.centre_qp1_pm1(p, q);
01142       break;
01143     default:
```

```
01144       break;
01145     }
01146     if (orig_p-orig_q > 4)
01147       while (p != orig_p)
01148         val.centre_pp4_qm4(p, q);
01149     if (orig_p-orig_q < -3)
01150       while (p != orig_p)
01151         val.centre_pm4_qp4(p, q);
01152
01153     // Return unfolded val
01154     return val.unfold(this->m_frame);
01155   }
01156
01158   template< typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T >
01159   class basis_table :
01160   public std::map< std::pair< const index_set<LO,HI>, const index_set<LO,HI> >,
01161                    Matrix_T* >
01162   {
01163   public:
01165     static auto basis() -> basis_table& { static basis_table b; return b;}
01166   private:
01170     friend class friend_for_private_destructor;
01171     // Enforce singleton
01172     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
01173     basis_table() = default;
01174     ~basis_table() = default;
01175   public:
01176     basis_table(const basis_table&) = delete;
01177     auto operator= (const basis_table&) -> basis_table& = delete;
01178   };
01179
01181   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01182   auto
01183   matrix_multi<Scalar_T,LO,HI,Tune_P>::
01184   basis_element(const index_set_t& ist) const -> const basis_matrix_t
01185   {
01186     using index_set_pair_t = std::pair<const index_set_t, const index_set_t>;
01187     const auto& unfolded_pair = index_set_pair_t(ist, this->m_frame);
01188
01189     using basis_table_t = basis_table<Scalar_T, LO, HI, basis_matrix_t>;
01190     auto& basis_cache = basis_table_t::basis();
01191
01192     const auto frame_count = this->m_frame.count();
01193     const auto use_cache = frame_count <= index_t(Tune_P::basis_max_count);
01194
01195     if (use_cache)
01196     {
01197       const auto basis_it = basis_cache.find(unfolded_pair);
01198       if (basis_it != basis_cache.end())
01199         return *(basis_it->second);
01200     }
01201     const auto folded_set = ist.fold(this->m_frame);
01202     const auto folded_frame = this->m_frame.fold();
01203     const auto& folded_pair = index_set_pair_t(folded_set, folded_frame);
01204     using basis_pair_t = std::pair<const index_set_pair_t, basis_matrix_t *>;
01205     if (use_cache)
01206     {
01207       const auto basis_it = basis_cache.find(folded_pair);
01208       if (basis_it != basis_cache.end())
01209       {
01210         auto* result_ptr = basis_it->second;
01211         basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01212         return *result_ptr;
01213       }
01214     }
01215     const auto folded_max = folded_frame.max();
01216     const auto folded_min = folded_frame.min();
01217     const auto p = std::max(folded_max,           index_t(0));
01218     const auto q = std::max(index_t(-folded_min), index_t(0));
01219     const auto* e = (gen::generator_table<basis_matrix_t>::generator())(p, q);
01220     const auto dim = matrix_index_t(1) « offset_level(p, q);
01221     auto result = matrix::unit<basis_matrix_t>(dim);
01222     for (auto
01223         k = folded_min;
01224         k <= folded_max;
01225         ++k)
01226       if (folded_set[k])
01227         result = matrix::mono_prod(result, e[k]);
01228     if (use_cache)
01229     {
01230       auto* result_ptr = new basis_matrix_t(result);
01231       basis_cache.insert(basis_pair_t(folded_pair, result_ptr));
01232       basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01233     }
01234     return result;
01235   }
01236
```

```
01238   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size
      >
01239   inline
01240   static
01241   auto
01242   pade_approx(
01243     const std::array<Scalar_T, Size>& numer,
01244     const std::array<Scalar_T, Size>& denom,
01245     const matrix_multi<Scalar_T,LO,HI,Tune_P>& X) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01246   {
01247     // Pade' approximation
01248     // Reference: [GW], Section 4.3, pp318-322
01249     // Reference: [GL], Section 11.3, p572-576.
01250
01251     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01252     using traits_t = numeric_traits<Scalar_T>;
01253
01254     if (X.isnan())
01255       return traits_t::NaN();
01256
01257     // Array size is assumed to be even
01258     const auto nbr_even_powers = Size/2 - 1;
01259
01260     // Create an array of even powers
01261     auto XX = std::vector<multivector_t>(nbr_even_powers);
01262     XX[0] = X * X;
01263     XX[1] = XX[0] * XX[0];
01264     for (auto
01265       k = size_t(2);
01266       k != nbr_even_powers;
01267       ++k)
01268       XX[k] = XX[k-2] * XX[1];
01269
01270     // Calculate numerator N and denominator D
01271     auto N = multivector_t(numer[1]);
01272     for (auto
01273         k = size_t(0);
01274         k != nbr_even_powers;
01275         ++k)
01276       N += XX[k] * numer[2*k + 3];
01277     N *= X;
01278     N += numer[0];
01279     for (auto
01280         k = size_t(0);
01281         k != nbr_even_powers;
01282         ++k)
01283       N += XX[k] * numer[2*k + 2];
01284     auto D = multivector_t(denom[1]);
01285     for (auto
01286         k = size_t(0);
01287         k != nbr_even_powers;
01288         ++k)
01289       D += XX[k] * denom[2*k + 3];
01290     D *= X;
01291     D += denom[0];
01292     for (auto
01293         k = size_t(0);
01294         k != nbr_even_powers;
01295         ++k)
01296       D += XX[k] * denom[2*k + 2];
01297     return N / D;
01298   }
01299
01301   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01302   inline
01303   static
01304   void
01305   db_step(matrix_multi<Scalar_T,LO,HI,Tune_P>& M, matrix_multi<Scalar_T,LO,HI,Tune_P>& Y)
01306   {
01307     // Reference: [CHKL]
01308     const auto& invM = inv(M);
01309     M = ((M + invM)/Scalar_T(2) + Scalar_T(1)) / Scalar_T(2);
01310     Y *= (invM + Scalar_T(1)) / Scalar_T(2);
01311   }
01312
01314   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01315   static
01316   auto
01317   db_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01318           Scalar_T norm_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) -> const
      matrix_multi<Scalar_T,LO,HI,Tune_P>
01319   {
01320     // Reference: [CHKL]
01321     if (val == Scalar_T(0))
01322       return val;
01323
01324     static const auto sqrt_max_steps = Tune_P::db_sqrt_max_steps;
```

```
01325     auto M = val;
01326     auto Y = val;
01327
01328     for (auto
01329         step = 0;
01330         step != sqrt_max_steps && norm(M - Scalar_T(1)) > norm_tol;
01331         ++step)
01332     {
01333       if (Y.isnan())
01334         return numeric_traits<Scalar_T>::NaN();
01335       db_step(M, Y);
01336     }
01337     return Y;
01338   }
01339
01341   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01342   static
01343   auto
01344   cr_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01345           Scalar_T norm_Y_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) -> const
      matrix_multi<Scalar_T,LO,HI,Tune_P>
01346   {
01347     // Reference: [MB]
01348     if (val == Scalar_T(0))
01349       return val;
01350
01351     static const auto sqrt_max_steps = Tune_P::cr_sqrt_max_steps;
01352     auto Z = Scalar_T(2) * (Scalar_T(1) + val);
01353     auto Y = Scalar_T(1) - val;
01354     auto norm_Y = norm(Y);
01355     for (auto
01356         step = 0;
01357         step != sqrt_max_steps && norm_Y > norm_Y_tol;
01358         ++step)
01359     {
01360       const auto old_norm_Y = norm_Y;
01361       Y = (-Y / Z) * Y;
01362       norm_Y = norm(Y);
01363       if (Y.isnan() || (norm_Y > old_norm_Y * Scalar_T(2)))
01364         return numeric_traits<Scalar_T>::NaN();
01365
01366       Z += Y * Scalar_T(2);
01367     }
01368     return Z / Scalar_T(4);
01369   }
01370 }
01371
01372 namespace pade {
01374   // Reference: [Z], Pade1
01375   template< typename Scalar_T >
01376   struct pade_sqrt_numer
01377   {
01378     using array = std::array<Scalar_T, 14>;
01379     static const array numer;
01380   };
01381   template< typename Scalar_T >
01382   const typename pade_sqrt_numer<Scalar_T>::array pade_sqrt_numer<Scalar_T>::numer =
01383   {
01384         1.0,               27.0/4.0,          81.0/4.0,        2277.0/64.0,
01385     10395.0/256.0,      32319.0/1024.0,    8721.0/512.0,    26163.0/4096.0,
01386     53703.0/32768.0,    36465.0/131072.0,  3861.0/131072.0,  7371.0/4194304.0,
01387       819.0/16777216.0,    27.0/67108864.0
01388   };
01389
01391   // Reference: [Z], Pade1
01392   template< typename Scalar_T >
01393   struct pade_sqrt_denom
01394   {
01395     using array = std::array<Scalar_T, 14>;
01396     static const array denom;
01397   };
01398   template< typename Scalar_T >
01399   const typename pade_sqrt_denom<Scalar_T>::array pade_sqrt_denom<Scalar_T>::denom =
01400   {
01401         1.0,               25.0/4.0,          69.0/4.0,        1771.0/64.0,
01402      7315.0/256.0,       20349.0/1024.0,    4845.0/512.0,    12597.0/4096.0,
01403     21879.0/32768.0,     12155.0/131072.0,  1001.0/131072.0,  1365.0/4194304.0,
01404        91.0/16777216.0,     1.0/67108864.0
01405   };
01406
01407   template< >
01408   struct pade_sqrt_numer<float>
01409   {
01410     using array = std::array<float, 10>;
01411     static const array numer;
01412   };
01413   const typename pade_sqrt_numer<float>::array pade_sqrt_numer<float>::numer =
```

```
01414   {
01415       1.0,            19.0/4.0,        19.0/2.0,      665.0/64.0,
01416     1729.0/256.0,    2717.0/1024.0,    627.0/1024.0,   627.0/8192.0,
01417      285.0/65536.0,    19.0/262144.0
01418   };
01419   template< >
01420   struct pade_sqrt_denom<float>
01421   {
01422     using array = std::array<float, 10>;
01423     static const array denom;
01424   };
01425   const typename pade_sqrt_denom<float>::array pade_sqrt_denom<float>::denom =
01426   {
01427       1.0,            17.0/4.0,        15.0/2.0,      455.0/64.0,
01428     1001.0/256.0,    1287.0/1024.0,    231.0/1024.0,   165.0/8192.0,
01429       45.0/65536,       1.0/262144.0
01430   };
01431
01432   template< >
01433   struct pade_sqrt_numer<long double>
01434   {
01435     using array = std::array<long double, 18>;
01436     static const array numer;
01437   };
01438   const typename pade_sqrt_numer<long double>::array pade_sqrt_numer<long double>::numer =
01439   {
01440         1.0L,                   35.0L/4.0L,             35.0L,               5425.0L/64.0L,
01441     35525.0L/256.0L,        166257.0L/1024.0L,      143325.0L/1024.0L,     740025.0L/8192.0L,
01442   2877875.0L/65536.0L,     4206125.0L/262144.0L,    572033.0L/131072.0L,  1820105.0L/2097152.0L,
01443   1028755.0L/8388608.0L,    395675.0L/33554432.0L,   24225.0L/33554432.0L,   6783.0L/268435456.0L,
01444      1785.0L/4294967296.0L,     35.0L/17179869184.0L
01445   };
01446   template< >
01447   struct pade_sqrt_denom<long double>
01448   {
01449     using array = std::array<long double, 18>;
01450     static const array denom;
01451   };
01452   const typename pade_sqrt_denom<long double>::array pade_sqrt_denom<long double>::denom =
01453   {
01454         1.0L,                   33.0L/4.0L,             31.0L,               4495.0L/64.0L,
01455     27405.0L/256.0L,        118755.0L/1024.0L,       94185.0L/1024.0L,     444015.0L/8192.0L,
01456   1562275.0L/65536.0L,     2042975.0L/262144.0L,    245157.0L/131072.0L,   676039.0L/2097152.0L,
01457    323323.0L/8388608.0L,    101745.0L/33554432.0L,    4845.0L/33554432.0L,    969.0L/268435456.0L,
01458       153.0L/4294967296.0L,      1.0L/17179869184.0L
01459   };
01460
01461 #if defined(_GLUCAT_USE_QD)
01462   template< >
01463   struct pade_sqrt_numer<dd_real>
01464   {
01465     using array = std::array<dd_real, 22>;
01466     static const array numer;
01467   };
01468   const typename pade_sqrt_numer<dd_real>::array pade_sqrt_numer<dd_real>::numer =
01469   {
01470           dd_real("1"),                                dd_real("43")/dd_real("4"),
01471         dd_real("215")/dd_real("4"),                dd_real("10621")/dd_real("64"),
01472       dd_real("90687")/dd_real("256"),             dd_real("567987")/dd_real("1024"),
01473      dd_real("168861")/dd_real("256"),            dd_real("1246355")/dd_real("2048"),
01474     dd_real("7228859")/dd_real("16384"),         dd_real("16583853")/dd_real("65536"),
01475     dd_real("7538115")/dd_real("65536"),        dd_real("173376645")/dd_real("4194304"),
01476   dd_real("195747825")/dd_real("16777216"),     dd_real("171655785")/dd_real("67108864"),
01477    dd_real("14375115")/dd_real("33554432"),      dd_real("14375115")/dd_real("268435456"),
01478    dd_real("20764055")/dd_real("4294967296"),     dd_real("5167525")/dd_real("17179869184"),
01479      dd_real("206701")/dd_real("17179869184"),      dd_real("76153")/dd_real("274877906944"),
01480        dd_real("3311")/dd_real("1099511627776") ,     dd_real("43")/dd_real("4398046511104")
01481   };
01482   template< >
01483   struct pade_sqrt_denom<dd_real>
01484   {
01485     using array = std::array<dd_real, 22>;
01486     static const array denom;
01487   };
01488   const typename pade_sqrt_denom<dd_real>::array pade_sqrt_denom<dd_real>::denom =
01489   {
01490           dd_real("1"),                                dd_real("41")/dd_real("4"),
01491         dd_real("195")/dd_real("4"),                dd_real("9139")/dd_real("64"),
01492       dd_real("73815")/dd_real("256"),             dd_real("435897")/dd_real("1024"),
01493      dd_real("121737")/dd_real("256"),             dd_real("840565")/dd_real("2048"),
01494     dd_real("4539051")/dd_real("16384"),          dd_real("9641775")/dd_real("65536"),
01495     dd_real("4032015")/dd_real("65536"),         dd_real("84672315")/dd_real("4194304"),
01496    dd_real("86493225")/dd_real("16777216"),      dd_real("67863915")/dd_real("67108864"),
01497     dd_real("5014575")/dd_real("33554432"),       dd_real("4345965")/dd_real("268435456"),
01498     dd_real("5311735")/dd_real("4294967296"),     dd_real("1081575")/dd_real("17179869184"),
01499       dd_real("33649")/dd_real("17179869184"),       dd_real("8855")/dd_real("274877906944"),
01500         dd_real("231")/dd_real("1099511627776"),        dd_real("1")/dd_real("4398046511104")
```

```
01501   };
01502
01503   template< >
01504   struct pade_sqrt_numer<qd_real>
01505   {
01506     using array = std::array<qd_real, 34>;
01507     static const array numer;
01508   };
01509   const typename pade_sqrt_numer<qd_real>::array pade_sqrt_numer<qd_real>::numer =
01510   {
01511               qd_real("1"),                                                qd_real("67")/qd_real("4"),
01512             qd_real("134"),                                              qd_real("43617")/qd_real("64"),
01513           qd_real("633485")/qd_real("256"),                              qd_real("6992857")/qd_real("1024"),
01514         qd_real("15246721")/qd_real("1024"),                            qd_real("215632197")/qd_real("8192"),
01515       qd_real("2518145487")/qd_real("65536"),
      qd_real("12301285425")/qd_real("262144"),
01516       qd_real("6344873535")/qd_real("131072"),
      qd_real("89075432355")/qd_real("2097152"),
01517     qd_real("267226297065")/qd_real("8388608"),
      qd_real("687479618945")/qd_real("33554432"),
01518     qd_real("379874182975")/qd_real("33554432"),
      qd_real("1443521895305")/qd_real("268435456"),
01519   qd_real("9425348845815")/qd_real("4294967296"),
      qd_real("13195488384141")/qd_real("17179869184"),
01520     qd_real("987417498133")/qd_real("4294967296"),
      qd_real("8055248011085")/qd_real("137438953472"),
01521   qd_real("6958363175533")/qd_real("549755813888"),
      qd_real("5056698705201")/qd_real("2199023255552"),
01522     qd_real("766166470485")/qd_real("2199023255552"),
      qd_real("766166470485")/qd_real("17592186044416"),
01523     qd_real("623623871325")/qd_real("140737488355328"),
      qd_real("203123203803")/qd_real("562949953421312"),
01524       qd_real("6478601247")/qd_real("2814749767106656"),
      qd_real("5038912081")/qd_real("4503599627370496"),
01525        qd_real("719844583")/qd_real("18014398509481984"),
      qd_real("71853815")/qd_real("72057594037927936"),
01526         qd_real("1165197")/qd_real("72057594037927936"),
      qd_real("87703")/qd_real("576460752303423488"),
01527           qd_real("12529")/qd_real("18446744073709551616"),
      qd_real("67")/qd_real("737869762948382064640")
01528   };
01529   template< >
01530   struct pade_sqrt_denom<qd_real>
01531   {
01532     using array = std::array<qd_real, 34>;
01533     static const array denom;
01534   };
01535   const typename pade_sqrt_denom<qd_real>::array pade_sqrt_denom<qd_real>::denom =
01536   {
01537             qd_real("1"),                                                qd_real("65")/qd_real("4"),
01538           qd_real("126"),                                              qd_real("39711")/qd_real("64"),
01539         qd_real("557845")/qd_real("256"),                              qd_real("5949147")/qd_real("1024"),
01540       qd_real("12515965")/qd_real("1024"),                            qd_real("170574723")/qd_real("8192"),
01541     qd_real("1916797311")/qd_real("65536"),
      qd_real("8996462475")/qd_real("262144"),
01542     qd_real("4450881435")/qd_real("131072"),
      qd_real("59826782925")/qd_real("2097152"),
01543   qd_real("171503444385")/qd_real("8388608"),
      qd_real("420696483235")/qd_real("33554432"),
01544   qd_real("221120793075")/qd_real("33554432"),
      qd_real("797168807855")/qd_real("268435456"),
01545 qd_real("4923689695575")/qd_real("4294967296"),
      qd_real("6499270398159")/qd_real("17179869184"),
01546   qd_real("456864812569")/qd_real("4294967296"),
      qd_real("3486599885395")/qd_real("137438953472"),
01547 qd_real("2804116503573")/qd_real("549755813888"),
      qd_real("1886827875075")/qd_real("2199023255552"),
01548   qd_real("263012370465")/qd_real("2199023255552"),
      qd_real("240141729555")/qd_real("17592186044416"),
01549   qd_real("176848560525")/qd_real("140737488355328"),
      qd_real("51538723353")/qd_real("562949953421312"),
01550     qd_real("1450433115")/qd_real("2814749767106656"),
      qd_real("977699359")/qd_real("4503599627370496"),
01551       qd_real("118183439")/qd_real("18014398509481984"),
      qd_real("9652005")/qd_real("72057594037927936"),
01552         qd_real("121737")/qd_real("72057594037927936"),
      qd_real("6545")/qd_real("576460752303423488"),
01553           qd_real("561")/qd_real("18446744073709551616"),
      qd_real("1")/qd_real("737869762948382064640")
01554   };
01555 #endif
01556 }
01557
01558 namespace glucat
01559 {
01561   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01562   auto
```

```
01563   matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01564               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01565               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01566   {
01567     // Reference: [GW], Section 4.3, pp318-322
01568     // Reference: [GL], Section 11.3, p572-576
01569     // Reference: [Z], Pade1
01570
01571     using traits_t = numeric_traits<Scalar_T>;
01572
01573     if (val.isnan())
01574       return traits_t::NaN();
01575
01576     const auto scr_val = val.scalar();
01577     if (val == scr_val)
01578     {
01579       if (scr_val < Scalar_T(0))
01580         return i * traits_t::sqrt(-scr_val);
01581       else
01582         return traits_t::sqrt(scr_val);
01583     }
01584
01585     // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01586     const auto scale =
01587       (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < Scalar_T(1))
01588       ? scr_val
01589       : (scr_val < Scalar_T(0))
01590         ? -abs(val)
01591         :  abs(val);
01592     const auto sqrt_scale = traits_t::sqrt(traits_t::abs(scale));
01593     if (traits_t::isNaN_or_isInf(sqrt_scale))
01594       return traits_t::NaN();
01595
01596     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01597     auto rescale = multivector_t(sqrt_scale);
01598     if (scale < Scalar_T(0))
01599       rescale = i * sqrt_scale;
01600
01601     const auto& unitval = val / scale;
01602     static const auto max_norm = Scalar_T(1.0/4.0);
01603     auto use_approx_sqrt = true;
01604     auto use_cr_sqrt = false;
01605     auto scaled_result = multivector_t();
01606 #if defined(_GLUCAT_USE_EIGENVALUES)
01607     static const auto sqrt_2 = traits_t::sqrt(Scalar_T(2));
01608     if (level == 0)
01609     {
01610       // What kind of eigenvalues does the matrix contain?
01611       const auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
01612       const index_t next_level =
01613         (genus.m_is_singular)
01614         ? level
01615         : level + 1;
01616       switch (genus.m_eig_case)
01617       {
01618       case matrix::neg_real_eigs:
01619         scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01620         use_approx_sqrt = false;
01621         break;
01622       case matrix::both_eigs:
01623         {
01624           const auto safe_arg = genus.m_safe_arg;
01625           scaled_result = matrix_sqrt(exp(i*safe_arg) * unitval, i, next_level) * exp(-i*safe_arg /
      Scalar_T(2));
01626         }
01627         use_approx_sqrt = false;
01628         break;
01629       default:
01630         break;
01631       }
01632       use_cr_sqrt = genus.m_is_singular;
01633     }
01634 #endif
01635     if (use_approx_sqrt)
01636     {
01637       scaled_result =
01638         (norm(unitval - Scalar_T(1)) < max_norm)
01639           // Pade' approximation of square root
01640         ? pade_approx(pade::pade_sqrt_numer<Scalar_T>::numer,
01641                       pade::pade_sqrt_denom<Scalar_T>::denom,
01642                       unitval - Scalar_T(1))
01643           // Product form of Denman-Beavers square root iteration
01644         : (use_cr_sqrt)
01645           ? cr_sqrt(unitval)
01646           : db_sqrt(unitval);
01647     }
01648     return (scaled_result.isnan() ||
```

```
01649         !approx_equal(pow(scaled_result, 2), unitval))
01650       ? traits_t::NaN()
01651       : scaled_result * rescale;
01652   }
01653
01655   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01656   auto
01657   sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01658   {
01659     // Reference: [GW], Section 4.3, pp318-322
01660     // Reference: [GL], Section 11.3, p572-576
01661     // Reference: [Z], Pade1
01662
01663     using traits_t = numeric_traits<Scalar_T>;
01664
01665     if (val.isnan())
01666       return traits_t::NaN();
01667
01668     check_complex(val, i, prechecked);
01669
01670     switch (Tune_P::function_precision)
01671     {
01672     case precision_demoted:
01673       {
01674         using demoted_scalar_t = typename traits_t::demoted::type;
01675         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
01676
01677         const auto& demoted_val = demoted_multivector_t(val);
01678         const auto& demoted_i = demoted_multivector_t(i);
01679
01680         return matrix_sqrt(demoted_val, demoted_i, 0);
01681       }
01682       break;
01683     case precision_promoted:
01684       {
01685         using promoted_scalar_t = typename traits_t::promoted::type;
01686         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
01687
01688         const auto& promoted_val = promoted_multivector_t(val);
01689         const auto& promoted_i = promoted_multivector_t(i);
01690
01691         return matrix_sqrt(promoted_val, promoted_i, 0);
01692       }
01693       break;
01694     default:
01695       return matrix_sqrt(val, i, 0);
01696     }
01697   }
01698 }
01699
01700 namespace pade {
01702   // Reference: [Z], Pade1
01703   template< typename Scalar_T >
01704   struct pade_log_numer
01705   {
01706     using array = std::array<Scalar_T, 14>;
01707     static const array numer;
01708   };
01709   template< typename Scalar_T >
01710   const typename pade_log_numer<Scalar_T>::array pade_log_numer<Scalar_T>::numer =
01711   {
01712         0.0,                      1.0,                      6.0,            4741.0/300.0,
01713      1441.0/60.0,            107091.0/4600.0,           8638.0/575.0,    263111.0/40250.0,
01714    153081.0/80500.0,         395243.0/1101240.0,      28549.0/688275.0, 605453.0/228813200.0,
01715    785633.0/10296594000.0, 1145993.0/1873980108000.0
01716   };
01717
01719   // Reference: [Z], Pade1
01720   template< typename Scalar_T >
01721   struct pade_log_denom
01722   {
01723     using array = std::array<Scalar_T, 14>;
01724     static const array denom;
01725   };
01726   template< typename Scalar_T >
01727   const typename pade_log_denom<Scalar_T>::array pade_log_denom<Scalar_T>::denom =
01728   {
01729         1.0,                    13.0/2.0,             468.0/25.0,        1573.0/50.0,
01730      1573.0/46.0,            11583.0/460.0,         10296.0/805.0,       2574.0/575.0,
01731     11583.0/10925.0,           143.0/874.0,           572.0/37145.0,      117.0/148580.0,
01732        13.0/742900.0,            1.0/10400600.0
01733   };
01734
01735   template< >
01736   struct pade_log_numer<float>
01737   {
```

```
01738     using array = std::array<float, 10>;
01739     static const array numer;
01740   };
01741   const typename pade_log_numer<float>::array pade_log_numer<float>::numer =
01742   {
01743       0.0,            1.0,             4.0,       1337.0/204.0,
01744     385.0/68.0,    1879.0/680.0,     193.0/255.0,  197.0/1820.0,
01745     419.0/61880.0, 7129.0/61261200.0
01746   };
01747   template< >
01748   struct pade_log_denom<float>
01749   {
01750     using array = std::array<float, 10>;
01751     static const array denom;
01752   };
01753   const typename pade_log_denom<float>::array pade_log_denom<float>::denom =
01754   {
01755       1.0,            9.0/2.0,       144.0/17.0,   147.0/17.0,
01756     441.0/85.0,      63.0/34.0,       84.0/221.0,    9.0/221.0,
01757       9.0/4862.0,     1.0/48620.0
01758   };
01759
01760   template< >
01761   struct pade_log_numer<long double>
01762   {
01763     using array = std::array<long double, 18>;
01764     static const array numer;
01765   };
01766   const typename pade_log_numer<long double>::array pade_log_numer<long double>::numer =
01767   {
01768         0.0L,                        1.0L,                             8.0L,
      3835.0L/132.0L,
01769       8365.0L/132.0L,         11363807.0L/122760.0L,            162981.0L/1705.0L,
      9036157.0L/125860.0L,
01770   18009875.0L/453096.0L,      44211925.0L/2718576.0L,          4149566.0L/849555.0L,
      16973929.0L/16020180.0L,
01771     172459.0L/1068012.0L,    116317061.0L/7025382936.0L,      19679783.0L/18441630207.0L,
      23763863.0L/614721006900.0L,
01772      50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
01773   };
01774   template< >
01775   struct pade_log_denom<long double>
01776   {
01777     using array = std::array<long double, 18>;
01778     static const array denom;
01779   };
01780   const typename pade_log_denom<long double>::array pade_log_denom<long double>::denom =
01781   {
01782         1.0L,                       17.0L/2.0L,                   1088.0L/33.0L,
      850.0L/11.0L,
01783      41650.0L/341.0L,           140777.0L/1023.0L,             1126216.0L/9889.0L,
      63206.0L/899.0L,
01784     790075.0L/24273.0L,          60775.0L/5394.0L,               38896.0L/13485.0L,
      21658.0L/40455.0L,
01785      21658.0L/310155.0L,          4165.0L/682341.0L,               680.0L/2047023.0L,
      34.0L/3411705.0L,
01786        17.0L/129644790.0L,          1.0L/2333606220
01787   };
01788 #if defined(_GLUCAT_USE_QD)
01789   template< >
01790   struct pade_log_numer<dd_real>
01791   {
01792     using array = std::array<dd_real, 22>;
01793     static const array numer;
01794   };
01795   const typename pade_log_numer<dd_real>::array pade_log_numer<dd_real>::numer =
01796   {
01797          dd_real("0"),                                    dd_real("1"),
01798         dd_real("10"),                                 dd_real("22781")/dd_real("492"),
01799       dd_real("21603")/dd_real("164"),               dd_real("5492649")/dd_real("21320"),
01800      dd_real("978724")/dd_real("2665"),              dd_real("4191605")/dd_real("10619"),
01801    dd_real("12874933")/dd_real("39442"),           dd_real("11473457")/dd_real("54612"),
01802     dd_real("2406734")/dd_real("22755"),          dd_real("166770367")/dd_real("4004880"),
01803    dd_real("30653165")/dd_real("2402928"),        dd_real("647746389")/dd_real("215195552"),
01804    dd_real("25346331")/dd_real("47074027"),       dd_real("278270613")/dd_real("3900419380"),
01805   dd_real("105689791")/dd_real("15601677520"),    dd_real("606046475")/dd_real("1379188292768"),
01806      dd_real("969715")/dd_real("53502994116"),     dd_real("11098301")/dd_real("26204577562592"),
01807      dd_real("118999")/dd_real("26204577562592"), dd_real("18858053")/dd_real("1392249205900512960")
01808   };
01809   template< >
01810   struct pade_log_denom<dd_real>
01811   {
01812     using array = std::array<dd_real, 22>;
01813     static const array denom;
01814   };
01815   const typename pade_log_denom<dd_real>::array pade_log_denom<dd_real>::denom =
01816   {
```

```
01817           dd_real("1"),                                   dd_real("21")/dd_real("2"),
01818         dd_real("2100")/dd_real("41"),                  dd_real("12635")/dd_real("82"),
01819       dd_real("341145")/dd_real("1066"),              dd_real("1037799")/dd_real("2132"),
01820     dd_real("11069856")/dd_real("19721"),             dd_real("9883800")/dd_real("19721"),
01821      dd_real("6918660")/dd_real("19721"),              dd_real("293930")/dd_real("1517"),
01822      dd_real("1410864")/dd_real("16687"),              dd_real("88179")/dd_real("3034"),
01823       dd_real("734825")/dd_real("94054"),              dd_real("305235")/dd_real("188108"),
01824       dd_real("348840")/dd_real("1363783"),             dd_real("40698")/dd_real("1363783"),
01825         dd_real("6783")/dd_real("2727566"),             dd_real("9975")/dd_real("70916716"),
01826          dd_real("266")/dd_real("53187537"),               dd_real("7")/dd_real("70916716"),
01827            dd_real("7")/dd_real("8155422340"),             dd_real("1")/dd_real("538257874440")
01828   };
01829
01830   template< >
01831   struct pade_log_numer<qd_real>
01832   {
01833     using array = std::array<qd_real, 34>;
01834     static const array numer;
01835   };
01836   const typename pade_log_numer<qd_real>::array pade_log_numer<qd_real>::numer =
01837   {
01838                 qd_real("0"),                                                                  qd_real("1"),
01839                 qd_real("16"),
      qd_real("95201")/qd_real("780"),
01840               qd_real("30721")/qd_real("52"),
      qd_real("7416257")/qd_real("3640"),
01841             qd_real("1039099")/qd_real("195"),
      qd_real("6097772319")/qd_real("555100"),
01842           qd_real("1564058073")/qd_real("85400"),
      qd_real("30404640205")/qd_real("1209264"),
01843           qd_real("725351278")/qd_real("25193"),
      qd_real("4092322670789")/qd_real("147429436"),
01844       qd_real("4559713849589")/qd_real("201040140"),
      qd_real("5049361751189")/qd_real("320023080"),
01845         qd_real("74979677195")/qd_real("8000577"),
      qd_real("16569850691873")/qd_real("3481514244"),
01846       qd_real("1065906022369")/qd_real("515779888"),
      qd_real("335956770855841")/qd_real("438412904800"),
01847   qd_real("1462444287585964")/qd_real("6041877844275"),
      qd_real("397242326339851")/qd_real("6122436215532"),
01848     qd_real("64211291334131")/qd_real("4373168725380"),
      qd_real("142322343550859")/qd_real("51080680851480"),
01849    qd_real("154355972958659")/qd_real("351179680853925"),
      qd_real("167483568676259")/qd_real("293713914896010 0"),
01850      qd_real("4230788929433")/qd_real("704913395750424"),
      qd_real("197968763176019")/qd_real("392923948371995600"),
01851     qd_real("10537522306718")/qd_real("3192507080522464 25"),
      qd_real("236648286272519")/qd_real("144249197475035425500"),
01852    qd_real("260715545088119")/qd_real("43755589900760745735 00"),
      qd_real("289596255666839")/qd_real("19287464028255336719 9880"),
01853      qd_real("8802625510547")/qd_real("36163995052978756349 9775"),
      qd_real("373831661521439")/qd_real("16592040930306653413 36967700"),
01854    qd_real("446033437968239")/qd_real("46457714604858629557 4350956000"),
      qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
01855       };
01856   template< >
01857   struct pade_log_denom<qd_real>
01858   {
01859     using array = std::array<qd_real, 34>;
01860     static const array denom;
01861   };
01862   const typename pade_log_denom<qd_real>::array pade_log_denom<qd_real>::denom =
01863   {
01864                 qd_real("1"),
      qd_real("33")/qd_real("2"),
01865               qd_real("8448")/qd_real("65"),
      qd_real("42284")/qd_real("65"),
01866             qd_real("211420")/qd_real("91"),
      qd_real("573562")/qd_real("91"),
01867           qd_real("32119472")/qd_real("2379"),
      qd_real("92917044")/qd_real("3965"),
01868           qd_real("603960786")/qd_real("17995"),
      qd_real("144626625")/qd_real("3599"),
01869         qd_real("2776831200")/qd_real("68381"),
      qd_real("16692542100")/qd_real("478667"),
01870        qd_real("12241197540")/qd_real("478667"),
      qd_real("1098569010")/qd_real("68381"),
01871        qd_real("31387686000")/qd_real("3624193"),
      qd_real("9939433900")/qd_real("2479711"),
01872        qd_real("67091178825")/qd_real("42155087"),
      qd_real("2683647153")/qd_real("4959422"),
01873        qd_real("19083713088")/qd_real("121505839"),
      qd_real("4708152900")/qd_real("121505839"),
01874          qd_real("941630580")/qd_real("116546417"),
      qd_real("88704330")/qd_real("62755763"),
01875           qd_real("12902448")/qd_real("62755763"),
      qd_real("1542684")/qd_real("62755763"),
```

```
01876           qd_real("6427850")/qd_real("2698497809"),
      qd_real("3471039")/qd_real("18889484663"),
01877           qd_real("8544096")/qd_real("774468871183"),
      qd_real("39556")/qd_real("79027435835"),
01878            qd_real("118668")/qd_real("7191496660985"),
      qd_real("10230")/qd_real("27327687311743"),
01879              qd_real("5456")/qd_real("1011124430534491"),
      qd_real("44")/qd_real("1011124430534491"),
01880               qd_real("11")/qd_real("70778710137414370"),
      qd_real("1")/qd_real("72194284340162657 40")
01881   };
01882 #endif
01883 }
01884
01885 namespace glucat{
01887   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01888   static
01889   auto
01890   pade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
      matrix_multi<Scalar_T,LO,HI,Tune_P>
01891   {
01892     // Reference: [GW], Section 4.3, pp318-322
01893     // Reference: [CHKL]
01894     // Reference: [GL], Section 11.3, p572-576
01895     // Reference: [Z], Pade1
01896
01897     using traits_t = numeric_traits<Scalar_T>;
01898     if (val == Scalar_T(0) || val.isnan())
01899       return traits_t::NaN();
01900     else
01901       return pade_approx(pade::pade_log_numer<Scalar_T>::numer,
01902                          pade::pade_log_denom<Scalar_T>::denom,
01903                          val - Scalar_T(1));
01904   }
01905
01907   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01908   static
01909   auto
01910   cascade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
      matrix_multi<Scalar_T,LO,HI,Tune_P>
01911   {
01912     // Reference: [CHKL]
01913     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01914     using traits_t = numeric_traits<Scalar_T>;
01915     if (val == Scalar_T(0) || val.isnan())
01916       return traits_t::NaN();
01917
01918     using limits_t = std::numeric_limits<Scalar_T>;
01919     static const auto epsilon = limits_t::epsilon();
01920     static const auto max_inner_norm = traits_t::pow(epsilon, 2);
01921     static const auto max_outer_norm = Scalar_T(6.0/limits_t::digits);
01922     auto Y = val;
01923     auto E = multivector_t(Scalar_T(0));
01924     Scalar_T norm_Y_1;
01925     auto pow_2_outer_step = Scalar_T(1);
01926     auto pow_4_outer_step = Scalar_T(1);
01927     int outer_step;
01928     for (outer_step = 0, norm_Y_1 = norm(Y - Scalar_T(1));
01929         outer_step != Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm;
01930         ++outer_step,    norm_Y_1 = norm(Y - Scalar_T(1)))
01931     {
01932       if (Y == Scalar_T(0) || Y.isnan())
01933         return traits_t::NaN();
01934
01935       // Incomplete product form of Denman-Beavers square root iteration
01936       auto M = Y;
01937       for (auto
01938           inner_step = 0;
01939           inner_step != Tune_P::log_max_inner_steps &&
01940             norm(M - Scalar_T(1)) * pow_4_outer_step > max_inner_norm;
01941           ++inner_step)
01942         db_step(M, Y);
01943
01944       E += (M - Scalar_T(1)) * pow_2_outer_step;
01945       pow_2_outer_step *= Scalar_T(2);
01946       pow_4_outer_step *= Scalar_T(4);
01947     }
01948     if (outer_step == Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm)
01949       return traits_t::NaN();
01950     else
01951       return pade_log(Y) * pow_2_outer_step - E;
01952   }
01953
01955   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01956   auto
01957   matrix_log( const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01958               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
```

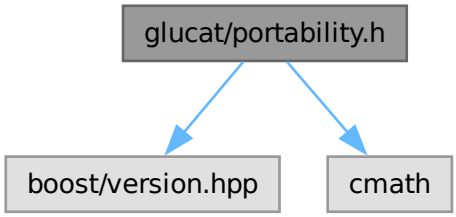
```
01959               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01960   {
01961     // Scaled incomplete square root cascade and scaled Pade' approximation of log
01962     // Reference: [CHKL]
01963
01964     using traits_t = numeric_traits<Scalar_T>;
01965     if (val == Scalar_T(0) || val.isnan())
01966       return traits_t::NaN();
01967
01968     static const auto pi = traits_t::pi();
01969     const auto scr_val = val.scalar();
01970     if (val == scr_val)
01971     {
01972       if (scr_val < Scalar_T(0))
01973         return i * pi + traits_t::log(-scr_val);
01974       else
01975         return traits_t::log(scr_val);
01976     }
01977
01978     // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01979     const auto max_norm = Scalar_T(1.0/9.0);
01980     const auto scale =
01981       (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < max_norm)
01982       ? scr_val
01983       : (scr_val < Scalar_T(0))
01984         ? -abs(val)
01985         :  abs(val);
01986     if (scale == Scalar_T(0))
01987       return traits_t::NaN();
01988
01989     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01990     const auto log_scale = traits_t::log(traits_t::abs(scale));
01991     auto rescale = multivector_t(log_scale);
01992     if (scale < Scalar_T(0))
01993       rescale = i * pi + log_scale;
01994     const auto unitval = val/scale;
01995     if (inv(unitval).isnan())
01996       return traits_t::NaN();
01997
01998 #if defined(_GLUCAT_USE_EIGENVALUES)
01999     auto scaled_result = multivector_t();
02000     if (level == 0)
02001     {
02002       // What kind of eigenvalues does the matrix contain?
02003       auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
02004       switch (genus.m_eig_case)
02005       {
02006       case matrix::neg_real_eigs:
02007         scaled_result = matrix_log(-i * unitval, i, level + 1) + i * pi/Scalar_T(2);
02008         break;
02009       case matrix::both_eigs:
02010         {
02011           const Scalar_T safe_arg = genus.m_safe_arg;
02012           scaled_result = matrix_log(exp(i*safe_arg) * unitval, i, level + 1) - i * safe_arg;
02013         }
02014         break;
02015       default:
02016         scaled_result = cascade_log(unitval);
02017         break;
02018       }
02019     }
02020     else
02021       scaled_result = cascade_log(unitval);
02022 #else
02023     auto scaled_result = cascade_log(unitval);
02024 #endif
02025     return (scaled_result.isnan())
02026       ? traits_t::NaN()
02027       : scaled_result + rescale;
02028   }
02029
02031   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02032   auto
02033   log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
      bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02034   {
02035     using traits_t = numeric_traits<Scalar_T>;
02036
02037     if (val == Scalar_T(0) || val.isnan())
02038       return traits_t::NaN();
02039
02040     check_complex(val, i, prechecked);
02041
02042     switch (Tune_P::function_precision)
02043     {
02044     case precision_demoted:
02045       {
```

```
02046         using demoted_scalar_t = typename traits_t::demoted::type;
02047         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02048
02049         const auto& demoted_val = demoted_multivector_t(val);
02050         const auto& demoted_i = demoted_multivector_t(i);
02051
02052         return matrix_log(demoted_val, demoted_i, 0);
02053       }
02054       break;
02055     case precision_promoted:
02056       {
02057         using promoted_scalar_t = typename traits_t::promoted::type;
02058         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02059
02060         const auto& promoted_val = promoted_multivector_t(val);
02061         const auto& promoted_i = promoted_multivector_t(i);
02062
02063         return matrix_log(promoted_val, promoted_i, 0);
02064       }
02065       break;
02066     default:
02067       return matrix_log(val, i, 0);
02068     }
02069   }
02070
02072   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02073   auto
02074   exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02075   {
02076     using traits_t = numeric_traits<Scalar_T>;
02077     if (val.isnan())
02078       return traits_t::NaN();
02079
02080     const auto scr_val = val.scalar();
02081     if (val == scr_val)
02082       return traits_t::exp(scr_val);
02083
02084     switch (Tune_P::function_precision)
02085     {
02086     case precision_demoted:
02087       {
02088         using demoted_scalar_t = typename traits_t::demoted::type;
02089         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02090
02091         const auto& demoted_val = demoted_multivector_t(val);
02092         return clifford_exp(demoted_val);
02093       }
02094       break;
02095     case precision_promoted:
02096       {
02097         using promoted_scalar_t = typename traits_t::promoted::type;
02098         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02099
02100         const auto& promoted_val = promoted_multivector_t(val);
02101         return clifford_exp(promoted_val);
02102       }
02103       break;
02104     default:
02105       return clifford_exp(val);
02106     }
02107   }
02108 }
02109 #endif  // _GLUCAT_MATRIX_MULTI_IMP_H
```

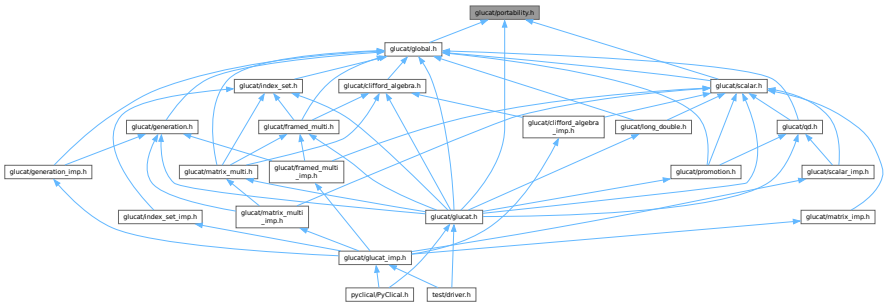
9.39 glucat/portability.h File Reference

```
#include <boost/version.hpp>
#include <cmath>
```

Include dependency graph for portability.h:



This graph shows which files directly or indirectly include this file:



Macros

- #define _GLUCAT_ISNAN(x)
- #define _GLUCAT_ISINF(x)
- #define UBLAS_ABS abs
- #define UBLAS_SQRT sqrt

9.39.1 Macro Definition Documentation

9.39.1.1 _GLUCAT_ISINF

```
#define _GLUCAT_ISINF(
                x)
```

Value:

```
(!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
```

Definition at line 43 of file portability.h.

Referenced by glucat::numeric_traits< Scalar_T >::isInf().

9.39.1.2 _GLUCAT_ISNAN

```
#define _GLUCAT_ISNAN(
              x)
```

Value:

```
(x != x)
```

Definition at line 42 of file portability.h.

Referenced by glucat::numeric_traits< Scalar_T >::isNaN().

9.39.1.3 UBLAS_ABS

```
#define UBLAS_ABS abs
```

Definition at line 51 of file portability.h.

9.39.1.4 UBLAS_SQRT

```
#define UBLAS_SQRT sqrt
```

Definition at line 52 of file portability.h.

9.40 portability.h

Go to the documentation of this file.

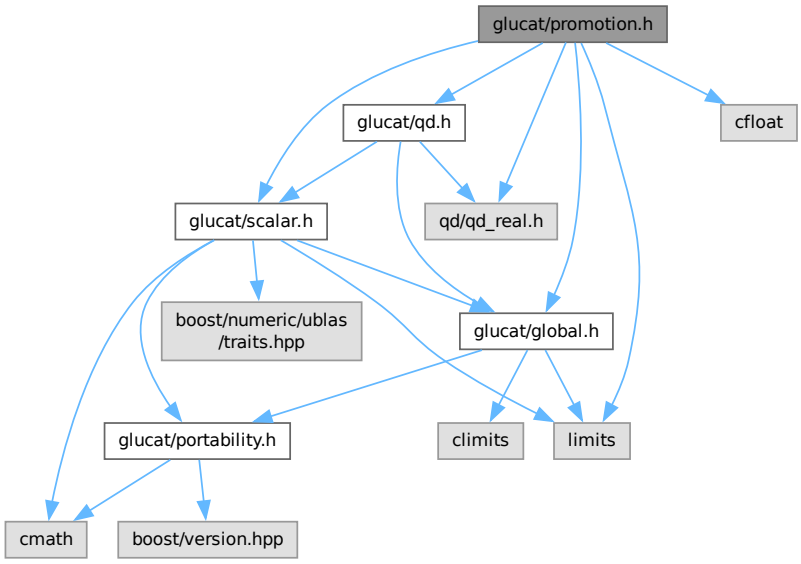
```
00001 #ifndef _GLUCAT_PORTABILITY_H
00002 #define _GLUCAT_PORTABILITY_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     portability.h : Work around non-standard compilers and libraries
00006                             -------------------
00007     begin                : Sun 2001-08-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include <boost/version.hpp>
00035 #include <cmath>
00036
00037 // Workaround for isnan and isinf
```

```
00038 #if __cplusplus > 199711L
00039 # define _GLUCAT_ISNAN(x) (std::isnan(x))
00040 # define _GLUCAT_ISINF(x) (std::isinf(x))
00041 #else
00042 # define _GLUCAT_ISNAN(x) (x != x)
00043 # define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
00044 #endif
00045
00046 // Workaround for abs and sqrt
00047 #if BOOST_VERSION >= 103400
00048 # define UBLAS_ABS  type_abs
00049 # define UBLAS_SQRT type_sqrt
00050 #else
00051 # define UBLAS_ABS  abs
00052 # define UBLAS_SQRT sqrt
00053 #endif
00054
00055 // Use with Cygwin gcc to obtain __WORDSIZE
00056 #if defined(HAVE_BITS_WORDSIZE_H)
00057 # include <bits/wordsize.h>
00058 #endif
00059
00060 #endif // _GLUCAT_PORTABILITY_H
```

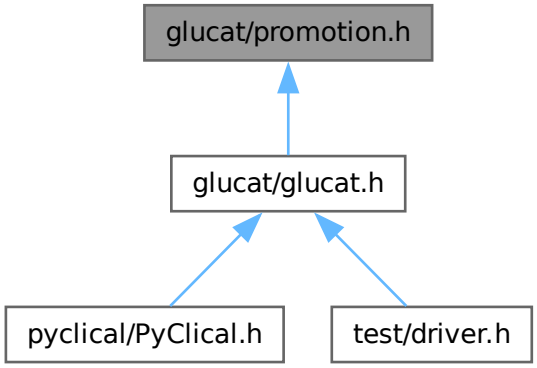
9.41 glucat/promotion.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <cfloat>
#include <limits>
#include <qd/qd_real.h>
```

Include dependency graph for promotion.h:



This graph shows which files directly or indirectly include this file:



Classes

- struct glucat::numeric_traits< Scalar_T >::promoted

  Extra traits which extend numeric limits.
- struct glucat::numeric_traits< Scalar_T >::demoted

  Demoted type for long double.

Namespaces

- namespace glucat

9.42 promotion.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_PROMOTION_H
00002 #define _GLUCAT_PROMOTION_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     promotion.h : Define promotion and demotion for specific scalar types
00006                           -------------------
00007     begin                : 2021-11-13
00008     copyright            : (C) 2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
```

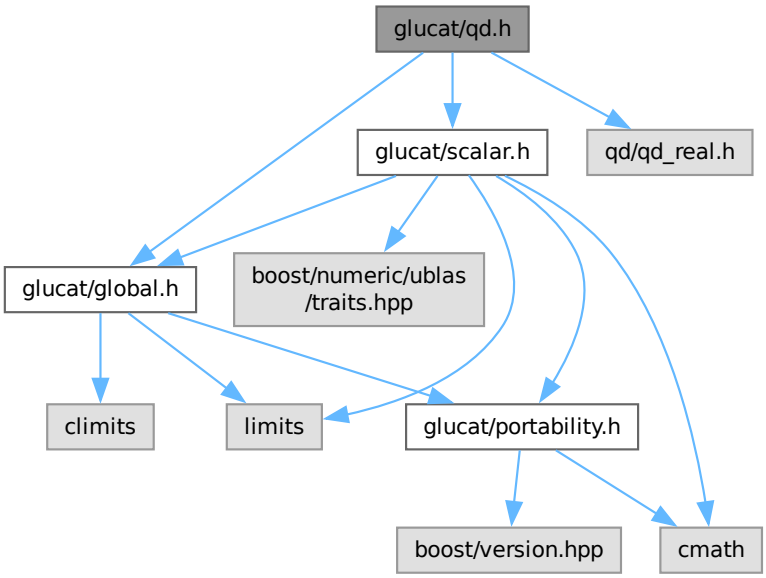
```
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ********************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ********************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036 #include "glucat/qd.h"
00037
00038 #include <cfloat>
00039 #include <limits>
00040
00041 #if defined(_GLUCAT_USE_QD)
00042 # include <qd/qd_real.h>
00043 #endif
00044
00045 namespace glucat
00046 {
00048   // Reference: [AA], 2.4, p. 30-31
00049
00050 #if !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00051
00052 # if DBL_MANT_DIG < LDBL_MANT_DIG
00053
00055   template<>
00056   struct
00057   numeric_traits<double>::
00058   promoted {using type = long double;};
00059
00061   template<>
00062   struct
00063   numeric_traits<long double>::
00064   demoted {using type = double;};
00065
00066 # else
00067
00069   template<>
00070   struct
00071   numeric_traits<double>::
00072   promoted {using type = double;};
00073
00075   template<>
00076   struct
00077   numeric_traits<long double>::
00078   demoted {using type = float;};
00079
00080 # endif // DBL_MANT_DIG < LDBL_MANT_DIG
00081
00083   template<>
00084   struct
00085   numeric_traits<long double>::
00086   promoted {using type = long double;};
00087
00088 #else
00089
00090 # if (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00091
00093   template<>
00094   struct
00095   numeric_traits<double>::
00096   promoted {using type = long double;};
00097
00099   template<>
00100   struct
00101   numeric_traits<long double>::
00102   demoted {using type = double;};
00103
00105   template<>
00106   struct
00107   numeric_traits<long double>::
00108   promoted {using type = dd_real;};
00109
00111   template<>
00112   struct
00113   numeric_traits<dd_real>::
00114   demoted {using type = long double;};
00115
00117   template<>
00118   struct
00119   numeric_traits<dd_real>::
00120   promoted {using type = qd_real;};
00121
00123   template<>
00124   struct
```

```
00125   numeric_traits<qd_real>::
00126   demoted {using type = dd_real;};
00127
00128 # elif (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00129
00131   template<>
00132   struct
00133   numeric_traits<double>::
00134   promoted {using type = dd_real;};
00135
00137   template<>
00138   struct
00139   numeric_traits<long double>::
00140   demoted {using type = float;};
00141
00143   template<>
00144   struct
00145   numeric_traits<long double>::
00146   promoted {using type = dd_real;};
00147
00149   template<>
00150   struct
00151   numeric_traits<dd_real>::
00152   demoted {using type = double;};
00153
00155   template<>
00156   struct
00157   numeric_traits<dd_real>::
00158   promoted {using type = qd_real;};
00159
00161   template<>
00162   struct
00163   numeric_traits<qd_real>::
00164   demoted {using type = dd_real;};
00165
00166 # else
00167
00169   template<>
00170   struct
00171   numeric_traits<double>::
00172   promoted {using type = dd_real;};
00173
00175   template<>
00176   struct
00177   numeric_traits<dd_real>::
00178   demoted {using type = double;};
00179
00181   template<>
00182   struct
00183   numeric_traits<dd_real>::
00184   promoted {using type = long double;};
00185
00187   template<>
00188   struct
00189   numeric_traits<long double>::
00190   demoted {using type = dd_real;};
00191
00193   template<>
00194   struct
00195   numeric_traits<long double>::
00196   promoted {using type = qd_real;};
00197
00199   template<>
00200   struct
00201   numeric_traits<qd_real>::
00202   demoted {using type = long double;};
00203
00204 # endif // (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00205
00207   template<>
00208   struct
00209   numeric_traits<qd_real>::
00210   promoted {using type = qd_real;};
00211
00212 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00213
00214 } // namespace glucat
00215
00216 #endif // _GLUCAT_PROMOTION_H
```

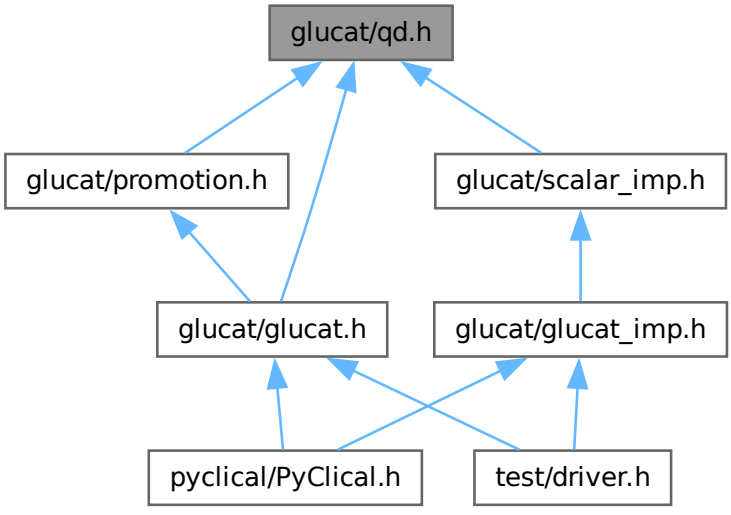
9.43 glucat/qd.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
#include <qd/qd_real.h>
```

Include dependency graph for qd.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

9.44 qd.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_QD_H
00002 #define _GLUCAT_QD_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     qd.h : Define functions for dd_real and qd_real as scalar_t
00006                             -------------------
00007     begin                : 2010-03-23
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ***************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 #if defined(_GLUCAT_USE_QD)
00038 # include <qd/qd_real.h>
00039 #endif
00040
00041 namespace glucat
00042 {
00044   // Reference: [AA], 2.4, p. 30-31
00045
00046 #if defined(_GLUCAT_USE_QD) && defined(QD_API)
00047
00049 # define _GLUCAT_QD_F(_T, _F) \
00050   template<>                  \
00051   inline                      \
00052   auto                        \
00053   numeric_traits<_T>::        \
00054   _F(const _T& val) -> _T \
00055   { return ::_F(val); }
00056
00058   template<>
00059   inline
00060   auto
00061   numeric_traits<dd_real>::
00062   isNaN(const dd_real& val) -> bool
00063   { return val.isnan(); }
00064
00066   template<>
00067   inline
00068   auto
00069   numeric_traits<dd_real>::
00070   isInf(const dd_real& val) -> bool
00071   { return val.isinf(); }
00072
00074   template<>
00075   inline
00076   auto
00077   numeric_traits<dd_real>::
00078   isNaN_or_isInf(const dd_real& val) -> bool
```

```
00079   { return val.isnan() || val.isinf(); }
00080
00082   template<>
00083   inline
00084   auto
00085   numeric_traits<dd_real>::
00086   to_int(const dd_real& val) -> int
00087   { return ::to_int(val); }
00088
00090   template<>
00091   inline
00092   auto
00093   numeric_traits<dd_real>::
00094   to_double(const dd_real& val) -> double
00095   { return ::to_double(val); }
00096
00098   template<>
00099   inline
00100   auto
00101   numeric_traits<dd_real>::
00102   fmod(const dd_real& lhs, const dd_real& rhs) -> dd_real
00103   { return ::fmod(lhs, rhs); }
00104
00106   template<>
00107   inline
00108   auto
00109   numeric_traits<dd_real>::
00110   pow(const dd_real& val, int n) -> dd_real
00111   {
00112     if (val == dd_real(0))
00113     {
00114       return
00115         (n < 0)
00116         ? NaN()
00117         : (n == 0)
00118           ? dd_real(1)
00119           : dd_real(0);
00120     }
00121     auto result = dd_real(1);
00122     auto power =
00123       (n < 0)
00124       ? dd_real(1)/val
00125       : val;
00126     for (auto
00127         k = std::abs(n);
00128         k != 0;
00129         k /= 2)
00130     {
00131       if (k % 2)
00132         result *= power;
00133       power *= power;
00134     }
00135     return result;
00136   }
00137
00139   template<>
00140   inline
00141   auto
00142   numeric_traits<dd_real>::
00143   pi() -> dd_real
00144   { return dd_real::_pi; }
00145
00147   template<>
00148   inline
00149   auto
00150   numeric_traits<dd_real>::
00151   ln_2() -> dd_real
00152   { return dd_real::_log2; }
00153
00155   _GLUCAT_QD_F(dd_real, exp)
00156
00157
00158   _GLUCAT_QD_F(dd_real, log)
00159
00160
00161   _GLUCAT_QD_F(dd_real, cos)
00162
00163
00164   _GLUCAT_QD_F(dd_real, acos)
00165
00166
00167   _GLUCAT_QD_F(dd_real, cosh)
00168
00169
00170   _GLUCAT_QD_F(dd_real, sin)
00171
00172
```

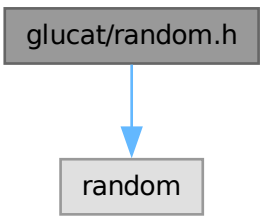
```
00173   _GLUCAT_QD_F(dd_real, asin)
00174
00175
00176   _GLUCAT_QD_F(dd_real, sinh)
00177
00178
00179   _GLUCAT_QD_F(dd_real, tan)
00180
00181
00182   _GLUCAT_QD_F(dd_real, atan)
00183
00184
00185   _GLUCAT_QD_F(dd_real, tanh)
00186
00187
00188   template<>
00189   inline
00190   auto
00191   numeric_traits<qd_real>::
00192   isNaN(const qd_real& val) -> bool
00193   { return val.isnan(); }
00194
00196   template<>
00197   inline
00198   auto
00199   numeric_traits<qd_real>::
00200   isInf(const qd_real& val) -> bool
00201   { return val.isinf(); }
00202
00204   template<>
00205   inline
00206   auto
00207   numeric_traits<qd_real>::
00208   isNaN_or_isInf(const qd_real& val) -> bool
00209   { return val.isnan() || val.isinf(); }
00210
00212   template<>
00213   inline
00214   auto
00215   numeric_traits<qd_real>::
00216   to_int(const qd_real& val) -> int
00217   { return ::to_int(val); }
00218
00220   template<>
00221   inline
00222   auto
00223   numeric_traits<qd_real>::
00224   to_double(const qd_real& val) -> double
00225   { return ::to_double(val); }
00226
00228   template<>
00229   inline
00230   auto
00231   numeric_traits<qd_real>::
00232   fmod(const qd_real& lhs, const qd_real& rhs) -> qd_real
00233   { return ::fmod(lhs, rhs); }
00234
00236   template<>
00237   inline
00238   auto
00239   numeric_traits<qd_real>::
00240   pow(const qd_real& val, int n) -> qd_real
00241   {
00242     if (val == qd_real(0))
00243     {
00244       return
00245         (n < 0)
00246         ? NaN()
00247         : (n == 0)
00248           ? qd_real(1)
00249           : qd_real(0);
00250     }
00251     auto result = qd_real(1);
00252     auto power =
00253       (n < 0)
00254       ? qd_real(1)/val
00255       : val;
00256     for (auto
00257         k = std::abs(n);
00258         k != 0;
00259         k /= 2)
00260     {
00261       if (k % 2)
00262         result *= power;
00263       power *= power;
00264     }
00265     return result;
```

```
00266   }
00267
00269   template<>
00270   inline
00271   auto
00272   numeric_traits<qd_real>::
00273   pi() -> qd_real
00274   { return qd_real::_pi; }
00275
00277   template<>
00278   inline
00279   auto
00280   numeric_traits<qd_real>::
00281   ln_2() -> qd_real
00282   { return qd_real::_log2; }
00283
00285   _GLUCAT_QD_F(qd_real, exp)
00286
00287
00288   _GLUCAT_QD_F(qd_real, log)
00289
00290
00291   _GLUCAT_QD_F(qd_real, cos)
00292
00293
00294   _GLUCAT_QD_F(qd_real, acos)
00295
00296
00297   _GLUCAT_QD_F(qd_real, cosh)
00298
00299
00300   _GLUCAT_QD_F(qd_real, sin)
00301
00302
00303   _GLUCAT_QD_F(qd_real, asin)
00304
00305
00306   _GLUCAT_QD_F(qd_real, sinh)
00307
00308
00309   _GLUCAT_QD_F(qd_real, tan)
00310
00311
00312   _GLUCAT_QD_F(qd_real, atan)
00313
00314
00315   _GLUCAT_QD_F(qd_real, tanh)
00316
00317 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00318
00319 } // namespace glucat
00320
00321 #endif // _GLUCAT_QD_H
```

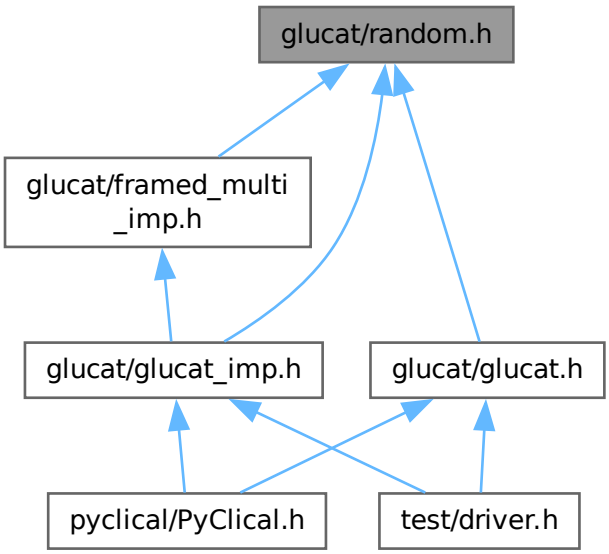
9.45 glucat/random.h File Reference

`#include <random>`

Include dependency graph for random.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::random_generator< Scalar_T >

  Random number generator with single instance per Scalar_T.

Namespaces

- namespace glucat

9.46 random.h

Go to the documentation of this file.

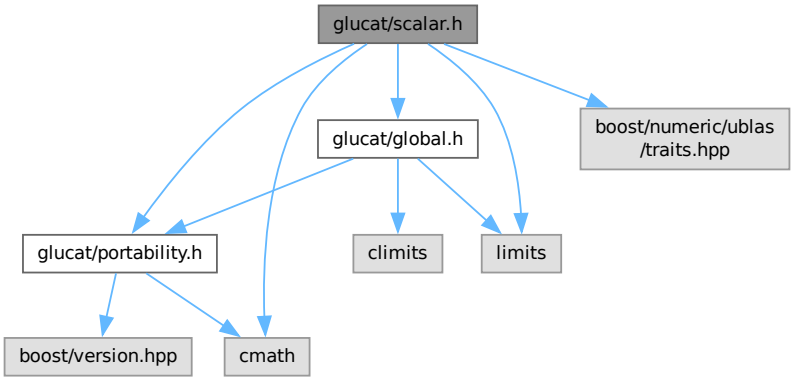
```
00001 #ifndef _GLUCAT_RANDOM_H
00002 #define _GLUCAT_RANDOM_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     random.h : Random number generator with single instance per Scalar_T
00006                             -------------------
00007     begin                : 2010-03-28
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
```

```
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ************************************************************************/
00033
00034 #include <random>
00035
00036 namespace glucat
00037 {
00039   // Enforce singleton
00040   // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00041   template< typename Scalar_T >
00042   class random_generator
00043   {
00044   private:
00048     friend class friend_for_private_destructor;
00049   public:
00051     static auto generator() -> random_generator& { static random_generator g; return g;}
00052     random_generator(const random_generator&) = delete;
00053     auto operator= (const random_generator&) -> random_generator& = delete;
00054   private:
00055     static const unsigned long seed = 19590921UL;
00056
00057     std::mt19937 uint_gen;
00058     std::uniform_real_distribution<double> uniform_dist;
00059     std::normal_distribution<double> normal_dist;
00060
00061     random_generator() :
00062     uint_gen(), uniform_dist(0.0, 1.0), normal_dist(0.0, 1.0)
00063     { this->uint_gen.seed(seed); }
00064
00065     ~random_generator() = default;
00066
00067   public:
00068     auto uniform() -> Scalar_T
00069     { return Scalar_T(this->uniform_dist(this->uint_gen)); }
00070     auto normal() -> Scalar_T
00071     { return Scalar_T(this->normal_dist(this->uint_gen)); }
00072   };
00073 }
00074
00075 #endif // _GLUCAT_RANDOM_H
```

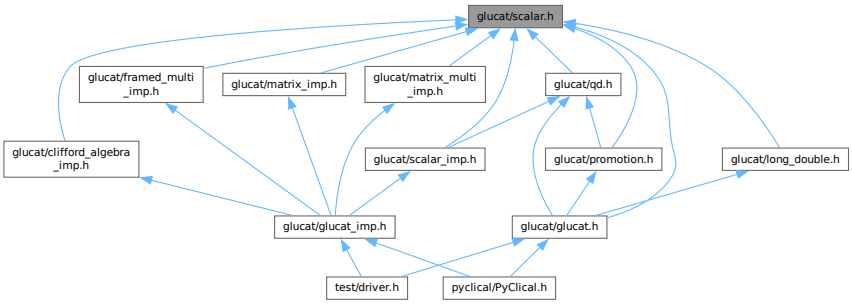
9.47 glucat/scalar.h File Reference

```
#include "glucat/portability.h"
#include "glucat/global.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>
```

Include dependency graph for scalar.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::numeric_traits< Scalar_T >

  Extra traits which extend numeric limits.
- struct glucat::numeric_traits< Scalar_T >::promoted

  Extra traits which extend numeric limits.
- struct glucat::numeric_traits< Scalar_T >::demoted

  Demoted type for long double.

Namespaces

- namespace glucat

Functions

- template<typename Scalar_T>
  auto glucat::log2 (const Scalar_T &x) -> Scalar_T

  Log base 2 of scalar.

9.48 scalar.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_SCALAR_H
00002 #define _GLUCAT_SCALAR_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar.h : Define functions for scalar_t
00006                             -------------------
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ***************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/portability.h"
00035 #include "glucat/global.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00045   // Reference: [AA], 2.4, p. 30-31
00046   template< typename Scalar_T >
00047   class numeric_traits
00048   {
00049   private:
00051     inline
00052     static
00053     auto
00054     isInf(const Scalar_T& val, bool_to_type<false>) -> bool
00055     { return false; }
00056
00058     inline
00059     static
00060     auto
00061     isInf(const Scalar_T& val, bool_to_type<true>) -> bool
00062     { return _GLUCAT_ISINF(val); }
00063
00065     inline
00066     static
00067     auto
00068     isNaN(const Scalar_T& val, bool_to_type<false>) -> bool
00069     { return false; }
00070
00072     inline
00073     static
00074     auto
00075     isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00076     { return _GLUCAT_ISNAN(val); }
00077
00078   public:
00080     inline
00081     static
00082     auto
00083     isInf(const Scalar_T& val) -> bool
00084     {
00085       return isInf(val,
00086              bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00087     }
00088
```

```
00090     inline
00091     static
00092     auto
00093     isNaN(const Scalar_T& val) -> bool
00094     {
00095       return isNaN(val,
00096              bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() );
00097     }
00098
00100     inline
00101     static
00102     auto
00103     isNaN_or_isInf(const Scalar_T& val) -> bool
00104     {
00105       return isNaN(val,
00106              bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() )
00107           || isInf(val,
00108              bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00109     }
00110
00112     inline
00113     static
00114     auto
00115     NaN() -> Scalar_T
00116     {
00117       return std::numeric_limits<Scalar_T>::has_quiet_NaN
00118            ? std::numeric_limits<Scalar_T>::quiet_NaN()
00119            : Scalar_T(std::log(0.0));
00120     }
00121
00123     inline
00124     static
00125     auto
00126     to_int(const Scalar_T& val) -> int
00127     { return static_cast<int>(val); }
00128
00130     inline
00131     static
00132     auto
00133     to_double(const Scalar_T& val) -> double
00134     { return static_cast<double>(val); }
00135
00137     template <typename Other_Scalar_T >
00138     inline
00139     static
00140     auto
00141     to_scalar_t(const Other_Scalar_T& val) -> Scalar_T
00142     { return static_cast<Scalar_T>(val); }
00143
00145     struct promoted {using type = double;};
00146
00148     struct demoted {using type = float;};
00149
00151     inline
00152     static
00153     auto
00154     fmod(const Scalar_T& lhs, const Scalar_T& rhs) -> Scalar_T
00155     { return std::fmod(lhs, rhs); }
00156
00158     inline
00159     static
00160     auto
00161     conj(const Scalar_T& val) -> Scalar_T
00162     { return val; }
00163
00165     inline
00166     static
00167     auto
00168     real(const Scalar_T& val) -> Scalar_T
00169     { return val; }
00170
00172     inline
00173     static
00174     auto
00175     imag(const Scalar_T& val) -> Scalar_T
00176     { return Scalar_T(0); }
00177
00179     inline
00180     static
00181     auto
00182     abs(const Scalar_T& val) -> Scalar_T
00183     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_ABS(val); }
00184
00186     inline
00187     static
00188     auto
00189     pi() -> Scalar_T
```

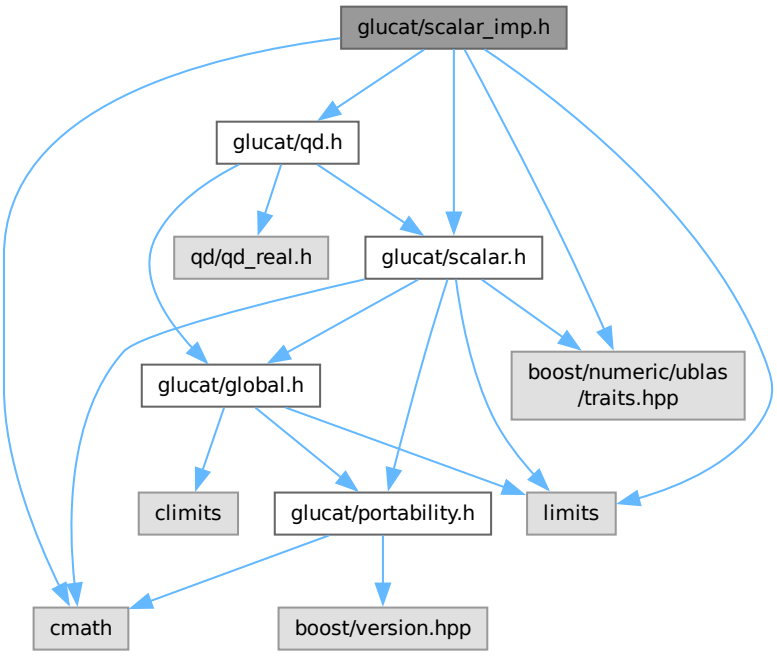
```
00190     { return Scalar_T(3.14159265358979323); }
00191
00193     inline
00194     static
00195     auto
00196     ln_2() -> Scalar_T
00197     { return Scalar_T(0.693147180559945309); }
00198
00200     inline
00201     static
00202     auto
00203     pow(const Scalar_T& val, int n) -> Scalar_T
00204     { return std::pow(val, n); }
00205
00207     inline
00208     static
00209     auto
00210     sqrt(const Scalar_T& val) -> Scalar_T
00211     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_SQRT(val); }
00212
00214     inline
00215     static
00216     auto
00217     exp(const Scalar_T& val) -> Scalar_T
00218     { return std::exp(val); }
00219
00221     inline
00222     static
00223     auto
00224     log(const Scalar_T& val) -> Scalar_T
00225     { return std::log(val); }
00226
00228     inline
00229     static
00230     auto
00231     log2(const Scalar_T& val) -> Scalar_T
00232     { return log(val)/ln_2(); }
00233
00235     inline
00236     static
00237     auto
00238     cos(const Scalar_T& val) -> Scalar_T
00239     { return std::cos(val); }
00240
00242     inline
00243     static
00244     auto
00245     acos(const Scalar_T& val) -> Scalar_T
00246     { return std::acos(val); }
00247
00249     inline
00250     static
00251     auto
00252     cosh(const Scalar_T& val) -> Scalar_T
00253     { return std::cosh(val); }
00254
00256     inline
00257     static
00258     auto
00259     sin(const Scalar_T& val) -> Scalar_T
00260     { return std::sin(val); }
00261
00263     inline
00264     static
00265     auto
00266     asin(const Scalar_T& val) -> Scalar_T
00267     { return std::asin(val); }
00268
00270     inline
00271     static
00272     auto
00273     sinh(const Scalar_T& val) -> Scalar_T
00274     { return std::sinh(val); }
00275
00277     inline
00278     static
00279     auto
00280     tan(const Scalar_T& val) -> Scalar_T
00281     { return std::tan(val); }
00282
00284     inline
00285     static
00286     auto
00287     atan(const Scalar_T& val) -> Scalar_T
00288     { return std::atan(val); }
00289
00291     inline
```

```
00292     static
00293     auto
00294     tanh(const Scalar_T& val) -> Scalar_T
00295     { return std::tanh(val); }
00296
00297   };
00298
00300   template< typename Scalar_T >
00301   inline
00302   auto
00303   log2(const Scalar_T& x) -> Scalar_T
00304   { return numeric_traits<Scalar_T>::log2(x); }
00305 }
00306
00307 #endif // _GLUCAT_SCALAR_H
```

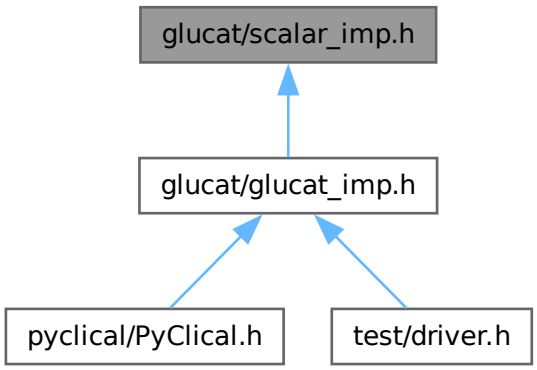
9.49 glucat/scalar_imp.h File Reference

```
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>
```

Include dependency graph for scalar_imp.h:



This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Functions

- template<typename Scalar_T>
  auto glucat::to_promote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::promoted::type

  Cast to promote.
- template<typename Scalar_T>
  auto glucat::to_demote (const Scalar_T &val) -> typename numeric_traits< Scalar_T >::demoted::type

  Cast to demote.

9.50 scalar_imp.h

Go to the documentation of this file.

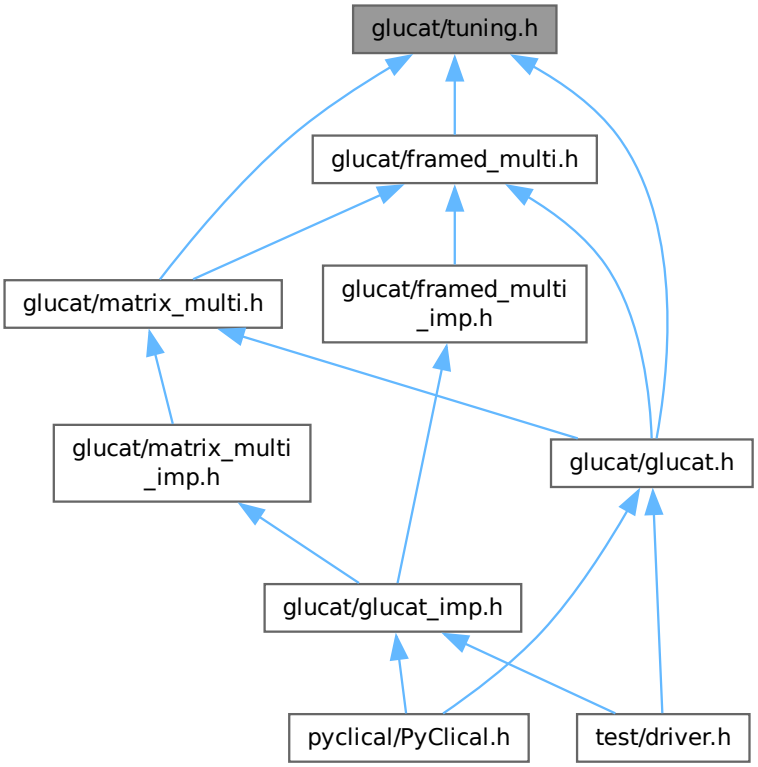
```
00001 #ifndef _GLUCAT_SCALAR_IMP_H
00002 #define _GLUCAT_SCALAR_IMP_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar_imp.h : Define functions for scalar_t
00006                             -------------------
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2014 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
```

```
00023
00024  ****************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030  ****************************************************************************
00031  See also Arvind Raja's original header comments and references in glucat.h
00032  ****************************************************************************/
00033
00034 #include "glucat/scalar.h"
00035 #include "glucat/qd.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00045   // Reference: [AA], 2.4, p. 30-31
00046
00048   template< >
00049   template< typename Other_Scalar_T >
00050   inline
00051   auto
00052   numeric_traits<float>::
00053   to_scalar_t(const Other_Scalar_T& val) -> float
00054   { return static_cast<float>(numeric_traits<Other_Scalar_T>::to_double(val)); }
00055
00057   template< >
00058   template< typename Other_Scalar_T >
00059   inline
00060   auto
00061   numeric_traits<double>::
00062   to_scalar_t(const Other_Scalar_T& val) -> double
00063   { return numeric_traits<Other_Scalar_T>::to_double(val); }
00064
00065 #if defined(_GLUCAT_USE_QD)
00067   template< >
00068   template< >
00069   inline
00070   auto
00071   numeric_traits<long double>::
00072   to_scalar_t(const dd_real& val) -> long double
00073   { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00074
00076   template< >
00077   template< >
00078   inline
00079   auto
00080   numeric_traits<long double>::
00081   to_scalar_t(const qd_real& val) -> long double
00082   { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00083
00085   template< >
00086   template< >
00087   inline
00088   auto
00089   numeric_traits<dd_real>::
00090   to_scalar_t(const long double& val) -> dd_real
00091   { return {double(val),double(val - static_cast<long double>(double(val)))}; }
00092
00094   template< >
00095   template< >
00096   inline
00097   auto
00098   numeric_traits<dd_real>::
00099   to_scalar_t(const qd_real& val) -> dd_real
00100   { return {val.x[0],val.x[1]}; }
00101
00103   template< >
00104   template< >
00105   inline
00106   auto
00107   numeric_traits<qd_real>::
00108   to_scalar_t(const long double& val) -> qd_real
00109   { return {double(val),double(val - static_cast<long double>(double(val))),0.0,0.0}; }
00110
00112   template< >
00113   template< >
00114   inline
00115   auto
00116   numeric_traits<qd_real>::
00117   to_scalar_t(const dd_real& val) -> qd_real
00118   { return {val.x[0],val.x[1],0.0,0.0}; }
```

```
00119 #endif
00120
00122   template< typename Scalar_T >
00123   inline
00124   auto
00125   to_promote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::promoted::type
00126   {
00127     using promoted_scalar_t = typename numeric_traits<Scalar_T>::promoted::type;
00128     return numeric_traits<promoted_scalar_t>::to_scalar_t(val);
00129   }
00130
00132   template< typename Scalar_T >
00133   inline
00134   auto
00135   to_demote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::demoted::type
00136   {
00137     using demoted_scalar_t = typename numeric_traits<Scalar_T>::demoted::type;
00138     return numeric_traits<demoted_scalar_t>::to_scalar_t(val);
00139   }
00140 }
00141
00142 #endif // _GLUCAT_SCALAR_IMP_H
```

9.51 glucat/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



Functions

- _GLUCAT_CTAssert (std::numeric_limits< unsigned int >::radix==2, CannotSetThresholds) namespace glucat

9.51.1 Function Documentation

9.51.1.1 _GLUCAT_CTAssert()

```
_GLUCAT_CTAssert (
            std::numeric_limits< unsigned int >::radix  = = 2,
            CannotSetThresholds )
```

Base class for policies

Precision policy

Tuning policy

Minimum index count needed to invoke matrix multiplication algorithm

Maximum steps of iterative refinement in division algorithm

Maximum number of steps in cyclic reduction square root iteration

Maximum number of steps in Denman-Beavers square root iteration

Maximum number of incomplete square roots in cascade log algorithm

Maximum number of steps in incomplete square root within cascade log algorithm

Maximum index count of folded frames in basis cache

Minimum map size needed to invoke generalized FFT

Minimum matrix dimension needed to invoke inverse generalized FFT

Minimum size needed for to invoke faster products algorithms

Denominator of proportion of different bits allowed in approximate equality

Extra number of different bits allowed in approximate equality

Precision used for exp, log and sqrt functions

Definition at line 35 of file tuning.h.

References _GLUCAT_CTAssert.

9.52 tuning.h

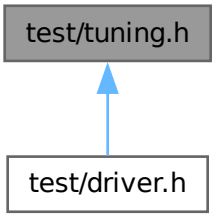
Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TUNING_H
00002 #define GLUCAT_TUNING_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Policy classes to control tuning
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 // If radix of int is not 2, we can't easily set thresholds
00035 _GLUCAT_CTAssert(std::numeric_limits<unsigned int>::radix == 2, CannotSetThresholds)
00036
00037 namespace glucat
00038 {
00040   struct policy{};
00041
00043   enum precision_t
00044   {
00045     precision_demoted,
00046     precision_same,
00047     precision_promoted
00048   };
00049
00050   // Tuning policy default constants
00051   const unsigned int Tuning_Default_Mult_Matrix_Threshold   =       8;
00052   const unsigned int Tuning_Default_Div_Max_Steps           =       4;
00053   const unsigned int Tuning_Default_CR_Sqrt_Max_Steps       =     256;
00054   const unsigned int Tuning_Default_DB_Sqrt_Max_Steps       =     256;
00055   const unsigned int Tuning_Default_Log_Max_Outer_Steps     =     256;
00056   const unsigned int Tuning_Default_Log_Max_Inner_Steps     =      32;
00057   const unsigned int Tuning_Default_Basis_Max_Count         =      12;
00058   const unsigned int Tuning_Default_Fast_Size_Threshold     = 1 «  6;
00059   const unsigned int Tuning_Default_Inv_Fast_Dim_Threshold  = 1 «  3;
00060   const unsigned int Tuning_Default_Products_Size_Threshold = 1 « 22;
00061   const unsigned int Tuning_Default_Denom_Different_Bits    =       8;
00062   const unsigned int Tuning_Default_Extra_Different_Bits    =       8;
00063   const precision_t  Tuning_Default_Function_Precision     = precision_same;
00064
00066   template
00067   <
00068     unsigned int Mult_Matrix_Threshold   = Tuning_Default_Mult_Matrix_Threshold,
00069     unsigned int Div_Max_Steps           = Tuning_Default_Div_Max_Steps,
00070     unsigned int CR_Sqrt_Max_Steps       = Tuning_Default_CR_Sqrt_Max_Steps,
00071     unsigned int DB_Sqrt_Max_Steps       = Tuning_Default_DB_Sqrt_Max_Steps,
00072     unsigned int Log_Max_Outer_Steps     = Tuning_Default_Log_Max_Outer_Steps,
00073     unsigned int Log_Max_Inner_Steps     = Tuning_Default_Log_Max_Inner_Steps,
00074     unsigned int Basis_Max_Count         = Tuning_Default_Basis_Max_Count,
00075     unsigned int Fast_Size_Threshold     = Tuning_Default_Fast_Size_Threshold,
00076     unsigned int Inv_Fast_Dim_Threshold  = Tuning_Default_Inv_Fast_Dim_Threshold,
00077     unsigned int Products_Size_Threshold = Tuning_Default_Products_Size_Threshold,
00078     unsigned int Denom_Different_Bits    = Tuning_Default_Denom_Different_Bits,
00079     unsigned int Extra_Different_Bits    = Tuning_Default_Extra_Different_Bits,
00080     precision_t  Function_Precision      = Tuning_Default_Function_Precision
00081   >
00082   struct tuning : policy
00083   {
00084     using tune_p = tuning
00085     <
```

```
00086       Mult_Matrix_Threshold,
00087       Div_Max_Steps,
00088       CR_Sqrt_Max_Steps,
00089       DB_Sqrt_Max_Steps,
00090       Log_Max_Outer_Steps,
00091       Log_Max_Inner_Steps,
00092       Basis_Max_Count,
00093       Fast_Size_Threshold,
00094       Inv_Fast_Dim_Threshold,
00095       Products_Size_Threshold,
00096       Denom_Different_Bits,
00097       Extra_Different_Bits,
00098       Function_Precision
00099     >;
00100   // Tuning for multiplication
00102     enum { mult_matrix_threshold = Mult_Matrix_Threshold };
00103   // Tuning for division
00105     enum { div_max_steps = Div_Max_Steps };
00106   // Tuning for sqrt
00108     enum { cr_sqrt_max_steps = CR_Sqrt_Max_Steps };
00110     enum { db_sqrt_max_steps = DB_Sqrt_Max_Steps };
00111   // Tuning for log
00113     enum { log_max_outer_steps = Log_Max_Outer_Steps };
00115     enum { log_max_inner_steps = Log_Max_Inner_Steps };
00116   // Tuning for basis cache
00118     enum { basis_max_count = Basis_Max_Count };
00119   // Tuning for FFT
00121     enum { fast_size_threshold = Fast_Size_Threshold };
00123     enum { inv_fast_dim_threshold = Inv_Fast_Dim_Threshold };
00124   // Tuning for products (other than geometric product)
00126     enum { products_size_threshold = Products_Size_Threshold };
00127   // Tuning for precision of exp, log and sqrt functions
00129     enum { denom_different_bits = Denom_Different_Bits };
00131     enum { extra_different_bits = Extra_Different_Bits };
00133     static const precision_t function_precision = Function_Precision;
00134   };
00135
00136   using tuning_demoted = tuning
00137     <
00138       Tuning_Default_Mult_Matrix_Threshold,
00139       Tuning_Default_Div_Max_Steps,
00140       Tuning_Default_CR_Sqrt_Max_Steps,
00141       Tuning_Default_DB_Sqrt_Max_Steps,
00142       Tuning_Default_Log_Max_Outer_Steps,
00143       Tuning_Default_Log_Max_Inner_Steps,
00144       Tuning_Default_Basis_Max_Count,
00145       Tuning_Default_Fast_Size_Threshold,
00146       Tuning_Default_Inv_Fast_Dim_Threshold,
00147       Tuning_Default_Products_Size_Threshold,
00148       Tuning_Default_Denom_Different_Bits,
00149       Tuning_Default_Extra_Different_Bits,
00150       precision_demoted
00151     >;
00152
00153   using tuning_promoted = tuning
00154     <
00155       Tuning_Default_Mult_Matrix_Threshold,
00156       Tuning_Default_Div_Max_Steps,
00157       Tuning_Default_CR_Sqrt_Max_Steps,
00158       Tuning_Default_DB_Sqrt_Max_Steps,
00159       Tuning_Default_Log_Max_Outer_Steps,
00160       Tuning_Default_Log_Max_Inner_Steps,
00161       Tuning_Default_Basis_Max_Count,
00162       Tuning_Default_Fast_Size_Threshold,
00163       Tuning_Default_Inv_Fast_Dim_Threshold,
00164       Tuning_Default_Products_Size_Threshold,
00165       Tuning_Default_Denom_Different_Bits,
00166       Tuning_Default_Extra_Different_Bits,
00167       precision_promoted
00168     >;
00169 }
00170
00171 #endif // GLUCAT_TUNING_H
```

9.53 test/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Typedefs

- using glucat::tuning_slow
- using glucat::tuning_naive
- using glucat::tuning_fast

Variables

- const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
- const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning_Int_Digits
- const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
- const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
- const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
- const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
- const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
- const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
- const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
- const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
- const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
- const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
- const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
- const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
- const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0

9.54 tuning.h

Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TEST_TUNING_H
00002 #define GLUCAT_TEST_TUNING_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Class definitions to control test tuning
00006                             -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 namespace glucat
00035 {
00036   const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits;
00037   const unsigned int Tuning_Max_Threshold = 1 « Tuning_Int_Digits;
00038
00039   // Specific tuning policy constants and tuning policies
00040
00041   const unsigned int Tuning_Slow_Mult_Matrix_Threshold   = Tuning_Max_Threshold;
00042   const unsigned int Tuning_Slow_Basis_Max_Count         =       0;
00043   const unsigned int Tuning_Slow_Fast_Size_Threshold     = Tuning_Max_Threshold;
00044   const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold  = Tuning_Max_Threshold;
00045   const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold;
00046
00047   using tuning_slow = tuning
00048     <
00049       Tuning_Slow_Mult_Matrix_Threshold,
00050       Tuning_Default_Div_Max_Steps,
00051       Tuning_Default_CR_Sqrt_Max_Steps,
00052       Tuning_Default_DB_Sqrt_Max_Steps,
00053       Tuning_Default_Log_Max_Outer_Steps,
00054       Tuning_Default_Log_Max_Inner_Steps,
00055       Tuning_Slow_Basis_Max_Count,
00056       Tuning_Slow_Fast_Size_Threshold,
00057       Tuning_Slow_Inv_Fast_Dim_Threshold,
00058       Tuning_Slow_Products_Size_Threshold,
00059       Tuning_Default_Denom_Different_Bits,
00060       Tuning_Default_Extra_Different_Bits,
00061       Tuning_Default_Function_Precision
00062     >;
00063
00064   const unsigned int Tuning_Naive_Mult_Matrix_Threshold   =       0;
00065   const unsigned int Tuning_Naive_Basis_Max_Count         = Tuning_Max_Threshold;
00066   const unsigned int Tuning_Naive_Fast_Size_Threshold     = Tuning_Max_Threshold;
00067   const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold  = Tuning_Max_Threshold;
00068
00069   using tuning_naive = tuning
00070     <
00071       Tuning_Naive_Mult_Matrix_Threshold,
00072       Tuning_Default_Div_Max_Steps,
00073       Tuning_Default_CR_Sqrt_Max_Steps,
00074       Tuning_Default_DB_Sqrt_Max_Steps,
00075       Tuning_Default_Log_Max_Outer_Steps,
00076       Tuning_Default_Log_Max_Inner_Steps,
00077       Tuning_Naive_Basis_Max_Count,
00078       Tuning_Naive_Fast_Size_Threshold,
00079       Tuning_Naive_Inv_Fast_Dim_Threshold,
00080       Tuning_Default_Products_Size_Threshold,
00081       Tuning_Default_Denom_Different_Bits,
00082       Tuning_Default_Extra_Different_Bits,
```

```
00083       Tuning_Default_Function_Precision
00084     >;
00085
00086   const unsigned int Tuning_Fast_Mult_Matrix_Threshold   =       0;
00087   const unsigned int Tuning_Fast_Div_Max_Steps           =       0;
00088   const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps       =     256;
00089   const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps       =     256;
00090   const unsigned int Tuning_Fast_Log_Max_Outer_Steps     =      16;
00091   const unsigned int Tuning_Fast_Log_Max_Inner_Steps     =       8;
00092   const unsigned int Tuning_Fast_Basis_Max_Count         =       1;
00093   const unsigned int Tuning_Fast_Fast_Size_Threshold     =       0;
00094   const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold  =       0;
00095   const unsigned int Tuning_Fast_Products_Size_Threshold =       0;
00096
00097   using tuning_fast = tuning
00098     <
00099       Tuning_Fast_Mult_Matrix_Threshold,
00100       Tuning_Fast_Div_Max_Steps,
00101       Tuning_Fast_CR_Sqrt_Max_Steps,
00102       Tuning_Fast_DB_Sqrt_Max_Steps,
00103       Tuning_Fast_Log_Max_Outer_Steps,
00104       Tuning_Fast_Log_Max_Inner_Steps,
00105       Tuning_Fast_Basis_Max_Count,
00106       Tuning_Fast_Fast_Size_Threshold,
00107       Tuning_Fast_Inv_Fast_Dim_Threshold,
00108       Tuning_Fast_Products_Size_Threshold,
00109       Tuning_Default_Denom_Different_Bits,
00110       Tuning_Default_Extra_Different_Bits,
00111       Tuning_Default_Function_Precision
00112     >;
00113 }
00114 #endif // GLUCAT_TEST_TUNING_H
```

9.55 pyclical/glucat.pxd File Reference

Namespaces

- namespace glucat

9.56 glucat.pxd

Go to the documentation of this file.

```
00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #           Generic library of universal Clifford algebra templates
00006 #
00007 # glucat.pxd: Basic Cython definitions
00008 #             corresponding to C++ definitions from PyClical.h.
00009 # Kept as a separate module from PyClical.pxd to avoid namespace clashes.
00010 #
00011 #    copyright            : (C) 2008-2012 by Paul C. Leopardi
00012 #
00013 #    This library is free software: you can redistribute it and/or modify
00014 #    it under the terms of the GNU Lesser General Public License as published
00015 #    by the Free Software Foundation, either version 3 of the License, or
00016 #    (at your option) any later version.
00017 #
00018 #    This library is distributed in the hope that it will be useful,
00019 #    but WITHOUT ANY WARRANTY; without even the implied warranty of
00020 #    MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00021 #    GNU Lesser General Public License for more details.
00022 #
00023 #    You should have received a copy of the GNU Lesser General Public License
00024 #    along with this library.  If not, see <http://www.gnu.org/licenses/>.
00025
00026 from libcpp.vector cimport vector
00027
00028 cdef extern from "PyClical.h":
00029
00030     cdef cppclass String:
00031         char* c_str()
00032
```

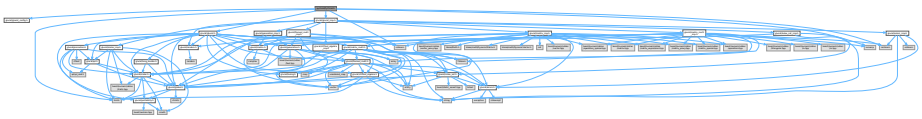
```
00033     cdef cppclass IndexSet:
00034         IndexSet ()
00035         IndexSet (IndexSet Ist) except+
00036         IndexSet (int idx) except+
00037         IndexSet (char* str) except+
00038         inline bint operator==(IndexSet Rhs)
00039         inline bint operator!=(IndexSet Rhs)
00040         inline bint operator<(IndexSet Rhs)
00041         inline IndexSet invert "operator~"()
00042         inline bint getitem "operator[]"(int idx)
00043         inline IndexSet set()
00044         inline IndexSet set(int idx) except+
00045         inline IndexSet set(int idx, int val) except+
00046         inline IndexSet reset()
00047         inline IndexSet reset(int idx) except+
00048         int count()
00049         int count_pos()
00050         int count_neg()
00051         int min()
00052         int max()
00053         int sign_of_mult(IndexSet Rhs)
00054         int sign_of_square()
00055         int hash_fn()
00056
00057     int compare(IndexSet Lhs, IndexSet Rhs)
00058     int min_neg(IndexSet Ist)
00059     int max_pos(IndexSet Ist)
00060
00061     ctypedef double scalar_t
00062
00063     cdef cppclass Clifford:
00064         Clifford ()
00065         Clifford (Clifford Clf) except+
00066         Clifford (Clifford Clf, IndexSet ist) except+
00067         Clifford (scalar_t scr) except+
00068         Clifford (char* str) except+
00069         Clifford (IndexSet ist, scalar_t scr) except+
00070         Clifford (vector[scalar_t] vec, IndexSet ist) except+
00071         bint     operator==(Clifford Rhs)
00072         bint     operator!=(Clifford Rhs)
00073         Clifford neg "operator-"()
00074         scalar_t getitem "operator[]"(IndexSet Ist)
00075         Clifford call "operator()"(int grade)
00076         scalar_t scalar()
00077         Clifford pure()
00078         Clifford even()
00079         Clifford odd()
00080         vector[scalar_t] vector_part()
00081         vector[scalar_t] vector_part(IndexSet frm) except+
00082         Clifford involute()
00083         Clifford reverse()
00084         Clifford conj()
00085         Clifford random(IndexSet Ist, scalar_t fill)
00086         scalar_t norm()
00087         scalar_t quad()
00088         IndexSet frame()
00089         scalar_t max_abs()
00090         Clifford inv()
00091         Clifford pow(int m)
00092         Clifford outer_pow(int m)
00093         Clifford truncated(scalar_t limit)
00094         bint     isinf()
00095         bint     isnan()
00096         void     write(char* msg)
00097
00098     scalar_t error_squared_tol(Clifford Clf)
00099     scalar_t error_squared(Clifford Lhs, Clifford Rhs, scalar_t threshold)
00100     bint     approx_equal(Clifford Lhs, Clifford Rhs, scalar_t threshold, scalar_t tol)
00101     scalar_t scalar(Clifford Clf)
00102     scalar_t real(Clifford Clf)
00103     scalar_t imag(Clifford Clf)
00104     Clifford pure(Clifford Clf)
00105     Clifford even(Clifford Clf)
00106     Clifford odd(Clifford Clf)
00107     Clifford involute(Clifford Clf)
00108     Clifford reverse(Clifford Clf)
00109     Clifford conj(Clifford Clf)
00110     scalar_t norm(Clifford Clf)
00111     scalar_t abs(Clifford Clf)
00112     scalar_t max_abs(Clifford Clf)
00113     scalar_t quad(Clifford Clf)
00114     Clifford inv(Clifford Clf)
00115     Clifford pow(Clifford Clf,int m)
00116     Clifford outer_pow(Clifford Clf,int m)
00117
00118     Clifford complexifier(Clifford Clf)
00119     Clifford sqrt(Clifford Clf, Clifford I) except+
```

```
00120     Clifford sqrt(Clifford Clf)
00121     Clifford exp(Clifford Clf)
00122     Clifford log(Clifford Clf, Clifford I) except+
00123     Clifford log(Clifford Clf)
00124     Clifford cos(Clifford Clf, Clifford I) except+
00125     Clifford cos(Clifford Clf)
00126     Clifford acos(Clifford Clf, Clifford I) except+
00127     Clifford acos(Clifford Clf)
00128     Clifford cosh(Clifford Clf)
00129     Clifford acosh(Clifford Clf, Clifford I) except+
00130     Clifford acosh(Clifford Clf)
00131     Clifford sin(Clifford Clf, Clifford I) except+
00132     Clifford sin(Clifford Clf)
00133     Clifford asin(Clifford Clf, Clifford I) except+
00134     Clifford asin(Clifford Clf)
00135     Clifford sinh(Clifford Clf)
00136     Clifford asinh(Clifford Clf, Clifford I) except+
00137     Clifford asinh(Clifford Clf)
00138     Clifford tan(Clifford Clf, Clifford I) except+
00139     Clifford tan(Clifford Clf)
00140     Clifford atan(Clifford Clf, Clifford I) except+
00141     Clifford atan(Clifford Clf)
00142     Clifford tanh(Clifford Clf)
00143     Clifford atanh(Clifford Clf, Clifford I) except+
00144     Clifford atanh(Clifford Clf)
00145
00146 cdef extern from "PyClical.h" namespace "cga3":
00147     Clifford agc3(Clifford Clf)
00148     Clifford cga3(Clifford Clf)
00149     Clifford cga3std(Clifford Clf)
```

9.57 pyclical/PyClical.h File Reference

```
#include "glucat/glucat_config.h"
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include <iostream>
#include <sstream>
#include <iomanip>
#include <limits>
```

Include dependency graph for PyClical.h:



Namespaces

- namespace cga3

  Definitions for 3D Conformal Geometric Algebra [DL].

Typedefs

- using String = std::string
- using IndexSet = index_set<lo_ndx, hi_ndx>
- using scalar_t = double
- using Clifford = matrix_multi<scalar_t,lo_ndx, hi_ndx,tuning_promoted>

Functions

- template<typename Scalar_T>
  PyObject * PyFloat_FromDouble (Scalar_T v)
- template<typename Index_Set_T>
  String index_set_to_repr (const Index_Set_T &ist)

  The “official” string representation of Index_Set_T ist.
- template<typename Index_Set_T>
  String index_set_to_str (const Index_Set_T &ist)

  The "informal" string representation of Index_Set_T ist.
- template<typename Multivector_T>
  String clifford_to_repr (const Multivector_T &mv)

  The “official” string representation of Multivector_T mv.
- template<typename Multivector_T>
  String clifford_to_str (const Multivector_T &mv)

  The "informal" string representation of Multivector_T mv.
- template<typename Multivector_T>
  Multivector_T cga3::cga3 (const Multivector_T &x)

  Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].
- template<typename Multivector_T>
  Multivector_T cga3::cga3std (const Multivector_T &X)

  Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].
- template<typename Multivector_T>
  Multivector_T cga3::agc3 (const Multivector_T &X)

  Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Variables

- String glucat_package_version = GLUCAT_PACKAGE_VERSION
- const index_t lo_ndx = DEFAULT_LO
- const index_t hi_ndx = DEFAULT_HI
- const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()

9.57.1 Typedef Documentation

9.57.1.1 Clifford

```
using Clifford = matrix_multi<scalar_t,lo_ndx, hi_ndx,tuning_promoted>
```

Definition at line 148 of file PyClical.h.

9.57.1.2 IndexSet

```
using IndexSet = index_set<lo_ndx, hi_ndx>
```

Definition at line 145 of file PyClical.h.

9.57.1.3 scalar_t

```
using scalar_t = double
```

Definition at line 147 of file PyClical.h.

9.57.1.4 String

```
using String = std::string
```

Definition at line 51 of file PyClical.h.

9.57.2 Function Documentation

9.57.2.1 clifford_to_repr()

```
template<typename Multivector_T>
String clifford_to_repr (
            const Multivector_T & mv)  [inline]
```

The “official” string representation of Multivector_T mv.

Definition at line 75 of file PyClical.h.

References clifford_to_repr().

Referenced by PyClical.clifford::__repr__(), and clifford_to_repr().

9.57.2.2 clifford_to_str()

```
template<typename Multivector_T>
String clifford_to_str (
            const Multivector_T & mv)  [inline]
```

The "informal" string representation of Multivector_T mv.

Definition at line 86 of file PyClical.h.

References glucat::abs(), and clifford_to_str().

Referenced by PyClical.clifford::__str__(), and clifford_to_str().

9.57.2.3 index_set_to_repr()

```
template<typename Index_Set_T>
String index_set_to_repr (
            const Index_Set_T & ist)  [inline]
```

The “official” string representation of Index_Set_T ist.

Definition at line 57 of file PyClical.h.

References index_set_to_repr().

Referenced by PyClical.index_set::__repr__(), and index_set_to_repr().

9.57.2.4 index_set_to_str()

```
template<typename Index_Set_T>
String index_set_to_str (
            const Index_Set_T & ist)  [inline]
```

The "informal" string representation of Index_Set_T ist.

Definition at line 66 of file PyClical.h.

References index_set_to_str().

Referenced by PyClical.index_set::__str__(), and index_set_to_str().

9.57.2.5 PyFloat_FromDouble()

```
template<typename Scalar_T>
PyObject * PyFloat_FromDouble (
            Scalar_T v)  [inline]
```

Create a PyFloatObject object from Scalar_T v. Needed because Scalar_T might not be the same as double.

Definition at line 45 of file PyClical.h.

References PyFloat_FromDouble(), and glucat::numeric_traits< Scalar_T >::to_double().

Referenced by PyFloat_FromDouble().

9.57.3 Variable Documentation

9.57.3.1 epsilon

```
const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()
```

Definition at line 150 of file PyClical.h.

Referenced by glucat::cascade_log(), and glucat::matrix::classify_eigenvalues().

9.57.3.2 glucat_package_version

```
String glucat_package_version = GLUCAT_PACKAGE_VERSION
```

Definition at line 53 of file PyClical.h.

9.57.3.3 hi_ndx

```
const index_t hi_ndx = DEFAULT_HI
```

Definition at line 144 of file PyClical.h.

9.57.3.4 lo_ndx

```
const index_t lo_ndx = DEFAULT_LO
```

Definition at line 143 of file PyClical.h.

9.58 PyClical.h

Go to the documentation of this file.

```
00001 /***************************************************************************
00002     GluCat : Generic library of universal Clifford algebra templates
00003     PyClical.h : C++ definitions needed by PyClical
00004                              -------------------
00005     copyright            : (C) 2008-2021 by Paul C. Leopardi
00006  ***************************************************************************
00007
00008     This library is free software: you can redistribute it and/or modify
00009     it under the terms of the GNU Lesser General Public License as published
00010     by the Free Software Foundation, either version 3 of the License, or
00011     (at your option) any later version.
00012
00013     This library is distributed in the hope that it will be useful,
00014     but WITHOUT ANY WARRANTY; without even the implied warranty of
00015     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00016     GNU Lesser General Public License for more details.
00017
00018     You should have received a copy of the GNU Lesser General Public License
00019     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00020
00021  ***************************************************************************
00022  This library is based on a prototype written by Arvind Raja and was
00023  licensed under the LGPL with permission of the author. See Arvind Raja,
00024  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00025  in Ablamowicz, Lounesto and Parra (eds.)
00026  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00027  ***************************************************************************
00028      See also Arvind Raja's original header comments in glucat/glucat.h
00029  ***************************************************************************/
00030 // References for algorithms:
00031 // [DL]:
00032 // C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00033
00034 #include "glucat/glucat_config.h"
00035 #include "glucat/glucat.h"
00036 #include "glucat/glucat_imp.h"
00037 #include <iostream>
00038 #include <sstream>
00039 #include <iomanip>
00040 #include <limits>
00041
00044 template <typename Scalar_T>
00045 inline PyObject* PyFloat_FromDouble(Scalar_T v)
00046 { return ::PyFloat_FromDouble(glucat::numeric_traits<Scalar_T>::to_double(v)); }
00047
00048
00049 // String representations for use by PyClical Python classes.
00050
00051 using String = std::string;
00052
00053 String glucat_package_version = GLUCAT_PACKAGE_VERSION;
00054
00056 template<typename Index_Set_T>
00057 inline String index_set_to_repr(const Index_Set_T& ist)
00058 {
00059   std::ostringstream os;
00060   os « "index_set(" « ist « ")";
00061   return os.str();
00062 }
00063
00065 template<typename Index_Set_T>
00066 inline String index_set_to_str(const Index_Set_T& ist)
00067 {
00068   std::ostringstream os;
00069   os « ist;
00070   return os.str();
00071 }
00072
00074 template<typename Multivector_T>
```

```
00075 inline String clifford_to_repr(const Multivector_T& mv)
00076 {
00077   using scalar_t = typename Multivector_T::scalar_t;
00078   std::ostringstream os;
00079   os « std::setprecision(std::numeric_limits<scalar_t>::digits10 + 1);
00080   os « "clifford(\"" « mv « "\")";
00081   return os.str();
00082 }
00083
00085 template<typename Multivector_T>
00086 inline String clifford_to_str(const Multivector_T& mv)
00087 {
00088   using scalar_t = typename Multivector_T::scalar_t;
00089   std::ostringstream os;
00090   if (abs(mv) < std::numeric_limits<scalar_t>::epsilon())
00091     os « 0.0;
00092   else
00093     os « std::setprecision(4) « mv.truncated(scalar_t(1.0e-4));
00094   return os.str();
00095 }
00096
00097
00099 namespace cga3
00100 {
00102   template<typename Multivector_T>
00103   inline Multivector_T cga3(const Multivector_T& x)
00104   {
00105     using cl = Multivector_T;
00106     using ist = typename cl::index_set_t;
00107     static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00108
00109     return (cl(ist(4)) - x) * ninf3 * (x - cl(ist(4)));
00110   }
00111
00113   template<typename Multivector_T>
00114   inline Multivector_T cga3std(const Multivector_T& X)
00115   {
00116     using cl = Multivector_T;
00117     using ist = typename cl::index_set_t;
00118     using scalar_t = typename cl::scalar_t;
00119     static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00120
00121     return scalar_t(-2.0) * X / (X & ninf3);
00122   }
00123
00125   template<typename Multivector_T>
00126   inline Multivector_T agc3(const Multivector_T& X)
00127   {
00128     using cl = Multivector_T;
00129     using ist = typename cl::index_set_t;
00130     using scalar_t = typename cl::scalar_t;
00131
00132     const cl& cga3stdX = cga3std(X);
00133     return (cl(ist(1))*cga3stdX[ist(1)] +
00134             cl(ist(2))*cga3stdX[ist(2)] +
00135             cl(ist(3))*cga3stdX[ist(3)]) / scalar_t(2.0);
00136   }
00137 }
00138
00139
00140 // Specifications of the IndexSet and Clifford C++ classes for use with PyClical.
00141
00142 using namespace glucat;
00143 const index_t lo_ndx = DEFAULT_LO;
00144 const index_t hi_ndx = DEFAULT_HI;
00145 using IndexSet = index_set<lo_ndx, hi_ndx>;
00146
00147 using scalar_t = double;
00148 using Clifford = matrix_multi<scalar_t,lo_ndx, hi_ndx,tuning_promoted>;
00149
00150 const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon();
00151
00152 // Do not warn about unused values. This affects clang++ as well as g++.
00153
00154 #pragma GCC diagnostic ignored "-Wunused-value"
00155
00156 #if defined(__clang__)
00157 // Do not warn about unused functions. The affects clang++ only.
00158
00159 # pragma clang diagnostic ignored "-Wunused-function"
00160
00161 // Do not warn about unneeded internal declarations. The affects clang++ only.
00162
00163 # pragma clang diagnostic ignored "-Wunneeded-internal-declaration"
00164 #endif
```

9.59 pyclical/PyClical.pxd File Reference

Namespaces

- namespace PyClical

9.60 PyClical.pxd

Go to the documentation of this file.

```
00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #           Generic library of universal Clifford algebra templates
00006 #
00007 # PyClical.pxd: Basic Cython definitions for PyClical
00008 #               corresponding to C++ definitions from PyClical.h.
00009 #
00010 #    copyright            : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #    This library is free software: you can redistribute it and/or modify
00013 #    it under the terms of the GNU Lesser General Public License as published
00014 #    by the Free Software Foundation, either version 3 of the License, or
00015 #    (at your option) any later version.
00016 #
00017 #    This library is distributed in the hope that it will be useful,
00018 #    but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #    MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00020 #    GNU Lesser General Public License for more details.
00021 #
00022 #    You should have received a copy of the GNU Lesser General Public License
00023 #    along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025 cimport glucat
00026 from glucat cimport IndexSet, String, Clifford, scalar_t, vector
00027 from libcpp.string cimport string
00028
00029 cdef extern from "PyClical.h":
00030     string glucat_package_version
00031
00032     IndexSet operator&(IndexSet Lhs, IndexSet Rhs)
00033     IndexSet operator|(IndexSet Lhs, IndexSet Rhs)
00034     IndexSet operator^(IndexSet Lhs, IndexSet Rhs)
00035
00036     string index_set_to_repr(IndexSet& Ist)
00037     string index_set_to_str(IndexSet& Ist)
00038
00039     Clifford operator+(Clifford Lhs, Clifford Rhs)
00040     Clifford operator-(Clifford Lhs, Clifford Rhs)
00041     Clifford operator*(Clifford Lhs, Clifford Rhs)
00042     Clifford operator&(Clifford Lhs, Clifford Rhs)
00043     Clifford operator%(Clifford Lhs, Clifford Rhs)
00044     Clifford operator^(Clifford Lhs, Clifford Rhs)
00045     Clifford operator/(Clifford Lhs, Clifford Rhs)
00046     Clifford operator|(Clifford Lhs, Clifford Rhs)
00047
00048     string clifford_to_repr(Clifford& Clf)
00049     string clifford_to_str(Clifford& Clf)
00050
00051     const scalar_t epsilon
```

9.61 pyclical/PyClical.pyx File Reference

Classes

- class PyClical.index_set
- class PyClical.clifford

Namespaces

- namespace PyClical

Functions

- PyClical.index_set_hidden_doctests ()
- PyClical.clifford_hidden_doctests ()
- PyClical.e (obj)
- PyClical.istpq (p, q)
- PyClical._test ()

Variables

- PyClical.__version__ = str(glucat_package_version,'utf-8')
- PyClical.lhs
- PyClical.rhs
- PyClical.threshold = error_squared_tol(rhs) if threshold is None else threshold
- PyClical.None
- PyClical.tol = error_squared_tol(rhs) if tol is None else tol
- PyClical.obj
- PyClical.i
- PyClical.ixt
- PyClical.fill
- PyClical.scalar_epsilon = epsilon
- float PyClical.pi = atan(clifford(1.0)) ∗ 4.0
- float PyClical.tau = atan(clifford(1.0)) ∗ 8.0
- PyClical.cl = clifford
- PyClical.ist = index_set
- PyClical.ninf3 = e(4) + e(-1)
- PyClical.nbar3 = e(4) - e(-1)

9.62 PyClical.pyx

Go to the documentation of this file.

```
00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 # distutils: language = c++
00004 #
00005 # PyClical: Python interface to GluCat:
00006 #           Generic library of universal Clifford algebra templates
00007 #
00008 # PyClical.pyx: Cython definitions visible from Python.
00009 #
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00011 #
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00021 #
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00023 #    along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025 # References for definitions:
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00026 # [DL]:
00027 # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00028
00029 import math
00030 import numbers
00031 import collections
00032
00033 from PyClical cimport *
00034
00035 __version__ = str(glucat_package_version,'utf-8')
00036
00037 # Forward reference
00038 cdef class index_set
00039
00040 cdef inline IndexSet toIndexSet(obj):
00041     """
00042     Return the C++ IndexSet instance wrapped by index_set(obj).
00043     """
00044     return index_set(obj).instance[0]
00045
00046 cdef class index_set:
00047     """
00048     Python class index_set wraps C++ class IndexSet.
00049     """
00050     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
00051
00052     cdef inline wrap(index_set self, IndexSet other):
00053         """
00054         Wrap an instance of the C++ class IndexSet.
00055         """
00056         self.instance[0] = other
00057         return self
00058
00059     cdef inline IndexSet unwrap(index_set self):
00060         """
00061         Return the wrapped C++ IndexSet instance.
00062         """
00063         return self.instance[0]
00064
00065     cpdef copy(index_set self):
00066         """
00067         Copy this index_set object.
00068
00069         >> s=index_set(1); t=s.copy(); print(t)
00070         {1}
00071         """
00072         return index_set(self)
00073
00074     def __cinit__(self, other = 0):
00075         """
00076         Construct an object of type index_set.
00077
00078         >> print(index_set(1))
00079         {1}
00080         >> print(index_set({1,2}))
00081         {1,2}
00082         >> print(index_set(index_set({1,2})))
00083         {1,2}
00084         >> print(index_set({1,2}))
00085         {1,2}
00086         >> print(index_set({1,2,1}))
00087         {1,2}
00088         >> print(index_set("{1,2,1}"))
00089         {1,2}
00090         >> print(index_set(""))
00091         {}
00092         """
00093         error_msg_prefix = "Cannot initialize index_set object from"
00094         if   isinstance(other, index_set):
00095             self.instance = new IndexSet((<index_set>other).unwrap())
00096         elif isinstance(other, numbers.Integral):
00097             self.instance = new IndexSet(<int>other)
00098         elif isinstance(other, (set, frozenset)):
00099             try:
00100                 self.instance = new IndexSet()
00101                 for idx in other:
00102                     self[idx] = True
00103             except IndexError:
00104                 raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
00105             except (RuntimeError, TypeError):
00106                 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
00107         elif isinstance(other, str):
00108             try:
00109                 bother = other.encode("UTF-8")
00110                 self.instance = new IndexSet(<char *>bother)
00111             except RuntimeError:
00112                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
```

```
00113         else:
00114             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00115 
00116     def __dealloc__(self):
00117         """
00118         Clean up by deallocating the instance of C++ class IndexSet.
00119         """
00120         del self.instance
00121 
00122     def __richcmp__(lhs, rhs, int op):
00123         """
00124         Compare two objects of class index_set.
00125 
00126         >> index_set(1) == index_set({1})
00127         True
00128         >> index_set({1}) != index_set({1})
00129         False
00130         >> index_set({1}) != index_set({2})
00131         True
00132         >> index_set({1}) == index_set({2})
00133         False
00134         >> index_set({1}) < index_set({2})
00135         True
00136         >> index_set({1}) <= index_set({2})
00137         True
00138         >> index_set({1}) > index_set({2})
00139         False
00140         >> index_set({1}) >= index_set({2})
00141         False
00142         """
00143         if (lhs is None) or (rhs is None):
00144             eq = bool(lhs is rhs)
00145             if op == 2: # ==
00146                 return eq
00147             elif op == 3: # !=
00148                 return not eq
00149             else:
00150                 if op == 0: # <
00151                     return False
00152                 elif op == 1: # <=
00153                     return eq
00154                 elif op == 4: # >
00155                     return False
00156                 elif op == 5: # >=
00157                     return eq
00158                 else:
00159                     return NotImplemented
00160         else:
00161             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
00162             if op == 2: # ==
00163                 return eq
00164             elif op == 3: # !=
00165                 return not eq
00166             else:
00167                 lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
00168                 if op == 0: # <
00169                     return lt
00170                 elif op == 1: # <=
00171                     return lt or eq
00172                 elif op == 4: # >
00173                     return not (lt or eq)
00174                 elif op == 5: # >=
00175                     return not lt
00176                 else:
00177                     return NotImplemented
00178 
00179     def __setitem__(self, idx, val):
00180         """
00181         Set the value of an index_set object at index idx to value val.
00182 
00183         >> s=index_set({1}); s[2] = True; print(s)
00184         {1,2}
00185         >> s=index_set({1,2}); s[1] = False; print(s)
00186         {2}
00187         """
00188         self.instance.set(idx, val)
00189         return
00190 
00191     def __getitem__(self, idx):
00192         """
00193         Get the value of an index_set object at an index.
00194 
00195         >> index_set({1})[1]
00196         True
00197         >> index_set({1})[2]
00198         False
00199         >> index_set({2})[-1]
```

```
00200         False
00201         >> index_set({2})[1]
00202         False
00203         >> index_set({2})[2]
00204         True
00205         >> index_set({2})[33]
00206         False
00207         """
00208         return self.instance.getitem(idx)
00209
00210     def __contains__(self, idx):
00211         """
00212         Check that an index_set object contains the index idx: idx in self.
00213
00214         >> 1 in index_set({1})
00215         True
00216         >> 2 in index_set({1})
00217         False
00218         >> -1 in index_set({2})
00219         False
00220         >> 1 in index_set({2})
00221         False
00222         >> 2 in index_set({2})
00223         True
00224         >> 33 in index_set({2})
00225         False
00226         """
00227         return self.instance.getitem(idx)
00228
00229     def __iter__(self):
00230         """
00231         Iterate over the indices of an index_set.
00232
00233         >> for i in index_set({-3,4,7}):print(i, end=",")
00234         -3,4,7,
00235         """
00236         for idx in range(self.min(), self.max()+1):
00237             if idx in self:
00238                 yield idx
00239
00240     def __invert__(self):
00241         """
00242         Set complement: not.
00243
00244         >>
      print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))
00245
      {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}
00246         """
00247         return index_set().wrap( self.instance.invert() )
00248
00249     def __xor__(lhs, rhs):
00250         """
00251         Symmetric set difference: exclusive or.
00252
00253         >> print(index_set({1}) ^ index_set({2}))
00254         {1,2}
00255         >> print(index_set({1,2}) ^ index_set({2}))
00256         {1}
00257         """
00258         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
00259
00260     def __ixor__(self, rhs):
00261         """
00262         Symmetric set difference: exclusive or.
00263
00264         >> x = index_set({1}); x ^= index_set({2}); print(x)
00265         {1,2}
00266         >> x = index_set({1,2}); x ^= index_set({2}); print(x)
00267         {1}
00268         """
00269         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
00270
00271     def __and__(lhs, rhs):
00272         """
00273         Set intersection: and.
00274
00275         >> print(index_set({1}) & index_set({2}))
00276         {}
00277         >> print(index_set({1,2}) & index_set({2}))
00278         {2}
00279         """
00280         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
00281
00282     def __iand__(self, rhs):
00283         """
00284         Set intersection: and.
```

```
00285
00286         >> x = index_set({1}); x &= index_set({2}); print(x)
00287         {}
00288         >> x = index_set({1,2}); x &= index_set({2}); print(x)
00289         {2}
00290         """
00291         return self.wrap( self.unwrap() & toIndexSet(rhs) )
00292
00293     def __or__(lhs, rhs):
00294         """
00295         Set union: or.
00296
00297         >> print(index_set({1}) | index_set({2}))
00298         {1,2}
00299         >> print(index_set({1,2}) | index_set({2}))
00300         {1,2}
00301         """
00302         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
00303
00304     def __ior__(self, rhs):
00305         """
00306         Set union: or.
00307
00308         >> x = index_set({1}); x |= index_set({2}); print(x)
00309         {1,2}
00310         >> x = index_set({1,2}); x |= index_set({2}); print(x)
00311         {1,2}
00312         """
00313         return self.wrap( self.unwrap() | toIndexSet(rhs) )
00314
00315     def count(self):
00316         """
00317         Cardinality: Number of indices included in set.
00318
00319         >> index_set({-1,1,2}).count()
00320         3
00321         """
00322         return self.instance.count()
00323
00324     def count_neg(self):
00325         """
00326         Number of negative indices included in set.
00327
00328         >> index_set({-1,1,2}).count_neg()
00329         1
00330         """
00331         return self.instance.count_neg()
00332
00333     def count_pos(self):
00334         """
00335         Number of positive indices included in set.
00336
00337         >> index_set({-1,1,2}).count_pos()
00338         2
00339         """
00340         return self.instance.count_pos()
00341
00342     def min(self):
00343         """
00344         Minimum member.
00345
00346         >> index_set({-1,1,2}).min()
00347         -1
00348         """
00349         return self.instance.min()
00350
00351     def max(self):
00352         """
00353         Maximum member.
00354
00355         >> index_set({-1,1,2}).max()
00356         2
00357         """
00358         return self.instance.max()
00359
00360     def hash_fn(self):
00361         """
00362         Hash function.
00363         """
00364         return self.instance.hash_fn()
00365
00366     def sign_of_mult(self, rhs):
00367         """
00368         Sign of geometric product of two Clifford basis elements.
00369
00370         >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
00371         1
```

```
00372         """
00373         return self.instance.sign_of_mult(toIndexSet(rhs))
00374
00375     def sign_of_square(self):
00376         """
00377         Sign of geometric square of a Clifford basis element.
00378
00379         >> s = index_set({1,2}); s.sign_of_square()
00380         -1
00381         """
00382         return self.instance.sign_of_square()
00383
00384     def __repr__(self):
00385         """
00386         The "official" string representation of self.
00387
00388         >> index_set({1,2}).__repr__()
00389         'index_set({1,2})'
00390         >> repr(index_set({1,2}))
00391         'index_set({1,2})'
00392         """
00393         return index_set_to_repr( self.unwrap() ).decode()
00394
00395     def __str__(self):
00396         """
00397         The "informal" string representation of self.
00398
00399         >> index_set({1,2}).__str__()
00400         '{1,2}'
00401         >> str(index_set({1,2}))
00402         '{1,2}'
00403         """
00404         return index_set_to_str( self.unwrap() ).decode()
00405
00406 def index_set_hidden_doctests():
00407     """
00408     Tests for functions that Doctest cannot see.
00409
00410     For index_set.__cinit__: Construct index_set.
00411
00412     >> print(index_set(1))
00413     {1}
00414     >> print(index_set({1,2}))
00415     {1,2}
00416     >> print(index_set(index_set({1,2})))
00417     {1,2}
00418     >> print(index_set({1,2}))
00419     {1,2}
00420     >> print(index_set({1,2,1}))
00421     {1,2}
00422     >> print(index_set({1,2,1}))
00423     {1,2}
00424     >> print(index_set(""))
00425     {}
00426     >> print(index_set("{"))
00427     Traceback (most recent call last):
00428     ...
00429     ValueError: Cannot initialize index_set object from invalid string '{'.
00430     >> print(index_set("{1"))
00431     Traceback (most recent call last):
00432     ...
00433     ValueError: Cannot initialize index_set object from invalid string '{1'.
00434     >> print(index_set("{1,2,100}"))
00435     Traceback (most recent call last):
00436     ...
00437     ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
00438     >> print(index_set({1,2,100}))
00439     Traceback (most recent call last):
00440     ...
00441     IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
00442     >> print(index_set([1,2]))
00443     Traceback (most recent call last):
00444     ...
00445     TypeError: Cannot initialize index_set object from <class 'list'>.
00446
00447     For index_set.__richcmp__: Compare two objects of class index_set.
00448
00449     >> index_set(1) == index_set({1})
00450     True
00451     >> index_set({1}) != index_set({1})
00452     False
00453     >> index_set({1}) != index_set({2})
00454     True
00455     >> index_set({1}) == index_set({2})
00456     False
00457     >> index_set({1}) < index_set({2})
00458     True
```

```
00459     >> index_set({1}) <= index_set({2})
00460     True
00461     >> index_set({1}) > index_set({2})
00462     False
00463     >> index_set({1}) >= index_set({2})
00464     False
00465     >> None == index_set({1,2})
00466     False
00467     >> None != index_set({1,2})
00468     True
00469     >> None < index_set({1,2})
00470     False
00471     >> None <= index_set({1,2})
00472     False
00473     >> None > index_set({1,2})
00474     False
00475     >> None >= index_set({1,2})
00476     False
00477     >> index_set({1,2}) == None
00478     False
00479     >> index_set({1,2}) != None
00480     True
00481     >> index_set({1,2}) < None
00482     False
00483     >> index_set({1,2}) <= None
00484     False
00485     >> index_set({1,2}) > None
00486     False
00487     >> index_set({1,2}) >= None
00488     False
00489     """
00490     return
00491
00492 cpdef inline compare(lhs,rhs):
00493     """
00494     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
00495     -1 if a<b, +1 if a>b, 0 if a==b.
00496
00497     >> compare(index_set({1,2}),index_set({-1,3}))
00498     -1
00499     >> compare(index_set({-1,4}),index_set({-1,3}))
00500     1
00501     """
00502     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
00503
00504 cpdef inline min_neg(obj):
00505     """
00506     Minimum negative index, or 0 if none.
00507
00508     >> min_neg(index_set({1,2}))
00509     0
00510     """
00511     return glucat.min_neg( toIndexSet(obj) )
00512
00513 cpdef inline max_pos(obj):
00514     """
00515     Maximum positive index, or 0 if none.
00516
00517     >> max_pos(index_set({1,2}))
00518     2
00519     """
00520     return glucat.max_pos( toIndexSet(obj) )
00521
00522 cdef inline vector[scalar_t] list_to_vector(lst):
00523      """
00524      Create a C++ std:vector[scalar_t] from an iterable Python object.
00525      """
00526      cdef vector[scalar_t] v
00527      for s in lst:
00528          v.push_back(<scalar_t>s)
00529      return v
00530
00531 # Forward reference.
00532 cdef class clifford
00533
00534 cdef inline Clifford toClifford(obj):
00535     return clifford(obj).instance[0]
00536
00537 cdef class clifford:
00538     """
00539     Python class clifford wraps C++ class Clifford.
00540     """
00541     cdef Clifford *instance # Wrapped instance of C++ class Clifford.
00542
00543     cdef inline wrap(clifford self, Clifford other):
00544         """
00545         Wrap an instance of the C++ class Clifford.
```

```
00546         """
00547         self.instance[0] = other
00548         return self
00549 
00550     cdef inline Clifford unwrap(clifford self):
00551         """
00552         Return the wrapped C++ Clifford instance.
00553         """
00554         return self.instance[0]
00555 
00556     cpdef copy(clifford self):
00557         """
00558         Copy this clifford object.
00559 
00560         >> x=clifford("1{2}"); y=x.copy(); print(y)
00561         {2}
00562         """
00563         return clifford(self)
00564 
00565     def __cinit__(self, other = 0, ixt = None):
00566         """
00567         Construct an object of type clifford.
00568 
00569         >> print(clifford(2))
00570         2
00571         >> print(clifford(2.0))
00572         2
00573         >> print(clifford(1.0e-1))
00574         0.1
00575         >> print(clifford("2"))
00576         2
00577         >> print(clifford("2{1,2,3}"))
00578         2{1,2,3}
00579         >> print(clifford(clifford("2{1,2,3}")))
00580         2{1,2,3}
00581         >> print(clifford("-{1}"))
00582         -{1}
00583         >> print(clifford(2,index_set({1,2})))
00584         2{1,2}
00585         >> print(clifford([2,3],index_set({1,2})))
00586         2{1}+3{2}
00587         """
00588         error_msg_prefix = "Cannot initialize clifford object from"
00589         if ixt is None:
00590             try:
00591                 if   isinstance(other, clifford):
00592                     self.instance = new Clifford((<clifford>other).unwrap())
00593                 elif isinstance(other, index_set):
00594                     self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
00595                 elif isinstance(other, numbers.Real):
00596                     self.instance = new Clifford(<scalar_t>other)
00597                 elif isinstance(other, str):
00598                     try:
00599                         bother = other.encode("UTF-8")
00600                         self.instance = new Clifford(<char *>bother)
00601                     except RuntimeError:
00602                         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00603                 else:
00604                     raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00605             except RuntimeError as err:
00606                 raise ValueError(error_msg_prefix + " " + str(type(other))
00607                                                   + " value " + repr(other) + ":"
00608                                                   + "\n\t" + str(err))
00609         elif isinstance(ixt, index_set):
00610             if   isinstance(other, numbers.Real):
00611                 self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
00612             elif isinstance(other, collections.abc.Sequence):
00613                 self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
00614             else:
00615                 raise TypeError(error_msg_prefix + " (" + str(type(other))
00616                                                  + ", " + repr(ixt) + ").")
00617         else:
00618             raise TypeError(error_msg_prefix + " (" + str(type(other))
00619                                              + ", " + str(type(ixt)) + ").")
00620 
00621     def __dealloc__(self):
00622         """
00623         Clean up by deallocating the instance of C++ class Clifford.
00624         """
00625         del self.instance
00626 
00627     def __contains__(self, x):
00628         """
00629         Not applicable.
00630 
00631         >> x=clifford(index_set({-3,4,7})); -3 in x
00632         Traceback (most recent call last):
```

```
00633           ...
00634         TypeError: Not applicable.
00635         """
00636         raise TypeError("Not applicable.")
00637 
00638     def __iter__(self):
00639         """
00640         Not applicable.
00641 
00642         »> for a in clifford(index_set({-3,4,7})):print(a, end=",")
00643         Traceback (most recent call last):
00644           ...
00645         TypeError: Not applicable.
00646         """
00647         raise TypeError("Not applicable.")
00648 
00649     def reframe(self, ixt):
00650         """
00651         Put self into a larger frame, containing the union of self.frame() and index set ixt.
00652         This can be used to make multiplication faster, by multiplying within a common frame.
00653 
00654         »> clifford("2+3{1}").reframe(index_set({1,2,3}))
00655         clifford("2+3{1}")
00656         »> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() ==
    (s|t);
00657         True
00658         """
00659         error_msg_prefix = "Cannot reframe"
00660         if isinstance(ixt, index_set):
00661             try:
00662                 result = clifford()
00663                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
00664             except RuntimeError as err:
00665                 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
00666                                                               + str(ixt) + ":"
00667                                                               + "\n\t" + str(err))
00668         else:
00669             raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
00670         return result
00671 
00672     def __richcmp__(lhs, rhs, int op):
00673         """
00674         Compare objects of type clifford.
00675 
00676         »> clifford("{1}") == clifford("1{1}")
00677         True
00678         »> clifford("{1}") != clifford("1.0{1}")
00679         False
00680         »> clifford("{1}") != clifford("1.0")
00681         True
00682         »> clifford("{1,2}") == None
00683         False
00684         »> clifford("{1,2}") != None
00685         True
00686         »> None == clifford("{1,2}")
00687         False
00688         »> None != clifford("{1,2}")
00689         True
00690         """
00691         if op == 2: # ==
00692             if (lhs is None) or (rhs is None):
00693                 return bool(lhs is rhs)
00694             else:
00695                 return bool( toClifford(lhs) == toClifford(rhs) )
00696         elif op == 3: # !=
00697             if (lhs is None) or (rhs is None):
00698                 return not bool(lhs is rhs)
00699             else:
00700                 return bool( toClifford(lhs) != toClifford(rhs) )
00701         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
00702             raise TypeError("This comparison operator is not implemented for "
00703                             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
00704         else:
00705             return NotImplemented
00706 
00707     def __getitem__(self, ixt):
00708         """
00709         Subscripting: map from index set to scalar coordinate.
00710 
00711         »> clifford("{1}")[index_set(1)]
00712         1.0
00713         »> clifford("{1}")[index_set({1})]
00714         1.0
00715         »> clifford("{1}")[index_set({1,2})]
00716         0.0
00717         »> clifford("2{1,2}")[index_set({1,2})]
00718         2.0
```

```
00719         """
00720         return self.instance.getitem(toIndexSet(ixt))
00721
00722     def __neg__(self):
00723         """
00724         Unary -.
00725
00726         >> print(-clifford("{1}"))
00727         -{1}
00728         """
00729         return clifford().wrap( self.instance.neg() )
00730
00731     def __pos__(self):
00732         """
00733         Unary +.
00734
00735         >> print(+clifford("{1}"))
00736         {1}
00737         """
00738         return clifford(self)
00739
00740     def __add__(lhs, rhs):
00741         """
00742         Geometric sum.
00743
00744         >> print(clifford(1) + clifford("{2}"))
00745         1+{2}
00746         >> print(clifford("{1}") + clifford("{2}"))
00747         {1}+{2}
00748         """
00749         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
00750
00751     def __radd__(rhs, lhs):
00752         """
00753         Geometric sum.
00754
00755         >> print(1 + clifford("{2}"))
00756         1+{2}
00757         """
00758         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
00759
00760     def __iadd__(self, rhs):
00761         """
00762         Geometric sum.
00763
00764         >> x = clifford(1); x += clifford("{2}"); print(x)
00765         1+{2}
00766         """
00767         return self.wrap( self.unwrap() + toClifford(rhs) )
00768
00769     def __sub__(lhs, rhs):
00770         """
00771         Geometric difference.
00772
00773         >> print(clifford(1) - clifford("{2}"))
00774         1-{2}
00775         >> print(clifford("{1}") - clifford("{2}"))
00776         {1}-{2}
00777         """
00778         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
00779
00780     def __rsub__(rhs, lhs):
00781         """
00782         Geometric difference.
00783
00784         >> print(1 - clifford("{2}"))
00785         1-{2}
00786         """
00787         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
00788
00789     def __isub__(self, rhs):
00790         """
00791         Geometric difference.
00792
00793         >> x = clifford(1); x -= clifford("{2}"); print(x)
00794         1-{2}
00795         """
00796         return self.wrap( self.unwrap() - toClifford(rhs) )
00797
00798     def __mul__(lhs, rhs):
00799         """
00800         Geometric product.
00801
00802         >> print(clifford("{1}") * clifford("{2}"))
00803         {1,2}
00804         >> print(clifford(2) * clifford("{2}"))
00805         2{2}
```

```
00806         >> print(clifford("{1}") * clifford("{1,2}"))
00807         {2}
00808         """
00809         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
00810 
00811     def __rmul__(rhs, lhs):
00812         """
00813         Geometric product.
00814 
00815         >> print(2 * clifford("{2}"))
00816         2{2}
00817         """
00818         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
00819 
00820     def __imul__(self, rhs):
00821         """
00822         Geometric product.
00823 
00824         >> x = clifford(2); x *= clifford("{2}"); print(x)
00825         2{2}
00826         >> x = clifford("{1}"); x *= clifford("{2}"); print(x)
00827         {1,2}
00828         >> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
00829         {2}
00830         """
00831         return self.wrap( self.unwrap() * toClifford(rhs) )
00832 
00833     def __mod__(lhs, rhs):
00834         """
00835         Contraction.
00836 
00837         >> print(clifford("{1}") % clifford("{2}"))
00838         0
00839         >> print(clifford(2) % clifford("{2}"))
00840         2{2}
00841         >> print(clifford("{1}") % clifford("{1}"))
00842         1
00843         >> print(clifford("{1}") % clifford("{1,2}"))
00844         {2}
00845         """
00846         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
00847 
00848     def __rmod__(rhs, lhs):
00849         """
00850         Contraction.
00851 
00852         >> print(2 % clifford("{2}"))
00853         2{2}
00854         """
00855         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
00856 
00857     def __imod__(self, rhs):
00858         """
00859         Contraction.
00860 
00861         >> x = clifford("{1}"); x %= clifford("{2}"); print(x)
00862         0
00863         >> x = clifford(2); x %= clifford("{2}"); print(x)
00864         2{2}
00865         >> x = clifford("{1}"); x %= clifford("{1}"); print(x)
00866         1
00867         >> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
00868         {2}
00869         """
00870         return self.wrap( self.unwrap() % toClifford(rhs) )
00871 
00872     def __and__(lhs, rhs):
00873         """
00874         Inner product.
00875 
00876         >> print(clifford("{1}") & clifford("{2}"))
00877         0
00878         >> print(clifford(2) & clifford("{2}"))
00879         0
00880         >> print(clifford("{1}") & clifford("{1}"))
00881         1
00882         >> print(clifford("{1}") & clifford("{1,2}"))
00883         {2}
00884         """
00885         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
00886 
00887     def __rand__(rhs, lhs):
00888         """
00889         Inner product.
00890 
00891         >> print(2 & clifford("{2}"))
00892         0
```

```
00893         """
00894         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
00895 
00896     def __iand__(self, rhs):
00897         """
00898         Inner product.
00899 
00900         >> x = clifford("{1}"); x &= clifford("{2}"); print(x)
00901         0
00902         >> x = clifford(2); x &= clifford("{2}"); print(x)
00903         0
00904         >> x = clifford("{1}"); x &= clifford("{1}"); print(x)
00905         1
00906         >> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
00907         {2}
00908         """
00909         return self.wrap( self.unwrap() & toClifford(rhs) )
00910 
00911     def __xor__(lhs, rhs):
00912         """
00913         Outer product.
00914 
00915         >> print(clifford("{1}") ^ clifford("{2}"))
00916         {1,2}
00917         >> print(clifford(2) ^ clifford("{2}"))
00918         2{2}
00919         >> print(clifford("{1}") ^ clifford("{1}"))
00920         0
00921         >> print(clifford("{1}") ^ clifford("{1,2}"))
00922         0
00923         """
00924         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
00925 
00926     def __rxor__(rhs, lhs):
00927         """
00928         Outer product.
00929 
00930         >> print(2 ^ clifford("{2}"))
00931         2{2}
00932         """
00933         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
00934 
00935     def __ixor__(self, rhs):
00936         """
00937         Outer product.
00938 
00939         >> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
00940         {1,2}
00941         >> x = clifford(2); x ^= clifford("{2}"); print(x)
00942         2{2}
00943         >> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
00944         0
00945         >> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
00946         0
00947         """
00948         return self.wrap( self.unwrap() ^ toClifford(rhs) )
00949 
00950     def __truediv__(lhs, rhs):
00951         """
00952         Geometric quotient.
00953 
00954         >> print(clifford("{1}") / clifford("{2}"))
00955         {1,2}
00956         >> print(clifford(2) / clifford("{2}"))
00957         2{2}
00958         >> print(clifford("{1}") / clifford("{1}"))
00959         1
00960         >> print(clifford("{1}") / clifford("{1,2}"))
00961         -{2}
00962         """
00963         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
00964 
00965     def __rtruediv__(rhs, lhs):
00966         """
00967         Geometric quotient.
00968 
00969         >> print(2 / clifford("{2}"))
00970         2{2}
00971         """
00972         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
00973 
00974     def __idiv__(self, rhs):
00975         """
00976         Geometric quotient.
00977 
00978         >> x = clifford("{1}"); x /= clifford("{2}"); print(x)
00979         {1,2}
```

```
00980         >> x = clifford(2); x /= clifford("{2}"); print(x)
00981         2{2}
00982         >> x = clifford("{1}"); x /= clifford("{1}"); print(x)
00983         1
00984         >> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
00985         -{2}
00986         """
00987         return self.wrap( self.unwrap() / toClifford(rhs) )
00988
00989     def inv(self):
00990         """
00991         Geometric multiplicative inverse.
00992
00993         >> x = clifford("{1}"); print(x.inv())
00994         {1}
00995         >> x = clifford(2); print(x.inv())
00996         0.5
00997         >> x = clifford("{1,2}"); print(x.inv())
00998         -{1,2}
00999         """
01000         return clifford().wrap( self.instance.inv() )
01001
01002     def __or__(lhs, rhs):
01003         """
01004         Transform left hand side, using right hand side as a transformation.
01005
01006         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
01007         -{1}
01008         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
01009         -{1}
01010         """
01011         return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
01012
01013     def __ior__(self, rhs):
01014         """
01015         Transform left hand side, using right hand side as a transformation.
01016
01017         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
01018         -{1}
01019         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
01020         -{1}
01021         """
01022         return self.wrap( self.unwrap() | toClifford(rhs) )
01023
01024     def __pow__(self, m, dummy):
01025         """
01026         Power: self to the m.
01027
01028         >> x=clifford("{1}"); print(x ** 2)
01029         1
01030         >> x=clifford("2"); print(x ** 2)
01031         4
01032         >> x=clifford("2+{1}"); print(x ** 0)
01033         1
01034         >> x=clifford("2+{1}"); print(x ** 1)
01035         2+{1}
01036         >> x=clifford("2+{1}"); print(x ** 2)
01037         5+4{1}
01038         >> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
01039         1
01040         """
01041         return pow(self, m)
01042
01043     def pow(self, m):
01044         """
01045         Power: self to the m.
01046
01047         >> x=clifford("{1}"); print(x.pow(2))
01048         1
01049         >> x=clifford("2"); print(x.pow(2))
01050         4
01051         >> x=clifford("2+{1}"); print(x.pow(0))
01052         1
01053         >> x=clifford("2+{1}"); print(x.pow(1))
01054         2+{1}
01055         >> x=clifford("2+{1}"); print(x.pow(2))
01056         5+4{1}
01057         >> print(clifford("1+{1}+{1,2}").pow(3))
01058         1+3{1}+3{1,2}
01059         >> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
01060         1
01061         """
01062         if isinstance(m, numbers.Integral):
01063             return clifford().wrap( self.instance.pow(m) )
01064         else:
01065             return exp(m * log(self))
01066
```

```
01067     def outer_pow(self, m):
01068         """
01069         Outer product power.
01070 
01071         »> x=clifford("2+{1}"); print(x.outer_pow(0))
01072         1
01073         »> x=clifford("2+{1}"); print(x.outer_pow(1))
01074         2+{1}
01075         »> x=clifford("2+{1}"); print(x.outer_pow(2))
01076         4+4{1}
01077         »> print(clifford("1+{1}+{1,2}").outer_pow(3))
01078         1+3{1}+3{1,2}
01079 
01080         """
01081         return clifford().wrap( self.instance.outer_pow(m) )
01082 
01083     def __call__(self, grade):
01084         """
01085         Pure grade-vector part.
01086 
01087         »> print(clifford("{1}")(1))
01088         {1}
01089         »> print(clifford("{1}")(0))
01090         0
01091         »> print(clifford("1+{1}+{1,2}")(0))
01092         1
01093         »> print(clifford("1+{1}+{1,2}")(1))
01094         {1}
01095         »> print(clifford("1+{1}+{1,2}")(2))
01096         {1,2}
01097         »> print(clifford("1+{1}+{1,2}")(3))
01098         0
01099         """
01100         return clifford().wrap( self.instance.call(grade) )
01101 
01102     def scalar(self):
01103         """
01104         Scalar part.
01105 
01106         »> clifford("1+{1}+{1,2}").scalar()
01107         1.0
01108         »> clifford("{1,2}").scalar()
01109         0.0
01110         """
01111         return self.instance.scalar()
01112 
01113     def pure(self):
01114         """
01115         Pure part.
01116 
01117         »> print(clifford("1+{1}+{1,2}").pure())
01118         {1}+{1,2}
01119         »> print(clifford("{1,2}").pure())
01120         {1,2}
01121         """
01122         return clifford().wrap( self.instance.pure() )
01123 
01124     def even(self):
01125         """
01126         Even part of multivector, sum of even grade terms.
01127 
01128         »> print(clifford("1+{1}+{1,2}").even())
01129         1+{1,2}
01130         """
01131         return clifford().wrap( self.instance.even() )
01132 
01133     def odd(self):
01134         """
01135         Odd part of multivector, sum of odd grade terms.
01136 
01137         »> print(clifford("1+{1}+{1,2}").odd())
01138         {1}
01139         """
01140         return clifford().wrap( self.instance.odd() )
01141 
01142     def vector_part(self, frm = None):
01143         """
01144         Vector part of multivector, as a Python list, with respect to frm.
01145 
01146         »> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
01147         [2.0, 3.0]
01148         »> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
01149         [0.0, 2.0, 3.0]
01150         """
01151         error_msg_prefix = "Cannot take vector part of "
01152         cdef vector[scalar_t] vec
01153         cdef int n
```

```
01154         cdef int i
01155         try:
01156             if frm is None:
01157                 vec = self.instance.vector_part()
01158             else:
01159                 vec = self.instance.vector_part((<index_set>frm).unwrap())
01160             n = vec.size()
01161             lst = [0.0]*n
01162             for i in xrange(n):
01163                 lst[i] = vec[i]
01164             return lst
01165         except RuntimeError as err:
01166             raise ValueError(error_msg_prefix + str(self) + " using invalid "
01167                                               + repr(frm) + " as frame:\n\t"
01168                                               + str(err))
01169 
01170     def involute(self):
01171         """
01172         Main involution, each {i} is replaced by -{i} in each term,
01173         eg. clifford("{1}") -> -clifford("{1}").
01174 
01175         >> print(clifford("{1}").involute())
01176         -{1}
01177         >> print((clifford("{2}") * clifford("{1}")).involute())
01178         -{1,2}
01179         >> print((clifford("{1}") * clifford("{2}")).involute())
01180         {1,2}
01181         >> print(clifford("1+{1}+{1,2}").involute())
01182         1-{1}+{1,2}
01183         """
01184         return clifford().wrap( self.instance.involute() )
01185 
01186     def reverse(self):
01187         """
01188         Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
01189 
01190         >> print(clifford("{1}").reverse())
01191         {1}
01192         >> print((clifford("{2}") * clifford("{1}")).reverse())
01193         {1,2}
01194         >> print((clifford("{1}") * clifford("{2}")).reverse())
01195         -{1,2}
01196         >> print(clifford("1+{1}+{1,2}").reverse())
01197         1+{1}-{1,2}
01198         """
01199         return clifford().wrap( self.instance.reverse() )
01200 
01201     def conj(self):
01202         """
01203         Conjugation, reverse o involute == involute o reverse.
01204 
01205         >> print((clifford("{1}")).conj())
01206         -{1}
01207         >> print((clifford("{2}") * clifford("{1}")).conj())
01208         {1,2}
01209         >> print((clifford("{1}") * clifford("{2}")).conj())
01210         -{1,2}
01211         >> print(clifford("1+{1}+{1,2}").conj())
01212         1-{1}-{1,2}
01213         """
01214         return clifford().wrap( self.instance.conj() )
01215 
01216     def quad(self):
01217         """
01218         Quadratic form == (rev(x)*x)(0).
01219 
01220         >> print(clifford("1+{1}+{1,2}").quad())
01221         3.0
01222         >> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
01223         2.0
01224         """
01225         return self.instance.quad()
01226 
01227     def norm(self):
01228         """
01229         Norm == sum of squares of coordinates.
01230 
01231         >> clifford("1+{1}+{1,2}").norm()
01232         3.0
01233         >> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
01234         4.0
01235         """
01236         return self.instance.norm()
01237 
01238     def abs(self):
01239         """
01240         Absolute value: square root of norm.
```

```
01241
01242         >> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
01243         2.0
01244         """
01245         return glucat.abs( self.unwrap() )
01246
01247     def max_abs(self):
01248         """
01249         Maximum of absolute values of components of multivector: multivector infinity norm.
01250
01251         >> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
01252         1.0
01253         >> clifford("3+2{1}+{1,2}").max_abs()
01254         3.0
01255         """
01256         return self.instance.max_abs()
01257
01258     def truncated(self, limit):
01259         """
01260         Remove all terms of self with relative size smaller than limit.
01261
01262         >> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
01263         clifford("100000000")
01264         >> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
01265         clifford("10000+{1}")
01266         """
01267         return clifford().wrap( self.instance.truncated(limit) )
01268
01269     def isinf(self):
01270         """
01271         Check if a multivector contains any infinite values.
01272
01273         >> clifford().isinf()
01274         False
01275         """
01276         return self.instance.isnan()
01277
01278     def isnan(self):
01279         """
01280         Check if a multivector contains any IEEE NaN values.
01281
01282         >> clifford().isnan()
01283         False
01284         """
01285         return self.instance.isnan()
01286
01287     def frame(self):
01288         """
01289         Subalgebra generated by all generators of terms of given multivector.
01290
01291         >> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
01292         {-2,-1,1,2,7}
01293         >> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
01294         <class 'PyClical.index_set'>
01295         """
01296         return index_set().wrap( self.instance.frame() )
01297
01298     def __repr__(self):
01299         """
01300         The "official" string representation of self.
01301
01302         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
01303         'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
01304         """
01305         return clifford_to_repr( self.unwrap() ).decode()
01306
01307     def __str__(self):
01308         """
01309         The "informal" string representation of self.
01310
01311         >> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
01312         '1+3{-1}+2{1,2}+4{-2,7}'
01313         """
01314         return clifford_to_str( self.unwrap() ).decode()
01315
01316 def clifford_hidden_doctests():
01317     """
01318     Tests for functions that Doctest cannot see.
01319
01320     For clifford.__cinit__: Construct an object of type clifford.
01321
01322     >> print(clifford(2))
01323     2
01324     >> print(clifford(2.0))
01325     2
01326     >> print(clifford(1.0e-1))
01327     0.1
```

```
01328     >> print(clifford("2"))
01329     2
01330     >> print(clifford("2{1,2,3}"))
01331     2{1,2,3}
01332     >> print(clifford(clifford("2{1,2,3}")))
01333     2{1,2,3}
01334     >> print(clifford("-{1}"))
01335     -{1}
01336     >> print(clifford(2,index_set({1,2})))
01337     2{1,2}
01338     >> print(clifford([2,3],index_set({1,2})))
01339     2{1}+3{2}
01340     >> print(clifford([1,2]))
01341     Traceback (most recent call last):
01342       ...
01343     TypeError: Cannot initialize clifford object from <class 'list'>.
01344     >> print(clifford(None))
01345     Traceback (most recent call last):
01346       ...
01347     TypeError: Cannot initialize clifford object from <class 'NoneType'>.
01348     >> print(clifford(None,[1,2]))
01349     Traceback (most recent call last):
01350       ...
01351     TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
01352     >> print(clifford([1,2],[1,2]))
01353     Traceback (most recent call last):
01354       ...
01355     TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
01356     >> print(clifford(""))
01357     Traceback (most recent call last):
01358       ...
01359     ValueError: Cannot initialize clifford object from invalid string ".
01360     >> print(clifford("{"))
01361     Traceback (most recent call last):
01362       ...
01363     ValueError: Cannot initialize clifford object from invalid string '{'.
01364     >> print(clifford("{1"))
01365     Traceback (most recent call last):
01366       ...
01367     ValueError: Cannot initialize clifford object from invalid string '{1'.
01368     >> print(clifford("+"))
01369     Traceback (most recent call last):
01370       ...
01371     ValueError: Cannot initialize clifford object from invalid string '+'.
01372     >> print(clifford("-"))
01373     Traceback (most recent call last):
01374       ...
01375     ValueError: Cannot initialize clifford object from invalid string '-'.
01376     >> print(clifford("{1}+"))
01377     Traceback (most recent call last):
01378       ...
01379     ValueError: Cannot initialize clifford object from invalid string '{1}+'.
01380
01381     For clifford.__richcmp__: Compare objects of type clifford.
01382
01383     >> clifford("{1}") == clifford("1{1}")
01384     True
01385     >> clifford("{1}") != clifford("1.0{1}")
01386     False
01387     >> clifford("{1}") != clifford("1.0")
01388     True
01389     >> clifford("{1,2}") == None
01390     False
01391     >> clifford("{1,2}") != None
01392     True
01393     >> None == clifford("{1,2}")
01394     False
01395     >> None != clifford("{1,2}")
01396     True
01397     """
01398     return
01399
01400 cpdef inline error_squared_tol(obj):
01401     """
01402     Quadratic norm error tolerance relative to a specific multivector.
01403
01404     >> print(error_squared_tol(clifford("{1}")) * 3.0 - error_squared_tol(clifford("1{1}-2{2}+3{3}")))
01405     0.0
01406     """
01407     return glucat.error_squared_tol(toClifford(obj))
01408
01409 cpdef inline error_squared(lhs, rhs, threshold):
01410     """
01411     Relative or absolute error using the quadratic norm.
01412
01413     >> err2=scalar_epsilon*scalar_epsilon
01414
```

```
01415     >> print(error_squared(clifford("{1}"), clifford("1{1}"), err2))
01416     0.0
01417     >> print(error_squared(clifford("1{1}-3{2}+4{3}"), clifford("{1}"), err2))
01418     25.0
01419     """
01420     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
01421
01422 cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
01423     """
01424     Test for approximate equality of multivectors.
01425
01426     >> err2=scalar_epsilon*scalar_epsilon
01427
01428     >> print(approx_equal(clifford("{1}"), clifford("1{1}")))
01429     True
01430     >> print(approx_equal(clifford("1{1}-3{2}+4{3}"), clifford("{1}")))
01431     False
01432     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+0.001"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01433     False
01434     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+1.0e-30"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01435     True
01436     """
01437     threshold = error_squared_tol(rhs) if threshold is None else threshold
01438     tol       = error_squared_tol(rhs) if tol       is None else tol
01439     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold, <scalar_t>tol)
01440
01441 cpdef inline inv(obj):
01442     """
01443     Geometric multiplicative inverse.
01444
01445     >> print(inv(clifford("{1}")))
01446     {1}
01447     >> print(inv(clifford("{-1}")))
01448     -{-1}
01449     >> print(inv(clifford("{-2,-1}")))
01450     -{-2,-1}
01451     >> print(inv(clifford("{-1}+{1}")))
01452     nan
01453     """
01454     return clifford(obj).inv()
01455
01456 cpdef inline scalar(obj):
01457     """
01458     Scalar part.
01459
01460     >> scalar(clifford("1+{1}+{1,2}"))
01461     1.0
01462     >> scalar(clifford("{1,2}"))
01463     0.0
01464     """
01465     return clifford(obj).scalar()
01466
01467 cpdef inline real(obj):
01468     """
01469     Real part: synonym for scalar part.
01470
01471     >> real(clifford("1+{1}+{1,2}"))
01472     1.0
01473     >> real(clifford("{1,2}"))
01474     0.0
01475     """
01476     return clifford(obj).scalar()
01477
01478 cpdef inline imag(obj):
01479     """
01480     Imaginary part: deprecated (always 0).
01481
01482     >> imag(clifford("1+{1}+{1,2}"))
01483     0.0
01484     >> imag(clifford("{1,2}"))
01485     0.0
01486     """
01487     return 0.0
01488
01489 cpdef inline pure(obj):
01490     """
01491     Pure part
01492
01493     >> print(pure(clifford("1+{1}+{1,2}")))
01494     {1}+{1,2}
01495     >> print(pure(clifford("{1,2}")))
01496     {1,2}
01497     """
01498     return clifford(obj).pure()
01499
01500 cpdef inline even(obj):
01501     """
```

```
01502     Even part of multivector, sum of even grade terms.
01503
01504     >> print(even(clifford("1+{1}+{1,2}")))
01505     1+{1,2}
01506     """
01507     return clifford(obj).even()
01508
01509 cpdef inline odd(obj):
01510     """
01511     Odd part of multivector, sum of odd grade terms.
01512
01513     >> print(odd(clifford("1+{1}+{1,2}")))
01514     {1}
01515     """
01516     return clifford(obj).odd()
01517
01518 cpdef inline involute(obj):
01519     """
01520     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
01521
01522     >> print(involute(clifford("{1}")))
01523     -{1}
01524     >> print(involute(clifford("{2}") * clifford("{1}")))
01525     -{1,2}
01526     >> print(involute(clifford("{1}") * clifford("{2}")))
01527     {1,2}
01528     >> print(involute(clifford("1+{1}+{1,2}")))
01529     1-{1}+{1,2}
01530     """
01531     return clifford(obj).involute()
01532
01533 cpdef inline reverse(obj):
01534     """
01535     Reversion, eg. {1}*{2} -> {2}*{1}
01536
01537     >> print(reverse(clifford("{1}")))
01538     {1}
01539     >> print(reverse(clifford("{2}") * clifford("{1}")))
01540     {1,2}
01541     >> print(reverse(clifford("{1}") * clifford("{2}")))
01542     -{1,2}
01543     >> print(reverse(clifford("1+{1}+{1,2}")))
01544     1+{1}-{1,2}
01545     """
01546     return clifford(obj).reverse()
01547
01548 cpdef inline conj(obj):
01549     """
01550     Conjugation, reverse o involute == involute o reverse.
01551
01552     >> print(conj(clifford("{1}")))
01553     -{1}
01554     >> print(conj(clifford("{2}") * clifford("{1}")))
01555     {1,2}
01556     >> print(conj(clifford("{1}") * clifford("{2}")))
01557     -{1,2}
01558     >> print(conj(clifford("1+{1}+{1,2}")))
01559     1-{1}-{1,2}
01560     """
01561     return clifford(obj).conj()
01562
01563 cpdef inline quad(obj):
01564     """
01565     Quadratic form == (rev(x)*x)(0).
01566
01567     >> print(quad(clifford("1+{1}+{1,2}")))
01568     3.0
01569     >> print(quad(clifford("1+{-1}+{1,2}+{1,2,3}")))
01570     2.0
01571     """
01572     return clifford(obj).quad()
01573
01574 cpdef inline norm(obj):
01575     """
01576     norm == sum of squares of coordinates.
01577
01578     >> norm(clifford("1+{1}+{1,2}"))
01579     3.0
01580     >> norm(clifford("1+{-1}+{1,2}+{1,2,3}"))
01581     4.0
01582     """
01583     return clifford(obj).norm()
01584
01585 cpdef inline abs(obj):
01586     """
01587     Absolute value of multivector: multivector 2-norm.
01588
```

```
01589     >> abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01590     2.0
01591     """
01592     return glucat.abs(toClifford(obj))
01593
01594 cpdef inline max_abs(obj):
01595     """
01596     Maximum absolute value of coordinates multivector: multivector infinity-norm.
01597
01598     >> max_abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01599     1.0
01600     >> max_abs(clifford("3+2{1}+{1,2}"))
01601     3.0
01602
01603     """
01604     return glucat.max_abs(toClifford(obj))
01605
01606 cpdef inline pow(obj, m):
01607     """
01608     Integer power of multivector: obj to the m.
01609
01610     >> x=clifford("{1}"); print(pow(x,2))
01611     1
01612     >> x=clifford("2"); print(pow(x,2))
01613     4
01614     >> x=clifford("2+{1}"); print(pow(x,0))
01615     1
01616     >> x=clifford("2+{1}"); print(pow(x,1))
01617     2+{1}
01618     >> x=clifford("2+{1}"); print(pow(x,2))
01619     5+4{1}
01620     >> print(pow(clifford("1+{1}+{1,2}"),3))
01621     1+3{1}+3{1,2}
01622     >> i=clifford("{1,2}"); print(exp(pi/2) * pow(i, i))
01623     1
01624     """
01625     try:
01626         math.pow(obj, m)
01627     except:
01628         return clifford(obj).pow(m)
01629
01630 cpdef inline outer_pow(obj, m):
01631     """
01632     Outer product power of multivector.
01633
01634     >> print(outer_pow(clifford("1+{1}+{1,2}"),3))
01635     1+3{1}+3{1,2}
01636     """
01637     return clifford(obj).outer_pow(m)
01638
01639 cpdef inline complexifier(obj):
01640     """
01641     Square root of -1 which commutes with all members of the frame of the given multivector.
01642
01643     >> print(complexifier(clifford(index_set({1}))))
01644     {1,2,3}
01645     >> print(complexifier(clifford(index_set({-1}))))
01646     {-1}
01647     >> print(complexifier(index_set({1})))
01648     {1,2,3}
01649     >> print(complexifier(index_set({-1})))
01650     {-1}
01651     """
01652     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
01653
01654 cpdef inline sqrt(obj, i = None):
01655     """
01656     Square root of multivector with optional complexifier.
01657
01658     >> print(sqrt(-1))
01659     {-1}
01660     >> print(sqrt(clifford("2{-1}")))
01661     1+{-1}
01662     >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)
01663     {1,2,3}
01664     -1
01665     >> j=sqrt(-1,"{1,2,3}"); print(j); print(j*j)
01666     {1,2,3}
01667     -1
01668     """
01669     if not (i is None):
01670         return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
01671     else:
01672         try:
01673             return math.sqrt(obj)
01674         except:
01675             return clifford().wrap( glucat.sqrt(toClifford(obj)) )
```

```
01676
01677 cpdef inline exp(obj):
01678     """
01679     Exponential of multivector.
01680
01681     >> x=clifford("{1,2}") * pi/4; print(exp(x))
01682     0.7071+0.7071{1,2}
01683     >> x=clifford("{1,2}") * pi/2; print(exp(x))
01684     {1,2}
01685     """
01686     try:
01687         return math.exp(obj)
01688     except:
01689         return clifford().wrap( glucat.exp(toClifford(obj)) )
01690
01691 cpdef inline log(obj,i = None):
01692     """
01693     Natural logarithm of multivector with optional complexifier.
01694
01695     >> x=clifford("{-1}"); print((log(x,"{-1}") * 2/pi))
01696     {-1}
01697     >> x=clifford("{1,2}"); print((log(x,"{1,2,3}") * 2/pi))
01698     {1,2}
01699     >> x=clifford("{1,2}"); print((log(x) * 2/pi))
01700     {1,2}
01701     >> x=clifford("{1,2}"); print((log(x,"{1,2}") * 2/pi))
01702     Traceback (most recent call last):
01703     ...
01704     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
01705     """
01706     if not (i is None):
01707         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
01708     else:
01709         try:
01710             return math.log(obj)
01711         except:
01712             return clifford().wrap( glucat.log(toClifford(obj)) )
01713
01714 cpdef inline cos(obj,i = None):
01715     """
01716     Cosine of multivector with optional complexifier.
01717
01718     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01719     {1,2}
01720     >> x=clifford("{1,2}"); print(cos(acos(x)))
01721     {1,2}
01722     """
01723     if not (i is None):
01724         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
01725     else:
01726         try:
01727             return math.cos(obj)
01728         except:
01729             return clifford().wrap( glucat.cos(toClifford(obj)) )
01730
01731 cpdef inline acos(obj,i = None):
01732     """
01733     Inverse cosine of multivector with optional complexifier.
01734
01735     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01736     {1,2}
01737     >> x=clifford("{1,2}"); print(cos(acos(x),"{-1,1,2,3,4}"))
01738     {1,2}
01739     >> print(acos(0) / pi)
01740     0.5
01741     >> x=clifford("{1,2}"); print(cos(acos(x)))
01742     {1,2}
01743     """
01744     if not (i is None):
01745         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
01746     else:
01747         try:
01748             return math.acos(obj)
01749         except:
01750             return clifford().wrap( glucat.acos(toClifford(obj)) )
01751
01752 cpdef inline cosh(obj):
01753     """
01754     Hyperbolic cosine of multivector.
01755
01756     >> x=clifford("{1,2}") * pi; print(cosh(x))
01757     -1
01758     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01759     {1,2,3}
01760     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01761     {1,2}
01762     """
```

```
01763     try:
01764         return math.cosh(obj)
01765     except:
01766         return clifford().wrap( glucat.cosh(toClifford(obj)) )
01767
01768 cpdef inline acosh(obj,i = None):
01769     """
01770     Inverse hyperbolic cosine of multivector with optional complexifier.
01771
01772     >> print(acosh(0,"{-2,-1,1}"))
01773     1.571{-2,-1,1}
01774     >> x=clifford("{1,2,3}"); print(cosh(acosh(x,"{-1,1,2,3,4}")))
01775     {1,2,3}
01776     >> print(acosh(0))
01777     1.571{-1}
01778     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01779     {1,2,3}
01780     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01781     {1,2}
01782     """
01783     if not (i is None):
01784         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
01785     else:
01786         try:
01787             return math.acosh(obj)
01788         except:
01789             return clifford().wrap( glucat.acosh(toClifford(obj)) )
01790
01791 cpdef inline sin(obj,i = None):
01792     """
01793     Sine of multivector with optional complexifier.
01794
01795     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),s))
01796     {-1}
01797     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01798     {-1}
01799     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01800     {1,2,3}
01801     """
01802     if not (i is None):
01803         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
01804     else:
01805         try:
01806             return math.sin(obj)
01807         except:
01808             return clifford().wrap( glucat.sin(toClifford(obj)) )
01809
01810 cpdef inline asin(obj,i = None):
01811     """
01812     Inverse sine of multivector with optional complexifier.
01813
01814     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),s))
01815     {-1}
01816     >> s="{-1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01817     {-1}
01818     >> print(asin(1) / pi)
01819     0.5
01820     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01821     {1,2,3}
01822     """
01823     if not (i is None):
01824         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
01825     else:
01826         try:
01827             return math.asin(obj)
01828         except:
01829             return clifford().wrap( glucat.asin(toClifford(obj)) )
01830
01831 cpdef inline sinh(obj):
01832     """
01833     Hyperbolic sine of multivector.
01834
01835     >> x=clifford("{1,2}") * pi/2; print(sinh(x))
01836     {1,2}
01837     >> x=clifford("{1,2}") * pi/6; print(sinh(x))
01838     0.5{1,2}
01839     """
01840     try:
01841         return math.sinh(obj)
01842     except:
01843         return clifford().wrap( glucat.sinh(toClifford(obj)) )
01844
01845 cpdef inline asinh(obj,i = None):
01846     """
01847     Inverse hyperbolic sine of multivector with optional complexifier.
01848
01849     >> x=clifford("{1,2}"); print(asinh(x,"{1,2,3}") * 2/pi)
```

```
01850     {1,2}
01851     >> x=clifford("{1,2}"); print(asinh(x) * 2/pi)
01852     {1,2}
01853     >> x=clifford("{1,2}") / 2; print(asinh(x) * 6/pi)
01854     {1,2}
01855     """
01856     if not (i is None):
01857         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
01858     else:
01859         try:
01860             return math.asinh(obj)
01861         except:
01862             return clifford().wrap( glucat.asinh(toClifford(obj)) )
01863
01864 cpdef inline tan(obj,i = None):
01865     """
01866     Tangent of multivector with optional complexifier.
01867
01868     >> x=clifford("{1,2}"); print(tan(x,"{1,2,3}"))
01869     0.7616{1,2}
01870     >> x=clifford("{1,2}"); print(tan(x))
01871     0.7616{1,2}
01872     """
01873     if not (i is None):
01874         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
01875     else:
01876         try:
01877             return math.tan(obj)
01878         except:
01879             return clifford().wrap( glucat.tan(toClifford(obj)) )
01880
01881 cpdef inline atan(obj,i = None):
01882     """
01883     Inverse tangent of multivector with optional complexifier.
01884
01885     >> s=index_set({1,2,3}); x=clifford("{1}"); print(tan(atan(x,s),s))
01886     {1}
01887     >> x=clifford("{1}"); print(tan(atan(x)))
01888     {1}
01889     """
01890     if not (i is None):
01891         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
01892     else:
01893         try:
01894             return math.atan(obj)
01895         except:
01896             return clifford().wrap( glucat.atan(toClifford(obj)) )
01897
01898 cpdef inline tanh(obj):
01899     """
01900     Hyperbolic tangent of multivector.
01901
01902     >> x=clifford("{1,2}") * pi/4; print(tanh(x))
01903     {1,2}
01904     """
01905     try:
01906         return math.tanh(obj)
01907     except:
01908         return clifford().wrap( glucat.tanh(toClifford(obj)) )
01909
01910 cpdef inline atanh(obj,i = None):
01911     """
01912     Inverse hyperbolic tangent of multivector with optional complexifier.
01913
01914     >> s=index_set({1,2,3}); x=clifford("{1,2}"); print(tanh(atanh(x,s)))
01915     {1,2}
01916     >> x=clifford("{1,2}"); print(tanh(atanh(x)))
01917     {1,2}
01918     """
01919     if not (i is None):
01920         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
01921     else:
01922         try:
01923             return math.atanh(obj)
01924         except:
01925             return clifford().wrap( glucat.atanh(toClifford(obj)) )
01926
01927 cpdef inline random_clifford(index_set ixt, fill = 1.0):
01928     """
01929     Random multivector within a frame.
01930
01931     >> print(random_clifford(index_set({-3,-1,2})).frame())
01932     {-3,-1,2}
01933     """
01934     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
01935
01936 cpdef inline cga3(obj):
```

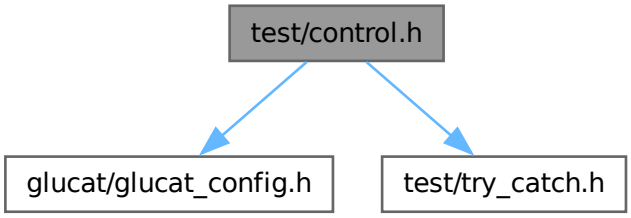
```
01937     """
01938     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
      definition.
01939
01940     >> x=clifford("2{1}+9{2}+{3}"); print(cga3(x))
01941     87{-1}+4{1}+18{2}+2{3}+85{4}
01942     """
01943     return clifford().wrap( glucat.cga3(toClifford(obj)) )
01944
01945 cpdef inline cga3std(obj):
01946     """
01947     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
01948
01949     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x)))
01950     87{-1}+4{1}+18{2}+2{3}+85{4}
01951     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x))-cga3(x))
01952     0
01953     """
01954     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
01955
01956 cpdef inline agc3(obj):
01957     """
01958     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
01959
01960     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x)))
01961     2{1}+9{2}+{3}
01962     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x))-x)
01963     0
01964     """
01965     return clifford().wrap( glucat.agc3(toClifford(obj)) )
01966
01967 # Some abbreviations.
01968 scalar_epsilon = epsilon
01969
01970 pi = atan(clifford(1.0)) * 4.0
01971 tau = atan(clifford(1.0)) * 8.0
01972
01973 cl = clifford
01974 """
01975 Abbreviation for clifford.
01976
01977 >> print(cl(2))
01978 2
01979 >> print(cl(2.0))
01980 2
01981 >> print(cl(5.0e-1))
01982 0.5
01983 >> print(cl("2"))
01984 2
01985 >> print(cl("2{1,2,3}"))
01986 2{1,2,3}
01987 >> print(cl(cl("2{1,2,3}")))
01988 2{1,2,3}
01989 """
01990
01991 ist = index_set
01992 """
01993 Abbreviation for index_set.
01994
01995 >> print(ist("{1,2,3}"))
01996 {1,2,3}
01997 """
01998
01999 def e(obj):
02000     """
02001     Abbreviation for clifford(index_set(obj)).
02002
02003     >> print(e(1))
02004     {1}
02005     >> print(e(-1))
02006     {-1}
02007     >> print(e(0))
02008     1
02009     """
02010     return clifford(index_set(obj))
02011
02012 def istpq(p, q):
02013     """
02014     Abbreviation for index_set({-q,...p}).
02015
02016     >> print(istpq(2,3))
02017     {-3,-2,-1,1,2}
02018     """
02019     return index_set(set(range(-q,p+1)))
02020
02021 ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
02022 nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
```

```
02023
02024 # Doctest interface.
02025 def _test():
02026     import PyClical, doctest
02027     return doctest.testmod(PyClical)
02028
02029 if __name__ == "__main__":
02030     _test()
```

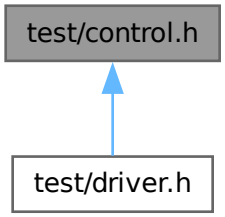
9.63 test/control.h File Reference

```
#include "glucat/glucat_config.h"
#include "test/try_catch.h"
```

Include dependency graph for control.h:



This graph shows which files directly or indirectly include this file:



Classes

- class glucat::control_t

  Parameters to control tests.

Namespaces

- namespace glucat

9.64 control.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_CONTROL_H
00002 #define _GLUCAT_CONTROL_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     control.h : Define and set parameters to control tests
00006                             -------------------
00007     begin                : 2010-04-21
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
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00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033 #include "glucat/glucat_config.h"
00034 #include "test/try_catch.h"
00035
00036 namespace glucat
00037 {
00039   class control_t
00040   {
00041   private:
00043     bool m_valid;
00044     bool valid() const
00045     { return m_valid; }
00046
00048     bool m_catch_exceptions;
00049     bool catch_exceptions() const
00050     { return m_catch_exceptions; }
00051
00053     static bool m_verbose_output;
00054
00056     control_t(int argc, char ** argv);
00057     // Enforce singleton
00058     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00059     control_t() = default;
00060     ~control_t() = default;
00061     control_t(const control_t&) = delete;
00062     control_t& operator= (const control_t&) = delete;
00063
00067     friend class friend_for_private_destructor;
00068   public:
00071     static const control_t& control(int argc, char ** argv)
00072     { static const control_t c(argc, argv); return c; }
00073
00075     int call(intfn f) const;
00077     int call(intintfn f, int arg) const;
00078
00080     static bool verbose()
00081     { return m_verbose_output; }
00082   };
00083
00085   bool control_t::m_verbose_output = false;
00086
00088   control_t::
00089   control_t(int argc, char ** argv)
00090   : m_valid(true), m_catch_exceptions(true)
00091   {
00092     bool print_help = false;
00093     const std::string& arg_0_str = argv[0];
00094     const std::string program_name = arg_0_str.substr(arg_0_str.find_last_of('/')+1);
00095     for (int arg_ndx = 1; arg_ndx < argc; ++arg_ndx)
00096     {
00097       const std::string& arg_str = argv[arg_ndx];
```

```
00098       bool valid = false;
00099       if (arg_str.substr(0,2) == "--")
00100       {
00101         valid = true;
00102         const std::string& arg_name = arg_str.substr(2);
00103         if (arg_name == "help")
00104         {
00105           this->m_valid = false;
00106           print_help = true;
00107         }
00108         else if (arg_name == "verbose")
00109           this->m_verbose_output = true;
00110         else if (arg_name == "no-catch")
00111           this->m_catch_exceptions = false;
00112         else
00113           valid = false;
00114       }
00115       if (!valid)
00116       {
00117         std::cout « "Invalid argument: " « arg_str « std::endl;
00118         this->m_valid = false;
00119         print_help = true;
00120       }
00121     }
00122     if (print_help)
00123     {
00124       std::cout « program_name « " for " « GLUCAT_PACKAGE_NAME « " version " « GLUCAT_VERSION « ":" «
    std::endl;
00125       std::cout « "Usage:  " « program_name « " [option ...]" « std::endl;
00126       std::cout « "Options:" « std::endl;
00127       std::cout « "  --help      : Print this summary." « std::endl;
00128       std::cout « "  --no-catch  : Do not catch exceptions." « std::endl;
00129       std::cout « "  --verbose   : Produce more detailed test output." « std::endl;
00130     }
00131   }
00132
00134   inline
00135   int
00136   control_t::
00137   call(intfn f) const
00138   {
00139     if (valid())
00140       return (catch_exceptions())
00141         ? try_catch(f)
00142         : (*f)();
00143     else
00144       return 1;
00145   }
00146
00148   inline
00149   int
00150   control_t::
00151   call(intintfn f, int arg) const
00152   {
00153     if (valid())
00154       return (catch_exceptions())
00155         ? try_catch(f, arg)
00156         : (*f)(arg);
00157     else
00158       return 1;
00159   }
00160 }
00161 #endif // _GLUCAT_CONTROL_H
```

9.65 test/driver.h File Reference

```
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include "test/tuning.h"
#include "test/try_catch.h"
#include "test/control.h"
#include <cstdio>
```

Include dependency graph for driver.h:

9.66 driver.h

Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TEST_DRIVER_H
00002 #define GLUCAT_TEST_DRIVER_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     driver.h : Header for example and timing test driver
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
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00014     (at your option) any later version.
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00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 #include "glucat/glucat.h"
00035 #include "glucat/glucat_imp.h"
00036 #include "test/tuning.h"
00037 #include "test/try_catch.h"
00038 #include "test/control.h"
00039 #include <cstdio>
00040
00041 #endif // GLUCAT_TEST_DRIVER_H
```

9.67 test/timing.h File Reference

Namespaces

- namespace glucat
- namespace glucat::timing

Functions

- static double glucat::timing::elapsed (clock_t cpu_time)

  Elapsed time in milliseconds.

Variables

- const double glucat::timing::MS_PER_SEC = 1000.0

  Timing constant: milliseconds per second.
- const double glucat::timing::MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)

  Timing constant: milliseconds per clock.
- const int glucat::timing::EXTRA_TRIALS = 2

  Timing constant: trial expansion factor.

9.68 timing.h

Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TEST_TIMING_H
00002 #define GLUCAT_TEST_TIMING_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     timing.h : Common definitions for timing tests
00006                              -------------------
00007     begin                : Tue 2012-03-27
00008     copyright            : (C) 2012 by Paul C. Leopardi
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
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00016     This library is distributed in the hope that it will be useful,
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00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 namespace glucat
00035 {
00036   namespace timing
00037   {
00039     const double MS_PER_SEC = 1000.0;
00040
00042     const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC);
00043
00045     const int EXTRA_TRIALS = 2;
00046
00048     inline
00049     static
00050     double
00051     elapsed(clock_t cpu_time)
00052     { return double(clock() - cpu_time) * MS_PER_CLOCK; }
00053
00054   }
00055 }
00056 #endif // GLUCAT_TEST_TIMING_H
```
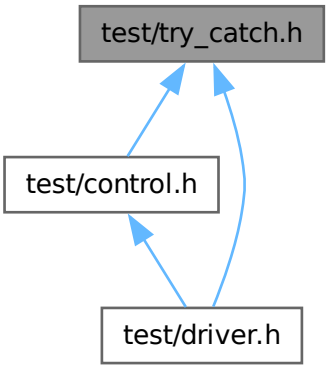

9.69 test/try_catch.h File Reference

This graph shows which files directly or indirectly include this file:



Namespaces

- namespace glucat

Typedefs

- typedef int(∗) glucat::intfn()

  For exception catching: pointer to function returning int.
- typedef int(∗) glucat::intintfn(int)

  For exception catching: pointer to function of int returning int.

Functions

- int glucat::try_catch (intfn f)

  Exception catching for functions returning int.
- int glucat::try_catch (intintfn f, int arg)

  Exception catching for functions of int returning int.

9.70 try_catch.h

Go to the documentation of this file.

```
00001 #ifndef _GLUCAT_TRY_CATCH_H
00002 #define _GLUCAT_TRY_CATCH_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     try_catch.h : Catch exceptions
00006                             -------------------
00007     begin                : Sun 2001-12-20
00008     copyright            : (C) 2001-2010 by Paul C. Leopardi
```

```
00009  ***************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
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00016     This library is distributed in the hope that it will be useful,
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00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ***************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ***************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ***************************************************************************/
00033
00034 namespace glucat
00035 {
00037   typedef int (*intfn)();
00038
00040   typedef int (*intintfn)(int);
00041
00043   int try_catch(intfn f);
00044
00046   int try_catch(intintfn f, int arg);
00047
00049   int try_catch(intfn f)
00050   {
00051     int result = 0;
00052     try
00053       { result = (*f)(); }
00054     catch (const glucat_error& e)
00055       { e.print_error_msg(); }
00056     catch (const std::bad_alloc& e)
00057       { std::cerr « "bad_alloc" « std::endl; }
00058     catch (...)
00059       { std::cerr « "unexpected exception" « std::endl; }
00060     return result;
00061   }
00062
00064   int try_catch(intintfn f, int arg)
00065   {
00066     int result = 0;
00067     try
00068       { result = (*f)(arg); }
00069     catch (const glucat_error& e)
00070       { e.print_error_msg(); }
00071     catch (const std::bad_alloc& e)
00072       { std::cerr « "bad_alloc" « std::endl; }
00073     catch (...)
00074       { std::cerr « "unexpected exception" « std::endl; }
00075     return result;
00076   }
00077 }
00078 #endif // _GLUCAT_TRY_CATCH_H
```

9.71 test/undefine.h File Reference

9.72 undefine.h

Go to the documentation of this file.

```
00001 #ifndef GLUCAT_TEST_UNDEFINE_H
00002 #define GLUCAT_TEST_UNDEFINE_H
00003 /***************************************************************************
00004     GluCat : Generic library of universal Clifford algebra templates
00005     undefine.h : Undefine preprocessor macro names that control test tuning
00006                              -------------------
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
```

```
00009  ********************************************************************************
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
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00016     This library is distributed in the hope that it will be useful,
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00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00023
00024  ********************************************************************************
00025  This library is based on a prototype written by Arvind Raja and was
00026  licensed under the LGPL with permission of the author. See Arvind Raja,
00027  "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028  in Ablamowicz, Lounesto and Parra (eds.)
00029  "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030  ********************************************************************************
00031      See also Arvind Raja's original header comments in glucat.h
00032  ********************************************************************************/
00033
00034 // Undefine tuning policy constants
00035 #undef _GLUCAT_TEST_TUNING_SLOW
00036 #undef _GLUCAT_TEST_TUNING_NAIVE
00037 #undef _GLUCAT_TEST_TUNING_FAST
00038 #undef _GLUCAT_TEST_TUNING_PROMOTED
00039 #undef _GLUCAT_TEST_TUNING_DEMOTED
00040
00041 #endif // GLUCAT_TEST_UNDEFINE_H
```

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